Research Article

Komal Chawla and Samit Ray-Chaudhuri*

Stress and strain concentration factors in orthotropic composites with hole under uniaxial tension

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Abstract: A systematic investigation is carried out on how different parameters influence stress and strain concentration factors (SCF and SNCF) in a composite plate with a hole under uniaxial tension. Flat and singly curved composite plates have been modelled in ANSYS 15.0. The governing parameter includes: (i) size, shape and eccentricity of hole, (ii) number of plies, (v) fiber orientation and (vi) plate curvature. It is observed that different parameters influence the SCF and SNCF with varying degrees. For example, SCF may be as high as 7.16 for a square shaped hole. Also, SCF and SNCF are found to be approximately same in most of the cases. Finally, simplified design formulas are developed for evaluation of SCF for a wide range of hole size, eccentricity and fiber orientation.

Keywords: Composite plate; Stress distribution; Stress concentration factor; Strain concentration factor; Fiber orientation; Sensitivity analysis

1 Introduction

Multi-functional materials such as carbon and glass fiber reinforced polymers (CFRP and GFRP) are gaining significant popularity in engineering applications. This is because these materials demonstrate superior properties compared to their constitutive components. This is evident from the wide usage of carbon and glass fiber composites in space, civil, mechanical and aerospace applications. Like other materials, these materials are also prone to damage under various operational loads. The damage in composite structure can happen due to increased stress or strain, which results in propagation of several micro-scale failures (*i.e.*, fiber/matrix debonding, fiber breakage, matrix cracking and delamination). Even sometimes,

debonding between fibre and matrix forms an interfacial crack, which leads to the stress concentration at crack tip.

In many applications such as aircraft and space structures, holes and discontinuities are unavoidable. Typically, in space applications, holes are necessary for passing electrical conduits etc. In addition, joint, connection and access also increase stress around the discontinuity. Stress concentration is the main concern at the jointed region of a composite T-joint, which is used in many offshore and marine applications. Figure 1 ([1]) shows section of a composite T-joint (CFRP), where delamination between the plies took place due to stress concentration at the jointed region that has already been weakened by the matrix rich pocket formation. Further, Figure 2 ([2]) shows the failure propagation in a composite laminate (CFRP) with embedded circular vasculature due to stress concentration at the circular region while subjected to axial compressional loading. It may be noted that holes in such composite laminates can be of any shape depending on the architectural as well as functional requirements. Also, holes of different sizes are often needed for connecting the structural components with bolted connection.

The ratio of the maximum stress at the hole edge to the nominal stress is known as the stress concentration factor (*i.e.*, SCF). Similarly, the ratio of the maximum strain at the hole edge to the nominal strain is termed as the strain concentration factor (*i.e.*, SNCF). These factors are often utilized for proper and safe design of structures with holes. Evaluation of stress concentration for isotropic material is straightforward but for orthotropic/anisotropic material, it is quite different because of directionality behavior. Classical solutions are available for stress concentration in an isotropic material such as a steel thin plate having a small circular hole at the middle and subjected to uniform tension. For a isotropic thin plate subjected to uniform tensile loading, the stress concentration factor has been derived as 3 and the direction of maximum stress is found to be transverse to the loading direction. For orthotropic materials, *e.g.*, laminated composites, the stress and strain concentration factors and the location of maximum stress depends on many parameters *i.e.*, fiber orientation, fiber distribution, stiffness ratio and loading direction. Also, uncertainty in material constants (resulting from intrinsic randomness of fabrication process or epistemic uncertain-

*Corresponding Author: Samit Ray-Chaudhuri: Associate Professor, Department of Civil Engineering, Indian Institute of Technology Kanpur, UP – 208016, India; Email: samitrc@iitk.ac.in

Komal Chawla: Ph.D. Candidate, Department of Civil Engineering, Indian Institute of Technology Kanpur, UP – 208016, India

Figure 1: Stress concentration in composite T-joint

Figure 2: Failure propagation in composite laminated plate (with embedded circular vasculature) due to stress concentration

ties involved in testing and measurements) may influence estimation of these factors. Further, evaluation of these factors (SCF and SNCF) will be more difficult in case of a structure with complex geometry.

The application of complex variable technique to solve a stress concentration problem can be found in [3]. [4] also used the same technique for the anisotropic plate solution. [5] analysed the tensile strength of composite laminates (with hole and crack) under uniaxial loading based on the stress distribution. It was found that discontinuities (circular hole and crack) and its size significantly influence the tensile stress and strength of isotropic and quasi-isotropic laminates. [6] further extended this study and validated the hole size effect with the experimental data. It was observed that, the stress distribution is more localized near the hole boundary in case of a smaller hole size. Stress distribution in an orthotropic plate with a circular hole was analysed by [7]. An approximate polynomial solution

obtained from the theory of elasticity was used. Further, these solutions were compared with the exact orthotropic solution. It was found that the stress concentration factor obtained from the approximate solution is comparable with the exact solution.

Fiber orientation effect on stress concentration in a unidirectional finite width laminate with a circular hole was studied by [8]. It was found that stress concentration is maximum when the fibers are aligned in the loading direction and minimum when the fibers are aligned in 45° to the loading direction. [9] also explained the effect of hole size and fibre orientation on the stress concentration factor for an anisotropic graphite epoxy composite plate with a circular hole. It was found that the stress concentration and its location depend on fiber orientation with respect to the loading direction. It was also observed that the stress concentration factor could be reduced drastically through the lamination process. [10] further extended this study

for stress concentration and failure criteria of anisotropic composite plates with a circular hole when subjected to tension and compression.

Stress concentration problem in cylindrical shells with a circular hole was solved by [11] under uniaxial tension and internal pressure. However, these solutions were valid only for a small value of curvature. Later, the influence of curvature on stress concentration in cylindrical shell with a circular cutout was investigated by [12] under axial tension, internal pressure and torsional loading using a perturbation approach. A single parameter β was introduced for different curvatures, which is as follows:

$$\beta^2 = a^2 \frac{[12(1 - \nu^2)]^{1/2}}{8Rt} \quad (1)$$

where a , ν , R and t are the hole size, Poisson's ratio, radius of curvature, and thickness of shell, respectively. Higher stress concentration factors were observed in cylindrical shell over the flat plate owing to curvature effect. Further, [13] observed similar results using variational approach. [14] proposed an empirical relationship for stress concentrations in (isotropic and orthotropic) plates and cylinders with a circular opening, and validated results with a finite element study. It was concluded that the stress concentration factors in case of orthotropic and isotropic plates (of finite width) depend on the dimension ratio. As per their study, if the dimension ratio is smaller than 0.5, the stress concentration factor for an orthotropic plate will be almost same with that of the isotropic plate.

[15] performed a three-dimensional finite element study to analyze the stress fields near notches of finite thickness elastic isotropic plates and compared the results with that of the planer fields (i.e., plane stress and plane strain). Plates with different thickness and notch configurations under uniaxial tension were considered. Stress concentration factor was found to be comparatively higher in case of the finite thickness plate over the planer case. [16] obtained an analytical solution for stress around the non-circular cutout in a composite plate. It was found that the stress concentration factor for perforated plate is affected by the material properties, fiber orientation and cutout parameters. [17] performed analytical and numerical studies to analyze the stresses around an irregular shape of hole in an orthotropic plate. Mapping functions were introduced for obtaining solutions for any hole shape and load conditions. Stress concentration was found to be less for angle laminates in comparison to cross-ply laminates under uniaxial loading conditions.

[18] performed experimental and numerical studies on stress concentration for different shapes of cutout. It was found that the order of maximum stress and strain con-

centration depends on the shape of the cut-out. [19] analysed the effect of special shaped cutout on stress concentration by using Lekhnitskii's solution and established a relationship between the non-circular cutout and a circular cutout with the help of a mapping function. [20] obtained the solution for stress distribution around polygonal holes in an infinite plate subjected to biaxial loading using the Muskhelishvili's complex variable method. It was observed that the stress concentration factor is higher in case of uniaxial loading in comparison to biaxial loading. [21] proposed a strong formulation finite element method (SFEM) approach and used this approach for solving stresses around inclusions in isotropic and composite structures under different loading conditions. The results obtained using the proposed approach are found to be in good agreement with the solutions found through a commercial FE software.

[22] performed a three-dimensional finite element simulation of finite-width orthotropic plates with a circular hole. They investigated the effect of varying orthotropy and hole radius-to-width ratio on the stress concentration factor and compared these results with the analytical solution. They observed that, with an increase in the orthotropy ratio, the stress concentration factor increases. Further, they also noticed that the finite element results are better converging with the analytical solutions for lower orthotropy ratio. [23] studied the effect of volume fraction, fiber angle and hole sizes on stress concentration around a circular hole in an orthotropic lamina under a unidirectional in-plane loading. It was found that the stress concentration (i) is not affected much by the volume fraction of fibers and (ii) it is maximum for 0° fiber orientation.

[24] investigated strain concentration in woven fabric composite by using strain mapping technique and compared with the theoretical model given by Lekhnitskiy. It was observed that the strain concentration is influenced by the loading direction and hole geometry. [25] performed experimental and numerical studies to evaluate the stress and strain concentrations in a composite plate with a circular hole when subjected to tensile loading. For experiments, they used electronic speckle pattern interferometer technique. By comparing the results with the Lekhnitskii's and numerical model, it was observed that, for 0° loading direction, stresses are comparable with analytical and numerical results. However, for an off-axis loading direction, stresses are found to be quite different. [26] performed a three-dimensional finite element study to analyze the stress and strain concentration in a finite isotropic, elastic plate with a circular hole. It was found that even for this elastic case, the stress and strain concentration factors are different. Further, the maximum stress and strain, and

their respective concentrated locations were found to be dependent on the plate thickness.

It is clear from the aforementioned literatures that various theoretical, numerical and experimental approaches have been proposed for evaluation of stress concentration in a composite laminate. As per authors' knowledge, only a few studies are available, which focus on evaluation of strain concentration. This may be due to the fact that, in elasticity problems, stress and strain concentration are generally in proportion. Moreover, stress and strain concentration, and their factors may not be same everywhere in plate except at some critical locations [26]. In addition, the above statements are applicable only when the plates are considered to be isotropic and homogeneous. For orthotropic and heterogeneous plates (*i.e.*, composite plates), the proportionality between the stress and strain concentration will be even more complex. For such type of plates, the proportionality depends on elastic constants, which in turn affected by many parameters (such as fiber orientation, fiber distribution, fibre volume fraction and stiffness ratio). Further, as per authors' knowledge, (i) studies are not available for the effect of uncertainties in elastic constants on stress and strain concentration factors, and (ii) a limited number of studies on stress concentration have been carried out so far, which deal with orthotropic cylindrical shell having different curvatures.

In this work, close form solutions are obtained for the stresses around holes of varying shapes in an orthotropic composite plate under a uniaxial tension. Further, a systematic numerical investigation has been carried out for flat as well as singly curved composite plates with a hole subjected to uniaxial tension. Applied stresses have been considered in linear elastic range for all cases. The objective of this study is to understand how stress and strain concentration factors are affected by: (i) size, shape and eccentricity of hole, (ii) number of plies, (iii) fiber orientation, and (iv) plate curvature. Further, a sensitivity analysis is carried out to understand influence of elastic constants on the values of these factors.

2 Complex variable theory

2.1 Mapping functions

Mapping functions are introduced for circular and non-circular hole shapes such as square, triangular and polygonal using conformal transformation inside the unit circle. The area on the inside or outside of a unit circle in z -plane can be represented by the function as: $z = w(\zeta)$, where ' ζ ' is

defined in terms of coordinates ρ and θ as: $\zeta = \rho e^{i\theta}$. For a unit circle (*i.e.*, $\rho = 1$), function ζ can be written as:

$$\zeta = \cos \theta + i \sin \theta \quad (2)$$

The mapping function $w(\zeta)$ for a regular polygon can be determined using the Schwarz-Christoffel integral as given below [27]:

$$z = w(\zeta) = R \left(\frac{1}{\zeta} + \sum_{j=1}^m b_j \zeta^{j n - 1} \right) \quad (3)$$

In Eq. (3), $b_1 = \left(\frac{2}{n(n-1)} \right)$, $b_2 = \left(\frac{n-2}{n^2(2n-1)} \right)$, $b_3 = \left(\frac{(n-2)(2n-2)}{3n^3(3n-1)} \right)$, $b_4 = \left(\frac{(n-2)(2n-2)(3n-2)}{12n^4(4n-1)} \right)$ etc. where n is the number of sides in a polygon; R is the size of polygon.

For $n = 3$ (*i.e.*, equilateral triangle), Eq. (3) can be written as:

$$w(\zeta) = R \left(\frac{1}{\zeta} + \frac{1}{3} \zeta^2 + \frac{1}{45} \zeta^5 + \frac{1}{162} \zeta^8 + \dots \right) \quad (4)$$

For a square hole *i.e.*, $n = 4$, Eq. (3) now can be solved as:

$$w(\zeta) = R \left(\frac{1}{\zeta} + \frac{1}{6} \zeta^3 + \frac{1}{56} \zeta^7 + \frac{1}{176} \zeta^{11} + \dots \right) \quad (5)$$

Similarly, for a pentagonal hole *i.e.*, $n = 5$, Eq. (3) becomes:

$$w(\zeta) = R \left(\frac{1}{\zeta} + \frac{1}{10} \zeta^4 + \frac{1}{75} \zeta^8 + \frac{8}{1750} \zeta^{14} + \dots \right) \quad (6)$$

Briefly, the mapping function for a regular polygon can be expressed as (if only first two terms are considered):

$$z = w(\zeta) = R \left(\frac{1}{\zeta} + \varepsilon \zeta^k \right) \quad (7)$$

where

$\varepsilon = \frac{a-b}{a+b}$, $R = \frac{a+b}{2}$, $k = 1$ for an elliptical hole; here a and b are the major and minor radii of an elliptical hole, respectively;

$\varepsilon = 0$, $k = 1$ for a circular hole; $\varepsilon = 1/3$, $k = 2$ for a triangular hole;

$\varepsilon = 1/6$, $k = 3$ for a square hole; $\varepsilon = 1/10$, $k = 4$ for a pentagonal hole

2.2 Stresses around hole

The stresses around a hole under a uniaxial tension ' σ_0 ' can be expressed as the summation of the basic stress state and the additional stresses localized around holes [27]:

$$\sigma_x = \sigma_0 \cos^2 \alpha + 2 \operatorname{Re} \left[s_1^2 \phi'_0(z_1) + s_2^2 \psi'_0(z_2) \right]; \quad (8)$$

$$\sigma_y = \sigma_0 \sin^2 \alpha + 2\operatorname{Re} [\phi'_0(z_1) + \psi'_0(z_2)];$$

$$\tau_{xy} = \sigma_0 \sin \alpha \cos \alpha + 2\operatorname{Re} [s_1 \phi'_0(z_1) + s_2 \psi'_0(z_2)]$$

where α is the loading direction; $s_1 = \alpha_1 + i\beta_1$ and $s_2 = \alpha_2 + i\beta_2$ with $\alpha_1, \alpha_2, \beta_1, \beta_2$ being anisotropic material constants, which can be determined by solving the following anisotropic characteristic equation:

$$a_{22}Ux + (2a_{12} + a_{66})Uxy + a_{11}Uy = 0 \quad (9)$$

In Eq. (9), U is the Airy stress function and a_{11}, a_{12}, a_{22} and a_{66} are the compliance constants related to the engineering material constants as given here: $a_{11} = \frac{1}{E_1}$, $a_{22} = \frac{1}{E_2}$, $a_{66} = \frac{1}{G_{12}}$ and $a_{12} = -\frac{\nu_{12}}{E_1}$ or $-\frac{\nu_{21}}{E_2}$. E_1 and E_2 are the modulus of elasticity along and across the fiber directions, respectively, G_{12} is the in plane shear modulus and ν_{21} is the Poisson's ratio.

In order to determine stresses around hole, the evaluation of functions $\phi_0(z_1)$ and $\psi_0(z_2)$ are required (see Eq. (8)). For further simplification, $\phi'_0(z_1)$ and $\psi'_0(z_2)$ can be expressed as:

$$\phi'_0(z_1) = \frac{\phi'_0(\zeta)}{w'_1(\zeta)}, \quad \psi'_0(z_2) = \frac{\psi'_0(\zeta)}{w'_2(\zeta)}, \quad (10)$$

with

$$w'_1(\zeta) = \frac{dz_1}{d\zeta}, \quad w'_2(\zeta) = \frac{dz_2}{d\zeta} \quad (11)$$

where z_1 and z_2 are two additional planes acquired from z -plane by affine transformation as [27]:

$$z_1 = x + s_1 y; \quad z_2 = x + s_2 y \quad (12)$$

where coordinates ' x ' and ' y ' can be written as follows:

$$x = 2\operatorname{Re}(z); \quad y = 2\operatorname{Im}(z) \quad (13)$$

Simplifying Eq. (13) using Eqs. (2) and (7), one can get:

$$x = R(\cos \theta + \varepsilon \cos n\theta); \quad y = -R(\sin \theta - \varepsilon \sin n\theta) \quad (14)$$

Considering:

$$\cos(n\theta) = \frac{1}{2} \left[\zeta^n + \frac{1}{\zeta^n} \right]; \quad (15)$$

$$\sin(n\theta) = -\frac{i}{2} \left[\zeta^n - \frac{1}{\zeta^n} \right]$$

Solving Eqs. (12–14), planes z_1 and z_2 can be written as:

$$z_1 = R[(\cos \theta + \varepsilon \cos n\theta) - s_1(\sin \theta - \varepsilon \sin n\theta)]; \quad (16)$$

$$z_2 = R[(\cos \theta + \varepsilon \cos n\theta) - s_2(\sin \theta - \varepsilon \sin n\theta)];$$

Further, simplifying Eqs. (15) and (16), planes z_1 and z_2 now can be expressed as:

$$z_1 = \frac{R}{2} \left(a_1 \zeta + \frac{b_1}{\zeta} + \varepsilon b_1 \zeta^n + \frac{a_1 \varepsilon}{\zeta^n} \right) \quad (17)$$

$$z_2 = \frac{R}{2} \left(a_2 \zeta + \frac{b_2}{\zeta} + \varepsilon b_2 \zeta^n + \frac{a_2 \varepsilon}{\zeta^n} \right)$$

where $a_j = 1 + i.s_j$; $b_j = 1 - i.s_j$ $j=1,2$.

2.2.1 Estimation of stress functions $\phi_0(\zeta)$ and $\psi_0(\zeta)$

The functions $\phi_0(\zeta)$ and $\psi_0(\zeta)$ can be evaluated from the expressions as given below [27]:

$$\phi_0(\zeta) = \frac{i}{4\pi(s_1 - s_2)} \int_{\gamma} (s_2 f_1^0 - f_2^0) \frac{\sigma + \zeta}{\sigma - \zeta} \frac{d\sigma}{\sigma}; \quad (18)$$

$$\psi_0(\zeta) = \frac{-i}{4\pi(s_1 - s_2)} \int_{\gamma} (s_1 f_1^0 - f_2^0) \frac{\sigma + \zeta}{\sigma - \zeta} \frac{d\sigma}{\sigma}$$

In Eq. (18), f_1^0 and f_2^0 can be estimated using the following expressions [27]:

$$f_1^0 = -2\operatorname{Re}\{B^* z_1 + (B^* + iC'^*)z_2\}; \quad (19)$$

$$f_2^0 = -2\operatorname{Re}\{B^* s_1 z_1 + s_2(B^* + iC'^*)z_2\}$$

where

$$B^* = \sigma_0 \frac{\cos^2 \alpha + (\alpha_2^2 + \beta_2^2) \sin^2 \alpha + \alpha_2 \sin 2\alpha}{2(\alpha_2 - \alpha_1)^2 + (\beta_2 - \beta_1)^2} \quad (20)$$

$$B'^* = \sigma_0 \frac{[(\alpha_1^2 - \beta_1^2) - 2\alpha_1 \alpha_2] \sin^2 \alpha - \cos^2 \alpha - \alpha_2 \sin 2\alpha}{2(\alpha_2 - \alpha_1)^2 + (\beta_2 - \beta_1)^2}$$

$$C'^* = \sigma_0 \left\{ \frac{(\alpha_1 - \alpha_2) \cos^2 \alpha + [\alpha_2(\alpha_1^2 - \beta_1^2)]}{2\beta_2[(\alpha_2 - \alpha_1)^2 + (\beta_2 - \beta_1)^2]} + \frac{-\alpha_1(\alpha_2^2 - \beta_2^2) \sin^2 \alpha + (\alpha_1^2 - \beta_1^2)}{2\beta_2[(\alpha_2 - \alpha_1)^2 + (\beta_2 - \beta_1)^2]} - \frac{(\alpha_2^2 - \beta_2^2) \sin \alpha \cos \alpha}{2\beta_2[(\alpha_2 - \alpha_1)^2 + (\beta_2 - \beta_1)^2]} \right\}$$

Considering:

$$\int_{\gamma} \sigma^k \frac{\sigma + \zeta}{\sigma - \zeta} \frac{d\sigma}{\sigma} = 4\pi i \zeta^k; \quad \int_{\gamma} \frac{1}{\sigma^k} \frac{\sigma + \zeta}{\sigma - \zeta} \frac{d\sigma}{\sigma} = 0 \quad (21)$$

Using Eqs. (17–21), the stress functions $\phi_0(\zeta)$ and $\psi_0(\zeta)$ can be solved as [27]:

$$\phi_0(\zeta) = \frac{\zeta}{(s_1 - s_2)} \{s_2(k_1 + \overline{k_2}) - (k_5 + \overline{k_6})\} \quad (22)$$

$$+ \frac{\zeta^n \varepsilon}{(s_1 - s_2)} \{s_2(k_2 + \overline{k_1}) - (k_6 + \overline{k_5})\};$$

$$\psi_0(\zeta) = \frac{\zeta}{(s_1 - s_2)} \{s_1(k_1 + \overline{k_2}) - (k_5 + \overline{k_6})\}$$

$$+ \frac{\zeta^n \varepsilon}{(s_1 - s_2)} \{s_1(k_2 + \overline{k_1}) - (k_6 + \overline{k_5})\}$$

where

$$k_1 = \frac{R}{2} [B^* a_1 + (B'^* + iC'^*)a_2]; \quad (23)$$

$$k_2 = \frac{R}{2} [B^* b_1 + (B'^* + iC'^*)b_2];$$

$$k_5 = \frac{R}{2} [B^* s_1 a_1 + s_2(B'^* + iC'^*)a_2];$$

$$k_6 = \frac{R}{2} [B^* s_1 b_1 + s_2(B'^* + iC'^*)b_2]$$

Table 1: Material properties considered for modelling

E_x	E_y	E_z	ν_{xy}	ν_{yz}	ν_{xz}	G_{xy}	G_{yz}	G_{xz}
$54 \times 10^9 (Pa)$	$18 \times 10^9 (Pa)$	$18 \times 10^9 (Pa)$	0.25	0.25	0.25	$9 \times 10^9 (Pa)$	$3.46 \times 10^9 (Pa)$	$9 \times 10^9 (Pa)$

2.2.2 Estimation of ‘ ζ ’ for different hole shapes

In order to evaluate $\phi_0(\zeta)$ and $\psi_0(\zeta)$, ‘ ζ ’ is estimated for different hole shapes as follows:

Elliptical hole: For an elliptical hole $\epsilon = \frac{a-b}{a+b}$; $n = 1$, using these constants and Eq. (17), one can obtain ‘ ζ ’ for functions $\phi_0(\zeta)$ and $\psi_0(\zeta)$ as:

$$\zeta = \left\{ \frac{z_j - \{z_j^2 - (a^2 + s_j^2 b^2)\}^{0.5}}{(a + i.s_j b)} \right\}; \quad j=1,2 \quad (24)$$

For a circular hole $a = b$, Eq. (24) becomes:

$$\zeta = \left\{ \frac{z_j - \{z_j^2 - a^2(1 + s_j^2)\}^{0.5}}{a(1 + i.s_j)} \right\}; \quad j=1,2 \quad (25)$$

Triangular hole: For a triangular hole: $\epsilon = 1/3$; $n = 2$. Putting these in Eq. (17), ‘ ζ ’ for functions $\phi_0(\zeta)$ and $\psi_0(\zeta)$ can be estimated on solving the fourth order quartic polynomial as follows:

$$\zeta^4 + \left(\frac{a_j}{b_j \epsilon} \right) \zeta^3 + \left(\frac{-2z_j}{R \epsilon b_j} \right) \zeta^2 + \frac{\zeta}{\epsilon} + \left(\frac{a_j}{b_j} \right) = 0; \quad (26)$$

$j = 1, 2$

Square hole: Similarly, for a square hole $\epsilon = 1/6$; $n = 3$. Using these constants in Eq. 17, ‘ ζ ’ for functions $\phi_0(\zeta)$ and $\psi_0(\zeta)$ can be obtained by solving sixth order sextic equation as given below:

$$\zeta^6 + \left(\frac{a_j}{b_j \epsilon_1} \right) \zeta^4 + \left(\frac{-2z_j}{R \epsilon_1 b_j} \right) \zeta^3 + \frac{1}{\epsilon_1} \zeta^2 + \left(\frac{a_j}{b_j} \right) = 0; \quad (27)$$

$j = 1, 2$

Pentagonal hole: For a pentagonal hole $\epsilon = 1/10$; $n = 4$, ‘ ζ ’ for functions $\phi_0(\zeta)$ and $\psi_0(\zeta)$ can be obtained by solving eight order polynomial as given below:

$$\zeta^8 + \left(\frac{a_j}{b_j \epsilon_1} \right) \zeta^5 + \left(\frac{-2z_j}{R \epsilon_1 b_j} \right) \zeta^4 + \frac{1}{\epsilon_1} \zeta^3 + \left(\frac{a_j}{b_j} \right) = 0; \quad (28)$$

$j = 1, 2$

Further, stress concentration factor (SCF) can be estimated as a ratio of the maximum stress around hole to the applied stress σ_0 .

3 Numerical investigation

Three-dimensional models of a flat and a singly curved composite plate of size 0.2m by 0.2m with a hole (as

Figure 3: Finite element model of (a) flat and (b) singly curved composite plates with hole

= 1, 2 shown in Figures 3a and 3b) have been developed in ANSYS APDL [28]. A 3-D four-node quadrilateral shell element (SHELL181, [28]) has been considered for modeling the plate. The material properties of glass-epoxy composite have been used for modeling the plate as given in Table 1 [29]. To keep the applied stress in linear elastic range, a uniaxial tensile load of 100 N has been applied to all nodes on one side of the plates while boundary conditions have been applied to the opposite side of the plates as shown in Figures 3a and 3b.

3.1 Mesh generation and mesh sensitivity analysis

Evaluation of mesh sensitivity is an important aspect for stress concentration related problems. This is because the

(a)

(b)

Figure 4: Free and mapped meshing (around hole) in (a) flat and (b) singly curved composite plates

accuracy of solution depends on the mesh density around the stress concentrator. For reasonable solution, the mesh should be more refined near the stress concentrators to capture the local effects. In this study, two regions are considered separately for meshing: one around the hole and the other for the remaining region as shown in Figures 4a and 4b. It is imperative that the two regions will have different level of stress or strain variations. Hence, a mapped meshing with quadratic element (higher precision) has been used for meshing the region around the hole and the remaining region has been meshed with free meshing (see Figures 4a and 4b). For mesh sensitivity analysis, several simulations have been run for more mesh refinement near

(a)

(b)

Figure 5: Mesh sensitivity plot for normalized: (a) maximum stress (σ_x) and (b) maximum strain

the circular hole. Typical mesh size has been varied from 5×10^{-3} m to 5×10^{-5} m.

For the flat plate with hole, Figures 5a and 5b show the mesh sensitivity of normalized maximum stress and strain with number of elements, respectively. The normalization has been done with respect to the maximum value of stress or strain within the plate. It can be observed from Figures 5a and 5b that the value of normalized stress and strain saturates (i.e., change is within 1%) for a number of elements of about 58,000. Hence, the refined mesh size has been taken as 5×10^{-5} i.e., corresponding to 58,000 el-

Table 2: Geometric configurations considered for parametric study

Parameters	Hole size (m)	Hole shape	Normalized eccentricity	Number of plies	Curvature (β)	Fiber orientation
Size of hole	0.0075, 0.015, 0.0225, 0.03	Circular	0	4	0	0°
Shape of hole	0.03	Elliptical, Circular, Pentagon, Square, Triangular (Same circumscribed radius)	0	4	0	0°
Eccentricity of hole	0.03	Circular	1, 1.5, 2, 2.5, 3.0	4	0	0°
Number of plies	0.03	Circular	0	4, 8, 12, 16	0	0°
Plate Curvature	0.03	Circular	0	1	0, 0.28, 0.40, 0.49, 0.57, 0.64, 0.70, 0.75, 0.80, 0.85	0°
Fiber orientation	0.0075, 0.015, 0.0225, 0.03	Elliptical, Circular, Pentagon, Square, Triangular (Same circumscribed radius)	0, 1, 1.5, 2, 2.5, 3.0	4, 8, 12, 16	0	0°, 45°, 90°

Table 3: Effect of hole size on stress and strain concentration factors

Hole Size (m)	SCF	SNCF
0.0075	5.66	5.66
0.015	4.52	4.50
0.0225	4.02	4.02
0.03	3.82	3.82

ements. Similar procedure has also been adopted for obtaining the refined mesh size in case of the singly curved plate.

3.2 Parameters considered

For parametric study, different plate parameters are considered while keeping the fiber orientation to 0°. At first, for studies on hole size, circular holes of diameter 0.0075m, 0.015m, 0.0225 and 0.03m are considered. Further, for studies related to shape of hole, eccentricity, number of plies and plate curvature, hole size is considered as 0.03m. Altogether, (i) six different hole shapes as shown in Figure 6, (ii) four different eccentricities, (iii) four different numbers of plies, and (iv) ten different curvatures (*i.e.*, β as defined in Eq. 1) are considered. The considered values of different parameters are given in Table 2. For better representation, eccentricity of the hole is normalized with respect to the hole diameter (*i.e.*, 0.03 m) as shown in Ta-

ble 2. Further, to see the effect of fiber orientation, studies are also conducted with flat laminated plates consisted of $[0]_4$, $[45/-45]_s$ and $[90]_4$ lay-ups as given in Table 2.

For elliptical hole, two cases have been studied with respect to the shape orientation. These are: (i) Elliptical I (*i.e.*, major axis is along the loading direction) and (ii) Elliptical II (*i.e.*, major axis is transverse to the loading direction) as shown in Figures 7a and 7b.

4 Results and discussions

At first, results are presented for laminated plates consisted of lay-up $[0]_4$. After that, different fiber orientations are considered. Then, the effect of plate curvature is studied. The following subsections provide results of this study.

4.1 Size of hole

Table 3 shows the effect of hole size on SCF and SNCF. It can be observed from Table 3 that, when the size of hole increases, the SCF and SNCF decreases. For example, SCF reduces from 5.66 for hole size of 0.0075m to 3.82 for hole size 0.03m. This can be explained from Figures 8a and 8b, where the normalized stress ($\sigma_x(y, 0)$) is obtained for different positions along the y-axis (away from the hole edge).

Figure 6: Different hole shapes considered for the study

Figure 7: Composite plate with a hole of (a) Elliptical I and (b) Elliptical II shape

Figure 8: (a) Stress distribution (σ_x) around hole and (b) zoomed view of the stress distribution (σ_x) near the vicinity of hole edge

Figure 8b shows a zoomed view of Figure 8a. The horizontal axis shows the distance from the hole edge. It can be noted from Figure 8a that the characteristic distance d_0

Figure 9: Schematic diagram of hole eccentricity with respect to loading direction (i.e., x-axis)

Figure 10: Stress distribution (σ_x) around hole for a normalized eccentricity of (a) zero and (b) 3.0

(as shown in Figure 18b) is same (i.e., 0.062) regardless of the hole size. The decrease in SCF with increase in hole size can now be explained as follows: as R increases, ϵ_1 increases for a given ' d_0 ' value (see Eq. A.5 in Appendix) resulting in decrease in SCF (see $\sigma_x(y, 0)/\sigma_0$ in Eq. A.4 given in Appendix).

Similar results are obtained for strain distribution around the hole. Also, SCF and SNCF are found to be same as expected. It may be noted that, these results are valid only when the hole size is smaller compared to the width of plate.

4.2 Eccentricity of hole

In this study, hole eccentricity ' e_y ' is considered in the transverse direction to the loading as shown in Figure 9. Table 4 shows the hole eccentricity effect on SCF and SNCF. It can be observed from this table that, both SCF and SNCF increase with increase in eccentricity. The reason can be understood from Figures 10a and 10b, which show the stress distribution around the hole for eccentricity of zero and of 3 times the radius of the hole. It can be observed that for a plate with a hole at zero eccentricity, the stress distribution around the hole edge is symmetrical (see Figure 10a) about the x-axis. However, with an increase in the hole eccentricity, stress distribution (about an axis parallel to the x-axis but passing through the center of hole) is found to be unsymmetrical (see Figure 10b). This can be better explained by scrutinizing the stress contours around the hole as shown in Figures 10a and 10b. For example, from Figure 10b, one can note that σ_x is much higher along A than B.

Table 4: Effect of hole eccentricity on stress and strain concentration factors

Normalized Eccentricity	SCF	SNCF
1	4.06	4.00
1.5	4.20	4.19
2	4.36	4.36
2.5	4.60	4.59
3	4.98	4.97

(a)

(b)

Figure 11: Effect of bluntness on: (a) stress concentration factor and (b) strain concentration factor

4.3 Shape of hole

To study the effect of hole shape on the stress and strain concentration factors, some common hole shapes have been analyzed as described earlier (see Figure 6). Table 5 shows these results. It can be observed from Table 5 that, Elliptical I shape shows the least stress and strain concentration factors amongst all considered hole shapes (*i.e.*,

2.96). However, for Elliptical II shape, the values of stress and strain concentration factors are significantly higher even that of the circular hole. These results of SCF are further validated using theoretical solutions as provided earlier (see Eqs. (8) and (22)). For this purpose, the anisotropic characteristic equation (see Eqs. (8) and (22)) has been solved considering the same material properties as assigned in the numerical study. The values of β_1 and β_2 have been found as 2.2104 and 0.7836, respectively. Putting these values in Eqs. (8) and (22), theoretical stress concentration factors have been calculated for the considered hole shapes as shown in parenthesis of Table 5.

From Table 5, it can also be observed that, for pentagonal, square and triangular holes, stress and strain concentration factors are higher compared to a circular and Elliptical I hole. Among these shapes, square hole shape has the maximum stress and strain concentration factors compared to the pentagonal and triangular. It may be noted that, these values also depend on the orientation of hole shape with respect to the loading direction. For square and pentagonal hole, sharp corners are symmetrically placed about the y-axis (see Figure 6). However, this is not the case for the triangular hole. Critical stress locations are also marked by solid dots in Figure 6. Among all these hole shapes, the increasing order of stress and strain concentration factor is as follows: Elliptical I, circular, triangular, Elliptical II, pentagon, square (Table 5). Further, it is also observed that SCF and SNCF are not same for the pentagonal, square and triangular hole shapes. However, as per authors knowledge, there is no theoretical formulations to explain these results.

In order to see how the bluntness of hole (*i.e.*, square, pentagon and triangular) affects the results of SCF and SNCF, Figures 11a and 11b are shown. In this study, bluntness is defined as by the radius ratio of hole *i.e.*, edge radius ' r ' to the circumscribed circle radius ' R ' (see Figure 12). It can be observed that, with an increase in bluntness of the hole, SCF and SNCF decreases. In addition, maximum effect of bluntness has been found for the square hole shape, which is pretty similar to the pentagonal hole shape. Further, the least effect has been found for the triangular hole shape.

4.4 Number of ply

For this case, the plate has been considered with a circular hole. It is observed that an increase in the number of plies does not affect SCF and SNCF. Since, the plates are considered to be flat (*i.e.*, absence of any curvature), it is obvious that in all the cases, the laminates have the same

Figure 12: Bluntness for different shapes of hole

Figure 13: Effect of fiber orientation on normalized: (a) stress (σ_x) and (b) strain around the circumference of hole

Table 5: Effect of hole shape on stress and strain concentration factors

Hole Shape	SCF (theoretical)	SNCF
Elliptical I	2.96(2.50)	2.96
Elliptical II	6.02(6.98)	6.02
Circular	3.82(3.99)	3.82
Pentagon	6.80(6.23)	7.27
Square	7.16(6.89)	7.66
Triangular	4.88(5.21)	5.01

state of stress at a given point on the plate, when the stress is normalized by the nominal stress.

4.5 Fiber orientation

To understand the effect of fiber orientation on SCF and SNCF, two additional flat laminated plates consisting of lay-ups $[45/-45]_s$ and $[90]_4$ have been studied. Figures 13a and 13b show the effect of fiber orientation on the normalized stress and strain around the circumference of the hole. The horizontal axis shows circumferential angle from the loading direction (see Figure 18c). The results of stress and strain around the circumferential path are obtained at an interval of 4° . For the ease of understanding, these points are joined by smooth curve. From the peaks of the curves, it is clear that the normalized stress and strain are maximum when the fibers are aligned in the loading direction and minimum when fibers are aligned

(a)

(b)

Figure 14: (a) Stress (σ_x) and (b) strain distribution around hole for 45° fiber orientation

at 45° to the loading direction. In addition, for 0° and 90° fiber orientations, the maximum stress occurs at a circumferential angle of 90° . However, for 45° fiber orientation, the peak (stress) occurs at around 100° circumferential angle. This implies that the stress distribution is not symmetric about the y-axis for 45° fiber orientation. This can also be observed from the antisymmetric stress distribution around the hole for 45° fiber orientation (see Figure 14a). The strain distribution around the hole is however found to be symmetric about the y-axis (Figures 13b and 14b).

For $[0]_4$ and $[90]_4$ lay-ups, laminates as well as holes are subjected to uniaxial local tensile state of stress. However, for $[0]_4$ lay-up, the stresses are much higher compared to $[90]_4$ lay-up because of the higher tensile strength

of the fiber in the loading direction. In case of $[90]_4$ lay-up, the load is applied in the direction transverse to the fiber direction, which causes separation between the fiber and matrix (transverse tension) as well as buckling of fiber (compressive stress due to Poisson's effect) around the hole. Since, the compressive strength is much lower in comparison to the tensile strength of a fibre, the peak stress around the hole is less in case of 90° fibre orientation. However, for $[45/-45]_s$ lay-up, the hole is subjected to a biaxial local tensile and shear stresses, which results in a reduced effect of peak stress near the vicinity of hole edge. Owing to this, the peak stress is lowest for $[45/-45]_s$ lay-up compared to $[0]_4$ and $[90]_4$ lay-ups.

The results obtained in Figures 13a are in coherence with the theoretical solution provided by [10] for anisotropic glass-epoxy composite plates. Stress concentration factors from the numerical study (considering the hole size 0.03 m) are then compared with the theoretical stress concentration factors at the hole boundary using Eq. A.6 (see Appendix). Table 6 shows that the stress concentration factors are comparable with the theoretical solution (see Eqs. A.6 and A.7e). However, for 90° fiber orientation, the theory underestimates the SCF by about 14.28%.

Table 6: Stress concentration factors for different fiber orientations

Fiber orientation	Numerical	Theoretical	% deviation
0°	3.82	3.84	0.523
45°	2.68	2.66	0.746
90°	3.22	2.76	14.28

Size of hole: To demonstrate how the hole size affects the SCF in a plate for different fiber orientations, Figures 15a and 15b are shown. For clarity of presentation, the points corresponding to different fiber orientations are joined by straight lines. It can be observed that from Figures 15a and 15b that, for 0° and 90° fiber orientations, stress as well as strain concentration factors are decreasing with the increase in the hole size as explained in Section 4.3 for $[0]_4$ lay-up. Whereas, for 45° fiber orientation, it is following the reverse trend. [23] explained that for 45° fiber orientation, both fibers as well as matrix are subjected to shear stress along with the tensile stress. In case of a larger hole size, more number of discontinuous fibers are subjected to the shear stress compared to the smaller size of hole.

(a)

(b)

Figure 15: Effect of hole size and fiber orientations on: (a) stress concentration factor and (b) strain concentration factor

Shape of hole: Table 7 shows the relative stress concentration factors for different shapes of hole and fiber orientations with respect to the circular hole shape having 0° fiber orientation. Relative stress concentration factors for different hole shapes are denoted as given here:

$$SCF_{eI} = \frac{SCF_{ellipticalI}}{SCF_{circular}} \quad (29a)$$

$$SCF_{eII} = \frac{SCF_{ellipticalII}}{SCF_{circular}} \quad (29b)$$

$$SCF_{Sq} = \frac{SCF_{square}}{SCF_{circular}} \quad (29c)$$

$$SCF_P = \frac{SCF_{pentagonal}}{SCF_{circular}} \quad (29d)$$

$$SCF_{Tr} = \frac{SCF_{triangular}}{SCF_{circular}} \quad (29e)$$

In Eq. 29, SCF_{eI} , SCF_{eII} , SCF_{Sq} , SCF_P and SCF_{Tr} are relative stress concentration factors for Elliptical I, Elliptical II, square, pentagonal and triangular hole shape, respectively. These can be used as safety factors for design of a laminated composite with the hole shapes used here. From Table 7, one can note that SCF is least for Elliptical I hole irrespective of fiber orientation and highest for the square hole except the 90° case, where it is maximum for the triangular hole. For any hole shape, the maximum value of SCF can be noted as much as twice that of the circular hole. Since, sharpness of a corner influences the values of SCF and SNCF, the variation of SCF and SNCF are studied with changing bluntness for pentagon, square and triangular hole for different lay-ups. As described earlier (see Figures 11a and 11b), bluntness reduces SCF and SNCF for 0° fiber orientation. The trend is also similar for other fiber orientations.

Table 7: Relative stress concentration factors for different shapes of hole

Relative stress concentration factor/Fiber orientation	0°	45°	90°
SCF_{eI}	0.774	0.575	0.748
SCF_{eII}	1.575	0.979	1.403
SCF_P	1.780	1.293	1.795
SCF_{Sq}	1.874	1.408	1.581
SCF_{Tr}	1.439	1.220	1.837

4.6 Plate curvature

In order to study the effect of plate curvature on SCF and SNCF, stresses around the circular hole in cylindrical plates are analysed. For this purpose, a singly curved cylindrical composite plate with a hole under uniaxial tension is analysed for varying plate curvature (See Figure 4b

Figure 16: Effect of plate curvature on: (a) stress concentration factor and (b) strain concentration factor

and Table 2). For comparison, isotropic steel plates are also analyzed along with the orthotropic (composite) plates as considered herein. Figure 16a shows the SCF for isotropic and orthotropic cylindrical plates for different curvatures (*i.e.*, β values as defined in Eq. 1). It may be noted that the results for SCF shown in Figure 16a for isotropic cylindrical plate are in good agreement with the theoretical solution derived by [12] for a cylindrical shell with a circular hole. From Figure 16a, it can be observed that SCF increases with an increase in curvature for both isotropic as well as orthotropic cylindrical plates. However, this increase in SCF with curvature is more pronounced in case of the orthotropic plate, specially for higher curvatures (β). The results for SNCF as shown in Figure 16b also follows similar trend as in for the SCF.

5 Sensitivity analysis

The effect of uncertainties in elastic material constants of composites namely E_1 , E_2 , ν_{12} and G_{12} on SCF and SNCF is evaluated here. For this purpose, a first order sensitivity analysis is performed wherein $\pm 10\%$ variation of elastic constants are considered from the mean value, *i.e.*, as given in Table 1. To get an insight, the case of composite plate with a circular hole of varying sizes but placed at the center is considered.

Figure 17a shows the tornado diagram for SCF with hole size 0.015 m. The vertical line in the middle of the tornado diagram indicates a SCF value of around 4.50, which corresponds to the SCF evaluated using the mean values of the elastic material constants. Further, the length of each horizontal bar represents the variation in SCF due to the variation in the respective material constants (*i.e.*, E_1 , E_2 , ν_{12} and G_{12}). Figures 17b and 17c provide the tornado diagrams for hole sizes of 0.0225 m and 0.03 m. One can observed from these figures that E_1 and G_{12} are the two most sensitive elastic material constants to govern SCF, particularly for smaller hole sizes. As the size of hole increases, E_2 also becomes significant. Similar results have also been found for SNCF, but not included here to save space.

6 Simplified formulas for SCF

In order to facilitate design of composites without performing extensive analysis, in this part, simplified formulas are developed to have an idea of SCF in composite plates with hole. Table 8 provides formulas for SCF with various hole sizes, shapes, eccentricities and fiber orientations. These formulas are developed by fitting first, second and third degree polynomials depending upon the nature of variation. For different fiber orientations, SCF can be evaluated for normalized hole size varying from 0.075

Table 8: Simplified formulas for SCF evaluation

Parameters/Fiber orientation	0°	45°	90°
Size of hole	$SCF_{dn0} = 106.67d_n^2 - 33.33d_n + 6.42$	$SCF_{dn45} = 7.73d_n + 1.436$	$SCF_{dn90} = 85.33d_n^2 - 28.8d_n + 5.7$
Hole eccentricity	$SCF_{en0^\circ} = 0.0481e_n^3 - 0.1241e_n^2 + 0.3254e_n + 3.82$	$SCF_{en45^\circ} \approx 0.75SCF_{en0^\circ}$	$SCF_{en90^\circ} \approx 0.85SCF_{en0^\circ}$
Hole Shape	$SCF_{EI0} = 0.774SCF_{C0}$	$SCF_{EI45} = 0.575SCF_{C0}$	$SCF_{EI90} = 0.748SCF_{C0}$
	$SCF_{EII0} = 1.575SCF_{C0}$	$SCF_{EII45} = 0.979SCF_{C0}$	$SCF_{EII90} = 1.403SCF_{C0}$
	$SCF_{P0} = 1.780SCF_{C0}$	$SCF_{P45} = 1.293SCF_{C0}$	$SCF_{P90} = 1.795SCF_{C0}$
	$SCF_{Sq0} = 1.874SCF_{C0}$	$SCF_{Sq45} = 1.408SCF_{C0}$	$SCF_{Sq90} = 1.581SCF_{C0}$
	$SCF_{Tr0} = 1.439SCF_{C0}$	$SCF_{Tr45} = 1.220SCF_{C0}$	$SCF_{Tr90} = 1.837SCF_{C0}$

[d_n = normalized hole size (hole size to plate width ratio)];

[e_n = normalized eccentricity (eccentricity e_y to hole size ratio)];

[$SCF_{dn\alpha^\circ}$ = SCF with respect to varying hole size for α° fiber orientation];

[$SCF_{en\alpha^\circ}$ = SCF with respect to varying eccentricity for α° fiber orientation];

[$SCF_{EI\alpha^\circ}$, $SCF_{C\alpha^\circ}$, $SCF_{EII\alpha^\circ}$, $SCF_{P\alpha^\circ}$, $SCF_{Sq\alpha^\circ}$ and $SCF_{Tr\alpha^\circ}$ are SCFs for Elliptical I, circular, Elliptical II, pentagonal, square and triangular];

[hole shapes, respectively for α° fiber orientation]

Figure 17: Tornado diagrams of SCF for a circular hole of size: (a) 0.015 m, (b) 0.0225 m, and (c) 0.030 m

Figure 18: (a) An infinite plate with a circular hole subjected to uniaxial tension, (b) stress distribution around hole, (c) a composite plate with hole subjected to uniaxial tension at an angle of α° from fiber direction

to 0.15 and normalized eccentricity ranging from 0 to 3. In addition, SCF can also be evaluated for plate with a hole of elliptical, circular, square, pentagon and triangu-

lar shapes. For example, a laminated plate with a circular hole at the center with normalized size of d_n and consisting of 0° fiber orientation plies, SCF can be evaluated from the expression as given in Table 8 i.e., $SCF_{dn0} = 106.67d_n^2 - 33.33d_n + 6.42$. Similarly, for a plate (0° fiber orientation) with a circular hole with normalized eccentricity of e_n , SCF can be evaluated from the expression i.e., $SCF_{en0^\circ} = 0.0481e_n^3 - 0.1241e_n^2 + 0.3254e_n + 3.82$. It may be noted that these formulas can be used for the parameter space (hole size, hole shape, hole eccentricity and fiber orientation) considered herein. Estimation outside this space requires further numerical validation.

7 Conclusions

A systematic parametric study is conducted here to understand how different governing parameters affect stress and strain concentration factors in a perforated composite plate under uniaxial tensile loading. The parameters considered include: (i) size of hole, (ii) shape of hole, (iii) eccentricity of hole, (iv) number of plies, and (v) fiber orientation. In addition, the effect of plate curvature is also studied. For this purpose, flat square and singly curved composite plates with hole subjected to uniaxial tensile loading are considered. [28] has been used for performing the analysis. While performing analysis, it is ensured that the applied stresses remain within the linear elastic range for all the cases. Three types of lay-up, i.e., $[0]_4$, $[45/-45]_s$ and $[90]_4$ have been considered to understand the fiber orientation effect along with other parameters. From these analyses, it has been found that various parameters influence the SCF and SNCF with varying degrees. Further, for validating the standard results, available analytical formulations have been used wherever applicable. A sensitivity analysis with a flat plate having a circular hole at the center shows that E_1 and G_{12} are the two most sensitive elastic material constants to govern SCF, particularly for smaller hole sizes. Some other important findings of this study are listed as follows:

- For the parameter space considered herein, it is noted that SCF and SNCF values are found to be approximately same with minor variations for holes having sharp corners. Hence, SCF and SNCF can be used equivalently for design purpose.
- SCF has been found to be maximum for 0° fiber orientation and minimum for 45° fiber orientation. In addition, stress concentration has been found to be maximum at 90° circumferential angle for 0° and 90° fiber orientations, and around 100° circumfer-

ential angle for 45° fiber orientation. As explained earlier, these variations arise due to change in stress distribution patterns around the hole.

- For 0° and 90° fiber orientations, SCF increases with decrease in the hole size. This is due to localization of stress and strain in the vicinity of hole edge. However, for 45° fiber orientation, the trend is opposite due to presence of significant shear stress around the hole.
- SCF has been found to be sensitive with hole shape and bluntness. The order of increasing SCF with respect to hole shape is Elliptical I, circular, triangular, Elliptical II, pentagonal and square. Also, increase in bluntness of a hole reduces the SCF. Further, SCF has been found to be maximum for 0° fiber orientation and minimum for 45° fiber orientation irrespective of hole shapes.
- SCF increases with an increase in hole eccentricity in the direction transverse to the loading. With increase in normalized eccentricity from 1 to 3, SCF increases by about 25%. This variation has been observed regardless of fiber orientation.
- For flat plates, SCF has been found to be independent to the number of plies in a laminate for the very obvious reason that all laminates have the same state of stress at a given point due to absence of any curvature.
- It has been observed that SCF increase with an increase in plate curvature. For example, in a specific case of plate curvature of $\beta=0.85$ (i.e., semi-circular) as given in Figure 16a, the value of SCF (i.e., 5.89) has been found to be almost 50% higher in comparison to the case of a flat plate (i.e., $\text{SCF} = 3.82$).

Finally, for the ease of designing, simplified formulas have been developed to evaluate SCF in a flat composite plate with hole for a wide range of parameters. These formulas may be used for the parameter space (hole size, hole shape, hole eccentricity and fiber orientation) considered herein. Extrapolation outside this parameter space requires validation through appropriate simulations.

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Appendix: Effect of hole size and fibre orientation on SCF

[7] derived approximate expressions for the normal stress distribution around a circular hole of radius R as shown in Figure 18a. The ratio of the maximum transverse stress component $\sigma_x(y, 0)$ (i.e., along the y-axis) to the applied stress σ_0 (i.e., along the x-direction) is defined here as the stress concentration factor and given by the following expression [7]:

$$\frac{\sigma_x(y, 0)}{\sigma_0} = 1 + \frac{1}{2}\varepsilon^{-2} + \frac{3}{2}\varepsilon^{-4} - \frac{(K_T - 3)}{2}(5\varepsilon^{-6} - 7\varepsilon^{-8}) \quad (\text{A.1})$$

In Eq. (A.1), $\varepsilon = \frac{y}{R}$; K_T is the anisotropic stress concentration factor. In case of a symmetric laminate with orthotropic in plane stiffness properties, K_T can be obtained from the following expression [7]:

$$K_T = 1 + \sqrt{\frac{2}{A_{22}} \sqrt{A_{11}A_{22} - A_{12}^2} + \frac{A_{11}A_{22} - A_{12}^2}{2A_{66}}} \quad (\text{A.2})$$

where A_{ij} is the element corresponding to the i^{th} row and j^{th} column of the orthotropic in-plane stiffness matrix of the laminate. At hole boundary, i.e., $y = R$ or $\varepsilon = 1$, Eq. (A.1) reduces to

$$\frac{\sigma_x(R, 0)}{\sigma_0} = K_T \quad (\text{A.3})$$

From Eq. (A.3), it may be noted that the stress concentration factor is independent to the hole size at the hole boundary, which is in contradiction to previous findings. [5] presented the following expression for stress distribution adjacent to a hole while accounting the effect of hole size at the hole boundary:

$$\frac{\sigma_x(R, 0)}{\sigma_0} = \frac{2}{2 + \varepsilon_1^2 + 3\varepsilon_1^4} \quad (\text{A.4})$$

where

$$\varepsilon_1 = \frac{1}{1 + \frac{d_0}{R}} \quad (\text{A.5})$$

and d_0 is the characteristic distance at which $\sigma_x(y = d_0, 0)$ reaches the value of nominal stress, i.e., σ_0 (see Figure 18b).

[10] studied the effect of fiber orientation on maximum stress and corresponding concentration factor. The following expression was presented for the circumferential stress at the edge of a circular cutout in an orthotropic plate subjected to uniaxial loading for any arbitrary fiber orientation α (see Figure 18c):

$$\frac{\sigma_{\alpha\theta}}{\sigma_0} = \left[\frac{P - Q - R}{S} \right] \quad (\text{A.6})$$

where

$$P = (1 + \gamma_1) \cdot (1 + \gamma_2) \cdot [1 + \gamma_1 + \gamma_2 - \gamma_1 \cdot \gamma_2 - 2 \cos 2(\theta - \alpha)] \quad (\text{A.7a})$$

$$Q = 4[\gamma_1 + \gamma_2 - (1 + \gamma_1 \cdot \gamma_2) \cdot \cos 2(\theta - \alpha)] \cdot \sin^2 \alpha \quad (\text{A.7b})$$

$$R = 4(\gamma_1 \cdot \gamma_2 - 1) \cdot \sin 2(\theta - \alpha) \cdot \sin \alpha \cdot \cos \alpha \quad (\text{A.7c})$$

$$S = [1 + \gamma_1^2 - 2 \cdot \gamma_1 \cdot \cos 2(\theta - \alpha)] \cdot [1 + \gamma_2^2 - 2 \cdot \gamma_2 \cdot \cos 2(\theta - \alpha)] \quad (\text{A.7d})$$

$$\gamma_1 = \frac{\sqrt{(\frac{E_2}{2G_{12}} - \nu_{21}) + \sqrt{(\frac{E_2}{2G_{12}} - \nu_{21})^2 - \frac{E_2}{E_1} - 1}}}{\sqrt{(\frac{E_2}{2G_{12}} - \nu_{21}) + \sqrt{(\frac{E_2}{2G_{12}} - \nu_{21})^2 - \frac{E_2}{E_1} + 1}}} \quad (\text{A.7e})$$

$$\gamma_2 = \frac{\sqrt{(\frac{E_2}{2G_{12}} - \nu_{21}) - \sqrt{(\frac{E_2}{2G_{12}} - \nu_{21})^2 - \frac{E_2}{E_1} - 1}}}{\sqrt{(\frac{E_2}{2G_{12}} - \nu_{21}) - \sqrt{(\frac{E_2}{2G_{12}} - \nu_{21})^2 - \frac{E_2}{E_1} + 1}}}$$

In Eq. (A.7), α is the angle of fibre orientation, θ is the circumferential angle.