Jead M. Macalisang, Mark L. Caay, Jayrold P. Arcede, and Randy L. Caga-anan*

Optimal Control for a COVID-19 Model Accounting for Symptomatic and Asymptomatic

<https://doi.org/10.1515/cmb-2020-0109>

Received September 1, 2020; accepted November 15, 2020

Abstract: Building on an SEIR-type model of COVID-19 where the infecteds are further divided into symptomatic and asymptomatic, a system incorporating the various possible interventions is formulated. Interventions, also referred to as controls, include transmission reduction (e.g., lockdown, social distancing, barrier gestures); testing/isolation on the exposed, symptomatic and asymptomatic compartments; and medical controls such as enhancing patients' medical care and increasing bed capacity. By considering the government's capacity, the best strategies for implementing the controls were obtained using optimal control theory. Results show that, if all the controls are to be used, the more able the government is, the more it should implement transmission reduction, testing, and enhancing patients' medical care without increasing hospital beds. However, if the government finds it very difficult to implement the controls for economic reasons, the best approach is to increase the hospital beds. Moreover, among the testing/isolation controls, testing/isolation in the exposed compartment is the least needed when there is significant transmission reduction control. Surprisingly, when there is no transmission reduction control, testing/isolation in the exposed should be optimal. Testing/isolation in the exposed could seemingly replace the transmission reduction control to yield a comparable result to that when the transmission reduction control is being implemented.

Keywords: COVID-19; optimal control

MSC: 92D30; 37N25; 34D20

1 Introduction

The COVID-19, since its outbreak in Wuhan, China, is now considered a major global health threat. The catastrophe is unprecedented. Not only that it affects at the economic level, but it also disrupts every facet of our society. As of 31st August 2020, worldwide cases hit 25 million, with almost 900,000 deaths [5]. The last time humanity faced such a global epidemic was in 1918. It was an H1N1 influenza virus of avian origin. With no vaccine available at that time, people learned to use non-pharmaceutical interventions (NPI) such as isolation, quarantine, good personal hygiene, use of disinfectants, and limitations of public gatherings, among others. Inspired by the lesson of using non-pharmaceutical interventions in 1918, in which the world also learned as a consequence from two US cities who responded differently in 1918 [6], countries are in a scramble to find the best fit according to their context.

According to [1], there are two fundamental strategies to combat this pandemic. First is by suppression, and second is by mitigation. Suppression means reducing the reproduction number below 1, thereby reducing

Jead M. Macalisang, Mark L. Caay: Department of Mathematics and Statistics, MSU-Iligan Institute of Technology, Iligan City, Philippines

Jayrold P. Arcede: Department of Mathematics, Caraga State University, Butuan City, Philippines

*Corresponding Author: Randy L. Caga-anan: Department of Mathematics and Statistics, MSU-Iligan Institute of Technology, Iligan City, Philippines

Mathematical Biology Research Cluster, Complex Systems Group, PRISM, MSU-Iligan Institute of Technology, Iligan City, Philippines, E-mail: randy.caga-anan@g.msuiit.edu.ph

the cases to lower levels and eventually dying out the disease. To do this, one needs to continually implement NPIs until the vaccine becomes available. However, this approach would be too costly for most governments. Otherwise, it will have a huge impact on the economy. Mitigation, on the other hand, used NPIs not to interrupt transmission completely but to reduce the health impact of an epidemic while waiting for the vaccine.

It was not a surprise that most governments implement the second strategy because of its practicality. However, the implementation of NPIs in most countries is a daunting task. Governments do not only consider the health implications but also the economic and social impact of the measures. We cite as an example the Philippines where NPIs had been adopted widely, i.e., isolation, social distancing, wearing of mask, closures on school and establishments, home quarantine, and good hygiene practice. However, after relaxing the intervention from the first implementation of Enhance Community Quarantine (ECQ) and travel restrictions for about three months, the virus continues to wreak havoc much stronger with infections more pronounced in high dense cities. Hence, the second implementation of a localized lockdown has been issued by the government [3].

In this paper, we take a similar view of that of [4] by considering some NPIs, namely, lockdown, testing, strengthening the healthcare system, while adding bed capacity as an additional control. We then compare and consider the implications of these NPIs by looking at the optimal level of implementation of what a government can do. However, we do not consider the economic and social dimensions as these complicate the model noting that decisions cannot solely rely on mathematical models. Instead, we believe that the most effective way to combat the virus is through interdisciplinary approaches and inclusive decision making. We also note that, to date, COVID-19 continues to evolve with reinfection to be a hot issue [8, 9]. Hence, as the days unfold, much remains to be understood about its nature of transmission. Therefore, we are left to calibrate best on NPIs while still waiting for a vaccine to come, with the assumption that initial vaccines will have high efficacy.

2 Mathematical model

2.1 Description of the model

We adopted the model created and calibrated for many countries at the start of the pandemic in [2]. The main compartments are Susceptible (S), Exposed (E), Symptomatic Infected (I_s), Asymptomatic Infected (I_a), Under Treatment (U), and Recovered (R). Here, we separate the U compartment into Hospitalized (H) and Quarantined (Q). We point out, however, that our model does not consider the effect of reinfection. Figure 1 shows the dynamics of the system and Table 1 describes the parameters.

From I_s , we distinguish the hospitalization rate c_s from the testing rate v_s . The hospitalization rate c_s is due to hospitalized symptomatic infected individuals not because of COVID-19 but because of other diseases. In reality, these infecteds with comorbidity are exacerbated by COVID-19 unknowingly. On the other hand, the testing rate v_s is due to hospitalized symptomatic infected individuals after testing positive for COVID-19. The testing/isolation parameters v_e , v_a , and v_s are part of the controls discussed in Section 3. The testing/isolation rate, v_a , can be materialized by having random testing. However, the rate v_e may be due to contact tracing and testing. It can also be due to quarantine efforts, especially those who have traveled from an infected area or those identified as being in close contact with a COVID-19 positive. The last-mentioned action may not involve testing, but it could effectively isolate those infected in the Exposed Compartment.

Figure 1: Flow chart of the model. Orange arrows represent the infection flow. Green arrows denote flow due to testing. Light blue arrows are for deaths.

The dynamics is governed by a system of seven ordinary differential equations (ODE) as follows

$$\begin{aligned}
 S'(t) &= -(\beta_e E + \beta_s I_s + \beta_a I_a) \frac{S}{N} \\
 E'(t) &= (\beta_e E + \beta_s I_s + \beta_a I_a) \frac{S}{N} - (\delta_e + v_e) E \\
 I_s'(t) &= f\delta_e E - \left(\gamma_s + \mu_s + (c_s + v_s) \left(1 - \frac{H}{\kappa_h} \right) \right) I_s \\
 I_a'(t) &= (1-f)\delta_e E - (\gamma_a + v_a) I_a \\
 H'(t) &= (c_s + v_s) \left(1 - \frac{H}{\kappa_h} \right) I_s - (\gamma_h + \mu_h) H \\
 Q'(t) &= v_e E + v_a I_a - \gamma_q Q \\
 R'(t) &= \gamma_s I_s + \gamma_a I_a + \gamma_h H + \gamma_q Q
 \end{aligned}$$

where κ_h is the carrying capacity of hospitals or the total hospital beds available for COVID-19 patients.

Note that the total living population at time t follows $N'(t) = -\mu_s I_s - \mu_h H$, while the total deaths at time t is computed by $D'(t) = \mu_s I_s + \mu_h H$. We are looking only at a relatively short period, so we do not include the natural birth and death rates.

2.2 Basic reproduction number

It is standard to check that the domain

$$\Omega = \left\{ (S, E, I_s, I_a, H, Q, R) \in \mathbb{R}_+^7; 0 \leq S + E + I_s + I_a + H + Q + R \leq N(0) \right\}$$

is positively invariant. Then, there exists a unique global in time solution $(S, E, I_s, I_a, H, Q, R)$ in $\mathcal{C}(\mathbb{R}_+; \Omega)$ as soon as the initial condition lives in Ω . Since the infected individuals are in E, I_s, I_a, H , and Q , the rate of appearance of new infections in each compartment ($\mathcal{F}$) and the rate of other transitions between all compartments ($\mathcal{V}$) are given by

$$\mathcal{F} = \begin{bmatrix} (\beta_e E + \beta_s I_s + \beta_a I_a) \frac{S}{N} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \text{ and}$$

$$\mathcal{V} = \begin{bmatrix} (\delta_e + v_e) E \\ \left(\gamma_s + \mu_s + (c_s + v_s) \left(1 - \frac{H}{K_h} \right) \right) I_s - f \delta_e E \\ (\gamma_a + v_a) I_a - (1-f) \delta_e E \\ (\gamma_h + \mu_h) H - (c_s + v_s) \left(1 - \frac{H}{K_h} \right) I_s \\ \gamma_q Q - v_e E - v_a I_a \end{bmatrix}$$

Evaluating the derivatives of $\mathcal{F}$ and $\mathcal{V}$ at the disease-free equilibrium

$$(S^*, 0, 0, 0, 0, 0, R^*), \text{ with } N^* = S^* + R^*,$$

lead to the following matrices F , V , and V^{-1} :

$$F = \begin{bmatrix} \frac{\beta_e S^*}{N^*} & \frac{\beta_s S^*}{N^*} & \frac{\beta_a S^*}{N^*} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} \delta_e + v_e & 0 & 0 & 0 & 0 \\ -f \delta_e & \xi & 0 & 0 & 0 \\ -(1-f) \delta_e & 0 & \gamma_a + v_a & 0 & 0 \\ 0 & -(c_s + v_s) & 0 & \gamma_h + \mu_h & 0 \\ -v_e & 0 & -v_a & 0 & \gamma_q \end{bmatrix}$$

$$V^{-1} = \begin{bmatrix} \frac{1}{\delta_e + v_e} & 0 & 0 & 0 & 0 \\ \frac{\delta_e f}{(\delta_e + v_e) \xi} & \frac{1}{\xi} & 0 & 0 & 0 \\ \frac{\delta_e (1-f)}{(\gamma_a + v_a)(\delta_e + v_e)} & 0 & \frac{1}{\gamma_a + v_a} & 0 & 0 \\ \frac{\delta_e f(c_s + v_s)}{(\gamma_h + \mu_h)(\delta_e + v_e) \xi} & \frac{c_s + v_s}{(\gamma_h + \mu_h) \xi} & 0 & \frac{1}{\gamma_h + \mu_h} & 0 \\ \frac{v_a(1-f) \delta_e + v_e(\gamma_a + v_a)}{(\gamma_q)(\delta_e + v_e)(\gamma_a + v_a)} & 0 & \frac{v_a}{(\gamma_a + v_a) \gamma_q} & 0 & \frac{1}{\gamma_q} \end{bmatrix}$$

where $\xi = \gamma_s + \mu_s + c_s + v_s$. The next generation matrix is then given by

$$FV^{-1} = \begin{bmatrix} \mathcal{A} & \frac{\beta_s S^*}{(\gamma_s + \mu_s) N^*} & \frac{\beta_a S^*}{(\gamma_a + v_a) N^*} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

where

$$\mathcal{A} = \frac{\beta_e S^*}{(\delta_e + v_e) N^*} + \frac{\beta_s f \delta_e S^*}{(\delta_e + v_e) \xi N^*} + \frac{\beta_a (1-f) \delta_e S^*}{(\gamma_a + v_a)(\delta_e + v_e) N^*}.$$

Hence, the basic reproduction number $\mathcal{R}_0$ for the disease-free equilibrium is given by

$$\mathcal{R}_0 = \left(\beta_e + \frac{\beta_s f \delta_e}{\xi} + \frac{\beta_a (1-f) \delta_e}{\gamma_a + v_a} \right) \left(\frac{S^*}{(\delta_e + v_e) N^*} \right).$$

The term $\frac{\beta_e}{\delta_e + v_e}$ represents the contact rate with the exposed during the average latency period $\frac{1}{\delta_e + v_e}$. The terms $\frac{\beta_s f}{\xi(1+\zeta)}$, $\frac{\beta_a(1-f)}{(\gamma_a + v_a)(1+\zeta)}$, where $\zeta = v_e/\delta_e$, represent the contact rates with the symptomatic and asymptomatic, respectively, during their respective infection periods that are minimized depending on how extensive the testing rate is in the exposed v_e , as can be seen in the factor $\frac{1}{1+\zeta}$. We note that δ_e is a fixed parameter in $\zeta = v_e/\delta_e$.

3 Optimal Control

3.1 Choice of controls

The controls used are transmission reduction (e.g. lockdown, social distancing, barrier gestures) denoted by $\omega(t)$; testing/isolation in the Exposed, Symptomatic, and Asymptomatic compartments given by $v_e(t)$, $v_s(t)$, and $v_a(t)$, respectively; enhancement of medical care to patients which can be mainly attained by pharmaceutical intervention given by $m(t)$; and increase of hospital beds given by $h(t)$. For any time t , the values of the controls are in the interval $[0, 1]$.

Increasing the transmission reduction control $\omega(t)$ means lowering the transmission of the virus from the susceptible to the exposed class. Lower values of $\omega(t)$ may mean that the transmission reduction control implemented is just the barrier gestures. In contrast, higher values of $\omega(t)$ may represent the general lockdown of a city, province, or country. Expanding the testing/isolation control means increasing the testing/isolation rates from the compartments E , I_s , I_a to H or Q . Improving medical care to patients implies increased recovery rate and low death rate from H .

We assume that we could increase the hospital beds to up to 3 times its original number κ_h only. We express this in our system by the term $(1 - \frac{2}{3}h(t))\frac{H}{\kappa_h}$.

Our system with the controls is given by

$$\begin{aligned} S'(t) &= -(1 - \omega(t))(\beta_e E + \beta_s I_s + \beta_a I_a) \frac{S}{N} \\ E'(t) &= (1 - \omega(t))(\beta_e E + \beta_s I_s + \beta_a I_a) \frac{S}{N} - (\delta_e + v_e(t))E \\ I_s'(t) &= f\delta_e E - \left(\gamma_s + \mu_s + (c_s + v_s(t)) \left(1 - \left(1 - \frac{2}{3}h(t) \right) \frac{H}{\kappa_h} \right) \right) I_s \\ I_a'(t) &= (1 - f)\delta_e E - (\gamma_a + v_a(t))I_a \\ H'(t) &= (c_s + v_s(t)) \left(1 - \left(1 - \frac{2}{3}h(t) \right) \frac{H}{\kappa_h} \right) I_s - \\ &\quad (\gamma_h(1 + m(t)) + \mu_h(1 - m(t)))H \\ Q'(t) &= v_e(t)E + v_a(t)I_a - \gamma_q Q \\ R'(t) &= \gamma_s I_s + \gamma_a I_a + \gamma_h H + \gamma_q Q \\ N'(t) &= -\mu_s I_s - \mu_h H \\ D'(t) &= \mu_s I_s + \mu_h H. \end{aligned}$$

3.2 Pontryagin's Maximum Principle

Our goal is to find control strategies, represented by control rates through time, such that the Exposed, Symptomatic, Asymptomatic, Hospitalized, Quarantined, and Death compartments along with the implementation

cost of controls are minimized. Therefore, the objective functional to be minimized is given by

$$J(\omega, v_e, v_s, v_a, m, h) = \int_0^T \left[E(t) + I_s(t) + I_a(t) + H(t) + Q(t) + D(t) \right. \\ \left. + \frac{A}{2} \omega^2(t) + \frac{B}{2} \sum_{i=e,a,s} v_i^2(t) + \frac{C}{2} m^2(t) + \frac{D}{2} h^2(t) \right] dt$$

and the corresponding Hamiltonian H_m by

$$H_m = E(t) + I_s(t) + I_a(t) + H(t) + Q(t) + D(t) + \frac{A}{2} \omega^2(t) + \frac{B}{2} \sum_{i=e,a,s} v_i^2(t) \\ + \frac{C}{2} m^2(t) + \frac{D}{2} h^2(t) + \sum_{i=1}^9 \lambda_i g_i,$$

where g_i is the right hand side of the differential equation of the i th state variable. Here we used a quadratic form for our controls to capture non-linear costs so that the differential equations arising from the optimization will have a known solution.

Applying Pontryagin's Maximum Principle, there exist adjoint variables $\lambda_1, \dots, \lambda_9$ which satisfy the following system of ordinary differential equations

$$\begin{aligned} \frac{\partial \lambda_1}{\partial t} &= (\lambda_1 - \lambda_2)(1 - \omega(t)) (\beta_e E + \beta_s I_s + \beta_a I_a) \left(\frac{N - S}{N^2} \right) \\ \frac{\partial \lambda_2}{\partial t} &= -1 + (\lambda_1 - \lambda_2)(1 - \omega(t)) \left(\beta_e \frac{S}{N} \right) + (\lambda_2 - \lambda_3 f - \lambda_4(1 - f)) \delta_e \\ &\quad + (\lambda_2 - \lambda_6) v_e(t) + (\lambda_2 - \lambda_1)(1 - \omega(t)) (\beta_e E + \beta_s I_s + \beta_a I_a) \left(\frac{S}{N^2} \right) \\ \frac{\partial \lambda_3}{\partial t} &= -1 + (\lambda_1 - \lambda_2)(1 - \omega(t)) \left(\beta_s \frac{S}{N} \right) + (\lambda_3 + \lambda_8 - \lambda_9) \mu_s \\ &\quad + (\lambda_3 - \lambda_5)(c_s + v_s(t)) \left(1 - \left(1 - \frac{2}{3} h(t) \right) \frac{H}{\kappa_h} \right) + (\lambda_3 - \lambda_7) \gamma_s \\ &\quad + (\lambda_2 - \lambda_1)(1 - \omega(t)) (\beta_e E + \beta_s I_s + \beta_a I_a) \left(\frac{S}{N^2} \right) \\ \frac{\partial \lambda_4}{\partial t} &= -1 + (\lambda_1 - \lambda_2)(1 - \omega(t)) \left(\beta_a \frac{S}{N} \right) + (\lambda_4 - \lambda_6) v_a(t) + (\lambda_4 - \lambda_7) \gamma_a \\ &\quad + (\lambda_2 - \lambda_1)(1 - \omega(t)) (\beta_e E + \beta_s I_s + \beta_a I_a) \left(\frac{S}{N^2} \right) \\ \frac{\partial \lambda_5}{\partial t} &= -1 + (\lambda_2 - \lambda_1)(1 - \omega(t)) (\beta_e E + \beta_s I_s + \beta_a I_a) \left(\frac{S}{N^2} \right) \\ &\quad + (\lambda_5 - \lambda_3)(c_s + v_s(t)) \left(1 - \frac{2}{3} h(t) \right) \frac{I_s}{\kappa_h} \\ &\quad + (\lambda_5(1 - m(t)) + \lambda_8 - \lambda_9) \mu_h + (\lambda_5(1 + m(t)) - \lambda_7) \gamma_h \\ \frac{\partial \lambda_6}{\partial t} &= -1 + (\lambda_2 - \lambda_1)(1 - \omega(t)) (\beta_e E + \beta_s I_s + \beta_a I_a) \left(\frac{S}{N^2} \right) \\ &\quad + (\lambda_6 - \lambda_7) \gamma_q \\ \frac{\partial \lambda_7}{\partial t} &= (\lambda_2 - \lambda_1)(1 - \omega(t)) (\beta_e E + \beta_s I_s + \beta_a I_a) \left(\frac{S}{N^2} \right) \\ \frac{\partial \lambda_8}{\partial t} &= (\lambda_2 - \lambda_1)(1 - \omega(t)) (\beta_e E + \beta_s I_s + \beta_a I_a) \left(\frac{S}{N^2} \right) \\ \frac{\partial \lambda_9}{\partial t} &= -1. \end{aligned}$$

Lastly, we proved the following theorem showing the form of our controls.

Theorem 3.1. *The optimal control variables are given by*

$$\begin{aligned}
 \omega(t) &= \max \left(0, \min \left(\frac{(\lambda_2 - \lambda_1)(\beta_e E + \beta_s I_s + \beta_a I_a)S}{AN}, 1 \right) \right) \\
 v_e(t) &= \max \left(0, \min \left(\frac{(\lambda_2 - \lambda_6)E}{B}, 1 \right) \right) \\
 v_a(t) &= \max \left(0, \min \left(\frac{(\lambda_4 - \lambda_6)I_a}{B}, 1 \right) \right) \\
 v_s(t) &= \max \left(0, \min \left(\frac{4((\lambda_3 - \lambda_5)HI_s)^2(\lambda_3 - \lambda_5)(\kappa_h I_s - HI_s)}{9\kappa_h^3 B^2 D - 4\kappa_h B((\lambda_3 - \lambda_5)HI_s)^2} \right. \right. \\
 &\quad \left. \left. + \frac{4((\lambda_3 - \lambda_5)HI_s)^2 \kappa_h c_s B}{9\kappa_h^3 B^2 D - 4\kappa_h B((\lambda_3 - \lambda_5)HI_s)^2} + \frac{(\lambda_3 - \lambda_5)(\kappa_h I_s - HI_s)}{\kappa_h B}, 1 \right) \right) \\
 m(t) &= \max \left(0, \min \left(\frac{\lambda_5(\gamma_h - \mu_h)H}{C}, 1 \right) \right) \\
 h(t) &= \max \left(0, \min \left(\frac{6(\lambda_3 - \lambda_5)^2 HI_s(\kappa_h I_s - HI_s)}{9\kappa_h^2 BD - 4((\lambda_3 - \lambda_5)HI_s)^2} \right. \right. \\
 &\quad \left. \left. + \frac{6(\lambda_3 - \lambda_5)\kappa_h c_s BHI_s}{9\kappa_h^2 BD - 4((\lambda_3 - \lambda_5)HI_s)^2}, 1 \right) \right).
 \end{aligned}$$

Proof. Optimal controls $\omega(t)$, $v_e(t)$, $v_a(t)$, $v_s(t)$, $m(t)$, and $h(t)$ are derived by the following optimality conditions:

$$\begin{aligned}
 \frac{\partial H_m}{\partial \omega(t)} &= \lambda_1 (\beta_e E + \beta_s I_s + \beta_a I_a) \frac{S}{N} - \lambda_2 (\beta_e E + \beta_s I_s + \beta_a I_a) \frac{S}{N} \\
 &\quad + A\omega(t) = 0 \\
 \frac{\partial H_m}{\partial v_e(t)} &= -\lambda_2 E + \lambda_6 E + Bv_e(t) = 0 \\
 \frac{\partial H_m}{\partial v_a(t)} &= -\lambda_4 I_a + \lambda_6 I_a + Bv_a(t) = 0 \\
 \frac{\partial H_m}{\partial v_s(t)} &= -\lambda_3 \left[1 - \left(1 - \frac{2}{3} h(t) \right) \frac{H}{\kappa_h} \right] I_s + \lambda_5 \left[1 - \left(1 - \frac{2}{3} h(t) \right) \frac{H}{\kappa_h} \right] I_s \\
 &\quad + Bv_s(t) = 0 \\
 \frac{\partial H_m}{\partial m(t)} &= \lambda_5 (\mu_h - \gamma_h) H + Cm(t) = 0 \\
 \frac{\partial H_m}{\partial h(t)} &= \lambda_5 (c_s + v_s(t)) \left(\frac{2H}{3\kappa_h} I_s \right) - \lambda_3 (c_s + v_s(t)) \left(\frac{2H}{3\kappa_h} I_s \right) + Dh(t) = 0.
 \end{aligned}$$

□

4 Simulations

The optimality system is numerically solved using the forward-backward sweep method discussed in [7]. Parameters are taken from [2], while the initial conditions are obtained from the Philippines Department of Health website <https://ncovtracker.doh.gov.ph>, on and for August 6, 2020, the start time of the simulations. The data taken is for NCR, Philippines, where most of the cases are, especially at the start of the country's pandemic. The simulations are for $T = 60$ days. Table 1 summarized these values.

Table 2 shows the summary of the total deaths, active symptomatics and asymptomatics by the end time of the simulations for the various control scenarios discussed in the following subsections.

A critical factor in the simulations is the weight of the controls given by A , B , C , and D . These weight constants are chosen to balance the effect of the compartments in the objective functional and the controls

Table 1: Parameters and initial conditions

Symbol	PARAMETERS Description	Value	INITIAL CONDITIONS	
			Compartment	Value
β_e	transmission from S to E from contact with E	0.223627	Susceptible	12,762,944
β_a	transmission from S to E from contact with I_a	0.1864638	Exposed	37,542
β_s	transmission from S to E from contact with I_s	0.3990195	Symptomatic	3,185
δ_e	latency rate	0.1433345	Asymptomatic	34,357
f	probability to become symptomatic	0.0848391	Hospitalized	4,411
$\gamma_a, \gamma_s, \gamma_q$	recovery rates	0.1227026	Quarantined	4,411
μ_s	death rate	0.0150899	Recovered	75,277
c_s	hospitalization rate from I_s	0.053	Total Death	1,490
κ_h	original number of hospital beds	6,086	Population	12,875,763

Table 2: Simulation results of active cases and total deaths by day 60 for the different scenarios.

SCENARIO	BY DAY 60		
	TOTAL DEATHS	ACTIVE SYMPTOMATIC	ACTIVE SYMPTOMATIC
No Control	99,389	81,743	1,11,477
With all the controls			
Low cost	2,194	1	15
Medium Cost	2,591	119	1,259
High Cost	10,582	12,087	111,485
Testing controls only			
Low Cost	3,377	127	860
High Cost	4,062	934	10,405
Testing and medical controls only			
Low Cost	2,796	118	817
High Cost	4,146	926	10,316

during the minimization process. The bigger these constants, the more costly it is to use the controls. We can use these weights to represent the capacity of an implementing unit. A not so able unit may find implementing the controls as costly, while those relatively wealthy may find it cheap. In our simulations, we use the value 10^2 when we assume that the controls are low cost, 10^4 for medium cost, and 10^6 for high cost.

4.1 Implementing all of the controls

In this section, we compute the best strategy (minimizing our objective functional) if all the controls are to be implemented simultaneously. We set the range of the controls as follows: $\omega(t) \in [0.1, 0.9]$ because wearing of face masks, social distancing, and localized lockdowns are normally being implemented in the Philippines but a total lockdown is not possible because essential works/services are not halted even at its hardest lockdown; $v_s(t) \in [0.05, 0.95]$ but $v_e(t), v_a(t) \in [0.05, 0.7]$ because there is no effective way of uncovering the individuals in the exposed and asymptomatic compartments for testing/isolation; $m(t) \in [0.05, 0.95]$; and $h(t) \in [0, 1]$.

As a baseline, we simulate first the case where we start with our initial conditions and let it run without any intervention. By day 60, the active cases and total deaths are shown in Table 2, while the changes in the compartments are shown in Figure 2 (A).

Figure 2: (A) The compartments through time when no control is used. (B), (C), and (D), show the optimal strategies if all controls are used with low, medium, and high cost, respectively.

Since the response of the government is also heavily influenced by its capacity to implement the controls, we solved for the optimal strategies considering three scenarios: the government is very able and willing to implement all of the controls (low cost); it is very difficult for them to implement all of the controls (high cost); and the moderate capacity of the government to implement the controls (medium cost). These optimal strategies corresponding to each of the scenarios are given in Figure 2 (B), (C), and (D). By day 60, the active cases and total deaths are also recorded in Table 2.

4.2 Testing and medical controls

In this section, we compute the best strategies in implementing the testing and medical controls with the transmission reduction control being held constant. We fixed $\omega(t)$ at 0.4, giving us 60% of the original transmission rate from the start of the pandemic, which we estimate as the current quarantine measure being implemented. In the first simulation, we compute the best options if we are to use testing only with no medical controls and, in the second, the combination of testing and medical controls. Just like in the previous section, we consider low cost and high cost situations. The results are shown in Figure 3.

4.3 Transmission reduction vs testing

This section will not consider the medical controls; instead, it will consider transmission reduction (which usually means lockdown in the Philippines) and testing/isolation only. Further, we assume that the government is very able to implement such control. In the first scenario, we compute the best strategy if we are

Figure 3: Optimal strategies for testing controls only (upper) and testing with medical controls (lower).

to combine testing/isolation and transmission reduction, with the transmission reduction control cannot be zero for any time ($\omega(t) \in [0.1, 0.9]$). In the second scenario, we compute the best strategy if there is no form of transmission reduction control whatsoever ($\omega(t) = 0$). The results are shown in Figure 4, and the compartment's values by day 60 are recorded in Table 3. Figure 4 left side shows the controls minimizing the objective functional and the right side shows its corresponding effect to the compartments.

Table 3: Values of the compartments in the objective functional by day 60 for the optimal strategies for testing with lockdown and testing without lockdown

COMPARTMENT	BY DAY 60	
	OPTIMAL STRATEGY	OPTIMAL STRATEGY
	TESTING & LOCKDOWN	TESTING & NO LOCKDOWN
Exposed	36	19
Total Deaths	2,634	2,432
Active Asymptomatic	2	1
Active Symptomatic	15	8
Hospitalized	6	3
Quarantined	142	96

Figure 4: For (A) and (B), the left figures show the optimal strategies given by the control rates through time, and the right figures show the corresponding effect of the optimal controls to the compartments through time.

5 Discussion

Section 4.1, show that the more able the government, the more it should implement transmission reduction control, strengthening its healthcare system, and testing of the symptomatic and asymptomatic compartments, as can be seen in Figure 2 (B). Moreover, increasing the hospital bed capacity is not much needed. The testing in the exposed compartment can be left to its minimum implementation (this could be explained by the fact that the lockdown is thereby decreasing the people entering the exposed compartment). However, in the case where the government finds it challenging to implement the controls (see Figure 2 (D)), then its best strategy is to implement the controls according to their allowable capacity. Nevertheless, increasing the hospital bed capacity at the maximum possible is needed. The reason for this is simply because active cases will rise significantly. As shown in Table 2, the increase in active cases and total deaths from low cost to medium, and medium cost to high cost is of order 10^2 .

In Section 4.2, Figure 3 (upper), fixing transmission reduction control to a constant, no medical controls, and in the low cost scenario, the best strategy is to test people in the symptomatic and asymptomatic compartments, leaving testing in the exposed at its minimum. However, in the high cost scenario, the best strategy is to prioritize testing in the asymptomatic compartment. This is understandable because the asymptomatic compartment has the highest probability of infecting more susceptible individuals. Even in combination with the medical controls, Figure 3 (lower), the above best strategy concerning testing is still valid. With respect to the medical controls, Figure 3 (lower, low cost scenario) tells us that if the government is able, it should prioritize strengthening the medical care system to its best. However, if it cannot afford to do so, its alternative strategy is to increase hospital bed capacity. However, the price to pay is the considerable difference in active cases and total deaths, as shown in Table 2.

Table 3 of Section 4.3, shows comparable results of two strategies - testing with transmission reduction control and testing without transmission reduction control. The one without transmission reduction control shows a slightly better result. To explain this, we look at the left side of Figure 4, showing the controls used. The main difference in (A) and (B) controls is the use of testing in the exposed compartment ($v_e(t)$). With transmission reduction control in (A), testing the exposed is not needed and kept to its minimum (this is maybe because transmission reduction control, such as lockdown, already minimizes the exposed compartment). However, in (B), it seems possible that the testing/isolation can replace the role of transmission reduction control in the exposed compartment, and it appears the effect is slightly better. This strategy has been effective for countries that utilized massive testing than lockdowns, as in South Korea, which apparently fared better than others who do not. Moreover, if we compare the total deaths by day 60, from Table 2 (Testing and medical controls only - Low Cost) and Table 3 (Testing and Lockdown), we can see that the one with lockdown is just slightly better, showing the greater impact of testing.

As a final note, we stress that the effect of implementing the strategies suggested by optimal control theory could be very substantial, as can be seen in the number of deaths in Table 2. This result is similar to one obtained by [4] where they also showed that the optimal strategy could strongly outperform even strategies like constant or cycling efforts. As for the limitation of this study, we point again that we have not yet considered the effect of reinfection in our model, as it is still a hot issue. Moreover, putting vaccination as an additional control would be an interesting study. Lastly, considering controls with spatial restrictions is a worthwhile direction.

Acknowledgements: The authors would like to thank the anonymous reviewers for their comments and suggestions that lead to the improvement of the paper. The first author was supported by the Department of Science and Technology of the Philippines.

Author's Statement: Conflict of interest: Authors state no conflict of interest.

References

- [1] R. M. Anderson, H. Heesterbeek, D. Klinkenberg, and T. D. Hollingsworth. How will country-based mitigation measures influence the course of the COVID-19 epidemic? *The Lancet*, 395(10228):931–934, Mar. 2020. ISSN 01406736. 10.1016/S0140-6736(20)30567-5. URL <https://linkinghub.elsevier.com/retrieve/pii/S0140673620305675>.
- [2] J. Arcede, R. Caga-anan, C. Q. Mentuda, and Y. Mammeri. Accounting for symptomatic and asymptomatic in a seir-type model of covid-19. *Math. Model. Nat. Phenom.*, 15(34), 2020. URL <https://doi.org/10.1051/mmnp/2020021>.
- [3] D. Bautista and M. L. Lopez. TIMELINE: How the Philippines is handling COVID-19, Apr. 2020. URL <https://www.cnnphilippines.com/news/2020/4/21/interactive-timeline-PH-handling-COVID-19.html>.
- [4] R. Djidjou-Demasse, Y. Michalakakis, M. Choisy, M. T. Sofonea, and S. Alizon. Optimal COVID-19 epidemic control until vaccine deployment. *medRxiv*, 2020. 10.1101/2020.04.02.20049189. URL <https://www.medrxiv.org/content/early/2020/05/15/2020.04.02.20049189>.
- [5] E. Dong, H. Du, and L. Gardner. An interactive web-based dashboard to track covid-19 in real time. *The Lancet Infectious Diseases*, 2020. URL [https://doi.org/10.1016/S1473-3099\(20\)30120-1](https://doi.org/10.1016/S1473-3099(20)30120-1).
- [6] S. Hearne. A Tale of Two Cities: The Catastrophic 1918 Flu Pandemic Can Guide City Policymakers Today, Mar. 2020. URL <https://www.cityhealth.org/blog/2018/12/5/a-tale-of-two-cities-the-catastrophic-1918-flu-pandemic-can-guide-city-policymakers-today>.
- [7] S. Lenhart and J. Workman. Optimal control applied to biological models. *CRC Press*, 2007.
- [8] J. Parry. Covid-19: Hong Kong scientists report first confirmed case of reinfection. *BMJ*, page m3340, Aug. 2020. ISSN 1756-1833. 10.1136/bmj.m3340. URL <http://www.bmj.com/lookup/doi/10.1136/bmj.m3340>.
- [9] K. K.-W. To, I. F.-N. Hung, J. D. Ip, A. W.-H. Chu, W.-M. Chan, A. R. Tam, C. H.-Y. Fong, S. Yuan, H.-W. Tsoi, A. C.-K. Ng, L. L.-Y. Lee, P. Wan, E. Tso, W.-K. To, D. Tsang, K.-H. Chan, J.-D. Huang, K.-H. Kok, V. C.-C. Cheng, and K.-Y. Yuen. COVID-19 re-infection by a phylogenetically distinct SARS-coronavirus-2 strain confirmed by whole genome sequencing. *Clinical Infectious Diseases*, 2020. ISSN 1058-4838. 10.1093/cid/ciaa1275. URL <https://doi.org/10.1093/cid/ciaa1275>.