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Ultimate Longitudinal Strength of Composite Ship Hulls

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Abstract: A simple analytical model to estimate the longitudinal strength of ship hulls in composite materials under buckling, material failure and ultimate collapse is presented in this paper. Ship hulls are regarded as assemblies of stiffened panels which idealized as group of plate-stiffener combinations. Ultimate strain of the plate-stiffener combination is predicted under buckling or material failure with composite beam-column theory. The effects of initial imperfection of ship hull and eccentricity of load are included. Corresponding longitudinal strengths of ship hull are derived in a straightforward method. A longitudinally framed ship hull made of symmetrically stacked unidirectional plies under sagging is analyzed. The results indicate that present analytical results have a good agreement with FEM method. The initial deflection of ship hull and eccentricity of load can dramatically reduce the bending capacity of ship hull. The proposed formulations provide a simple but useful tool for the longitudinal strength estimation in practical design.

Keywords: Composites; Ship hull; stiffened panel; Ultimate longitudinal strength; analytical model

1 Introduction

Laminated fiber reinforced composites have been widely used in marine and ship structures. The application of such materials in ship structures dates back to the late 1970s. Over the time, the usage of composite materials in ship construction continues to grow due to the improvement of design, fabrication and mechanical performance of advanced composites. Currently, some large vessels like frigates and passenger ships are made of laminated composite materials. There are now all-composite naval ships in service.

Longitudinal bending moment carried by ship hull girder increasing with the increasing length of composite ship hulls, the ship safety related to longitudinal ultimate strength becomes more important at design. According to traditional ship design rules, the longitudinal strength of the ship hull built of steel with length exceeding 60 m must be assessed. The stiffness of composite materials is generally low. Longitudinal deformation of ship hulls in composite materials is usually larger than that built of steel. Recent years, with the rapid development trend and momentum of construction of large-scale and super large composite ship hulls, the length of ship hulls in composite materials are increasing so that the study of longitudinal ultimate strength of composite ship becomes increasingly important and urgent.

The ultimate strength of steel ships has been widely investigated by many researchers. Caldwell [1] was the first who estimated the ultimate strength of steel ships employing the fully plastic bending theory of beams. In his method, the hull girder was divided into a number of panels and the collapse load of each panel was calculated through strength reduction factor due to buckling for compressive loading. However his model did not take into account the post-buckling strength of the panels. Smith [2] proposed an approach for calculation of the ultimate strength of ship hulls. He divided the ship's cross section into a series of stiffened panels and then performed a progressive collapse analysis under bending where the post-buckling behavior of the stiffened plates was considered. The collapse of a girder section is assumed to occur between two adjacent frames, being induced either by the inter-frame flexural beam-column collapse of panels under compression or by the inter-frame yielding of panels under tension. The behavior of the stiffened plates under compressive load can be characterized through so-called load-shortening curves. The resulting strains and stresses in the stiffened plates making up the cross section were calculated using the predefined load-shortening curves. Based on the general approach of Smith's method, Dow *et al.* [3] and Ueda and Yao [4] developed a method for determination of load-shortening curves of beam-column ele-

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ments, which similar to the Smith's beam-column method with the difference that a large portion of the hull girder plating with multiple stiffeners was considered. Viner [5] proposed an expression for calculating the ultimate bending moment assuming that the elastic behavior is maintained up to the point where the compression flange reach the collapse state resulting in immediate hull collapse. Khedmati [6] made attempts for derivation of the average stress-average strain relationships for stiffened plates subject to in-plane compression alone or in combination with other loads using analytical approaches. Frieze and Lin [7] expressed ultimate bending moment capacity of the ship hull as a function of a normalized ultimate strength of the compression flange. Paik and Mansour [8] presented an expression for predicting the ultimate strength of single and double-hull ships under vertical bending moments. Garbatov [9] performed hull girder ultimate strength verification according to the Class Society rules based on experimental results and the dimensional theory. Some other ultimate strength approaches can be found in [10–12].

Very few publications can be found in the literature on evaluating the ultimate strength of composite ships. Chen *et al.* [13] firstly tried to estimate the ultimate strength of composite ships. They proposed a simple analytical method for calculating the ultimate strength of composite vessels. But his model is too simple to handle the staking sequence of real composite materials and fail to give procedures to handle the composite hat-stiffened panels. Chen and Soares [14] conducted a calculation of ultimate strength of composite ships under bending moment with nonlinear finite element method. Two types of the failure modes were considered in their study; the panel buckling as well as fracture of the composite materials. Later, Chen and Soares [15] used Smith's method to calculate the ultimate strength of composite vessels. Morshedsoluk [16] applied a Coupled Beam Theory to calculate the ultimate strength of composite ships taking into account the effect of the superstructure. The behaviour of the composite panels in the ship structure is deduced from their mean stress-mean strain curves. The progressive failure method in conjunction with the nonlinear finite element method is used to calculate mean stress- mean strain curves. The efficiency of the composite superstructure in contribution to the ultimate bending strength of the composite ships is evaluated. Case studies showed that the length of superstructure has significant effect on the ultimate strength of the composite ship.

Application of composite materials to the construction of large ships is a relatively new and growing subject, which needs correct assessment of the ultimate strength of these types of the ships. Especially, a simple analytical method is needed for practical engineering at preliminary stage of ship design. For this purpose, an analytical approach to estimate the longitudinal strengths of ship hull in composite materials was developed in this paper. Ship hulls are modeled as assemblies of stiffened composite panels. Stiffened panels are idealized as a series of plate-stiffener combinations. Ultimate strain is predicted under buckling or material failure with beam-column theory, considering the effects of initial imperfection of ship hull and eccentricity of load. The corresponding longitudinal strengths of ship hulls are derived with a straightforward analytical method.

The primary modes of failure for ship hull girder is column or beam-column type collapse of plate-stiffener combination as a representative of stiffened panel. Two failure modes of beam-column in composite materials are considered in this paper. The first failure mode is buckling. The tensile and compressive strengths of FRP laminates commonly employed in hull design are approximately equal to the yield strength of mild steel, and the Young's modulus of such laminates are only 5-10% of that of steel. Thus buckling may be an important failure mode of ship hull in composite materials. The second failure mode is material failure. The material failure of a local area in ship hull may occur with the increase of longitudinal bending moment, which will influence structural performances. Consequently, it is necessary to take material failure as an important failure mode of ship hull in composite materials.

2 Bending stiffness of composite ship hulls

Longitudinal framed structure system is usually used for large ships. The deck plate, bilge plate and side plate are strengthened with a series of longitudinal stiffeners (including Stringer). They are all hat-stiffened laminates. Hull girder consists of a number of structural elements of stiffened plates (Figure 1). According to the Smith analysis method [17], each stiffener of the hull girder (including attached flange and plating) works almost independently from other stiffeners. Consequently, each plate-stiffener combination can be analyzed alone. The load capacity of structural element of stiffened panel and ship hull girder can be calculated with composite beam-column theories.

To determine the bending stiffness of ship hull girder, the section is divided into a set of representative plate-stiffener combination (Figure 2). The plate-stiffener combination is then divided into a set of slats [18]. Each strip can be calculated as a laminate.

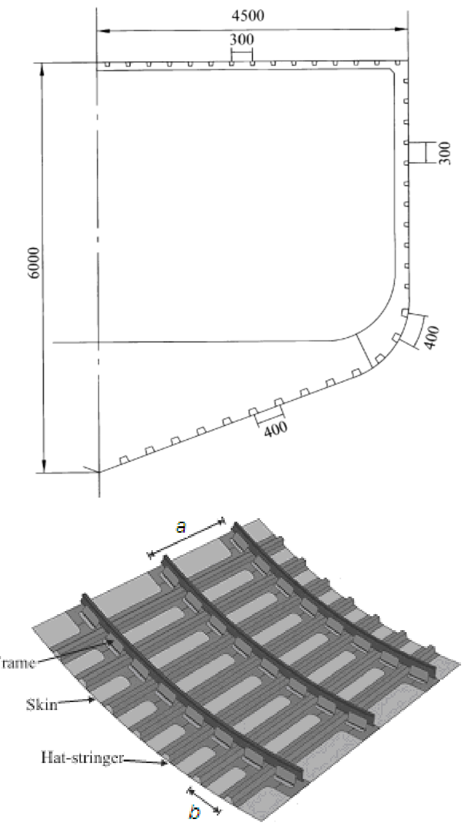

Figure 1: Hull girder section

2.1 Stiffness of symmetrical balanced laminate

Symmetrical balanced laminate presents orthotropic properties. The planar stiffness coefficients are

$$(Et)_{l1} = A_{11} - \frac{A_{12}^2}{A_{22}}; \quad (Et)_{l2} = A_{22} - \frac{A_{12}^2}{A_{11}}; \quad (Gt)_l = A_{66} \tag{1}$$

Where, $(Et)_{l1}$ is effective tensile/compressive stiffness coefficient at x direction; $(Et)_{l2}$ is effective tensile/compressive stiffness coefficient at y direction, $(Gt)_l$ is effective shear stiffness coefficient.

Bending stiffness coefficients of symmetrical balanced laminate are

$$D_{l1} = D_{11} - \frac{D_{12}^2}{D_{22}}; \quad D_{l2} = D_{22} - \frac{D_{12}^2}{D_{11}}; \quad D_{l12} = D_{66} \tag{2}$$

Where, D_{l1} is effective bending stiffness at x direction; D_{l2} is effective bending stiffness at y direction; D_{l12} is torsional stiffness.

Effective elastic modulus of symmetrical balanced laminate is

$$E_{l1} = \frac{1}{t}\left(A_{11} - \frac{A_{12}^2}{A_{22}}\right), \quad E_{l2} = \frac{1}{t}\left(A_{22} - \frac{A_{12}^2}{A_{11}}\right), \tag{3}$$

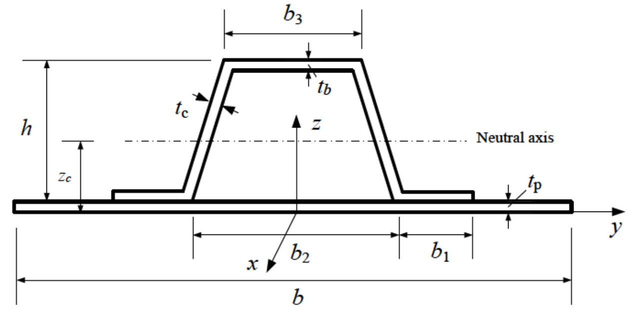

Figure 2: Section of a plate-stiffener combination

$$G_l = \frac{1}{t} A_{66}$$

Where, E_{l1} is effective tensile/compressive elastic modulus at x direction; E_{l2} is effective tensile/compressive elastic modulus at y direction; G_l is effective shear modulus.

2.2 Stiffness of laminate plate-stiffener combination

The stiffened panel is idealized as a number of plate-stiffener combination (stiffener including attached flange) or a beam-column. Typical section of a beam-column is shown in Figure 2.

Beam-column is divided into a set of strips. Assuming these strips to possess the same longitudinal strain, which are equal to that of stiffened laminate, we have

$$\varepsilon = \frac{P}{(EA)_{ps}} = \frac{P_i}{E_{l1i} A_i} \tag{4}$$

Where, P is axial force of beam-column; P_i is axial force of each slat, $(EA)_{ps}$ is tensile/compressive stiffness of beam-column; E_{l1i} is the effective elastic modulus of each slat; A_i is the sectional area of each strip.

$$P = \sum P_i = \frac{P}{(EA)_{ps}} \sum E_{l1i} A_i = \frac{P}{(EA)_{ps}} \sum (E_{l1i} t_i) b_i \tag{5}$$

Where t_i is thickness of strip; b_i is width of strip. The tensile/compressive stiffness of beam-column is

$$(EA)_{ps} = \sum (E_{l1i} t_i) b_i = \sum \left(A_{11i} - \frac{A_{12i}^2}{A_{22i}}\right) b_i = E_0 \sum \lambda_i b_i \tag{6}$$

Where, E_0 is the elastic modulus of an arbitrarily selected standard reference material and

$$\lambda_i = \left(A_{11i} - \frac{A_{12i}^2}{A_{22i}}\right) / E_0 \tag{7}$$

Similarly,

$$(GA)_{ps} = \sum A_{66i} b_i \tag{8}$$

The sectional area of beam-column is

$$A_{ps} = \sum A_i = \sum b_i t_i \tag{9}$$

The effective elastic modulus of beam-column is

$$E_{ps} = \frac{\sum \lambda_i b_i}{\sum b_i t_i} E_0 \tag{10}$$

Assuming the bending deformation of each strip keeps consistent with that of beam-column

$$\varphi = \frac{Ml}{(EI)_{ps}} = \frac{M_i l}{(EI)_{li}} \tag{11}$$

Where, M is the bending moment of beam-column, $(EI)_{ps}$ is the bending stiffness of beam-column; M_i is the bending moment of slat; $(EI)_{li}$ is the bending stiffness of slat; l is the length of beam-column; φ is the relative rotation of beam-column end sections.

$$M = \sum M_i = \frac{M}{(EI)_{ps}} \sum (EI)_{li} \tag{12}$$

The bending stiffness of beam-column is obtained from

$$(EI)_{ps} = \sum (EI)_{li} \tag{13}$$

At local reference coordinates system shown in Figure 2, the neutral axis of beam-column is determined from

$$z_{cps} = \frac{\sum E_{l1i} A_i z_i}{\sum E_{l1i} A_i} = \frac{\sum \left(A_{11i} - \frac{A_{12i}^2}{A_{22i}}\right) b_i z_i}{\sum \left(A_{11i} - \frac{A_{12i}^2}{A_{22i}}\right) b_i} = \frac{\sum \lambda_i b_i z_i}{\sum \lambda_i b_i} \tag{14}$$

Where, the z_{cps} is the coordinates of neutral axis of beam-column; z_i is the centroidal coordinate of each slat.

When the width of slat is parallel to the neutral axis of beam-column section, bending stiffness of the slat about its own neutral axis is in form of

$$(EI_y)'_{li} = \left(D_{11} - \frac{D_{12}^2}{D_{22}}\right)_i b_i \tag{15}$$

When the width of slat is at an inclined direction which is not parallel to the neutral axis of beam-column section, bending stiffness of the slat about its own neutral axis is expressed as

$$(EI_y)'_{li} = \frac{1}{12} E_{l1i} t_i (b_i \sin\alpha)^3 = \frac{1}{12}\left(A_{11} - \frac{A_{12}^2}{A_{22}}\right)_i (h_i)^3 = \frac{1}{12} E_0 \lambda_i (h_i)^3 \tag{16}$$

Where, α is the angle between the slat width and the neutral axis.

Bending stiffness of shifting neutral axis of slat to that of beam-column is

$$(EI_y)''_{li} = E_{l1i} t_i b_i (z_i - z_{cps})^2 = E_0 \lambda_i b_i (z_i - z_{cps})^2 \tag{17}$$

Bending stiffness of beam-column is expressed as

$$(EI_y)_{ps} = \sum [(EI)'_{li} + (EI)''_{li}] \tag{18}$$

Similarly, bending stiffness of beam-column about its symmetric axis is expressed as

$$(EI_z)_{ps} = \sum [(EI_z)'_{li} + (EI_z)''_{li}] \tag{19}$$

where,

$$(EI_z)'_{li} = \frac{1}{12}(Et)_{li}(b_i \cos\alpha)^3 = \frac{1}{12} E_0 \lambda_i (b_i \cos\alpha)^3 \tag{20}$$
$$(EI_z)''_{li} = (EA_i)_{li}(y_i)^2 = E_0 \lambda_i b_i (y_i)^2$$

2.3 Bending stiffness of composite ship hull girder

The sectional area of ship hull gird is sum of that of all beam-column

$$A = \sum A_{ps} \tag{21}$$

The composite beams are mainly distinguished from traditional beams in structure. Different types and specifications of reinforcement as well as different layup are utilized in different parts of a composite beam. Composite beam theory should be applied. Area conversion methods should be used to dealt with composite beams [19, 20], namely, an elastic modulus of an arbitrarily selected materials as standard reference modulus, denoted E_0, and cross-sectional area of other materials are transformed in the coefficient

$$\gamma_i = \frac{E_{psi}}{E_0} \tag{22}$$

The axial stiffness of ship hull gird is

$$(EA)_{gir} = \sum (EA)_{psi} = E_0 \sum (\gamma_i A_{psi}) \tag{23}$$

Setting a global reference coordinates at the bottom of ship hull, the neutral axis is determined as

$$z_{cgir} = \frac{\sum (EA)_{psi} z_{psi}}{\sum (EA)_{psi}} = \frac{\sum (\gamma_i A_{psi}) z_{psi}}{\sum (\gamma_i A_{psi})} \tag{24}$$

Z_{cgir}-centroidal coordinates of ship hull girder section.

Bending stiffness of ship hull girder is

$$(EI)_{gir} = D_{gir} = \sum \left[(EI_y)_{psi} + (EA)_{psi}(z_{psi} - z_{cgir})^2 \right] \quad (25)$$

The moment of inertia of ship hull girder section is

$$I_{gir} = \sum \left[\frac{(EI_y)_{psi}}{E_0} + (\gamma_i A_{psi}) \left(z_{psi} - z_{cgir} \right)^2 \right] \quad (26)$$

When the neutral axis of beam-column is not parallel to that of ship hull girder, the moment of inertia should be transformed.

$$(EI_y)_{ps} = \frac{(EI_y)_{ps} + (EI_z)_{ps}}{2} + \frac{(EI_y)_{ps} - (EI_z)_{ps}}{2} \cos 2\theta \quad (27)$$

Where, θ-orientation of the neutral axis of beam-column with respect to that of ship hull girder, which clockwise is positive.

3 Material failure

The lamina stresses in principal material direction under uniaxial stress are

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = [T_\sigma] \begin{Bmatrix} \sigma_x \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} m^2 \\ n^2 \\ -mn \end{Bmatrix} \sigma_x \quad (28)$$

Where, T_σ is transform matrix; $m = \cos\theta$, $n = \sin\theta$; θ is the orientation angle of lamina.

To identify the material failure of stiffened composite panels, the Tsai-Wu criterion [21] is adopted in this paper. Material failure is considered to have occurred as the following equation is satisfied at any lamina

$$F = F_i \sigma_i + F_{ij} \sigma_i \sigma_j = 1 \qquad i, j = 1, 2, 6 \quad (29)$$

where σ_i denotes the stress components referred to the principal material coordinates; F_i and F_{ij} are material strength coefficients.

Introducing Eq. (28) into Eq. (29), we have

$$\left(F_{11} m^4 + 2F_{12} m^2 n^2 + F_{22} n^4 + F_{66} m^2 n^2 - 2F_{16} m^3 n - 2F_{26} mn^3 \right) \sigma_{mf}^2 + (F_1 m^2 + F_2 n^2 - F_6 mn) \sigma_{mf} - 1 = 0 \quad (30)$$

Or simply

$$\phi_1 \sigma_{mf}^2 + \phi_2 \sigma_{mf} - 1 = 0 \quad (31)$$

Where

$$\phi_1 = F_{11} m^4 + 2F_{12} m^2 n^2 + F_{22} n^4 + F_{66} m^2 n^2 - 2F_{16} m^3 n - 2F_{26} mn^3 \quad (32)$$
$$\phi_2 = F_1 m^2 + F_2 n^2 - F_6 mn$$

The ultimate strength of composite stiffened panel under material failure is given by

$$\sigma_{mf} = \frac{-\phi_2 + \sqrt{\phi_2^2 + 4\phi_1}}{2} \quad (33)$$

The material failure strain is

$$\varepsilon_{mf} = \frac{\sigma_{mf}}{E_{ps}} \quad (34)$$

4 Column buckling

The column buckling load of plate-stiffener combination with uniform cross-section is given by

$$P_{cr} = \frac{\pi^2 (EI)_{ps}}{l^2} \quad (35)$$

The column buckling stress is

$$\sigma_{cr} = \frac{\pi^2 (EI)_{ps}}{A_{ps} l^2} \quad (36)$$

The critical buckling stress including shear deformation is in form of [22]

$$\sigma_{cr} = \frac{\frac{\pi^2 (EI)_{ps}}{A_{ps} (\mu l)^2}}{1 + \frac{\pi^2 (EI)_{ps}}{(GA)_{ps} (\mu l)^2}} \quad (37)$$

The ultimate buckling strain of stiffened composite panels is

$$\varepsilon_{bf} = \frac{\sigma_{cr}}{E_{ps}} \quad (38)$$

5 Initial imperfection and eccentricity of load

The initial defection of beam-column is assumed to be

$$w_1 = w_0 \sin \frac{\pi x}{l} \quad (39)$$

Where, w_0 is the amplitude of initial deflection. l is length of beam-column. x is longitudinal coordinates.

The total deflection is given by $w_t = w_1 + w$, where w is bending deflection.

The differential equation of beam-column is given by

$$(EI)_{ps}\frac{d^2w}{dx^2} + Pw = -P\left(w_0 \sin\frac{\pi x}{l} + e\right) \tag{40}$$

Where, P is axial compressive load; $(EI)_{ps} = D_{ps}$ is the flexural stiffness of plate-stiffener combination; e is the eccentricity of load.

Particular solution of Eq. (40) is given by

$$w_p = C\sin\frac{\pi x}{l} + D\cos\frac{\pi x}{l} - e \tag{41}$$

Substituting into Eq. (40), we obtain

$$\left[C\left(k^2 - \frac{\pi^2}{l^2}\right) + k^2 w_0\right]\sin\frac{\pi x}{l} + D\left(k^2 - \frac{\pi^2}{l^2}\right)\cos\frac{\pi x}{l} = 0$$

where, $k^2 = P/(EI)_{ps}$. Solving for C and D, we have

$$\begin{aligned} C &= w_0/\left(\frac{\pi^2}{k^2l^2} - 1\right) = w_0/\left(\frac{P_{cr}}{P} - 1\right)\\ &= \eta w_0/(1-\eta); \quad D = 0 \end{aligned}$$

Where, $\eta = P/P_{cr}$.

General solution of Eq. (40) is in form of

$$w = A\sin kx + B\cos kx + \eta w_0/(1-\eta)\sin\frac{\pi x}{l} - e \tag{42}$$

Boundary conditions are

$$x = 0,\ l \quad w = 0 \tag{43}$$

Substituting into Eq. (42), we get

$$A = \frac{e}{\sin kl} - e\cot kl; \quad B = e$$

Solution of Eq. (40) is written as

$$\begin{aligned} w &= \eta w_0/(1-\eta)\sin\frac{\pi x}{l} - e\cot\frac{kl}{2}\sin kx + e\cos kx - e\\ &= \eta w_0/(1-\eta)\sin\frac{\pi x}{l} + \frac{e}{\cos\frac{kl}{2}}\cos\left(kx - \frac{kl}{2}\right) - e \end{aligned}$$

Bending moment is determined as

$$\begin{aligned} M &= P(w + w_1 + e)\\ &= Pw_0/(1-\eta)\sin\frac{\pi x}{l} + \frac{Pe}{\cos\frac{kl}{2}}\cos\left(kx - \frac{kl}{2}\right) \end{aligned}$$

Maximum bending moment is

$$\begin{aligned} M_{\max} &= Pw_0/(1-\eta) + Pe + Pe\frac{k^2l^2}{8}\\ &= Pw_0/(1-\eta) + Pe + Pe\frac{\pi^2}{8}\frac{P}{P_{cr}} \end{aligned}$$

The maximum stress is

$$\begin{aligned} \sigma_{\max} &= \frac{P}{A} + \frac{M_{\max}}{W} = \frac{P}{A}\left(1 + \frac{M_{\max}}{\rho}\right)\\ &= \frac{P}{A}\left(1 + \frac{P_E w_0}{\rho(P_E - P)}\right) + \frac{e}{\rho} + \frac{\pi^2}{8\rho}\frac{Pe}{P_{cr}} \end{aligned} \tag{44}$$

with

$$W = \frac{(EI)_{ps}}{E_{l1}z_l}; \quad \rho = W/A$$

where z_l is the maximum vertical coordinates measured from neutral axis, A is the cross-sectional area.

Adopting the approximation:

$$\frac{P_{cr}}{P_{cr} - P} \approx 1 + \frac{P}{P_{cr}}$$

Substituting into Eq. (44) yields:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{e + w_0}{\rho} + \left(w_0 + \frac{\pi^2 e}{8}\right)\frac{P}{\rho P_{cr}}\right] \tag{45}$$

Ultimate strength is reached when material failure occurs:

$$\sigma_u\left[1 + \frac{e + w_0}{\rho} + \left(w_0 + \frac{\pi^2 e}{8}\right)\frac{\sigma_u}{\rho\sigma_{cr}}\right] = \sigma_{mf} \tag{46}$$

or simply

$$\sigma_u^2 + \mu\sigma_{cr}\sigma_u - \omega(\sigma_{cr})^2 = 0 \tag{47}$$

where,

$$\mu = \frac{\rho + e + w_0}{w_0 + \frac{\pi^2 e}{8}}; \quad \omega = \frac{\rho\sigma_{mf}/\sigma_{cr}}{w_0 + \frac{\pi^2 e}{8}} \tag{48}$$

The ultimate strength of composite stiffened panel with initial imperfection and eccentricity of load is given by

$$\sigma_u = \frac{-\mu + \sqrt{\mu^2 + 4\omega}}{2}\sigma_{cr} \tag{49}$$

The ultimate strain is

$$\varepsilon_{ime} = \frac{\sigma_u}{E_{ps}} \tag{50}$$

6 Longitudinal strength of composite ship hulls

To simplify the problem, the following hypotheses are made:

Table 1: Dimensions of stiffened composite panels at deck and side

Name	a	b	b_1	b_2	b_3	h	t_p	t_b	t_c
Size(mm)	600	300	18	62	50	60	4.1	3.5	2.1

Table 2: Dimensions of stiffened composite panels at bilge

Name	a	b	b_1	b_2	b_3	h	t_p	t_b	t_c
Size(mm)	600	400	38	124	100	90	7.2	6	3.1

1. The hull girder is assumed as an Euler-Navier beam. The transverse cross-sections of ship remain plane when subjected to bending moments and the influence of transverse restraint on longitudinal stress is negligible.
2. Longitudinal failure only occurs between two adjacent transverse frames.
3. The failure of each stiffened composite panel is assumed to occur individually and independently.

The normal strain ε varies linearly across the perpendicular direction of cross-section.

$$\varepsilon_i = z_i \kappa = z_i \frac{M}{D_{gir}} \tag{51}$$

where z is the coordinates measured from neutral axis; κ is the curvature; M is bending moment.

The ultimate strength of ship hull is defined as

$$M_u = \frac{D_{gir}\varepsilon_u}{z_{ps\,\max}} \tag{52}$$

7 Numerical example

A longitudinally framed structure ship hull built of fiber-reinforced plastic was considered. The cross-section of mid-ship is shown in Fig. 1. Breadth moulded is 9 m; Depth moulded is 6 m. The thicknesses of the plate are 4.1 mm at deck and side, and 7.2 mm at the bilge, respectively. The geometry of the stiffened composite panels at mid-ship section is shown in Fig. 2. The dimensions of the stiffened panels at deck and side are shown in Table 1. The dimensions at bilge are shown in Table 2. The skin of deck is made of 32 symmetrically stacked unidirectional plies [90/45/90/-45/0/45/90/90/-45/0/45/90/90/-45/0/-45]s. The measured ply thickness is 0.13 mm. The stacking sequence of stringer is [90/45/90/-45/0/45/90/90]s and the ply thickness is 0.13 mm. The head of the stiffener is reinforced and made of 17 plies [90/45/90/-45/0/45/90/90/-45/0/45/90/$\overline{90}$]s. The ship is made of one kind of composite material, and the material properties are shown in Table 3.

The strength coefficients for Tsai-Wu failure criterion can be computed from the failure strengths in Table 3.

When the ship is under sagging, the stiffened panels in the deck are in compression and those in the bilge are in tension. Tables 1 and 2 show the dimensions of stiffened panels at bilge are larger than those at deck. It is certain that the buckling and material failure of the stiffened panels at deck will be prior to that at bilge. So the first step is to calculate the buckling and material failure strain of the stiffened panels at deck.

The sectional properties of plate-stiffener combination and ship hull are computed and shown in table 4.

The plate-stiffener combination is assumed to be simply supported at the ends. Initial deflection w_0 is taken as 10 mm and eccentricity of load e as 5 mm, The buckling failure strain ε_{bf} and material failure strain ε_{mf} at deck and ultimate strain with initial imperfection and load eccentricity ε_{ime} are evaluated. Then the buckling failure strength M_{bf}, material failure strength M_{mf} and ultimate strength M_{ime} of ship hull under sagging are obtained from Eq. (52). The results from present analytical solution are showed in table 5. It indicates that the initial deflection of ship hull and the eccentricity of load can remarkably decrease the bending capacity of ship hull. The present analytical method has been correlated with simulations using the nonlinear finite element method described in [14]. The results indicate that it gives very similar results to that of the FEM.

8 Conclusions

An analytical model accounting for the effects of initial imperfection of ship hull and eccentricity of load is presented in this paper to estimate the longitudinal strength of ship hull in composite materials under buckling failure, material failure and ultimate collapse of the deck, based on

Table 3: Material properties of composite material

Property	Value
Elastic modulus in 1 principal material direction E_1	15.7 GPa
Elastic modulus in 2 principal material direction E_2	14.8 GPa
Poisson's ratios 12 ν_{12}	0.127
Shear modulus in 2-3 principal material plane G_{23}	0.34 GPa
Shear modulus in 1-3 principal material plane G_{13}	0.34 GPa
Shear modulus in 1-1 principal material plane G_{12}	0.34 GPa
Tensile strength in 1 principal material direction X_T	238 MPa
Compressive strength in 1 principal material direction X_C	204 MPa
Tensile strength in 2 principal material direction Y_T	210 MPa
Compressive strength in 2 principal material direction Y_C	224 MPa
Shear strength in 2-3 principal material plane R	23.5MPa
Shear strength in 1-3 principal material plane S	23.5MPa
Shear strength in 1-2 principal material plane T	104MPa

Table 4: Sectional properties of plate-stiffener combination and ship hull

Property	Value
Sectional area of plate-stiffener A_{ps}	1.73×10^{-3} m^2
Bending stiffness of plate-stiffener $(EI)_{ps}$	1.31×10^4 Nm^2
Shear stiffness of plate-stiffener $(GA)_{ps}$	6.5×10^5 N
Effective elastic modulus of plate-stiffener E_{ps}	15.5GPa
Neutral axis Coordinates of mid-ship hull z_{cgir}	2.917m
Flexural stiffness of hull girder D_{gir}	1.736×10^{10} Nm^2

Table 5: Results of failure strain and ultimate strength

	failure strain ($\times10^{-3}$)			ultimate strength ($\times10^7$Nm)		
	buckling	material failure	Imperfection	buckling	material failure	Imperfection
Present analytical	8.63	13.8	4.3	4.88	7.8	2.39
Finite element	9.4	14.2	4.6	5.31	8.03	2.43

accurate strain computation of stiffened composite panels with a simple analytical method. In the present model, detailed procedures to evaluate the stiffness of symmetrical balanced laminate, composite beam-columns and ship hull girder is described. Because of the special characteristics of composite material, the buckling and material failure may be the important failure modes of ship hull in composite materials. The Tsai-Wu criterion is adopted to identify the material failure of stiffened composite panels. Because of the lower stiffness of composite materials at transverse direction, the critical buckling stress including shear deformation is adopted. A longitudinally framed structure ship hull in composite materials made of symmetrically stacked unidirectional plies under sagging is analyzed as an example application. The longitudinal ultimate strength was investigated. The results show that the initial deflection of ship hull and the eccentricity of load can dramatically decrease the bending capacity of the ship hull. Owing to lack of information on ultimate strength of composite ship hulls, the present method was compared to the finite element simulation. The results indicate present analytical results have a good agreement with FEM methods and the present approach provides a simple but useful tool for the longitudinal strength estimation of ship hull in composite materials.

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