Research Article

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Small size and rotary inertia effects on the natural frequencies of carbon nanotubes

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Abstract: In the present paper, rotary inertia and small size effects on the free vibration response of single-walled carbon nanotubes are examined. The equations in motion and associated boundary conditions are obtained by using Hamilton's principle on the basis of modified couple stress and Rayleigh beam theories. The size effect is taken into account by modified couple stress theory while the rotary inertia effect is considered by Rayleigh beam theory. The resulting equations are analytically solved by implementing Navier's solution technique for pinned-pinned carbon nanotubes. Influences of slenderness ratio, length scale parameter and rotary inertia on the natural frequencies of single-walled carbon nanotubes are studied in detail.

Keywords: size dependency, carbon nanotube, non-classical continuum theory, rotary inertia effect, vibration

1 Introduction

In 1980s, measurement and modeling in micro- and nano-dimensions became possible due to the development of Atomic Force Microscope (AFM) and Scanning Electron Microscope (SEM). In 1991 and 1993, multi-walled and single-walled carbon nanotubes were discovered by Iijima [32] and Iijima and Ichihashi [34]. Carbon nanotubes (CNTs) have extraordinary physical, optic, thermal, electronic and mechanical properties. CNTs can conduct heat and electricity better than copper and they have high flexibility and elastic modulus. In fact, the characteristics of CNTs

are changeable according to the rolling condition of the graphene sheet to form CNTs as armchair, zigzag and chiral [10, 33, 53]. In recent times, CNTs have aroused great research interest of scientists, academicians, engineers and researchers due to these unique material properties and have been widely used as an essential structural element in applications of nano- and micro-electro mechanical systems (NEMS and MEMS) such as sensors, supercapacitors, actuators, nanofilters and transistors [39, 62].

In order to determine the mechanical properties of CNTs, atomistic modeling such as molecular dynamics simulation and continuum modeling like rod, beam and shell have been used. Unfortunately, the classical continuum models are not able to predict the mechanical characteristics of CNTs. Subsequently, various non-classical (higher-order) continuum theories have been introduced as couple stress theory [35, 43, 59], micropolar theory [22], nonlocal [23, 24] and strain gradient elasticity theories [1, 28, 29, 38, 42].

Static analysis of nano beams was investigated via the nonlocal Bernoulli-Euler model [47]. The bending response of micro/nano structures were studied based on nonlocal Bernoulli-Euler and Timoshenko beam models [60]. Four different nonlocal beam models were developed for static and dynamic analyses of nanobeams by Reddy [50]. Also, various studies have been made on static and dynamic responses of carbon nanotubes based on nonlocal elasticity theory [11, 40, 52]. On the other hand, nonlocal bending and vibration analyses of microtubules are examined with finite element method [16–18]. Free vibrational response of functionally graded embedded nanoplates lying on Pasternak elastic foundation is studied via a nonlocal refined trigonometric shear deformation plate theory [14]. Microstructure-dependent static and dynamic analyses of functionally graded nanobeams and nanoplates are investigated based on a nonlocal quasi-3D theory [13, 15]. On the other hand, thermal effects on buckling and free vibration responses of size-dependent functionally graded nanobeams are examined using nonlocal elasticity theory [19, 21]. Also, size-dependent vibration response of a rotating functionally graded nanobeam is studied based on first-order shear deformation beam and

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nonlocal elasticity theories [20]. Moreover, some another applications of nonlocal elasticity theory can be found in the some recent studies [12, 45, 48, 49, 63].

Modified couple stress theory is one of the most popular and useful higher-order continuum theory [61]. In the literature, there are some studies on the mechanical analyses of carbon nanotubes. Static bending and buckling responses of SWCNTs are examined based on modified couple stress and strain gradient elasticity theories in conjunction with various beam theories [2, 4, 7, 8]. Dynamic instability of embedded SWCNTs resting on a visco-Pasternak foundation is analyzed based on modified couple stress theory and trigonometric beam theory by Kolahchi and Moniri Bidgoli [36]. Fakhrabadi investigated the size-dependent electromechanical behaviors of actuated carbon nanotubes via modified couple stress theory in conjunction with homotopy perturbation method [25]. Also, nonlinear vibration analysis of actuated carbon nanotubes under a step DC voltage is performed based on modified couple stress theory [26]. Also, this theory and modified strain gradient theory have been widely employed to analyze static and dynamic behaviors of microbeams [3, 5, 6, 9, 27, 31, 37, 41, 44, 46, 51, 54–57]. Free vibration response of an embedded magneto-electro-elastic cylindrical nanoshell subjected to thermo-electro-magnetic loading is examined based on modified couple stress theory [30]. Also, free transverse vibration of microstructure-dependent cracked microbeam is studied via modified couple stress theory [58].

The objective of this article is to investigate the influences of rotary inertia and small size on the vibrational behavior of single-walled carbon nanotubes (SWCNTs). The equations of motion are derived on the basis of modified couple stress and Rayleigh beam theories. Analytical solution is performed for the size-dependent dynamic equations are by Navier's solution procedure. Influences of slenderness ratio, length scale parameter and rotary inertia on the natural frequencies of SWCNTs are studied in detail.

2 Main formulations and analytical solution

Modified couple stress theory includes only one additional material length scale parameter associated with symmetric rotation gradient tensor for linear elastic isotropic materials and the couple stress tensor is symmetric. According to this useful non-classical theory, the strain energy U

can be expressed as [61]

$$U = \frac{1}{2} \int_0^L \int_A \left(\sigma_{ij} \epsilon_{ij} + m_{ij}^s \chi_{ij}^s \right) dA dx \quad (1)$$

where σ_{ij} , ϵ_{ij} , m_{ij}^s and χ_{ij}^s indicate the components of the Cauchy stress tensor, the strain tensor, the symmetric couple stress tensor and the symmetric rotation gradient tensor, respectively and can be expressed as

$$\epsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \quad (2)$$

$$\chi_{ij}^s = \frac{1}{2} (\theta_{i,j} + \theta_{j,i}) \quad (3)$$

$$\theta_i = \frac{1}{2} e_{ijk} u_{k,j} \quad (4)$$

where u_i and θ_i represent the components of the displacement and the rotation vectors and e_{ijk} is the permutation symbol.

On the other hand, the components of the classical stress and the symmetric couple stress tensors can be expressed by [61]

$$\sigma_{ij} = \lambda \epsilon_{mm} \delta_{ij} + 2G \epsilon_{ij} \quad (5)$$

$$m_{ij}^s = 2Gl^2 \chi_{ij}^s \quad (6)$$

where δ_{ij} is the Kronecker delta and l is the material length scale parameter. Furthermore, λ and G are the Lamé constants and they can be defined in terms of elastic modulus (E) and Poisson's ratio (ν) as

$$\lambda = \frac{Ev}{(1+\nu)(1-2\nu)}, \quad G = \frac{E}{2(1+\nu)} \quad (7)$$

In the present study, it is notable that carbon nanotubes are modeled as an initially straight beam. A typical simply supported carbon nanotube and coordinate system are seen in Figure 1. The displacement fields for time-dependent deformations on the basis of Bernoulli-Euler beam theory can be written as

$$u_1(x, z, t) = -z \frac{\partial w(x, t)}{\partial x} \quad (8)$$

$$u_2(x, z, t) = 0 \quad (9)$$

$$u_3(x, z, t) = w(x, t) \quad (10)$$

where u_1 , u_2 and u_3 are the x , y and z components of the displacement vector, and w is the transverse displacement of any point on the midline of carbon nanotubes.

Figure 1: A typical simply supported carbon nanotube

Substituting Eqs. (8)-(10) into Eq. (2) yields the macro strain components as

$$\epsilon_{11} = \frac{\partial u_1}{\partial x} = -z \frac{\partial^2 w}{\partial x^2} \quad (11)$$

$$\epsilon_{12} = \epsilon_{21} = \frac{1}{2} \left(\frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x} \right) = 0 \quad (12)$$

$$\epsilon_{13} = \epsilon_{31} = \frac{1}{2} \left(\frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x} \right) = 0 \quad (13)$$

$$\epsilon_{23} = \epsilon_{32} = \frac{1}{2} \left(\frac{\partial u_2}{\partial z} + \frac{\partial u_3}{\partial y} \right) = 0 \quad (14)$$

$$\epsilon_{33} = \frac{\partial u_3}{\partial z} = 0 \quad (15)$$

Similarly, the components of rotation vectors are achieved by using Eq. (8)-(10) in Eq. (4) as

$$\theta_1 = \frac{1}{2} \left(e_{123} \frac{\partial u_3}{\partial y} + e_{132} \frac{\partial u_2}{\partial z} \right) = 0 \quad (16)$$

$$\theta_2 = \frac{1}{2} \left(e_{231} \frac{\partial u_1}{\partial z} + e_{213} \frac{\partial u_3}{\partial x} \right) = -\frac{\partial w}{\partial x} \quad (17)$$

$$\theta_3 = \frac{1}{2} \left(e_{312} \frac{\partial u_2}{\partial x} + e_{321} \frac{\partial u_1}{\partial y} \right) = 0 \quad (18)$$

and employing these components in Eq. (3) yields the all components of symmetric rotation gradient tensor as

$$\chi_{11}^s = \frac{1}{2} \left(2 \frac{\partial \theta_1}{\partial x} \right) = 0 \quad (19)$$

$$\chi_{12}^s = \frac{1}{2} \left(\frac{\partial \theta_1}{\partial y} + \frac{\partial \theta_2}{\partial x} \right) = -\frac{1}{2} \left(\frac{\partial^2 w}{\partial x^2} \right) \quad (20)$$

$$\chi_{13}^s = \frac{1}{2} \left(\frac{\partial \theta_1}{\partial z} + \frac{\partial \theta_3}{\partial x} \right) = 0 \quad (21)$$

$$\chi_{21}^s = \frac{1}{2} \left(\frac{\partial \theta_2}{\partial x} + \frac{\partial \theta_1}{\partial y} \right) = -\frac{1}{2} \left(\frac{\partial^2 w}{\partial x^2} \right) \quad (22)$$

$$\chi_{22}^s = \frac{1}{2} \left(2 \frac{\partial \theta_2}{\partial y} \right) = 0 \quad (23)$$

$$\chi_{23}^s = \frac{1}{2} \left(\frac{\partial \theta_2}{\partial z} + \frac{\partial \theta_3}{\partial y} \right) = 0 \quad (24)$$

$$\chi_{31}^s = \frac{1}{2} \left(\frac{\partial \theta_3}{\partial x} + \frac{\partial \theta_1}{\partial z} \right) = 0 \quad (25)$$

$$\chi_{32}^s = \frac{1}{2} \left(\frac{\partial \theta_2}{\partial z} + \frac{\partial \theta_3}{\partial y} \right) = 0 \quad (26)$$

$$\chi_{33}^s = \frac{1}{2} \left(2 \frac{\partial \theta_3}{\partial z} \right) = 0 \quad (27)$$

Use of the non-zero components of macro and micro deformation gradient tensors in Eqs. (5) and (6) we obtain the non-zero components of classical stress and symmetric couple stress tensors as (by neglecting Poisson effect)

$$\sigma_{11} = -Ez \frac{\partial^2 w}{\partial x^2} \quad (28)$$

$$m_{12}^s = m_{21}^s = -GI^2 \left(\frac{\partial^2 w}{\partial x^2} \right) \quad (29)$$

The kinetic energy of the carbon nanotube is defined as [56]

$$K = \frac{1}{2} \rho \int_0^L \int_A \left[\left(\frac{\partial u_1}{\partial t} \right)^2 + \left(\frac{\partial u_2}{\partial t} \right)^2 + \left(\frac{\partial u_3}{\partial t} \right)^2 \right] dA dx \quad (30)$$

where ρ is the mass density. The first variation of the kinetic energy on the time interval $[0, T]$ can be expressed in the presence of rotary inertia effect as

$$\int_0^T \delta T dt = \int_0^T \int_0^L \left[\rho A \left(\frac{\partial w}{\partial t} \frac{\partial \delta w}{\partial t} \right) + \rho I \left(\frac{\partial^2 w}{\partial x \partial t} \frac{\partial^2 \delta w}{\partial x \partial t} \right) \right] dx dt \quad (31)$$

where A is the cross-sectional area, I is the second moment of area. The governing differential equations and related boundary conditions in motion are obtained with rotary inertia effect by implementing Hamilton's principle together with some mathematical manipulations as

$$\left(EI + GAI^2 \right) \frac{\partial^4 w}{\partial x^4} + \rho A \frac{\partial^2 w}{\partial t^2} - \rho I \frac{\partial^4 w}{\partial x^2 \partial t^2} = 0 \quad (32)$$

$$-\rho I \frac{\partial^3 w}{\partial x \partial t^2} + (EI + GAl^2) \frac{\partial^3 w}{\partial x^3} + V = 0 \quad (33)$$

or $\delta w = 0$

$$-(EI + GAl^2) \frac{\partial^2 w}{\partial x^2} - M = 0 \quad \text{or} \quad \delta \left(\frac{\partial w}{\partial x} \right) = 0 \quad (34)$$

where V and M are shear force and moment at the end of the structure.

The following expansion of generalized displacement is implemented to analytically solve the free vibration problem of pinned-pinned SWCNTs as

$$w(x, t) = \sum_{n=1}^{\infty} W_n \sin \frac{n\pi x}{L} e^{i\omega_n t} \quad (35)$$

where W_n is the unknown Fourier coefficient, i is the well-known imaginary number, ω_n is the vibration frequency and t is time. Substituting Eq. (35) into Eq. (32) yields

$$\frac{n^4 \pi^4}{L^4} (EI + GAl^2) - \omega_n^2 \left(\rho A + \frac{n^2 \pi^2}{L^2} \rho I \right) = 0 \quad (36)$$

The natural frequency of CNTs with small size and rotary inertia effects is given as following

$$\omega_n^{\text{Rayleigh}} = \frac{n^2 \pi^2}{L^2} \sqrt{\frac{EI + GAl^2}{\rho A + \frac{n^2 \pi^2}{L^2} \rho I}} \quad (37)$$

It is notable that if the term of rotary (rotatory) inertia effect is neglected, the natural frequency of CNTs for size-dependent Bernoulli-Euler beam model will be achieved as [37]

$$\omega_n^{\text{Euler}} = \frac{n^2 \pi^2}{L^2} \sqrt{\frac{EI + GAl^2}{\rho A}} \quad (38)$$

3 Numerical results and discussions

In this section, some parametric results are presented to investigate the influences of length scale parameter, rotary inertia, mode numbers and slenderness ratio on the vibrational response of carbon nanotubes. The material and geometric properties are assumed as [52]: $E = 1$ TPa, $\nu = 0.19$, $D = 1$ nm. It should be noted that the results obtained by classical and modified couple stress theories are symbolized by CT and MCST, respectively.

Variations of the classical and non-classical frequency ratios against to slenderness ratio and mode numbers are illustrated in Figures 2 and 3, respectively. It is found that

Figure 2: Variation of the classical frequency ratio against to slenderness ratio and mode numbers

Figure 3: Variation of the non-classical frequency ratio against slenderness ratio and mode numbers

the ratio (frequencies of Euler-Bernoulli beam model-to-frequencies of Rayleigh beam model) is nearly equal to one for greater values of slenderness ratio (thickness-to-diameter ratio) while it tends to increase by decreasing the slenderness ratio. It can be said that this situation is more prominent for higher modes. Moreover, it is seen that Figures 2 and 3 are almost identical to each other and this situation can be interpreted as the material length scale parameter has no effect on this frequency ratio.

Figures 4 and 5 depict the influences of mode number and thickness-to-diameter ratio on the natural frequencies of carbon nanotube. It is observed from these figures that when the mode numbers increase, both the natural frequencies and the divergence between the natural frequencies of Bernoulli-Euler and Rayleigh beams also increase. In addition, it can be said that the natural frequencies pre-

Figure 4: Effects of mode number and thickness-to-diameter ratio on the classical natural frequencies of carbon nanotube

Figure 5: Effects of mode number and thickness-to-diameter ratio on the size-dependent natural frequencies of carbon nanotube

dicted by Bernoulli-Euler beam model are always bigger than those evaluated by Rayleigh beam model and also the natural frequencies based on MCST are greater than those based on CT.

Variations of the first four natural frequencies with respect to length scale parameter-to-diameter ratio for $L = 6D$ based on Bernoulli-Euler and Rayleigh beam models are shown in Figures 6 and 7, respectively. It is clearly seen from these figures that an increase in length scale parameter-to-diameter ratio leads to increase in the natural frequencies, especially in higher modes. It can also be observed that the natural frequencies obtained by Rayleigh beam model are smaller than those evaluated by Bernoulli-Euler beam model and this situation is more prominent for higher modes.

Figure 6: Variation of the first four natural frequencies with respect to length scale parameter-to-diameter ratio based on Bernoulli-Euler beam model for $L = 6D$

Figure 7: Variation of the first four natural frequencies with respect to length scale parameter-to-diameter ratio based on Rayleigh beam model for $L = 6D$

4 Conclusions

Effects of small size and rotary inertia on the free vibration response of single-walled carbon nanotubes are studied via modified couple stress and Rayleigh beam theories. Influences of additional material length scale parameter, mode numbers, slenderness ratio and rotary inertia on the natural frequencies of carbon nanotubes are investigated in detail. It is observed from the results that the rotary inertia term has a decreasing effect on the natural frequency values of carbon nanotubes. In addition, it can be said that this effect is more prominent for lower slenderness ratio and higher modes. Consequently, it can be emphasized that taking into account the rotary inertia effect is useful

and necessary for the dynamic analysis of shorter carbon nanotubes.

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