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Identification of crack location in metallic biomaterial cantilever beam subjected to moving load base on central difference approximation

Abstract: If not detected early, the cracks in structural components may ultimately result in the failure of the structure. This issue becomes even more critical when the component under investigation is a prosthesis placed in the human body. This study presents a crack location identification method based on the time domain in a cantilever beam of metallic biomaterials (CBMB). The absolute difference between the central difference approximation of the root mean square (RMS) of displacement of points on the cracked and uncracked beams was applied as a cracked location indicator. Captured time-domain data (displacement) at each node of the cracked and uncracked beams were processed into a central difference approximation of the RMS of displacement. Then, the crack could be detected by a sudden change of the cracked location indicator. The feasibility and effectiveness of the proposed method were validated by numerical simulations. The finite-element simulation of a CBMB with a transverse notch was analyzed in the numerical study. The notch or crack was detected along the beam under a moving load at various locations. A set of simulation experiments and numerical calculations was performed to determine whether the proposed identification method would accurately detect the location of a crack in a cantilever beam under a moving load compared to the location found by an exact solution method. The results showed that the proposed method was not only as able as the analytical method but also robust against noise: it was able to detect a crack precisely under 5% noise.

Keywords: crack identification, metallic biomaterials, central difference approximation, cantilever beam, moving load

1 Introduction

Time-dependent load, fluctuating and vibratory, plays a significant role in design and manufacturing in various industries. Branco et al. analyzed 3D crack propagation in relatively bone and denture, and adaptive finite element (FE) remeshing techniques have been widely adopted in the literature [1]. Zhang et al. [2] conducted numerical research on crack growth in a gear tooth using FRANC3D. Schöllmann et al. [3] developed a new crack growth method to simulate the propagation of cracks in knee levers using FRANC/FAM and ADAPCRACK3D. Despite its predictive functions, this type of numerical approach has disadvantages of being time-consuming and inadequate simulation accuracy due to frequent changes in mesh topology in the course of crack tip extension.

In industry, the vibration of parts of structures can cause multiple problems, which may be minor, e.g., the vehicle vibration caused by rough road surface or dry friction between the brake pad and wheel induces brake noise and self-excited vibration, which upon elevation to a larger scale disturbs riders, as reported in studies by...
Jearsiripongkul et al. [4] and Jearsiripongkul and Hochlehnert [5] or truly terrible vibrations, such as the breakdown of the Tacoma Narrows suspension bridge in high wind inducing catastrophically violent torsional vibration [3]. The vibration of parts of the structure is undesirable and must be eliminated from the system. Dimarogonas [6] was the first researcher who focused on vibration to identify cracks in shafts, providing a starting point for other researchers to develop a theory of vibration to detect a crack in a structure based on the idea that a crack in a system changes the physical and dynamic behavior of the system, which can be easily detected under vibration [7].

Early crack identification methods were based on frequency analysis of the vibration of structures and testing machines used for prosthesis life tests. Adams et al. [8], Stubbs et al. [9], Lee and Chung [10], and Liang et al. [11] used frequency analysis of vibration to indicate the existence of cracks in various types of structures. Later, Qiu et al. [12] stated that the presence of a crack, its location, and its depth change the natural frequency of a structure. Nevertheless, if the crack is at a nodal point (no displacement), the natural frequency of vibration does not change, which is a major disadvantage of the method based on natural frequency. Moreover, the method was too sophisticated since the vibration signal was measured in the time domain (displacement, velocity, and acceleration) and then transformed into a frequency domain.

Pandey et al. [13] presented a crack indication method based only on time-domain data. They used the peak of absolute differences in curvature mode shapes between the cracked and uncracked beams to detect the location of the crack. The curvature mode shapes were calculated by the displacement of the mode shape. Hamey et al. [14], Qiao et al. [15], Chandrashekar and Ganguli [16], and Frans et al. [17] developed other crack identification methods based on Pandey’s method [13]. On the other hand, Choia and Stubbsb [18], Rzeszucinski et al. [19], and Umesh and Ganguli [20] developed crack identification methods that were not based directly on Pandey’s method but still based on time-domain data.

Concerning moving load and crack identification systems, structures under a moving load, such as bridges, railways, sliding robot manipulators, and machine tools, undergo a larger deflection and higher stress than those under a static load of the same magnitude. Deflection refers to a function of both the time and speed of the load movement [21–25]. Chouiaykh et al. [26] and Roveri and Carcaterra [27] developed a crack identification method that uses the dynamic displacement of the structure under a moving load. Their results indicated that methods based on moving load were able to detect the location of a crack precisely. However, the methods were still too sophisticated since they were based on instant frequency.

This study aimed at developing a crack identification method based solely on time-domain vibration data and moving load to avoid a common problem for many engineers responsible for crack identification of their facilities, i.e., demanding and time-consuming tasks in encoding unnecessarily complicated mathematical calculation steps into a functioning and precise crack identification app for their intended structure. According to Asnaashari and Sinha [28], the complexity of the mathematical calculation steps for seeking a solution based on frequency–domain data was greater than that based on the time-domain data.

Various types of vibrations have different effects on cracks. We investigated the forced free vibration of a sandwich plate with an electrorheological fluid core layer and functionally graded face layers via the first-order shear theory. Thongchom et al. [29,30] investigated the tensile strength and modulus of elasticity nanocomposite and sound transmission loss of cylindrical sandwich shells.

Nanomaterials are very important in various industries today. Nanomaterials are used in nanocomposites to reinforce them. However, an important issue after adding additives is crack development. These nanomaterials include nanographene, carbon nanotubes, clay nanotubes, and nanosilica [31–33]. Thongchom et al. [33] studied the mechanical properties of carbon nanotubes.

2 Methods

In the following approach, the time domain in cantilever beam of metallic biomaterials was utilized to identify a crack location in the specimen. The main difference between the estimated root mean square (RMS) of displacement at the points on the cracked and uncracked beams was introduced as a cracked location indicator as follows.

2.1 Developed crack identification method based on time-domain analysis

A crack that appears in the structure of a beam is always accompanied by some changes in structural properties. Pandey et al. [13] found that a crack can be detected by the curvature mode shape of the beam. They introduced a verified relationship between the flexural stiffness and amplitude of curvature of natural modes of vibration at
a point in or on a common beam, which could be used to successfully detect a crack. The curvature at a point in or on the beam is obtained by the following equation:

$$\kappa = \frac{M}{EI},$$  \hspace{1cm} (1)

where \(\kappa\) is the curvature at that point, \(M\) is the bending moment at that section, \(E\) is the modulus of elasticity, and \(I\) is the second moment of the cross-sectional area. If a crack appears in a structure, it reduces the \(EI\) of the structure at the crack point, which makes the magnitude of the curvature greater at that point. This change in the curvature is local in nature, and accordingly, it can be used to detect a crack in a small, suspected region.

The magnitude of mode shape to estimate the curvature mode shape by central difference approximation is obtained by the following equation:

$$y_i'' = \frac{(y_{i+1} - y_{i-1})}{h^2},$$ \hspace{1cm} (2)

where \(y_i''\) and \(h\) represent the central difference approximation at node or point \(i\) and the length between node \(i\) and \(i + 1\). Also \(y_i\) represents the magnitude of mode shape at node \(i\); node number, \(i = 2..m - 1\); and \(m\) is total number of nodes.

The absolute difference between the central difference approximation at node or point \(i\) of the cracked and uncracked beams is used as a cracked location indicator.

$$\Delta y_i'' = |y_c'' - y_{uc}'|,$$ \hspace{1cm} (3)

where \(\Delta y_i'', y_c''\), and \(y_{uc}''\) represent cracked location indicator at node \(i\), the central difference approximation of the cracked beam at node \(i\), and the central difference approximation of an uncracked beam at node \(i\), respectively.

### 2.2 Main processes in the proposed method

The proposed crack identification method was modified from a previously published crack identification method by Kunla et al. [34]. The main modification was to use the RMS of displacement value instead of the average displacement value. This kind of use was supported by Mohammed et al. [35]. According to them, the RMS of the signal is important when the data range is between positive and negative values (such as sinusoids). This method consists of three processes: the RMS of displacement process, central difference approximation calculation process, and crack identification process. In the first process, the RMS of the vertical displacement of assigned points (nodes) on the beam is determined. In the second process, this set of the RMS of displacement value is taken as input and processed into a set of central difference approximation values of points on the beam. Finally, this set of central difference approximation values is inputted into and processed by the crack identification process. The output will be a graph of the magnitudes of central difference approximation at points on the beam. A sharp peak at any point in this graph indicates a crack at that point.

#### 2.3 Theoretical formulations of the main process

As indicated in Section 2.2.1, the proposed system includes three main processes. The operational procedure of each process is outlined in this subsection. The first process is the RMS process, consisting of two steps. In the first step, the time step and the total number of time steps for RMS displacement are calculated. The time of contact \(T\) on a beam depends on the speed of the moving load on the beam contact surface. A time step, \(\Delta T\), is calculated as follows:

$$\Delta T = \frac{T}{p \cdot n},$$ \hspace{1cm} (4)

where \(p\) is a specified number of intervals, and for each interval, \(n\) is the number of sub-intervals. The total number of time steps, \(N\), for a specified speed of moving load is estimated in the following equation:

$$N = p \cdot n.$$ \hspace{1cm} (5)

In the second step, the RMS displacement values of assigned nodes are calculated. The RMS value of the signal is the normalized second statistical moment of the signal (standard deviation). The RMS of displacement in this article is the RMS used in the calculation of the crest factor of a time-domain analysis to indicate a crack on a gearbox in a study by Mohammed et al. [35]

$$\text{RMS}_i = \sqrt{\frac{\sum_{j=1}^{N} [y_{ij}]^2}{N}},$$ \hspace{1cm} (6)

where \(\text{RMS}_i\) is the root mean square displacement value at node \(i\) for the whole time \(T\); \(y_{ij}\) is the displacement value at node \(i\) at time step \(j\); \(i\) is the node label, \(2,3,..,m-1\); \(j\) is the number of time steps along which the load moves from one fixed end to the other open-end of the beam, \(1,2,..,N\) (Figure 1).

The second process estimates the magnitudes of central difference approximation of the RMS of displacement value output obtained from the first process as follows:
the angle of twist; \( \phi \) is the starting time. \( w_i\) is the polar moment of inertia; \( I_{xy} \) and \( I_w \) are displacement of the cracked location indicator, as previously stated in Eq. (3), can be rewritten as follows:
\[
\Delta w_i'' = |w_i'' - w_{i,mc}''),
\]
where \( \Delta w_i'', w_i'', \) and \( w_{i,mc}'' \) are cracked location indicators at node or point \( i \), the central difference approximation of RMS of displacement of the cracked beam at node \( i \), and the central difference approximation of RMS of displacement of the uncracked beam at node \( i \), respectively.

### 2.4 Finite-element analysis (FEA) of beam vibration under moving load

FEA was conducted to evaluate the efficiency of the proposed method. Displacement results achieved from the FEA approach could be substituted into Eq. (8) to create a graph of the cracked location indicator versus position, of which the location of an abnormal peak would be identified as the crack location. In this subsection, the background of this kind of FEA is described. In the proposed method, vibration in or on every point (node) of the beam under moving load needs to be simulated with this numerical technique in combination with the parameters of the static beam. Consequently, the simulation needs to include a moving load to produce vibration. Using d’Alembert’s principle for three-dimensional problem, and neglecting coupling effects, inertial forces, and inertial torque, the system of equations becomes

\[
\begin{align*}
E_l \frac{\partial^4 z(x, t)}{\partial x^4} + E_{xy} \frac{\partial^4 y(x, t)}{\partial x^4} + \rho A \frac{\partial^2 z(x, t)}{\partial t^2} &= 0, \\
E_k \frac{\partial^4 y(x, t)}{\partial x^4} + E_{xy} \frac{\partial^4 z(x, t)}{\partial x^4} + \rho A \frac{\partial^2 y(x, t)}{\partial t^2} &= F\delta(x - vt), \\
E_k \frac{\partial^4 \varphi(x, t)}{\partial x^4} - GJ \frac{\partial^2 \varphi(x, t)}{\partial x^2} &= 0,
\end{align*}
\]
where \( E \) is the Young’s modulus of the beam; \( I_e \) and \( I_y \) are the second moment of inertia of the beam cross section about the \( x \)- and \( y \)-axis, respectively; \( \rho \) is the density; \( A \) is the area of the cross section; \( x \) is the point of interest on the beam; \( I_{xy} \) is the product moment of inertia; \( I_w \) is the warping constant; \( G \) is the shear modulus; \( \delta \) is the Dirac delta distribution; \( v \) is the speed of the load moving along the beam; \( y(x, t) \) and \( z(x, t) \) are displacement of the shear center in the \( y \) and \( z \) directions, respectively; and \( t \) is the starting time.

This study considered transverse vibration and symmetrical beam cross section. The product of the moment of inertia was zero \( (I_{xy} = 0) \). A simple beam model labeled with these variables is illustrated in Figure 2.

One end of the beam is fixed to a location \( (x = 0) \), while the other end \( (x = l) \) can move freely when a force is exerted on it; hence, the governing boundary conditions of a beam vibrating under the influence of a moving load are Eqs. (10)–(13) as follows:

\[
y(0, t) = 0,
\]
by Lin and Chang [36] were compared and are shown in Figure 3. Two verification schemes were employed: (i) verifying by using natural transverse frequency with Fourier analysis and (ii) verifying by using forced deflection responses at the free end of the cracked cantilever beam under moving load.

The first scheme was a model of a cracked cantilever beam with a modeled crack at $x_{1}/l \approx 0.3$ and a notch depth of 30% of the beam thickness; an impulse of 100 N load was applied on the free end for 0.001 s, as shown in Figure 4. Ansys’s transient analysis module was used to simulate the free vibration response of the beam under impulse load. The length of the beam, $L$, was 580 mm. A Newmark’s integration scheme was the solver, and the time increment was 0.0000219 s. The free vibration of the beam was calculated up to 0.1 s after the load had been applied. Then, the first three natural frequencies of the beam, calculated by applying a fast Fourier transform of acceleration at the mid-span of the beam, were compared with the analytical values obtained from the closed-form solution reported in the study of Lin and Chang [36]. Table 2 shows that the FE prediction was less than 0.2% different from the analytical result, indicating that the FE model and the transient analysis module were quite accurate in simulating the dynamic behavior of the beam.

The location of a crack detected by the proposed method was compared with that detected by the mode shape curvature method in the study by Pandey et al. [13]. The displacement calculation method, picked from a menu in Ansys, was the Newmark’s integration scheme with a fixed time increment of 0.00001 s.

### 2.5 Finite-element model and transient simulation

To verify, the FEA model and a cantilever beam with the same dimensions and mechanical properties conducted
Applied loads, as can be seen in Figure 1, are concentrated load \((F)\) which is applied at points of contact on the surface of the cantilever beam. Points are separated by 25 mm from adjacent points. The force moves from the left end to the right end of the beam at a speed of 30.9 m/s. The tested magnitudes of the moving load were 70, 80, and 90 N. For each magnitude, three runs were conducted. The goal was to specify which moving load magnitude would provide the most discriminating peak in the graph of cracked location indicator versus position coordinate (detailed in Section 2.2). The displacement of a point in and on a beam was dependent only on the concentrated load, \(F\), acting on the numbered nodes \(-1, 2, 3, \ldots, m\) – as indicated in Figure 1.

For the curvature mode shape method, the magnitude of the first displacement mode shape was used to estimate the curvature mode shape with the Modal Module in Ansys. The assigned settings for the beam were the same as in the evaluation of the proposed method.

A crack was assigned to be at one-third, the middle, and two-thirds of the beam length, and their locations were identified. Then, the same procedures were run but used the proposed method to identify the location, and the results were compared. As shown in Figure 5, plots (with \(10^{-5}\) scaling) of the three crack locations identified by the proposed method (for which a moving load of 70 N was applied) overlayed with plots of the three crack locations identified by Pandey’s curvature mode shape method.

### Table 2: Comparison of natural frequencies estimated by FE model and analytical solution

<table>
<thead>
<tr>
<th>Mode</th>
<th>Natural frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical ([36])</td>
</tr>
<tr>
<td>1</td>
<td>30.88</td>
</tr>
<tr>
<td>2</td>
<td>195.60</td>
</tr>
<tr>
<td>3</td>
<td>540.48</td>
</tr>
</tbody>
</table>

3 Verifying the accuracy of crack characterization in the presence of noise

It is necessary to prove the effectiveness of the method in the presence of white noise because most real-world data are contaminated with white noise. In this subsection, the performance of the proposed damage detection technique is investigated with the simulated white noise. The simulated white noise, generated from Gaussian distribution, encompassed a series of independent samples, which were expressed [38] by the following equation:

![Figure 4: Beam with meshes assigned for FEA.](image)

![Figure 5: Crack at \(x/l = 0.3\), at the middle of the beam, and \(x/l = 0.7\).](image)
where \( w_{\text{Noise}, i} \), \( w_i \), \( E_p \), \( N_{\text{Noise}} \), and \( \sigma(w) \) represent white noise pollution displacement at node or point \( i \), original displacement at node or point \( i \), percent of noise level, a standard normal distribution (Gaussian distribution) vector with zero mean value and unit standard deviation at node or point \( i \) and the standard deviation of the original displacements, respectively.

Noisy measurements were represented by introducing 5% of white noise into the signal in the form expressed by Eq. (14) and generated by MATLAB software.

### 4 Numerical results and discussions

Four main points are discussed in this section: crack identification based on the difference in the RMS of displacement of the uncracked and cracked beams; dependence of difference in central difference approximation of RMS of displacement on the magnitude of moving load; dependence of difference in central difference approximation of displacement on the magnitude of moving load with noise; comparison of crack locations identified by the proposed method and Pandey’s et al. mode shape method [13].

#### 4.1 Difference in RMS of displacement of the uncracked and cracked beams

The difference between the RMS of displacement of the cracked and uncracked beams under three magnitudes of moving load is shown in Figure 6, where a crack is positioned at \( x/l = 0.3, 0.5, \) and 0.7. As can be seen, all three curves in each panel show a peak or trough at the crack position, i.e., a crack can be detected visually and directly from these curves under any tested magnitudes of moving load. The magnitude of the difference between the RMS of displacement of the cracked and uncracked beams depended on the location of the crack.

The third panel shows a sharp trough occurring at the crack location and a shallower trough at another location, which is due to the small discontinuity in the RMS of displacement values, as shown in Figure 7. This result indicated that the peaks and troughs of RMS of displacement values versus the location of the cracked and uncracked beams were very sensitive to discontinuity.

#### 4.2 Dependence of difference in central difference approximation of RMS of displacement on the magnitude of moving load

As can be seen in Figure 6, the differences in the RMS of displacement of uncracked and cracked beams indicate the location of the crack. However, for a crack at \( x/l \approx 0.7 \), the difference in the RMS of displacement of the uncracked and cracked beams does not give a good indication of the crack location. Owing to changes in the stiffness of the beam at the crack, the plot of the difference in central difference approximation of RMS of displacement of the cracked beam and an uncracked beam under moving mass showed a sharp peak at the crack location on the cantilever beam. However, a plot of the position coordinate of the difference in central difference approximation of RMS values of displacement showed the crack location clearly.

Figure 6: Absolute difference in the magnitude of RMS of the cracked and uncracked beams (crack at \( x/l = 0.30, x/l = 0.50, \) and \( x/l = 0.70 \) under various moving loads).
Figure 8 illustrates the results of the absolute difference in the central difference approximation of RMS of displacement at a node of a cracked beam and an uncracked beam, evaluated by Eq. (8), and each of three wedge-shaped notch locations: a notch (or a crack) at $x/l = 0.3$, a notch at mid-span of the beam, and a notch at $x/l = 0.5$. The notch depth was 30% of the beam thickness. It can be seen that for all notch locations, varying the moving load from 70 to 90 N still provided easily distinguishable peaks at the same notch location, although a load of 90 N provided the highest peak because it led to more deflection or displacement.

At the crack point in or on the beam, the IE (flexural stiffness) was the lowest, and so the curvature (central difference approximation) at the cracked point was the highest, which was shown clearly as a sharp trough at the crack location.

It can be seen that for all crack locations, varying the moving load from 70 to 90 N still provided easily distinguishable peaks at the same notch location, although a load of 90 N provided the highest peak because it led to more deflection or displacement.

Notably, curvature was a linear function of the load; as the load increased, the curvature increased, and the trough at the crack location got sharper.
4.3 Dependence of the difference in central difference approximation of RMS displacement on the magnitude of moving load from a signal with noise

An evaluation of the effect of noisy measurements on damage detection was performed by introducing 5% noise, as expressed in Eq. (14), into the displacement signal simulated by Ansys.

Figure 9 provides the results achieved from the absolute difference of the central difference approximation of RMS of displacement at a node of a cracked beam and an uncracked beam. The results indicated the effectiveness of the proposed method in identifying the crack location under noisy conditions and under no-noise conditions. In short, the proposed method was robust against noise.

4.4 Comparison of crack locations identified by the proposed method and Pandey’s mode shape method

As shown in Figure 5, plots (with $10^{-5}$ scaling) of the three crack locations identified by the proposed method (for which a moving load of 70 N was applied) overlayed with plots of the three crack locations identified by Pandey’s curvature mode shape method. It can be easily seen that all the identified locations were equivalent to the same locations, implying that the proposed method could be applied as a reasonable alternative to Pandey’s analytical method without any significant errors in identifying a crack location in or on a cantilever beam.

5 Conclusions

In the recent research work, the crack identification method in a cantilever beam under a moving load was verified by numerical simulations, assuming a transverse surface crack extending uniformly along the width of the cantilever beam. Transient analysis with FEA software was utilized to simulate forced responses of an uncracked and a cracked cantilever beam under a moving load. The location of the notches for a crack was investigated. A period of procedure relying on measured time response was discussed.

A numerical procedure hinging on curvature mode shape and RMS was carried out for forced responses of cantilever beams under a moving load. Forced responses of cracked beams were examined with a varying moving load. The location of a crack was detected by a peak of the absolute difference between the central difference approximation of RMS of displacement of the cracked and uncracked beams. The effectiveness of the proposed damage identification scheme was positively verified by its ability to identify the same location of the simulated crack as the exact solution method had found. In addition, the results of the numerical studies indicated that the proposed method was not sensitive to noise, notably within its procedure of localizing the accurate damages under 5% white noise conditions. Accordingly, it has a great potential application with the following advantages: only the deflection parameters of the beam are needed to perform damage localization, and the peak amplitude of a crack location indicator varies based on the moving load. This method can be considered a simple, economical, and effective tool for nondestructive testing that does not require any sophisticated equipment.

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Data availability statement: Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.
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