Editorial

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Modern Problems of Numerical Analysis. On the Centenary of the Birth of Alexander Andreevich Samarskii

<https://doi.org/10.1515/cmam-2020-0108>

Received July 10, 2020; revised September 5, 2020; accepted September 5, 2020

Alexander Andreevich Samarskii (1919–2008) is one of the greats from the pleiad of Russian scientists who brought prominence, international recognition, and fame to the Soviet science. He is the founder of the mathematical modeling in Russia, the leading world expert in the field of computational mathematics, mathematical physics, theory of difference schemes, and numerical simulation of complex nonlinear systems. In 1999, A. A. Samarskii was awarded the State Prize for his work on the theory of difference schemes, [7].

Since receiving his Ph.D. in 1947 has held a series of appointments both at various universities and at the Russian Academy of Science. He has been Professor of M. V. Lomonosov Moscow State University and Keldysh Institute of Applied Mathematics of the Soviet Academy of Sciences for more than 50 years. In 1966, A. A. Samarskii was elected a corresponding member, and in 1976 – a full member of the Academy of Sciences of the USSR. He is a Hero of Socialist Labor, Laureate of the Lenin and State Prizes and M. V. Lomonosov Prize, and awardee of numerous medals and orders as a veteran of the World War II.

His early works were in the defense projects on the explosive power of nuclear, and later, of the hydrogen devices. A. A. Samarskii is the co-author of the scientific discovery “The effect of the T-layer”, which is listed in the State Register of Discoveries of the USSR – the first recorded event detected first through mathematical modeling and computer simulations, and only then in the real experiment. His cutting edge research in laser fusion, magnetic and radiation gas dynamics, high-power lasers, aerodynamics, nuclear energy, and plasma physics was the base for developing understanding of the principles and the organization of the concept of mathematical modeling and computational experiment.

A. A. Samarskii created the general theory of operator-difference schemes, [8], including stability of difference schemes for elliptic, [9] and time-dependent problems, [10]. The main trust of Samarskii’s scientific heritage is the theory of operator-difference schemes, [11]. After a proper discretization in space by finite differences or finite elements any initial value problem for time dependent linear partial differential equations (or systems) is transformed into Cauchy problem for a system of ordinary differential equations. This problem is set as an differential-operator equation in a proper finite dimensional Hilbert space. Time discretization leads to a operator-difference equation, which is studied by the developed by A. A. Samarskii theory of stability of such schemes. Fundamental result of Samarskii is the formulation of general and abstract criteria, namely, sufficient and necessary conditions, for the stability of two- and three-level schemes. These criteria are based on quite simple and easy to check operator inequalities. Using this approach, he developed the general theory of operator-difference schemes – the stability with respect to the initial data and the right-hand side and also stability with respect to problem coefficients (strong stability).

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A. A. Samarskii had formulated a general principle for improving the properties of the operator-difference schemes, namely, the principle of regularization based of small perturbations of the operators. This theory has various applications to construction and study of additive operator-difference methods, [11], splitting schemes, [13], analysis of iteration methods for operator equations, [12], and also numerical treatments of some inverse problems involving PDEs, [14].

Samarskii's research has a wide range of impact at the national and international level. He brought tremendous energy to his research pursuits, directing and collaborating with numerous students and colleagues. He has been an adviser, mentor, and most important, an inspiration to many scientists around the world. He has been a tireless champion of research and education, a man with a big heart one can count on to help, to share one's joy in happy times, and to support in difficult times. Among the students of A. A. Samarskii there are three academicians and two corresponding members of the Russian Academy of Sciences. A number of scientific schools were created not just at universities in Russia, but also in Belarus, Bulgaria, Georgia, Hungary, Lithuania, Ukraine, and Uzbekistan, with the encouragement, mentorship, and direct participation of A. A. Samarskii.

In 1989, A. A. Samarskii founded and became the chief editor of the new journal "Mathematical modeling". In addition, Alexander Andreevich was a founding Editor of the Journal of Computational Methods in Applied Mathematics and a member of the editorial boards of six other journals.

A. A. Samarskii authored and co-authored a large number of monographs and textbooks that have become reference books for specialists in mathematics, physics, computational science, and engineering, e.g., [9–12, 14]. The joint with A. N. Tikhonov book "Equations of Mathematical Physics", [16], was reprinted many times, has been translated into thirteen languages and has long since become a classic textbook.

Everyone who had contact or interaction with Alexander Andreevich has fond memories of his friendly but focused approach in research opportunities, scientific discussions, and joint research. Moreover, his brilliance, generosity, and great personality inspired many of his colleagues and students. The unconditional support of young scientists made Alexander Andreevich preferred and highly sought advisor, mentor, and collaborator. His vigorous research program and leadership attracted many young people from Russia and East Europe, he directed more than 100 candidates (PhD), of whom more than 40 became doctor of sciences.

This Special Issue of CMAM is dedicated to Samarskii's 100th anniversary. The presented works, written by Samarskii's former students and/or collaborators and also scholars doing their research in numerical analysis are related to his multifaceted scientific research and represent to some extent a continuation and further development of his ideas and use of the techniques devised by him.

The non-stationary convection-diffusion equation is a basic mathematical model of continuum mechanics, on which key aspects of numerical analysis are worked out. In [17] V. Thomee has obtained fundamental results in approximating the solutions of such problems by the finite element method. He investigates the accuracy of an approximate solution for the convection-diffusion equation with the standard finite element approximation in space and applying lumping the mass. Inhomogeneous two-level in time schemes are used in which the diffusion part is taken on the upper time level, and the convective part - on the lower one. Taking into account the different scaling of convection and diffusion processes, the calculation of convective transfer is carried out on a finer grid in time.

Modern computational methods for solving multidimensional non-stationary problems are based on space-time finite element approximations. New results by using this technique for computing the approximate solution of parabolic initial-boundary value problems are obtained by U. Langer and A. Schafelner in [5]. The considered problem has variable coefficients and right-hand side that are sufficiently smooth in subdomains, but have jumps across their boundaries, that results in low regularity of the solution. The main result of this work is obtaining a priori and a posteriori error estimates for problems with non-smooth solutions. The adaptive mesh refinement is achieved through design of proper indicators for the posterior error of the approximate solution. The analysis of the results of numerical experiments for a set of model problems with various features is given.

Problems of numerical analysis of boundary value problems for quasilinear parabolic equations of second order (containing mixed derivatives) are discussed by P. Matus, D. Poliakov and L. M. Hieu in [6]. An

approximation of the Dirichlet problem in a rectangle with finite-differences on a uniform grid under fairly general assumptions about the nonlinear dependence on the solution of the principal coefficient is considered. The convergence analysis is based on the principle of maximum and two-sided a priori estimates of the solutions of the continuous and discrete problems. Estimates for the rate of convergence of an approximate solution to a generalized solution of the differential problem are established.

Numerical treatment of the deformed state of a thin shell, described by the nonlinear Budiansky–Sanders model, is presented by M. Karchevsky in [3]. For this class of boundary value problems, the construction, the analysis, and the justification of the corresponding computational algorithm has been carried out. The study begins with a discussion on the solvability problem for the corresponding variational problem. The approximation is carried out by the means of mixed finite elements, conditions for the solvability of the corresponding discrete problem for a sufficiently small mesh-size are formulated, and estimates of the accuracy of an approximate solution are obtained. To solve a system of nonlinear equations, a two-level iterative process is proposed and investigated.

Applied mathematical models of electrostatics of biomolecules are considered by J. Kraus, S. Nakov and S. Repin in [4]. The computational domain consists of three subdomains, in which the solution is determined from a nonlinear elliptic equation when formulating the interface conditions on the common boundaries of the subdomains. The main issues of such problems are related to the right-hand side, which is a linear combination of delta functions. A full investigation of the proposed computational technology based on finite element approximation has been performed. Solvability of the problem is established by a special additive two- and three-term representation of the solution. The key point of the study is obtaining fully computable posterior estimates. The theoretical results of the work are illustrated by numerical experiments with three different test problems.

Many processes in continuum mechanics are characterized by a significant predominance of convective transport over diffusion. This behavior of the solution is modeled by singularly perturbed convection-diffusion problems due to a small parameter at the highest derivatives with respect to the space. An accurate computational method should take into account the main features of such problems, namely, the presence of boundary layers. In particular, it is necessary to ensure the monotonicity of the approximate solution. Key results on the construction and study of difference schemes for singularly perturbed boundary value problems belong to G. I. Shishkin. The success in devising such methods is achieved by separating the singular part of the solution and constructing an approximate solution on a substantially inhomogeneous computational grid. G. Shishkin and L. Shishkina in [15] present monotone approximations of singularly perturbed second order parabolic equation that have second order of accuracy in space and the first order of accuracy in time. Uniform with respect to the small parameter convergence of the approximate solution is established and a comparison with the classical Samarskii scheme is given.

In the recent decade there is an increased interest in non-classical boundary value problems with fractional derivatives. A BVP for the second order ordinary differential equation supplemented by Riemann-Liouville fractional derivative is considered by I. Gavriluk and V. Makarov in [2]. The problem correctness in a corresponding weighted Sobolev spaces is established by transforming the boundary value problem to an equivalent integral equation. Estimates for the error of grid approximations of solutions subject boundary conditions are obtained. Analogous results were obtained for the Goursat problem with a partial differential operator with Riemann-Liouville fractional derivatives.

A. DeliĆ, B. S. Jovanović and S. Živanović [1] consider problems with fractional time derivatives. The authors study an initial-boundary value problem for a generalization of the classical telegraph equation and including two terms with Riemann–Liouville fractional derivatives. The existence of a weak solution of the differential problem in a corresponding Sobolev spaces is established. A finite-difference scheme is constructed and estimates of the convergence rate of the approximate solution are obtained. Theoretical results are illustrated by a number of test calculations.

After a space discretization of time-dependent mathematical models, one needs to solve the Cauchy problem for the resulted systems of nonlinear ordinary differential equations. To improve the accuracy of an approximate solution at the expense of calculations with various time steps, the Richardson extrapolation is used. In [19], Z. Zlatev and co-authors consider new variants of Richardson extrapolation. In addition to

improving the accuracy of the numerical solution of the Cauchy problem for the ODE system, a better stability of the used approximations is established. Theoretical analysis of computational algorithms is supplemented by a meaningful analysis of test calculations.

As a general rule, the implicit approximations in time of time-dependent problems involving partial differential equations ensures stability of the numerical solution. For computing the solution on the next time level various iterative method is most often used. In [18] P. Vabishchevich considered two-level schemes for the first-order evolution equation and devised a class of iterative methods that preserve the stability of the approximate solution at each iteration. The study of time approximations and iterative methods is carried out by the general theory of stability (correctness) of operator-difference schemes developed by Samarskii, [11].

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