Research Article

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A numerical evaluation on nonlinear dynamic response of sandwich plates with partially rectangular skin/core debonding

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Abstract: As one of the most dangerous defects in the sandwich panel, debonding could significantly degrade load carrying capacity and affect dynamic behaviour. The present work dealt with debonding detection of the rectangular clamped hybrid sandwich plate by using ABAQUS software. The influence of various damage ratios on the linear and nonlinear dynamic responses has been studied. The finite element model was initially validated by comparing the modal response with the experimental test. Rectangular debonding was detected by comparing dynamic responses of free and forced vibrations between intact and debonded models. A wide range of driving frequency excitation corresponding to transient and harmonic concentrated loads was implemented to highlight nonlinear behaviour in the intermittent contact in the debonded models. The results showed that debonding existence contributed to the natural frequency reduction and modes shape change. The numerical results revealed that debonding affected both the steady-state and impulse responses of the debonded models. Using the obtained responses, it was detected that the contact in the debonded region altered the dynamic global response of the debonded models. The finding provided the potential debonding diagnostic in ship structure using vibration-based structural health monitoring.

Keywords: debonding detection, hybrid sandwich, finite element analysis, dynamic contact, forced vibration, marine.

1 Introduction

As one of the unique thin-walled structures, sandwich material is broadly used in shipbuilding industries due to combining high performance with a lightweight design. Sandwich panels are profitable during construction and reparation because they can reduce the weight by eliminating the stiffeners compared to the stiffened plate structures. However, weight reduction often leads to an increase in vibration problems occurring under manufacture and service conditions. Increased vibration can cause a variety of structural damage. Face sheet/core debonding is the serious damage that can reduce the stiffness [1, 2, 3] and threaten the safety and service life [4].

The difficulty in controlling the appropriate bonding during the manufacturing stage is the main problem [5, 6], in which the debonding location is often invisible in the interface layer between two adjacent surfaces [7]. Debonding has resulted from manufacturing failure when a small region's adhesive layers have not been sufficiently bonded. Moreover, this early-stage damage may propagate to create larger debonding, altering the vibration characteristic and behaviour. The debonding can lead to frequency reduction, leading to structural failure in the lower mode [8, 9]. Since bonding quality during manufacture determines the performance and integrity of the structure, an effective and efficient debonding detection in the early stage is necessary to assess structural health and performance.

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Assessment of the structure using vibration-based structural health monitoring (VSHM) is widely used by examining the changes in measured vibration response [10]. The basic principle is comparing the change of dynamic characteristics between intact and debonded structures such as natural frequencies [11, 12, 13], mode shapes [14], frequency response function (FRF) curvatures [15], modal damping, and time or frequency domain data [16, 17]. Recently reported VSHM studies based on linear and nonlinear vibration analysis in the composite sandwich were comprehensively presented. Burlayenko and Sadowski investigated the influence of debonding ratio, location, and type on free vibration response [18]. It showed that the ratio, location, and type of debonding influence the vibration behaviour of the composite sandwich. Moreover, the previous studies found that debonding is easily detected in higher mode than lower mode [19, 20, 21], but it is challenging to get higher modes in the practical experiment due to restriction of excitation magnitude. Consequently, nonlinear vibration utilising time-domain data of structural responses of the composite sandwich is more popular. Compared with modal analysis, investigating the damage based on time-domain dynamic responses is easier [22], making the result more reliable [23]. Several studies have been reviewed in detail. Burlayenko and Sadowski have created an explicit 3D finite element model (FEM) to examine the transient dynamic response of an impacted sandwich panel with debonding [24]. Moreover, damage detection using an implicit dynamic has been reviewed in [25] to investigate the responses of the debonded sandwich subjected to harmonic forces. A recent development study of debonding identification based on the different types of analysis in the composite sandwich is comprehensively reviewed by Burlayenko and Sadowski [26].

Although a majority of previous investigations have been reviewed to explore the dynamic behaviour of the composite sandwich, there is limited study in the comprehensive assessment of dynamic behaviour and nonlinear characteristic of hybrid sandwich panels containing metal/polymer combinations as part of ship structure due to debonding problems. The issue is crucial since the mechanical properties of two adjacent layers of hybrid sandwich have higher dissimilar properties, leading to initiate a debonding problem more severe. There is an important issue investigating the response characteristic of debonding problems due to the free and forced vibration of this type of sandwich under different loading scenarios. Moreover, there is a limited study concerning the validity and correctness of proposed debonding modelling of hybrid sandwiches with the response obtained from the experimental test.

To address this issue, the numerical damage assessment of a hybrid sandwich containing a steel faceplate and polymer core was developed in this study. The essential contribution of this work is to study the nonlinear effect of debonding contact between the faceplate and core due to free and forced vibrations. An initial study was performed by comparing the numerical model using ABAQUS software [27] with the experimental test to check the correctness of debonding modelling. The three-dimensional model of the clamped sandwich plate with rectangular shape debonded zone was firstly conducted free vibration analysis to extract natural frequency. The general dynamic based on explicit time-stepping procedure due to transient and harmonic load was then undertaken to check the effect of dynamic response due to contact dynamic under various debonding ratios. The structure of this paper is constructed in sections as follows. Section 1 presents the introduction and objective of the study. The theoretical backgrounds of model discretization, contact modelling, and finite element analysis are reported in Section 2. Section 3 offers the Experimental test preparation and procedure, and Section 4 reports the result of the free and forced vibration test. In the last, a few concluding remark and summary is provided in Section 5.

2 Numerical modelling of sandwich panels

2.1 Aspects of model discretization

In the aspect of model discretization, there are several schemes available for modelling sandwich and laminate structure. They are classified into equivalent single layer (ESL), layerwise (LW), and continuum-based three-dimensional (3-D) elasticity theories. ESL theory is simply efficient for general analysis with computationally efficient, but interlaminar stress and transverse compressibility are ignored [26]. In order to alleviate the shear-locking phenomenon in shell/plate, mixed interpolation tensorial component technique (MITC) technique with Carrera Unified Formulation (CUF) framework is proposed by Kumar *et al.* [28] to perform modal analysis of delaminated composite shell structures. Using this strategy, the shell element has 9-nodes and allows displacement distribution along with the thickness of the multilayered shell. The implementation of refined and advanced delamination model of CUF shell model using various theories and different ordered theories was studied. Moreover, using 2D MITC finite element based on the CUF framework, Legendre-like polynomial expansion

sions are introduced by Pagani *et al.* [29] to implement ESL, LW, and variable kinematics theories.

A nonlinear pattern in the height of the core during deformation/ core transverse compressibility should be accounted for modelling modern sandwich panels. This makes it an obstacle to handle this modelling using ESL 2D elements. In this regard, quasi three-dimensional model or layerwise (LW) scheme is preferable due to accommodating core compressibility with effective computational cost. Solid-brick elements for the core and continuum shell elements for faceplates were used in this case.

2.2 Sandwich configuration and debonding geometry

Ferry Ro-Ro 6300 Gross Tonnage (GT) stern ramp door was used as a reference model, as previously described in [21]. The sandwich panel geometry of the stern ramp door is illustrated in detail in Figure 1. The steel faceplate and resin/clamshell core material with dimensions 300 mm in length and width are modelled. The configuration of the sandwich panel is 4 mm in the upper and lower faceplate thickness ($h_t = h_b$) and 20 mm in core material thickness (h_c). The sandwich thickness configuration and debonding geometry are presented in Table 1. The material constants of the sandwich panel used for debonding detection are shown in Table 2 and previously reported in [11, 21].

Figure 1: Sandwich geometry with debonding [21].

A total of five models was created, as presented in Table 1. Debonding was modelled using the FEM by single rectangular damage at the centre of the top adhesive layer, as illus-

Table 1: Sandwich configuration and debonding geometry.

Models	$h_t-h_c-h_b$ (mm)	$D_1 = D_2$ (mm)	T (mm)
Intact (0%)	4-20-4	0	0
5% debonding ratio	4-20-4	67.08	0.00004
10% debonding ratio	4-20-4	94.90	0.00004
20% debonding ratio	4-20-4	134.2	0.00004
30% debonding ratio	4-20-4	164.3	0.00004

Table 2: Material constants of faceplates and core sandwich material.

Parameters	Faceplate	Core material
ρ (kg m^{-3})	7850	1465
E (Pa)	2.1×10^{11}	4.4×10^9
G (Pa)	8.07×10^{10}	1.69×10^9
σ_y (MPa)	275	24.8
σ_u (MPa)	292	24.8
ν	0.3	0.3

trated in Figure 1. The rectangular debonded region at the middle span was presented by a small artificial gap ($t = 0.1\% \times h_c$) between the upper faceplate and the core. No treatment of either geometrical entities or material constants is applied in the debonded area to guarantee a physically real case condition. The ratio of debonding was represented by a damage parameter ($D = A_D/A_T \times 100$) expressing the percentage of the region of debonding (A_D) to the whole region of the interface layer ($A_T = D_1 \times D_2$). In rectangular debonding, the length and width of the debonding have similar values ($D_1 = D_2$). The model was modelled by a solid/shell layerwise (LW) approach in the model discretisation, as previously recommended [30, 31]. In terms of the element type, the faceplate was modelled using a continuum shell element (SC8R), and the core material was modelled using a solid element (3CD8R). Meanwhile, the clamped-free-clamped-free (CFCF) boundary condition, as illustrated in Figure 1, was applied.

2.3 Theoretical basis of modal analysis

Free vibration analysis was used to investigate structural dynamic behaviour. In free vibration, there was no external load causing the motion. The motion has resulted from initial conditions, such as an initial displacement from an equilibrium position. The equation of motion for forced vibration can be assumed in the form [32].

$$[M][\ddot{U}] + [C][\dot{U}] + [K][U] = [F] \quad (1)$$

where M , C , and K are mass, damping, and stiffness matrices. U is nodal points displacements, and $\dot{U}$ and $\ddot{U}$ are their time derivatives referring to nodal velocities and acceleration. For free vibration analysis, the damping of the structure can be neglected. Thus, the equation can be written in Eq. (2).

$$[M][\ddot{U}] + [K][U] \quad (2)$$

where Eq. (1) shortens to an eigenvalue problem. The governing equation for free vibration analysis can be written in Eq. (3).

$$[K] - \lambda_i[M][\phi_i] = 0 \quad (3)$$

where λ_i is eigenvalue and ϕ_i is eigenvector corresponding to the eigenvalues. The eigenvalues associated with the natural frequency can be written in Eq. (4), as follows:

$$f_i = \sqrt{\lambda_i/2\pi} \quad (4)$$

The analysis was conducted to present an insight into the oscillation response between intact and face sheet/core debonded models. The free vibration analysis in the first six modes using ABAQUS/Standard was carried using the Lanczos method to extract eigenvalues. The modal responses were used as a damage index to detect the debonding problem.

2.4 Descriptions of explicit time-stepping procedure

The explicit procedure accomplishes a large number of time increments. A set of a nonlinear differential equation of motion with the assumption of linear elasticity and small deformation is written as [33]:

$$M\ddot{U}(t) + C\dot{U}(t) + KU(t) = F^{ext}(U(t)) - F^{cont}(U(t)) \quad (5)$$

where $\ddot{U}(t)$, $\dot{U}(t)$ and $U(t)$ is the global vectors of unknown accelerations, velocities, and displacements, respectively, at each instant of time. F^{ext} is the vector of the external force, and F^{cont} is the vector of contact forces. Then, M , C , and K are the global mass, damping, and stiffness matrices serially. Eq. (5) is discretised in a time domain. The time interval $[0, T]$ is divided into subintervals in the form $[0, T] = \cup_{i=0}^{L-1} [t_i, t_{i+1}]$, where $t_i < t_{i+1}$, and $t_0 = 0$, $t_L = 0$. So, Eq. (5) can only be found in a finite number of time steps. For example, the time increment is $\Delta t_{i+1} = t_{i+1} - t_i$.

Then, the accelerations, velocities, and displacements in this time increment are expressed by $\ddot{U}_{i+1}$, $\dot{U}_{i+1}$, U_{i+1} , respectively. Therefore, Eq. (5) at $t_{i+1} = t_i + \Delta t_{i+1}$ is expressed in the form:

$$M\ddot{U}_{i+1} + C\dot{U}_{i+1} + KU_{i+1} = F_{i+1}^{ext} - F_{i+1}^{cont} \quad (6)$$

$U_0 = \bar{U}$ and $\dot{U}_0 = \bar{V}$ are the assumption given at the initial boundary conditions.

The explicit algorithms determine Eq. (6) using central difference formulation without iterations and tangent stiffness matrix by explicitly advancing the kinematic state known from a previous increment to the next one [34]. Initially, the accelerations at the beginning of each increment Δt_{i+1} are calculated. Then, Eq. (6) is rewritten:

$$\ddot{U}_i = M^{-1} (F_i^{ext} - F_i^{cont}) \quad (7)$$

where F_i^{ext} is the vector of the external nodal forces at time t_i , and F_i is the sum of nodal internal $F_i^{int} = KU_i$, damping $F_i^{damp} = C\dot{U}_i$ and contact force F_i^{cont} which are updated during the previous time increment Δt_i .

Then, the accelerations computed at t_i are used to advance the velocity solution to $t_i + \frac{1}{2}\Delta t_{i+1}$ and the displacement solution to $t_i + \Delta t_{i+1}$ as follows:

$$\dot{U}_{i+1} = \dot{U}_{i-\frac{1}{2}} + \frac{\Delta t_{i+1} + \Delta t_i}{2} \ddot{U}_i \quad (8)$$

$$U_{i+1} = U_i + \Delta t_{i+1} \dot{U}_{i+\frac{1}{2}} \quad (9)$$

The initial half-step lagging velocity $\dot{U}_{.1/2}$ is calculated from the initial velocity assuming the initial acceleration. The computational time in the explicit analysis linearly increases with problem capacity, and the explicit procedure is only stable if the time increment in Eq. (6) is smaller than the stability limit of the central difference operator [34]. An approximation to the stability limit (Δt_{crit}) can be formulated as follows:

$$\Delta t_{crit} \approx \frac{L_e}{c_d} \quad (10)$$

Where L_e is the smallest element dimension in the mesh and c_d is dilatational wave speed.

To gain a deeper investigation of debonding diagnostic, the comparison of dynamic behaviour due to debonding problem was investigated using general dynamic analysis. First, after analysing the modal analysis of the debonded model for evaluating its natural frequencies, the FE models were subjected to the concentrated harmonic load at the upper faceplate. The investigations were developed in ABAQUS/Explicit that incorporated the explicit integration

algorithm for a general dynamic analysis with contact described in Section 2.5. The models with rectangular debonding were loaded with a concentrated harmonic force $F(t) = F_0 \sin \Omega t$ subjected at the central point of the upper faceplate. The dynamic response of the model was investigated for fixed excitation amplitude $F_0 = 100$ N by varying the excitation frequency (Ω).

In this respect, four different driving frequencies ($\eta=1/3$, $\eta=1/2$, $\eta=3/4$, and $\eta=2$) were analysed so that the applied load can highlight dynamic phenomena that occur in the model. The driving frequency (η) is the ratio between the excitation frequency (Ω) and the first natural frequency on the intact model panel (f_0), i.e., $\eta = \Omega / f_0$. Comparing dynamic responses between intact and debonded models were investigated in the different four different measurement points at the upper faceplate. The dynamic responses corresponding to transverse displacement, velocity, and phase portrait were measured in both the inside debonding (N1 and N2) and outside debonding area (N3 and N4), as illustrated in Figure 2b.

Further, the effect of debonding ratio due to transient load will be analysed using ABAQUS/Explicit. To check the responses of debonded model, the clamped sandwich models on both sides were subjected to the concentrated force at the centre point on the upper faceplate. The period of the force applied was set in such a way that it was shorter than the analysis time (1/10 of the total analysis time).

$$F(t) = \begin{cases} F_0, & 0 \leq t \leq t^* \\ 0, & t > t^* \end{cases} \quad (11)$$

The amplitude of $F_0 = 50$ kN and the time $t^* = 0.05$ s. The dynamic responses corresponding to time histories of displacement, velocity, acceleration, and displacement trendline within the upper faceplate of the model were compared between intact and debonded models.

In this case, the damping ratio was estimated at 1% of the critical damping. Therefore, the coefficients of the Rayleigh damping matrix C were determined for each desired frequency range. The damping material in the system was defined by the C matrix represented by Rayleigh damping with the following formula:

$$C = \alpha M + \beta K \quad (12)$$

Where α and β according to damping ratio can be calculated as follows:

$$\xi_n = \frac{\alpha}{2\omega_n} + \frac{\beta\omega_n}{2} \quad (13)$$

2.5 Descriptions of contact formulation

The fundamental features of the contact formulation employed in the current study to perform dynamic FEA were described briefly. The vector of F^{cont} are presented by normal t_N and tangential t_T components of a contact traction vector. The normal g_N and tangential g_T gap functions, which describe the relative motions of contacting surfaces in the normal and tangential directions, can be formulated in Eqs. (14) and (15) [35, 36]:

$$g_N = (\mathbf{x}^- - \bar{\mathbf{x}}^+) \cdot \bar{\mathbf{n}}^+ \quad (14)$$

$$g_T = g_{T\alpha} \bar{\mathbf{a}}^{\alpha+}, g_{T\alpha} = (\mathbf{x}^- - \bar{\mathbf{x}}^+) \cdot \bar{\mathbf{a}}^{\alpha+} \quad (15)$$

where $\mathbf{x}^-$ is slave surface point and $\bar{\mathbf{x}}^+(\bar{\xi}^1, \bar{\xi}^2)$ is an orthogonal projection on the master surface parameterized by $\mathbf{x}^\alpha$ ($\alpha = 1, 2$) and $\bar{\mathbf{n}}^+$ is the unit vector normal to the master surface and $\bar{\mathbf{a}}_\alpha$ ($\alpha = 1, 2$) is the tangent base vectors at the point $\bar{\mathbf{x}}^+$. The rate of the tangential gap function at this point may be calculated in the geometrically linear case as:

$$\dot{g}_T = \dot{\xi}^\alpha \bar{\mathbf{a}}_\alpha^+ = \dot{g}_{T\alpha} \bar{\mathbf{a}}_\alpha^+ \text{ with } \dot{g}_{T\alpha} = (\dot{\mathbf{x}}^- - \dot{\mathbf{x}}^+) \cdot \bar{\mathbf{a}}_\alpha^+ = \alpha_{\alpha\beta} \dot{\xi}^\beta \quad (16)$$

where $\alpha_{\alpha\beta} = \bar{\mathbf{a}}_\alpha^+ \cdot \bar{\mathbf{a}}_\beta^+$ is the metric tensor at $\bar{\mathbf{x}}^+$. The impenetrability criteria known as the Karush–Kuhn–Tucker inequalities can therefore be formulated as follows [37]:

$$t_N \leq 0, \quad g_N \geq 0, \text{ and } t_N g_N = 0 \quad (17)$$

where t_N is the scalar quantity of the normal contact pressure, i.e., $t_N = t_N \bar{\mathbf{n}}^+$. The friction conditions that arise in tangential directions can be expressed in Eq. (18).

$$\|t_T\| \leq \tau_{\text{crit}}, \|g_T\| \geq 0, (\|t_T\| - \tau_{\text{crit}}) \|g_T\| = 0 \quad (18)$$

where τ_{crit} is a threshold of tangential contact traction when a tangential slip occurs. The Coulomb friction law defines $\tau_{\text{crit}} = \mu t_N$ where μ is the coefficient of friction. Using an analogy between plasticity and friction leads to the Eq. (18) along with loading–unloading conditions can be expressed in Eqs. (19) and (20).

$$\dot{g}_T^{\text{slip}} = \dot{\gamma} \frac{\partial \phi(t_T)}{\partial t_T} = \dot{\gamma} \frac{t_T}{\|t_T\|} \quad (19)$$

$$\phi \leq 0, \dot{\gamma} \geq 0, \text{ and } \phi \dot{\gamma} = 0 \quad (20)$$

where $\dot{\gamma}$ is the slip rate parameter, and the potential function is expressed as $\phi(t_T) = \|t_T\| - \mu t_N$.

In this case, between the surface in part 1 and part 2 within the debonded zone (Figure 2a), a contact formula was developed. The surface-to-surface contact approach in terms of master and slave formulation in the ABAQUS/Explicit was utilised for modelling the debonding interface. The formulation of pure master-slave contact pair was applied due to high dissimilar mechanical constants. Further, in this case, small-sliding displacement kinematics to describe small oscillations of the interacting surfaces was assumed. The computational cost of using a small sliding kinematic is less expensive than a finite sliding formulation [38].

The constitutive behaviour of the contact in the normal direction between two adjacent surfaces in the debonded region was governed by the hard contact model. It is assumed that the surfaces transmitted no contact pressure unless the nodes of the master surface contacted the slave surface, and penetration was not allowed between them. In the case of tangential direction, the contact was developed by the isotropic Coulomb friction model. The penalty parameter was automatically computed to provide some portion of reversible tangential motion specified by a user.

Figure 2: a) Details of intermittent contact modelling in debonded region b) four different measurement points (N) in the upper faceplate.

3 Experimental modal analysis (EMA)

3.1 Material and specimen manufacturing

The Sandwich panel is consisted of relatively thin but has high stiffness faceplates and a thick core with relatively low-density soft material [39]. The compressive and tensile stresses are mainly carried by the faceplates, while the core carries transverse shear stresses [40]. In this study, the steel faceplate and UPR/ clamshell core were used. The

core material has been previously developed and tested [41] based on the Det Norske Veritas-Lloyd's Register (DNV-GL) standard [42].

The casting process manufactured the sandwich material. The first process was cleaning and drying the cavity of faceplate surfaces and releasing them from surface rust. The minimum surface roughness of 60 microns was created on the bonding surfaces before injection. The second process was a mechanical blending of the core, and the mixture was injected into the steel mould. In the final stage, the blending core was cured, preferably up to 24 hours. After removal from the mould, the core was visually examined from surface defects. For debonded models, the debonding region was created by inserting the Teflon tape on the specimen mould. The debonding geometry and thickness of four debonded models are created based on Table 1. The assembly of the debonded model is presented in Figure 3.

Figure 3: Assembly of the debonded specimen.

3.2 Experimental setup and procedure

EMA was conducted on the intact and 30% debonded specimens to obtain natural frequencies experimentally. The time-domain response was obtained from the impact load input with the instrumentation setup illustrated in Figure 4. The clamped model of both sides was excited by the impact hammer at the centre of the plate. Simultaneously, the natural frequency was measured from the accelerometer located at the centre of the upper faceplate in three different measurement points, see Figure 5. The range of accelerometer frequency was 1 – 2000 Hz. The accelerometers are affixed with the help of adhesive on the surface of the upper faceplate to fix the position [43]. Before taking measurements, it is necessary to measure the response of the accelerometer and impact hammer by calibrating it to determine the sensitivity. The time-domain response was transformed using a Fast Fourier Transform (FFT) into the frequency domain. The peaks on the frequency domain are

identified as the natural frequency of the models. The result of the experimental natural frequency of intact and debonded specimens will be compared with that of numerical results.

Figure 4: Experimental vibration test setup.

4 Numerical results and discussion

4.1 Convergence study and validation test

To be confident in the result, mesh convergence was initially studied to get an accurate solution at the minimum element sizes required to obtain optimum CPU time [44, 45]. In this analysis, the general mesh size between 0.02 m and 0.006 m was evaluated. The convergence study was performed by comparing the first three natural frequencies and mesh sizes of the intact model. All the geometries, material properties, and the applied boundary condition of the intact model are similar to Section 2.2. Figure 5 shows the convergence study of the intact model in the first three modes analysed in the mesh sizes between 0.02 – 0.006 m. It can be concluded that mesh element size 0.008 m with 7220 elements ensures sufficient accuracy with moderate computational efficiency.

Figure 5: Mesh convergence of the intact model in the first three modes.

The accuracy and efficiency of the proposed numerical modelling are crucial to study and compare with experimental data [46]. The results of natural frequencies between numerical methods and experimental vibration tests on the intact model and the 30% debonding model are presented in Table 3. As seen in the result, it shows that the largest error is 11.22% in mode 1 in the intact model. The comparison results show a small error rate ($< 15\%$), so that it is assumed that the proposed numerical modelling of the sandwich plate has good accuracy.

4.2 Result of free vibration analysis

To explore the issue of debonding problems on the modal characteristics, the undamaged and damaged models containing debonding were investigated in the first six modes. The model discretisation was similar to the previous convergence study in Section 4.1. A total of five models containing different debonding ratios were analysed by ABAQUS/Standard. The natural frequency of the first six modes between intact and debonded models is illustrated in Figure 6a. As presented in Figure 6a, debonding reduces the frequencies of the debonded model compared to the intact plate, and the frequency reduction is different for each mode. The result demonstrates that debonding causes a significant frequency, especially 20% and 30% debonding ratio. However, the reduction of frequency in the small debonding ratio (5%) is practically no change.

Figure 6b presents the normalised frequency (ω/ω_0) as a function of the debonding ratio. The natural frequency of damaged models (ω) has been normalised with respect to the natural frequency of the intact model (ω_0). It can be mentioned that frequency changes are more rapid with the increase of mode number. However, the natural frequency changes do not exhibit a defined trend as the mode number increases. Moreover, one can also be analysed that the relatively small debonding ($D < 10\%$) does not almost change the frequency in the lower modes and only decreases the frequencies in the higher modes. It may be violated by the effect of the local thickening phenomenon [47], where the frequency of debonded model is higher than those of the intact model, for instance, in mode 3. It is also noted that mode 6 has the highest frequency reduction of all corresponding modes. Hence, its result can be summarised that the damage can influence natural frequencies and depend on the mode number. The frequency reduction increase due to a loss in stiffness and strength of the model is caused by initial discontinuity [11, 13, 17, 26].

In general, it can be analysed that the natural frequencies and associated mode shapes of the debonded models

Table 3: The natural frequency between experimental test (EMA) and numerical analysis (FEM).

Mode number	Intact model			Debonded model (30%)		
	FEM	EMA	Error (%)	FEM	EMA	Error (%)
Mode 1	986.05	875.4	11.22	875.79	784.2	10.46
Mode 2	1214.19	1095.2	9.80	1009.06	922.5	8.58
Mode 3	1862.67	1748.6	6.12	1104.06	1045.1	5.34

Figure 6: Comparison of a) natural frequency b) normalised natural frequency between intact and debonded model.

shift from the initial model. The eigenmode is introduced to understand the deformation of models. Figure 7 shows the eigenmodes between intact and 30% debonded models. It can be concluded that local oscillations in the debonded region and the global modes of the entire sandwich lead to changes in the mode shapes.

4.3 Result of forced dynamic analysis

Forced vibrations in terms of external harmonic excitation are of great importance as it raises in the practical field, which may cause severe damage [48]. The comparison of dynamic behaviour between the intact and debonded models was investigated using explicit dynamic analysis. Time history responses corresponding to transverse displacements, velocities, and phase portrait are firstly compared at four measurement points (see Figure 2b) at the upper faceplate with a different driving frequency $\eta=1/3$, $\eta=1/2$, $\eta=3/4$ dan $\eta=2$.

The debonding detection with frequency ratio (η) of $1/3$ leading to the driving frequency of about 2333 rad/s is firstly analysed due to concentrated transverse force load. Fig 8a and b present displacement and velocity-time histories of the models with and without debonding at N1. As shown, the existence of the interfacial debonding in the model significantly influences all evaluated time history

responses. The amplitudes of displacement and velocity responses of the debonded models are higher than that of the intact model. The larger the ratio of debonding, the larger the amplitude responses. It is caused by the stiffness reduction due to the existence of debonding, where the larger the debonding ratio, the larger the stiffness reduction. Moreover, the displacement and velocity-time responses of the intact sandwich are periodic motions with a pure sinusoidal that corresponds to its steady-state motion (see Figure 8a). In contrast, the response of debonded models has periodic displacement motions with the sinusoidal waveform with a modulated signal (see Figure 9b). The presence of contact motion in the area of debonding caused the modulated response. It can be found that the higher the debonding ratio, the bigger the modulated signal of the responses.

Analyzing in more detailed result, the comparison of phase portrait between intact and 5% debonding at different measurement points of the upper faceplates is presented in Figure 10. Analysing these plots, one can see that the phase plots in the point inside debonding region (N1 & N2) and the outside (N3 & N4) of the intact model are elliptical forms. However, the phase plots of the 5% debonded model measured in the point outside debonding (N3 & N4) are close to the elliptical form. In contrast, the phase plots in the inside debonding (N1 & N2) are significantly disturbed from the elliptical form. Hence, it can be assumed that the responses calculated in the outside

Figure 7: Eigenmodes between intact and debonded model (30%).

debonding region oscillate periodically with the driving frequency, but at the point in the debonding region, it experiences a general periodic motion which is resulted by superposition between driving frequency and motion of periodic contacts [38].

Further, the debonding diagnostics using phase portrait between intact and small debonding with driving frequency $1/3$ measured in the various points are depicted in Figure 9. From the result, it can be analysed that the difference of phase plot between intact and debonded model measured in the inside debonding area (N1 and N2) is more significant. The phase plot in the 5% debonding model has a larger size than the intact one. Meanwhile, the phase plots between the intact and debonded model in the outside debonding region have similar sizes, so the diagnostics of small debonding using phase plot measured at the points outside the debonding area (N3 and N4) is not sensitive.

Figure 8: Dynamic response of frequency ratio $\eta=1/3$ between intact and debonded models a) displacement b) velocity.

Figure 9: Comparison of phase portrait calculated at different measurement points at frequency ratio $\eta=1/3$.

Figure 10 shows the comparison of displacement and velocity responses at frequency ratio $1/2$ (3500 rad/s) at N1. As can be seen, the same phenomenon occurs where the presence of damage significantly influences time responses. The displacement and velocity-time responses of the damaged models are higher than the responses of the intact ones. Moreover, the signal of displacement and velocity of the intact model is steady-state motion with excitation frequency. Still, the displacement and transverse velocity response of debonded models is a periodic motion with a modulated signal that is much more complicated than that in the previous driving frequency. As shown, the phase plots of the debonded model are significantly disturbed by the elliptical form. The surfaces in the debonded zone run into an aperiodic contact manner due to such interactions between additional frequencies with the driving frequency.

The comparison of phase portrait with driving frequency $1/2$ is depicted in Figure 11. The phase portrait of both intact and 5% debonded models measured in the inside debonding region (N1 & N2) shows a larger size than that in the outside debonding region. The difference of

phase portrait between the intact and debonded model in the inside debonding region is more visible while the response in the outside debonding region has a similar size. The debonding detection using a phase plot with driving frequency $1/2$ measured at points outside the debonding area is not recommended.

Figure 10: Dynamic response of frequency ratio $\eta=1/2$ between intact and debonded models a) displacement b) velocity.

Figure 11: Comparison of phase portrait calculated at different measurement points at frequency ratio $\eta=1/2$.

The time responses between the intact and debonded models are further analysed for the frequency ratio η of $3/4$ with the same excitation amplitude. A comparison of time responses between intact and debonded models at the centre of debonding region (N1) is presented in Figure 12. It follows from these plots that the amplitudes of the debonded plate's responses are larger than those in the time signals of the intact plate. The bigger the debonding ratio, the larger the amplitude of time signals. Moreover, the time responses of the intact model are steady-state motion, but the displacement and transverse velocity response of debonded models is a periodic function with a more se-

vere modulated waveform than that in the previous driving frequency, as one can see that the modulated waveform of velocity response is more severe than that of displacement response.

Further, the comparison of phase portrait with driving frequency 5250 rad/s is depicted in Figure 13. The comparison of phase plots in the inside debonding shows a larger trajectory size than that in the outside debonding region. Moreover, the comparison of phase plots between intact and debonded models in the outside debonding region is more visible than the previous excitation frequency.

Figure 12: Dynamic response of frequency ratio $\eta=3/4$ between intact and debonded models a) displacement b) velocity.

Figure 13: Comparison of phase portrait calculated at various measurement points at frequency ratio $\eta=3/4$.

The investigation is further conducted when the frequency ratio is increased up to $\eta=2$, corresponding to driving frequency 14.000 rad/s, which falls between mode 4 and mode 5. The dynamic response of debonded models is changed noticeably from the previous result. Comparing the transverse displacements at N1 between intact and debonded models, the response of debonded models varies

as a quasi-periodic function with large amplitudes, and a signal is severely distorted by the existence of the higher driving frequency, see Figure 14a. However, the velocity response of this point compared to the intact model is characterized by the uncommonly large amplitude, distorted waveform, and loss of periodicity. It happens due to irregular contact interactions that occur between the surface in the damaged area [38]. To prove this phenomenon, the comparison of phase portrait at four various measurement points is drawn in Figure 15. The phase trajectory in the debonded models at all measurement points is distorted from the elliptical form (see Figure 15). Thus, the phase trajectory in the inside debonded region (N1 & N2) is irregular and includes frequencies excited by contact surfaces. Moreover, the dynamic behaviour of the point outside debonding region (N3 & N4) is assumed a quasi-periodic motion.

Regarding debonding detection, a comparison of phase portraits at the different debonding ratios with frequency ratio $\eta=2$ is presented in Figure 15. It can be found from the result that the higher the debonding ratio, the difference of phase portrait of the debonded model compared with intact one is more visible. It signifies that the higher the debonding ratio, the larger the amplitude of displacement and velocity. It is caused by higher stiffness loss in the debonding model with a high ratio.

Figure 14: Dynamic response of frequency ratio $\eta=2$ between intact and debonded models a) displacement b) velocity.

As the debonded models are excited by the concentrated harmonic force, the models begin to oscillate, so the debonded contact behaviour must be investigated in detail. Several deformation contours of vibrating models at different moments of time corresponding to its 'breathing' are presented in Figure 16a, i.e., fully closed at time 0.003 s and 0.017 s, partially open at 0.0377 s, and fully open at 0.0452 s. As shown during oscillations, the contact of debonded surfaces oscillates from close to open status and vice versa during time analysis. To certainly understand the contact behaviour of 10% debonding ratio between the debonded surfaces, the contours of both normal (CPRESS)

Figure 15: Comparison of phase portrait calculated at different measurement points at frequency ratio $\eta=2$.

and tangential (CSHEAR1 and CSHEAR2) contact stress at the time similar with Figure 16a are illustrated in Figure 16c-d. It can be shown when fully or partially closed contacts, the distribution of normal and shear contact forces appear at the debonded region to fulfilling stress continuity requirements. Otherwise, when the contact is fully open, no normal and shear contact forces appear in the debonded region and stress-free contact conditions are realised [33].

As a consequence of transition behaviour between the open and closed forms during oscillations, various normal and shear contact forces distributions during analysis time exist in the debonding region. The comparison of normal contact stress and contact shear stress evolution in debonded models is shown in Figure 17. As shown, the debonded models have higher normal and shear contact forces than the intact model. It can be analysed that with increasing debonding ratio, the magnitude of the contact forces increases. The evolution of the contact forces between intact and debonded models change with analysis time so that contact behaviour changes towards debonding ratio variation. Additionally, the magnitudes of normal contact forces are larger than the shear forces, as seen in Figure 17. It occurs due to contact surfaces in the debonding region interact with each other in the normal direction. However, sliding also occurs due to the rotation and lateral movement of the faceplate in the debonded surface. Thus, the contact forces arising in the debonding region are influenced by several factors, including inertial forces, the local deformation of the debonded region, and global deformation of the model caused by external excitation or a combination of all factors [33].

Figure 16: a) Deformed shapes of the debonded zone b) CPRESS c) CSHEAR1 d) CSHEAR2 at instants of time a) 0.0003 s, b) 0.017 s, c) 0.0377 s, d) 0.0452 s.

Figure 17: Comparison of a) normal contact force b) contact shear force of debonded models.

4.4 Result of transient dynamic analysis

To gain a deeper investigation of debonding diagnostic, the comparison of the dynamic behaviour of debonding

problems due to concentrated impulse load was performed with ABAQUS/Explicit. First, time history responses corresponding to transverse displacements, velocities, accelerations, and displacement trendline at point N1 (see Figure 2b) were compared. Figure 18 indicates the comparison of transient responses computed at the centre of debonding in the upper faceplate of both intact and debonded models. The intermittent contact causes a significant difference in transient responses in both waveforms and signal properties between intact and debonded models. As illustrated in Figure 18a, the waveform of the debonded model is more disturbed than that of the intact model because the dynamics of the debonded surfaces are modelled by intermittent contact. Further, the existence of debonding increases the amplitude of the transient responses. The bigger the ratio

of debonding, the higher the amplitude of the transient responses, as clearly seen in the displacement trendlines in Figure 18d. It occurs due to the result of superimposing between vibrational waves caused by external loads and those generated by contact interactions [25]. So, the effect of debonding due to transient loading of the sandwich models is significant.

The evolution of contact forces due to impulse loading in the time interval 0 - 0.002 s is illustrated in Figure 19, which describes the comparison of the normal and shear contact forces with various debonding ratios. As seen, debonding causes an increase both in the normal and shear contact forces. The magnitude of the normal and shear contact forces increases with the increase of debonding ratio. Moreover, the magnitude of the normal contact forces is higher than the shear forces due to the interaction of contacting surfaces in the debonded region occurs in the normal direction.

To present a better visualisation of both normal and shear contact force distribution due to impulse load, the visualisation of contact at instants of time of 10% debonded model is presented in Figure 20. The contact forces arising between the faceplate and the core si passing from closing to opening and vice versa. Figure 20a shows the visualisation of normal and shear contact forces when the model is fully closed at time 0.00045 s. Moreover, Figures 21b-c shows normal and shear contact force distributions when the debonded model is partially closed. As shown, the normal contact traction due to the contact-impact motion of the debonded surfaces changes the sliding mechanism described by the shear contact stress distributions.

5 Conclusions

The damage detection of the hybrid sandwich using free and forced vibration was studied in this work. The influence of dynamic behaviour on the debonding ratio using FE software ABAQUS can be drawn. The free vibration using the Lanczos method was used to extract eigenfrequencies and eigenmodes. To obtain an understanding of the dynamic behaviour of debonded models, both the transient dynamic analysis and the forced dynamic analysis were analysed using ABAQUS/Explicit.

The result shows that debonding causes natural frequency reduction and alters the mode shapes as well. It can be determined that the natural frequency decreases with the increase of the debonding ratio. The higher modes are found to be more sensitive to debonding existence. It can be found that the presence of local oscillations in the

Figure 18: Comparison of transient response between intact and debonded models a) displacement b) velocity c) acceleration d) displacement trendline.

Figure 19: Transient response of debonded models a) normal contact force b) contact shear force.

debonded region along with the global mode changes in the mode shapes of the debonded model.

Further, the numerical result of dynamic responses, both harmonic and impulse loading, shows that the debonding can be identified by comparing time responses between intact and debonded models. The dynamic responses analysed in the four different driving frequencies of the debonded models due to harmonic excitation depend on the intermittent contact. A variety of motions such as periodic, quasi-periodic, and irregular on the time responses and phase portraits in the debonded region are found. The dynamic responses of the debonded models are higher than that of the intact model. In terms of transient dynamic analysis, numerical results show that the presence of debonding affects the short-time response of debonded models. Using transient time responses, the magnitude of the time responses of the debonded models is higher than the intact model. To sum up, using the change of responses resulted from free and forced vibration analyses of both un-

Figure 20: Distributions of normal contact (CPRESS) and shear contact (CSHEAR1, and CSHEAR2) of debonded model at instants of time a) $t = 0.00045$ s b) $t = 0.00655$ s c) $t = 0.0358$ s.

damaged and damaged models, the debonding diagnostics can be implemented.

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