Research Article

Zhmagul Nuguzhinov, Omirkhan Khabidolda*, Zhetpisbai Bakirov, Syrlybek Zholmagambetov, Alexey Kurokhtin, and Daniyar Tokanov

Regression dependences in bending reinforced concrete beam with cracks

https://doi.org/10.1515/cls-2022-0182
Received Feb 07, 2022; accepted Aug 26, 2022

Abstract: The work is devoted to determining the stress parameters of flexible reinforced concrete beams with cracks. The problem is solved using LIRA-SAPR using beam finite elements, taking into account the nonlinear relationship between deformation and stress in concrete. In the course of solution, a step-by-step loading method is used with the use of an iterative process at each step. To obtain the dependence of the stress parameters on varied factors, a rational planning matrix for a multifactor computer simulation was compiled to determine the stress parameters in bent rectangular reinforced concrete beams with a crack. According to this plan, computer simulations were conducted for concrete beams of C20/25 and B32/40 class. The obtained dependences enable to evaluate the operability of the considered structural elements for both groups of limiting states. They can be used to determine the parameters of fracture mechanics and evaluate the crack resistance of a beam.

Keywords: bending crack; experimental design; physical nonlinearity; regression dependencies; reinforced concrete; stress state

1 Introduction

In the modern world, concrete is one of the most commonly used materials in construction. One of the main problems that are remaining relevant today is associated with long-term, technical maintenance and modernization of various designs. When designing reinforced concrete structures, it is essential to take into account the requirements for the limiting state during operation. Since the behavior during the destruction of reinforced concrete beams is non-linear, their prediction is usually performed on the basis of experimental tests, which are carried out only in critical places, due to various tests and equipment limitations [1].

A crack is one of the main types of defects whose behavior is being studied. The crack appearance significantly worsens the durability of reinforced concrete structures, creating preferred paths for the penetration of corrosion-inducing particles, such as water, oxygen, carbon dioxide, chloride, which leads to the rapid formation and spread of reinforcement corrosion and significantly reduces material service life [2–4]. Cracks may be caused by excessive reinforcement tension in prestressed structures, insufficient protective layer of concrete, concrete shrinkage, exposure to high temperature during welding of nodes of mating elements. In addition, their appearance can occur both at the manufacturing stage and at the operational stage. In bearing reinforced concrete structures, the occurrence and development of cracks occurs as a result of their deformations from the load effects, temperature fluctuations, uneven settlement of buildings and structures [5]. The width of the crack opening, which depends on the stress state in the cross section with the crack, is one of the criteria for ultimate states [6]. Since cracks exist in any reinforced concrete structures, the determination of the stress state in a section with a crack is very important for assessing the real state of an operated structure.

Many scientific papers are devoted to the study of crack formation conditions and estimates of their influence on strength and deformation. Studies of the cause of cracks were carried out by assessing the composition of concrete mixtures, analysis of the design and construction methods [7]. The study of the characteristics of tensile pressure, longitudinal and transverse expansion and cracking in concrete samples at various degrees of restriction describes a certain picture of the dynamics of crack growth [8]. Also, one of the causes of cracks in concrete is uneven corrosion

***Corresponding Author: Omirkhan Khabidolda:** Department of Algebra, Mathematical Logic and Geometry named after T. G. Mustafin, E. A. Buketov Karaganda State University, University St., 28, 100028, Karaganda, Kazakhstan; Email: khabidoldaom@rambler.ru, oka-kargtu@mail.ru
Zhmagul Nuguzhinov, Zhetpisbai Bakirov, Syrlybek Zholmagambetov, Alexey Kurokhtin, Daniyar Tokanov: Kazakhstan Multidisciplinary Institute of Reconstruction and Development, Karaganda State Technical University, Nursultan Nazarbaev Ave., 56, 100027, Karaganda, Kazakhstan

of the reinforcement [9] and when a load is connected to the process [10].

To predict the behavior of reinforced concrete beams experimentally, it is necessary to conduct numerous fracture tests, which are very expensive and not always accurate. Therefore, the development and use of numerical methods is a more rational method of forecasting from an economic and practical point of view.

The calculation of reinforced concrete structures with cracks is a rather complicated scientific problem. The substantiation of design models and the determination of the stress-strain state of beams with normal cracks along the axis are discussed [8, 11–13]. One of the main criteria for the ultimate state of reinforced concrete structures with cracks is the crack opening width [14–19], as well as the angle of inclination of the cracks, the magnitude of which affects the parameters of the stress state in the cross section [20, 21]. Numerous studies are devoted to issues of crack formation [22–24], calculation of crack resistance, and change in the rigidity of beams and deflections during cracking [25–27]. The results allow us to evaluate the bearing capacity of beams with cracks and crack resistance according to the criteria of the strength of fracture mechanics.

The use of calculation methods and probabilistic models [28–30] in predicting crack formation parameters and calculating the reliability of reinforced concrete structures with cracks according to various criteria are in good agreement with experimental data [31]. In addition, the probabilistic model of explicit cracking gives more accurate representations regarding the mechanical behavior of a reinforced concrete beam and provides accurate information about the cracking process of this type of material [28]. The use of computer programs in conjunction with the developed packages, including various models of strength properties of elementary structures, gives fairly good forecasting results comparable with experiments [32–34]. Thus, modeling the behavior of crack propagation in reinforced concrete structures under various operating conditions is a very urgent task in construction engineering.

The complexity of solving problems of determining the stress-strain state of flexible reinforced concrete beams is associated with a number of factors. Firstly, the relationship between stress and strain in concrete is not linear, therefore, the problem must be solved in a nonlinear formulation [9]. Secondly, the presence of reinforcing inclusions makes the task statically indeterminate initially [35, 36]. Thirdly, the initial crack length is unknown, and its appearance leads to disruption of adhesion of concrete with reinforcement along the length of the beam and a significant redistribution of stresses in the cross sections with the crack [19].

According to the literature analysis, the available analytical methods for calculating the stress state of cracked beams are approximate methods [37–39]. The accuracy of such methods depends on the accuracy of the initial hypotheses, which depends on the level of stress, and the accuracy of solving a nonlinear equation system, and the development of new universal software systems. The main disadvantage of numerical calculation methods is the difficulty in analyzing the final results in terms of managing the calculated value. From the decisions it is not clear which input parameters are needed, how much and in what direction to make changes to obtain the desired result. The complexity of these changes consists in multiple calculations with a step-by-step change of many variable parameters. In addition, the results obtained are difficult to use for a quantitative change in the properties of materials, structural parameters or load. To obtain an optimal plan, we were using a planning matrix based on orthogonal Latin squares [40, 41]. The obtained mathematical dependencies were compared with similar dependencies obtained by other methods. In particular, in [42] they used the method for predicting the shear strength of reinforced concrete beams reinforced with a polymer fiber, in [43] authors examined the applicability of regression based on support vectors for predicting the fracture and breaking load characteristics of high-strength and heavy-duty concrete beams. The effectiveness of using nonparametric regression analysis is also shown in Kusuma [44], Pudjisuryadi and Suprobo [45], and Tavio *et al.* [46]. Moreover, the prestresses in a reinforced concrete beam can be determined by an approximate analytical method in a linear or nonlinear formulation [47, 48].

1.1 Research significance

For a reasonable choice of parameters of reinforced concrete structures and control of load parameters, it is necessary to have analytical dependences of output values on variable parameters. The aim of this work was to develop a method for determining the stress parameters in reinforced concrete beams in a section with a crack. The new methodology is based on the planning and modeling of a multivariate experiment, followed by software processing of the results, in order to obtain the desired regression dependencies.

2 Methods

2.1 Experimental procedure

2.1.1 The method of numerical calculation of the stress state of a bent reinforced concrete element

As a starting position, a tabular or analytical description of the diagram of the deformation of materials: concrete and reinforcement is adopted. For concrete, the most often applied exponential law (normative strength) [26], which is laid down in the LIRA-SAPR program (Structural Engineering Analysis & Design Software), which can be written as follows:

$$\sigma = 1.1R_{\sigma n}\left[1 - \exp\left(-0.9\epsilon E_b/R_{bn}\right)\right] \quad (1)$$

where E, R_{bn} – modulus of elasticity and standard resistance of concrete.

Substituting normative resistance instead R_{bn} design resistance R_b, exponential law can be represented as follows:

$$\sigma_b = E_b\epsilon_b \exp\left(\epsilon_{rb}^{1/y}\gamma\epsilon_{rb}^{1/y}\right), \quad \gamma = \ln\left(R_b/E_b\epsilon_{rb}\right) \quad (2)$$

where ε_{rb} – concrete deformation corresponding to the maximum stress, the value of which is taken from the diagram or determined from the expression:

$$\epsilon_{rb} = R_{bn}\left(0.12 + 18.8/R_{bn}\right)\cdot 10^{-4}, \quad (3)$$
$$\epsilon_{rbt} = 1.5R_{bn}/E_b$$

Formula (2) can be represented in a more compact form:

$$\sigma_b = R_b\frac{kn - n^2}{1 + (k-2)n}, \quad n = \frac{\epsilon}{\epsilon_{rb}}, \quad k = \frac{E_b\epsilon_{rb}}{R_b} \quad (4)$$

For concrete, the parameters of these laws during compression and tension will be different, and for armature the piecewise-linear law is most often used. The diagram of reinforcement deformation under tension and compression is assumed to be the same [36].

When solving the problem numerically, the cross section of a reinforced concrete element is considered as a set of elementary areas within which stresses are considered uniformly distributed, and it is assumed that deformations along the section height vary linearly. Internal cross-sectional forces are determined by summing the forces over small areas within which average stress values are taken. The moment of internal forces is determined analogously by summing the moments of forces acting on elementary platforms.

With direct bending of the section, it is convenient to divide into elementary strips and individual reinforcing bars, their position is determined by the coordinate of the center of gravity of the strip or reinforcement, measured from the lower edge – y_{bi}, y_{sj}. We cut the beam along a dangerous section and draw up the equilibrium conditions for the cut-off part of the beam. In view of the foregoing, these conditions during bending will take the following form:

$$\sum_{i=1}^{n}\sigma_{bi}A_{bi} + \sum_{j=1}^{k}\sigma_{sj}A_{sj} = 0 \quad (5)$$

$$\sum_{i=1}^{n}\sigma_{bi}A_{bi}\left(y_0 = y_{bi}\right) + \sum_{j=1}^{k}\sigma_{sj}A_{sj}\left(y_0 = y_{sj}\right) - M = 0 \quad (6)$$

Where σ_{bi}, σ_{sj} are normal voltages on i elementary strip of concrete and j fixture; – area of elementary strips of concrete and j fixture; y_0 distance from the bottom edge to the center of gravity.

2.1.2 The algorithm for solving the problem

1. This algorithm provides a step-by-step method of sequential loads, at each stage of which an iterative process of calculating deformations in elementary areas is implemented.
2. The sequence of calculation of the stress state in the section consists of the following stages of execution:
3. Determination of the curvature of the axis of the bent element, according to the formula

$$\frac{1}{r} = \frac{M}{\sum_{i=1}^{n}E_{bi}A_{bi}(y_0 = y_{bi})^2 + \sum_{j=1}^{k}E_{sj}A_{sj}\left(y_0 = y_{sj}\right)^2} \quad (7)$$

At this stage of the first load stage, it is assumed that the deformation moduli are equal to the elastic moduli of the materials. Subsequently, they are replaced by secant modules. The moment corresponds to the load stage.

1. Determination of the deformation of elementary strips and individual reinforcement according to the fidelity of the hypothesis of flat sections

$$\epsilon_{bi} = \left(y_0 - y_{bi}\right)/r, \quad \epsilon_{sj} = \left(y_0 - y_{sj}\right)/r \quad (8)$$

2. Determination of stress from the strain diagram

$$\sigma_{bi} = f_b\left(\epsilon_{bi}\right) \quad (9)$$

3. Definition of secant modules of concrete and reinforcement

$$E_{bi} = \sigma_{bi}/\epsilon_{bi}, \quad E_{sj} = \sigma_{sj}/\epsilon_{sj} \quad (10)$$

4. If the limit value of the tensile stress in the concrete of the elementary strip is exceeded, it is identified as the formation of a crack in this strip and in the future, it is excluded from the calculation.
5. Verification of the equilibrium Eq. (4).
6. Verification of the criteria for ending the iterative process

$$\left|\epsilon_{bi} - \epsilon_{b,i-1}\right| \leq \delta, \quad \left|\epsilon_{sj} - \epsilon_{s,j-1}\right| \leq \delta \tag{11}$$

where δ is the convergence criteria.

7. If the convergence criterion is not fulfilled, the calculation process is repeated starting from point 1). Otherwise, the load on the next step is increased and the calculation is repeated.

The cycling of the calculation algorithm continues until the upper load value is reached.

When programming the algorithm, the finite element method is taken into account. For beam elements, the cross section is divided into strips, stresses are calculated by sections, where the finite elements are joined. For volumetric elements, the base areas of the finite elements are elementary areas, the stress calculation is performed on the cross-sections indicated in advance.

2.1.3 Algorithm for the numerical calculation of stress states

The implementation of the software algorithm was carried out in the certified LIRA-SAPR program, which is intended for normative calculations and the physicomechanical characteristics of concrete of various classes and grades are already included in the program in the laws of the deformation diagram. Using this program, we calculated the stress state of a pivotally supported bent reinforced concrete beam of rectangular cross section with reinforcement in the stretched zone (Figure 1).

In this approach, a nonlinear beam element with three degrees of freedom in a node was used.

The calculation consists of sequential operations.

1. Creating a geometric scheme of the beam and the division into finite elements.
2. Setting the boundary conditions.
3. Setting the stiffness parameters of the beam elements is carried out by the appropriate choice:
   (a) section sizes (from the standard list, the type of section used (beam) is selected and section dimensions are set);
   (b) material characteristics (the law of non-linear deformation of concrete is selected in the drop-down list (the exponential law – standard strength was chosen in this paper), and the parameters of this law (class and type of concrete) are selected;
   (c) the characteristics of the reinforcement (a piecewise-linear law was set in the form of a Prandtl diagram with an actual yield strength of MPa);
   (d) the type, location and area of reinforcement;
   (e) the type of crushing of the cross section and the number of strips;
   (f) type of finite element (nonlinear element 210 was chosen in the work);
   (g) assignment of these characteristics to finite elements.
4. The task of loads, where the place of application, the direction and magnitude of the load are indicated. In non-linear problems, the load number, calculation method (simple step), the number of load steps and the choice of intermediate results are additionally indicated.
5. The task is started for calculation.
6. Results review and analysis.

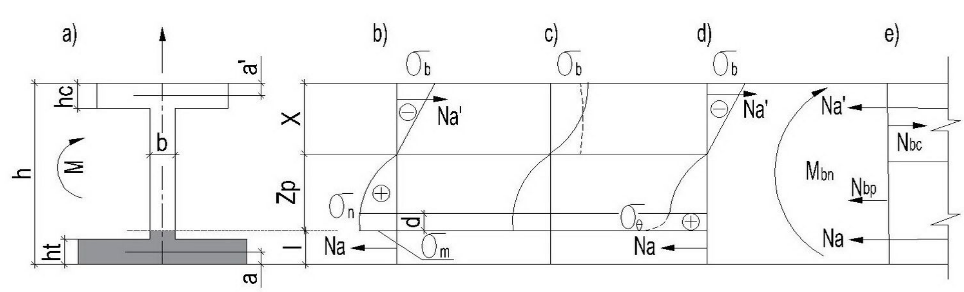

Figure 1: Stress distribution in the cross section of a curved beam with crack length l and compression zone height x: (a) beam cross section; (b,c) diagrams of nominal stresses in the beam section; (d) stress diagrams at the crack tip; (e) forces acting in the section. The crack zone is marked in grey

The calculated results show the diagrams of strains and stresses along the height of the cross section along with a Table 1 that shows the values of the greatest tensile and compressive strains, the maximum compressive and tensile stresses in concrete, the position of the zero line, and the crack length (in the case of presence), the width of its opening and the pitch of the cracks. In addition, the data of the force values in each reinforcement can be viewed, as well as the information on the internal forces in the beam sections, the displacement of various nodes, including the value of the maximum deflection in the beam.

2.2 Experiment planning and regression analysis technique

The stress state in the section with a crack is characterized by the maximum compressive stress in concrete σ_b, height of the compressed zone x, crack length l_m, and voltage in the reinforcement σ_s. Numerous calculations to determine the stress state in reinforced concrete beams during bending showed that the main factors affecting the parameters of the stress state in sections with cracks, in addition to concrete class, are section sizes h, b reinforcement percentage μ modulus of concrete elasticity E and bending moment M. To cover the entire range of changes in these parameters in real conditions, the following values are taken beyond the limits of change: $h = (10 \div 50)$ cm, $b = (5 \div 40)$ cm, $\mu = (0.5 \div 4.5)\%$, $E = (18000 - 30000)$ MPa for concrete class C20/25 (B25) and $E = (20500 - 36500)$ MPa for concrete C32/40 (B40).

Bending moment varies from moment to crack M_m until the destructive moment M_p. Since these moments depend on the size of the cross section, we choose the dimensionless load parameter as a variable parameter m, equal to the ratio of bending moment to destructive moment of a particular beam ($m = M/M_p$). When analyzing the stress state of cracked beams, this parameter should vary from the value M_m/M_p up to one.

2.3 Analytical procedure

Preliminary calculations were carried out to determine the limiting moments M_m and M_p at the smallest and largest section sizes. The destructive moment was determined by the standard calculation method, and a separate calculation method was developed to determine the moment by crack formation. These calculations showed that the ratio of ultimate moments varies from 0.15 to 0.34, depending on the percentage of reinforcement. To derive the regression dependences of the stress parameters on these factors, we carry out a multi-factor rationally planned computer simulation using the LIRA-SAPR program.

Thus, there are five variable factors. If there is no mutual influence between them or they can be neglected, then it is advisable to use rational design of the experiment [37–39]. The use of this method reduces the number of experiments compared to a full-factor experiment per L^{k-2} times, where k-number of factors, L-the number of levels of change. To obtain an optimal plan, a planning matrix based on orthogonal Latin squares is used [40, 41].

An $n \times n$ Latin square is a matrix whose elements are the elements of a set of n-elements such that each number occurs exactly once in each row and column. For all n, the following cyclic Latin square arises:

$$\begin{bmatrix} 1 & 2 & \dots & n \\ 2 & 3 & \dots & 1 \\ \vdots & \vdots & \dots & \dots \\ n & 1 & \dots & n-1 \end{bmatrix}$$

Two Latin squares (a_{ij}), (b_{ij}) are orthogonal if for all i, j the ordered pairs (a_{ij}, b_{ij}) are different. Orthogonal Latin squares are used in experiments that test multiple factors at the same time.

We will change the variable parameters at five equidistant levels. The values of the variable parameters at different levels are shown in Table 1.

According to the planning matrix, the experiment will consist of 25 experiments divided into five orthogonal

Table 1: The values of the variable parameters at different levels

Levels	Variable values					
	m	h, cm	b, cm	μ, %	E (C32/40), MPa	E (C20/25), MPa
1	0.2	10	5	0.5	20500	1800
2	0.38	20	14	1.5	24500	2100
3	0.56	30	23	2.5	28500	2400
4	0.74	40	32	3.5	32500	2700
5	0.92	50	41	4.5	36500	3000

squares. The first three squares are shown below:

1 : 1 2 3 4 5	6 : 1 3 5 2 4	11 : 1 4 2 5 3
2 : 2 3 4 5 1	7 : 2 4 1 3 5	12 : 2 5 3 1 4
3 : 3 4 5 1 2	8 : 3 5 2 4 1	13 : 3 1 4 2 5
4 : 4 5 1 2 3	9 : 4 1 3 5 2	14 : 4 2 5 3 1
5 : 5 1 2 3 4	10 : 5 2 4 1 3	15 : 5 3 1 4 2

Each row of the matrix represents one experiment, the number of which is indicated by the first digit, and subsequent numbers indicate the levels of the corresponding factor. The values of the varied parameters given in Table 1 correspond to these levels. The first row of the planning matrix shows in ascending order all levels. The remaining rows are obtained by increasing the level number by one on the column of the matrix in order from the last to the first number. The first line of the next square consists of the diagonal of the previous square.

Based on the results of a machine experiment, regression dependencies were constructed for the output parameters. For this, an unconventional method of regression analysis is used, which consists in the selection of particular (pair) dependencies from fifteen equations. This method allows one to describe with sufficient accuracy the majority of physical phenomena with a smooth change in functions. The unconventional method is implemented in the special ANETR program developed at KSTU.

The program first arranges the factors in order of decreasing effect on the value of the output parameter, then for each factor selects the corresponding equation from 15 types of equations embedded in the program. Moreover, to identify the influence of some factors, it may be necessary to neutralize the influence of potent factors. The program ends with the selection of a certain combination of particular equations for the formation of a common model. The dependences thus obtained are valid in the range of variation of the varied parameters.

The adequacy of the general model is estimated by the standard deviation (SD) of the calculated and experimental values of the output value, as well as the coefficient of multiple correlation of the model – R, with acceptable SD values less than 25%.

3 Experimental results and discussion

The stress state of a reinforced concrete beam of rectangular cross section is calculated using LIRA-SAPR using the rational planning matrix. As a result of the calculation, we determine the maximum compressive stress and deformation (σ_b, ε_c) in concrete, height of the concrete compressed zone (x), reinforcement force (N_a), maximum deflection in the beam f_m, as well as length l_m, disclosure width a_m and crack pitch h_m. They were determined for a section in a state of clean bending for concrete of class C20/25 and concrete C32/40. A fragment of the results table for concrete C20/25 is given in Table 2.

Regression dependencies will be determined for maximum compressive stress in concrete σ_b, for voltage in reinforcement $\sigma_b = N_a/A_s$, for crack length l_m, and the width of its disclosure. For this, the initial data of each experiment are entered into the ANETR program in the form of a 25×5 matrix and the values of the analyzed output parameter (response function) in the form of a 25×1 column vector.

Graphic dependences between the indicators σ_b, as well as N_a and ϵ_C are shown in Figures 2 and 3.

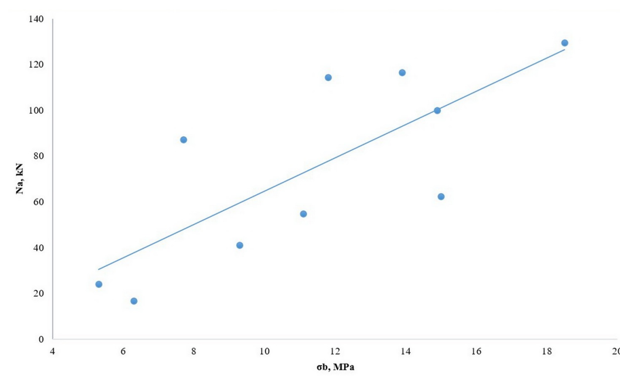

Figure 2: Dependence N_a from σ_b

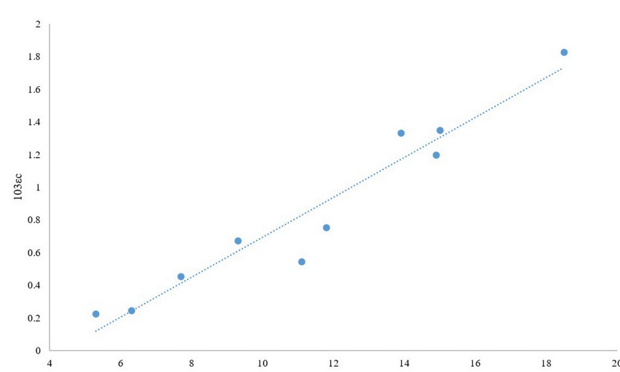

Figure 3: Dependence ϵ_c from σ_b

We present finally formed multidimensional models for the stress parameters in concrete C20/25. Regression

Table 2: Fragment of the result of the bending experiment for concrete C20/25

No	X, cm	σ_b, MPa	N_a, kN	l_m, cm	a_m, mm	h_m, cm	f_m, mm	$10^3\ \epsilon_c$
1	9.9	6.3	16.8	2.21	0.01	4.53	1.37	0.248
2	18.4	9.3	41.2	6.17	0.02	8.32	1.686	0.675
3	11.2	7.7	87.2	23.94	0.19	23.03	1.32	0.455
4	22	14.9	100	24.32	0.09	8.25	2.166	1.2
5	6.38	18.5	129.5	3.62	0.03		0.825	1.83
6	12.9	5.3	24	5.77	0.02	8.97	0.834	0.227
7	18.5	11.1	54.9	14.74	0.03	5.89	1.454	0.547
8	30.2	13.9	116.6	15.31	0.05	9.57	1.777	1.335
9	5.9	15	62.3	3.2	0.03	6.59	9.43	1.351
10	5.4	11.8	114.4	13.15	0.28	20.19	4.85	0.756

dependence for maximum compressive stress in concrete

$$\sigma_b = \frac{m}{0.035 + 0.02m} + 5.6 \cdot 10^{-4} h^2 - 0.033h + \frac{1.68}{b} - 0.7\mu^2 + 4.34\mu + 1.367 \cdot 10^{-8} E^2 - 6.41 \cdot 10^{-4} E + 2.92 \text{ (MPa)} \quad (12)$$

Variables should be substituted into this expression in those units in which they are indicated in Table 2. The results will also be in those units in which the output values are shown in Table 2.

The standard deviation of the model is 15.4%, and the coefficient of multiple correlations, which indicates a fairly high accuracy of the experiment and the completeness of the model coverage of all factors involved in the experiment.

The regression dependence for the voltage in the reinforcement has the form:

$$\sigma_s = 332.86m^{1.97} + 0.07h^2 + 3.523h - 3.463b - 39.52\mu - 6.24 \cdot 10^{-7} E^2 + 0.0305E - 212.2 \text{ (MPa)}, \quad (13)$$

$$SD = 33\%, \quad R = 0.92$$

The crack length is determined by the expression

$$l_m = 0.378h - \frac{2.653}{m} - 2.14 \cdot 10^{-3} b^2 + 0.097b - 2.665\mu + 1.1210^{-8} E^2 - 4.56 \cdot 10^{-4} E + 16.05 \text{ (cm)}, \quad (14)$$

$$SD = 24.9\%, \quad R = 0.952$$

Regression dependence for crack opening width

$$a_m = 0.183m - 8.73 \cdot 10^{-5} h^2 + 6.25 \cdot 10^{-3} h - 1.3210^{-4} b^2 + 6.78 \cdot 10^{-3} b - 0.154 lg\mu + 1.17 \cdot 10^3 E^{-1} - 0.189 \text{ (mm)} \quad (15)$$

Processing the results of similar computer simulations for concrete C32/40 gave the following regression dependencies:

$$\sigma_b = \frac{m}{0.036 + 0.009m} + 0.0029h^2 - 0.163h + \frac{4.31}{b} - 1.08\mu^2 + 6.61\mu + 5.1 \cdot 10^{-5} E^2 - 7.7 \text{ (MPa)} \quad (16)$$

$$\sigma_s + 238.9m - 0.044h^2 + 1.72h - 2.544b - 37\mu - 5.83 \cdot 10^{-7} E^2 + 0.035E - 391.6 \text{ (MPa)} \quad (17)$$

$$l_m = \frac{h}{2.742 - 1.28 \cdot 10^{-3} h} - \frac{3.37}{m} - \frac{4.81}{b} - 10jg\mu + 5.85 \cdot 10^{-5} E + 8.91 \text{ (cm)} \quad (18)$$

$$a_m = 0.153jgm - 5.63 \cdot 10^{-5} h^2 + 3.85 \cdot 10^{-3} h - \frac{0.233}{b} + \frac{0.77}{\mu} - \frac{159}{E} + 0.025 \text{ (mm)} \quad (19)$$

The standard deviations, the correlation coefficients of the models are approximately the same as for concrete C20/25.

The obtained regression dependences make it possible to determine the stress parameters of bent rectangular reinforced concrete beams with cracks. In addition, according to the data of the dependences, one can evaluate the bearing capacity of the structural elements under consideration. For this, expressions are used for the maximum stress in concrete and for the stress in the reinforcement. In order to evaluate the health of structures and the second group of limit states, it's recommended to use the expression for the width of the crack opening. According to the ANETR program, regression dependences can be obtained for the maximum deflection of the beam. The most significant is to obtain the dependence for determining the length of the crack. To do this, the resulting expression is equated to zero. This way the moment of crack formation can be found. Knowing the length of the crack, it is possible to construct a calculated model of a reinforced concrete beam with a crack in the form of a mathematical section and calculate the parameters of fracture mechanics to evaluate the crack resistance of the beam.

The materials presented in the article allow us to expand the scope of such regression dependencies. Using the described methodology, one can obtain regression dependences for other classes of concrete and reinforcement and for other sections. These dependences were obtained for reinforced concrete beams without prestressing. If it is necessary to calculate a pre-stressed beam for bending, then it can be performed analytically using the obtained regression dependencies.

A similar approach to the development of a predictive forecasting technique was used in other studies. For example, Marí *et al.* [42] presented a method for predicting the shear strength of reinforced concrete beams reinforced with a polymer fiber. The calculation equations proposed in the method take into account only those parameters that determine the shear strength, such as the tensile strength of concrete, the coefficient of longitudinal reinforcement, and the ratio between the elastic modulus of longitudinal reinforcement and concrete. The results of a comparative analysis with experimental data showed high accuracy of the predicted calculation method. In Yuvaraj *et al.* [43] the authors examined the applicability of regression based on support vectors for predicting the fracture and breaking load characteristics of high-strength and heavy-duty concrete beams. The developed support-vector regression models using MATLAB software showed values that are in good agreement with the experiment. The effectiveness of using nonparametric regression analysis is also shown in Kusuma [44], Pudjisuryadi and Suprobo [45], and Tavio *et al.* [46]. Therefore, the above results indicate the correctness of the approach chosen in this technique.

The prestresses in a reinforced concrete beam can be determined by an approximate analytical method in a linear or nonlinear formulation [47, 48]. The calculation of a prestressed reinforced concrete beam with a crack during bending can be performed using the approximate method based on the hypothesis of flat sections. To do this, according to the diagram of deformation through stress σ_b and R_{bt} the maximum (edge) deformation in concrete and the deformation at the crack tip during bending are determined. The same deformations are found under prestressing according to the hypothesis of flat sections. By adding these strains, a plot of the total strains along the section height is built. Through these deformations, the maximum stress in concrete is determined from the deformation diagram, then the height of the compressed zone and the length of the crack are determined by the hypothesis of flat sections. The stress in the valve can be determined by simply summing the stresses.

4 Conclusions

The presented work outlines a method for determining the stress parameters in a reinforced concrete beam during bending in a nonlinear setting. For its numerical implementation, an algorithm for solving the problem using LIRA-SAPR using non-linear beam elements is presented. A rational planning matrix for a multifactor computer simulation to determine the stress parameters in bent rectangular reinforced concrete beams with a crack has been compiled. Based on this planning matrix using LIRA-SAPR, machine experiments were carried out for concrete beams of class C20/25 and C32/40. By processing the experimental results under the ANETR unconventional regression analysis program, dependencies are obtained for determining the stress parameters of reinforced concrete beams with cracks made of concrete C20/25 and C32/40. In particularly, regression dependencies were determined for maximum compressive stress in concrete σ_b, for voltage in reinforcement $\sigma_b = N_a/A_s$, for crack length l_m, and the width of its disclosure.

The obtained dependences make it possible to evaluate the performance of the structural elements under consideration for both groups of limiting states, which can be used to determine the parameters of fracture mechanics and assess the crack resistance of the beam. The materials presented in the article allow us to expand the scope of such regression dependencies. Using the described methodology, one can obtain regression dependences for other classes of concrete and reinforcement and for other sections. These dependences were obtained for reinforced concrete beams without prestressing. If it is necessary to calculate a prestressed beam for bending, then it can be performed analytically using the obtained regression dependencies.

Conflict of interest: The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Author contributions: All authors have accepted responsibility for the entire content of this manuscript and approved its submission.

Funding: This research has no funding.

Data availability statement: Data will be available on request.

References

[1] Şahmaran M, Li VC. Engineered cementitious composites: can composites be accepted as crack-free concrete? Transp Res Rec. 2010;2164(1):1–8.
[2] Jang SY, Kim BS, Oh BH. Effect of crack width on chloride diffusion coefficients of concrete by steady-state migration tests. Cement Concr Res. 2011;41(1):9–19.
[3] Otieno MB, Alexander MG, Beushausen HD. Corrosion in cracked and uncracked concrete–influence of crack width, concrete quality and crack reopening. Mag Concr Res. 2010;62(6):393–404.
[4] Wiktor V, Jonkers HM. Quantification of crack-healing in novel bacteria-based self-healing concrete. Cement Concr Compos. 2011;33(7):763–70.
[5] Ohno K, Ohtsu M. Crack classification in concrete based on acoustic emission. Constr Build Mater. 2010;24(12):2339–46.
[6] Carpinteri A. Mechanical damage and crack growth in concrete: plastic collapse to brittle fracture. Berlin, Heidelberg: Springer Science & Business Media; 2012.
[7] Wright JR, Rajabipour F, Laman JA, Radlińska A. Causes of early age cracking on concrete bridge deck expansion joint repair sections. Adv Civ Eng. 2014;2014:103421.
[8] Kagimoto H, Yasuda Y, Kawamura M. ASR expansion, expansive pressure and cracking in concrete prisms under various degrees of restraint. Cement Concr Res. 2014;59:1–15.
[9] Jin L, Zhang R, Du X, Li Y. Investigation on the cracking behavior of concrete cover induced by corner located rebar corrosion. Eng Fail Anal. 2015;52:129–43.
[10] Li CQ, Yang ST. Prediction of concrete crack width under combined reinforcement corrosion and applied load. J Eng Mech. 2011;137(11):722–31.
[11] Chernin L, Val DV. Prediction of corrosion-induced cover cracking in reinforced concrete structures. Constr Build Mater. 2014;4:1854–69.
[12] Delyavskyy M, Opanasovych V, Bilash O. Bending by concentrated force of a cantilever strip having a through-thickness crack perpendicular to its axis. Appl Sci (Basel). 2020;10(6):20–37.
[13] Mohammadhassani M, Jumaat MZ, Jameel M. Experimental investigation to compare the modulus of rupture in high strength self-compacting concrete deep beams and high strength concrete normal beams. Constr Build Mater. 2012;30:265–73.
[14] Amin A, Gilbert RI. Instantaneous crack width calculation for steel fiber-reinforced concrete flexural members. ACI Struct J. 2018;115(2):535–43.
[15] Kwan AK, Ma FJ. Crack width analysis of reinforced concrete under direct tension by finite element method and crack queuing algorithm. Eng Struct. 2016;126:618–27.
[16] Ma FJ, Kwan AK. Crack width analysis of reinforced concrete members under flexure by finite element method and crack queuing algorithm. Eng Struct. 2015;105:209–19.
[17] Oliver-Leblond C, Delaplace A, Ragueneau F. Modelling of three-dimensional crack patterns in deep reinforced concrete structures. Eng Struct. 2015;83:176–86.
[18] Tan R, Eileraas K, Opkvitne O, Žirgulis G, Hendriks MA, Geiker M, et al. Experimental and theoretical investigation of crack width calculation methods for RC ties. Struct Concr. 2018;19(5):1436–47.
[19] Yang ST, Li KF, Li CQ. Numerical determination of concrete crack width for corrosion-affected concrete structures. Comput Struct. 2018;207:75–82.
[20] Fakhri M, Amoosoltani E, Aliha MR. Crack behavior analysis of roller compacted concrete mixtures containing reclaimed asphalt pavement and crumb rubber. Eng Fract Mech. 2017;180:3–59.
[21] Yamamoto Y, Nakamura H, Kuroda I, Furuya N. Crack propagation analysis of reinforced concrete wall under cyclic loading using RBSM. Eur J Environ Civ Eng. 2014;18(7):780–92.
[22] Deng Z, Wang Y, Yang H, Qian J. Research on crack behavior of recycled concrete beams under short-term loading. KSCE J Civ Eng. 2018;22(5):1763–70.
[23] Salnikov A, Kolchunov V, Yakovenko I. The computational model of spatial formation of cracks in reinforced concrete constructions in torsion with bending. Appl Mech Mater. 2015;725-6:784–9.
[24] Zhu H. Crack formation of steel reinforced concrete structure under stress in construction period. Frattura Integr Strutt. 2016;36(36):191–200.
[25] Khabidolda O, Bakirov ZB, Nuguzhinov ZS, Vatin N. Determining stress intensity factor in bending reinforced concrete beams. Bull Karaganda Univ-Mathemat. 2019;96(4):90–8.
[26] Nuguzhinov ZS, Bakirov ZB, Kurokhtin AY, Khabidolda O, Nuguzhinova A. Assessment of bending reinforced concrete beams crack resistance. IOP Conf Ser: Mater Sci Eng. 2019;690:012002.
[27] Sheng J, Yin SP, Xu SL, Jin ZY. Experimental and theoretical investigations on crack spacing and stiffness of textile-reinforced concrete–strengthened reinforced concrete beams. Adv Struct Eng. 2018;21(11):1696–707.
[28] Cohen M, Monteleone A, Potapenko S. Finite element analysis of intermediate crack debonding in fibre reinforced polymer strengthened reinforced concrete beams. Can J Civ Eng. 2018;45(10):840–51.
[29] Ma FJ, Kwan AK. Finite element analysis of concrete shrinkage cracks. Adv Struct Eng. 2018;21(10):1454–68.
[30] Rossi P, Daviau-Desnoyers D, Tailhan JL. Analysis of cracking in steel fibre-reinforced concrete (SFRC) structures in bending using probabilistic modelling. Struct Concr. 2015; 3(3):381–8.
[31] Bykov AA, Matveenko VP, Shardakov IN, Shestakov A. Shock wave method for monitoring crack repair processes in reinforced concrete structure. Mech Solids. 2017;52(4):378–83.
[32] Badiger NS, Malipatil KM. Parametric study on reinforced concrete beam using ANSYS. Civ Environ Res. 2014;6:88–94.
[33] Banjara NK, Ramanjaneyulu K. Experimental and numerical investigations on the performance evaluation of shear deficient and GFRP strengthened reinforced concrete beams. Constr Build Mater. 2017;137:520–34.
[34] Dahmani L, Khennane A, Kaci S. Crack identification in reinforced concrete beams using ANSYS software. Strength Mater. 2010;42(2):232–40.
[35] Biolzi L, Cattaneo S. Response of steel fiber reinforced high strength concrete beams: experiments and code predictions. Cement Concr Compos. 2017;77:1–13.
[36] Hsu TT. Unified theory of reinforced concrete. London: Routledge; 2017.

[37] Atkinson A, Donev A, Tobias R. Optimum experimental designs, with SAS. Oxford: Oxford University Press; 2007.
[38] Campbell DT, Stanley JC. Experimental and quasi-experimental designs for research. Cambridge: Ravenio Books; 2015.
[39] Kleijnen JP. Regression and Kriging metamodels with their experimental designs in simulation: A review. Eur J Oper Res. 2017;256(1):1–16.
[40] Naidu RR, Jampana P, Sastry CS. Deterministic compressed sensing matrices: construction via Euler squares and applications. IEEE Trans Signal Process. 2016;64(14):3566–75.
[41] Zhang L, Huang Q, Lin S, Abdel-Ghaffar K, Blake IF. Quasi-cyclic LDPC codes: an algebraic construction, rank analysis, and codes on Latin squares. IEEE Trans Commun. 2010;58(11):3126–39.
[42] Marí A, Cladera A, Oller E, Bairan J. Shear design of FRP reinforced concrete beams without transverse reinforcement. Compos B Eng. 2014;57:228–41.
[43] Yuvaraj P, Murthy AR, Iyer NR, Sekar SK, Samui P. Support vector regression based models to predict fracture characteristics of high strength and ultra high strength concrete beams. Eng Fract Mech. 2013;98:29–43.
[44] Kusuma B. Analytical model for axial stress-strain behavior of welded reinforcement grid confined concrete columns. J Asian Concr Fed. 2015;1:1–10.
[45] Pudjisuryadi P, Suprobo P. Analytical confining model of square reinforced concrete columns using external steel collars [dissertation]. Surabaya: Petra Christian University; 2013.
[46] Tavio T, Budiantara IN, Kusuma B. Spline nonparametric regression analysis of stress-strain curve of confined concrete. Civ Eng Dimens. 2008;10:14–27.
[47] Slobbe AT, Hendriks MA, Rots JG. Sequentially linear analysis of shear critical reinforced concrete beams without shear reinforcement. Finite Elem Anal Des. 2012;50:108–24.
[48] Vasudevan G, Kothandaraman S, Azhagarsamy S. Study on non-linear flexural behavior of reinforced concrete beams using ANSYS by discrete reinforcement modeling. Strength Mater. 2013;45(2):231–41.