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Detections and SIR simulations of the COVID-19 pandemic waves in Ukraine

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Abstract: Background. Unfortunately, the COVID-19 pandemic is still far from stabilizing. Of particular concern is the sharp increase in the number of diseases in June-July, September-October 2020 and February-March 2021. The causes and consequences of this sharp increase in the number of cases are still waiting for their researchers, but there is already an urgent need to assess the possible duration of the pandemic, the expected number of patients and deaths. Correct simulation of the infectious disease dynamics needs complicated mathematical models and many efforts for unknown parameters identification. Constant changes in the pandemic conditions (in particular, the peculiarities of quarantine and its violation, situations with testing and isolation of patients) cause various epidemic waves, lead to changes in the parameter values of the mathematical models.

Objective. In this article, pandemic waves in Ukraine will be detected, calculated and discussed. The estimations for durations and final sizes of the epidemic waves will be presented.

Methods. We propose a simple method for the epidemic waves detection based on the differentiation of the smoothed number of cases. We use the generalized SIR (susceptible-infected-removed) model for the dynamics of the epidemic waves. The known exact solution of the SIR differential equations and statistical approach were used. We will use different data sets for accumulated number of cases in order to compare the results of simulations and predictions.

Results. Nine pandemic waves were detected in Ukraine and corresponding optimal values of the SIR model parameters were identified. The number of cases and the number of patients spreading the infection versus time were calculated. In particular, the pandemic in Ukraine probably began in January 2020. If current trends continue, the end of the pandemic should be expected no earlier than in summer 2021.

Conclusions. The differentiation of the smoothed number of cases, the SIR model and statistical approach to the parameter identification are helpful to select COVID-19 pandemic waves and make some reliable estimations and predictions. The obtained information will be useful to regulate the quarantine activities, to predict the medical and economic consequences of the pandemic.

Keywords: COVID-19 pandemic, COVID-19 epidemic in Ukraine, mathematical modeling of infection diseases, SIR model, parameter identification, statistical methods

MSC: 34A34, 62J05, 92C60

1 Introduction

Here we consider the COVID-19 pandemic dynamics in Ukraine with the use of official WHO data sets about the confirmed number of cases, [1]; Ukrainian national statistics (UNS) [2, 3] and COVID-19 Data Repository by the Center for Systems Science and Engineering (CSSE) at Johns Hopkins University (JHU), [4]. The classical SIR model, connecting the number of susceptible S , infected and spreading the infection I and removed R persons, was developed in [5, 6, 7]. The unknown parameters of this model can be estimated with the use of the cumulative number of cases $V=I+R$ and the statistics-based method of parameter identification [8, 9].

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This approach was used in [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20] to estimate the first waves of pandemic dynamics in Ukraine, the city of Kyiv, China, the Republic of Korea, Italy, Austria, Spain, Germany, France, the Republic of Moldova, the UK, USA and in the world. Usually the number of cases registered during the initial period of an epidemic is not reliable, since many infected persons are not detected. That is why the correct estimations of epidemic parameters can be done with the use of data sets obtained for later periods of the epidemic when the number of detected cases is closer to the real one. On the other hand, changes in quarantine conditions, human behavior, pathogen activity, weather etc. can cause changes in the course of the pandemic, namely the so-called epidemic waves. The mathematical simulation of these waves needs development of some criteria of the waves detection, modification of models and methods of parameter identification.

In this paper simple criteria for identification of next pandemic waves will be discussed. The SIR model and the parameter identification procedure will be modified in order to simulate next waves of the pandemic. The results of calculations for nine waves of the Covid-19 pandemic in Ukraine will be presented and discussed.

2 Materials and Methods

2.1 Data

The information regarding the accumulated numbers of confirmed COVID-19 cases V_j in Ukraine from WHO daily situation reports, [1]; UNS [2, 3] (denoted by V_j and V_{j1}) and JHU [4] (denoted V_{j2}) is presented in Tables 1-3. The corresponding moments of time t_j (measured in days, zero point corresponds to January 20, 2020) are also shown in Tables 1 and 2. All these data sets were used for the identification of the epidemic waves. For SIR simulations were used only some time periods corresponding to specific waves. Other values were used only for verifications of SIR simulations.

2.2 Epidemic waves detection

Changes in quarantine conditions, human behavior, pathogen activity, weather etc. can cause changes in the epidemic dynamics, namely the so-called epidemic waves. The simplest way of their detection is to find some changes in the dependences of the number of registered cases on time. Since the number of cases is random, its time dependence needs some smoothing. We can use the method proposed in [19, 20]:

$$\bar{V}_i = \frac{1}{7} \sum_{j=i-3}^{j=i+3} V_j, \quad (1)$$

and the derivatives of the smoothed values

$$\left. \frac{d\bar{V}}{dt} \right|_{t=t_i} \approx \frac{1}{2} (\bar{V}_{i+1} - \bar{V}_{i-1}) \quad (2)$$

$$\left. \frac{d^2\bar{V}}{dt^2} \right|_{t=t_i} \approx \bar{V}_{i+1} - 2\bar{V}_i + \bar{V}_{i-1} \quad (3)$$

in order to detect the changes in epidemics dynamics. Some mathematical background of this method can be found in [21, 22, 23, 24].

2.3 Generalized SIR model

The generalized SIR model for an infectious disease [20] relates the number of susceptible persons S (persons who are sensitive to the pathogen and **not protected**); the number of infected is I (persons who are sick and

Table 1: Official cumulative numbers of confirmed cases in Ukraine in spring and summer 2020, [1, 2, 3]

Day in April, May, and June 2020	Time in days t_j	Number of cases, V_j	Day in June and July 2020	Time in days t_j	Number of cases, V_j	Day in July and August 2020	Time in days t_j	Number of cases, V_j
29	100	10406	10	142	29070	22	184	61851
30	101	10861	11	143	29753	23	185	62823
1	102	11411	12	144	30506	24	186	63929
2	103	11913	13	145	31154	25	187	64849
3	104	12331	14	146	31810	26	188	65656
4	105	12697	15	147	32476	27	189	66575
5	106	13184	16	148	33234	28	190	67597
6	107	13691	17	149	34063	29	191	68794
7	108	14195	18	150	34984	30	192	69884
8	109	14710	19	151	35825	31	193	71056
9	110	15232	20	152	36560	1	194	72168
10	111	15648	21	153	37241	2	195	73158
11	112	16023	22	154	38074	3	196	74219
12	113	16425	23	155	39014	4	197	75490
13	114	16847	24	156	40009	5	198	76808
14	115	17330	25	157	41117	6	199	78261
15	116	17858	26	158	42065	7	200	79750
16	117	18291	27	159	42982	8	201	80949
17	118	18616	28	160	43628	9	202	81957
18	119	18876	29	161	44334	10	203	83115
19	120	19230	30	162	44998	11	204	84548
20	121	19706	1	163	45888	12	205	86140
21	122	20148	2	164	46763	13	206	87872
22	123	20580	3	165	47677	14	207	89719
23	124	20986	4	166	48500	15	208	91356
24	125	21245	5	167	49043	16	209	92820
25	126	21584	6	168	49607	17	210	94436
26	127	21905	7	169	50414	18	211	96403
27	128	22382	8	170	51224	19	212	98537
28	129	22811	9	171	52043	20	213	100643
29	130	23294	10	172	52843	21	214	102971
30	131	23672	11	173	53521	22	215	104958
31	132	24002	12	174	54133	23	216	106757
1	133	24340	13	175	54771	24	217	108415
2	134	24823	14	176	55607	25	218	110085
3	135	25411	15	177	56455	26	219	112059
4	136	25964	16	178	57264	27	220	114497
5	137	26514	17	179	58111	28	221	116978
6	138	26999	18	180	58842	29	222	119074
7	139	27462	19	181	59493	30	223	121215
8	140	27856	20	182	60166	31	224	123303
9	141	28381	21	183	60995			

Table 2: Official cumulative numbers of confirmed cases in Ukraine in autumn 2020, [2, 3]

Day in September and October 2020	Time in days t_j	Number of cases in Ukraine V_j	Day in October and November 2020	Time in days t_j	Number of cases in Ukraine V_j	Day in November and December 2020	Time in days t_j	Number of cases in Ukraine V_j
1	225	125798	8	262	250538	14	299	535857
2	226	128228	9	263	256266	15	300	545689
3	227	130951	10	264	261034	16	301	557657
4	228	133787	11	265	265454	17	302	570153
5	229	135894	12	266	270587	18	303	583510
6	230	138068	13	267	276177	19	304	598085
7	231	140479	14	268	281239	20	305	612665
8	232	143030	15	269	287231	21	306	624744
9	233	145612	16	270	293641	22	307	635689
10	234	148756	17	271	298872	23	308	647976
11	235	151859	18	272	303638	24	309	661858
12	236	154335	19	273	309107	25	310	677189
13	237	156797	20	274	315826	26	311	693407
14	238	159702	21	275	322879	27	312	709701
15	239	162660	22	276	330396	28	313	722679
16	240	166244	23	277	337410	29	314	732625
17	241	169472	24	278	343498	30	315	745123
18	242	172712	25	279	348924	1	316	758264
19	243	175678	26	280	355601	2	317	772760
20	244	178353	27	281	363075	3	318	787891
21	245	181237	28	282	370417	4	319	801716
22	246	184734	29	283	378729	5	320	813306
23	247	188106	30	284	387481	6	321	821947
24	248	191671	31	285	395440	7	322	832758
25	249	195504	1	286	402194	8	323	845343
26	250	198634	2	287	411093	9	324	858714
27	251	201305	3	288	420617	10	325	872228
28	252	204932	4	289	430467	11	326	885039
29	253	208959	5	290	440188	12	327	894215
30	254	213028	6	291	450934	13	328	900666
1	255	217661	7	292	460331	14	329	909082
2	256	222322	8	293	469018	15	330	919704
3	257	226462	9	294	479197	16	331	931751
4	258	230236	10	295	489808	17	332	944381
5	259	234584	11	296	500865	18	333	956123
6	260	239337	12	297	512652	19	334	964448
7	261	244734	13	298	525176	20	335	970993

Table 3: Cumulative numbers of confirmed Covid-19 cases in Ukraine in winter 2020-2021: V_{j1} -National statistics (UNS), [2, 3] and V_{j2} - according to COVID-19 Data Repository by the Center for Systems Science and Engineering (CSSE) at Johns Hopkins University (JHU), [4].

Day in December 2020	Number of cases, UNS, [2, 3] V_{j1}	Number of cases, JHU, [4] V_{j2}	Day in January 2021	Number of cases, UNS, [2, 3] V_{j1}	Number of cases, JHU, [4] V_{j2}
1	758264	765117	1	1069517	1096855
2	772760	778560	2	1074093	1102256
3	787891	793372	3	1078251	1107137
4	801716	808828	4	1083585	1111631
5	813306	822985	5	1090496	1117256
6	821947	834913	6	1099493	1124482
7	832758	843898	7	1105169	1133802
8	845343	855054	8	1110015	1139800
9	858714	867991	9	1115026	1144943
10	872228	881727	10	1119314	1150265
11	885039	895620	11	1124430	1154850
12	894215	908839	12	1130839	1160243
13	900666	918444	13	1138764	1166958
14	909082	925321	14	1146963	1175343
15	919704	934161	15	1154692	1183963
16	931751	945218	16	1160682	1192114
17	944381	957692	17	1163716	1198512
18	956123	970758	18	1167655	1201894
19	964448	982937	19	1172038	1206125
20	970993	991700	20	1177621	1210854
21	979506	998678	21	1182969	1216780
22	989642	1007627	22	1187897	1222459
23	1001132	1018199	23	1191812	1227723
24	1012167	1030125	24	1194328	1231965
25	1019876	1041583	25	1197107	1234772
26	1025989	1049717	26	1200883	1237810
27	1030374	1056265	27	1206412	1241863
28	1037362	1061074	28	1211593	1247674
29	1045348	1068476	29	1216278	1253127
30	1055047	1076880	30	1219455	1258093
31	1064479	1086997	31	1221485	1261546

spread the infection; please don't confuse with the number of still ill persons, so known active cases) and the number of removed R (persons who **no longer spread the infection**; this number is the sum of isolated, recovered, dead, and infected people who left the region):

$$\frac{dS}{dt} = -\alpha_i SI \quad (4)$$

$$\frac{dI}{dt} = \alpha_i SI - \rho_i I, \quad (5)$$

$$\frac{dR}{dt} = \rho_i I. \quad (6)$$

Parameters α_i and ρ_i are supposed to be constant for every wave of the epidemics, i.e. for the time periods $t_i^* \leq t \leq t_{i+1}^*$, $i = 1, 2, 3, \dots$

Parameters α_i show how quick the susceptible persons become infected (see (4)). Large values of this parameter correspond to severe epidemics with many victims. These parameters accumulate many characteristics. First they show how strong (virulent) is the pathogen and what is the way of its spreading. Parameters α_i accumulate also the frequency of contacts and the way of contacting. In order to decrease the values of α_i , we have to minimize the number of our contacts and change our contacting habits. For example, we have to avoid the public places and use masks there, minimize or cancel traveling. We have to change our contact habits: to avoid handshakes and kisses. First, all these simple things are very useful to protect yourself. In addition, if most people follow these recommendations, we have chance to diminish the values of parameter α_i and reduce the negative effects of the pandemic.

The parameters ρ_i characterize the patient removal rates, since eq. (6) demonstrates the increase rate for R . The inverse values $1/\rho_i$ are the estimations for time of spreading the infection τ . So, we are interested in increasing the values of parameters ρ_i and decreasing $1/\rho_i$. People and public authorities should work on this and organize immediate isolation of suspicious cases.

Since the derivative $d(S + I + R)/dt$ is equal to zero (it follows from summarizing Eqs. (4)–(6)), the sum

$$N_i = S + I + R \quad (7)$$

must be constant for every wave and is not the volume of population (see also [20]).

2.4 Analytical solution of SIR equations

To determine the initial conditions for the set of equations (4)–(6), let us suppose that at the beginning of every epidemic wave t_i^* :

$$I(t_i^*) = I_i, \quad R(t_i^*) = R_i, \quad S(t_i^*) = N_i - I_i - R_i \quad (8)$$

In particular, when the first wave of the epidemic starts with one infected person, the initial conditions (8) can be written as follows:

$$I(t_1^*) = 1, \quad R(t_1^*) = 0, \quad S(t_1^*) = N_1 - 1. \quad (9)$$

Equations (9) were used in [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20] to simulate the first waves of the COVID-19 pandemic in different countries.

It follows from (4) and (5) that

$$\frac{dI}{dS} = \frac{v_i}{S} - 1, \quad v_i = \frac{\rho_i}{\alpha_i} \quad (10)$$

Integration of (10) with the initial conditions (8) yields:

$$I = v_i \ln S - S + N_i - R_i - v_i \ln(N_i - I_i - R_i) \quad (11)$$

It follows from (11) that function I has a maximum at $S = v_i$ and tends to zero at infinity. The corresponding number of susceptible persons at infinity $S_{i\infty} > 0$ can be calculated from a non-linear equation

$$S_{i\infty} = (N_i - I_i - R_i)e^{\frac{S_{i\infty} - N_i - R_i}{v_i}} \quad (12)$$

Formula (12) follows from (11) at $I=0$.

As in [8] we solve (4)–(6) by introducing the function $V(t) = I(t) + R(t)$, corresponding to the number of victims or cumulative confirmed number of cases. It follows from (5)–(7) and (11) that:

$$\frac{dV}{dt} = \alpha_i SI = \alpha_i (N_i - V) [v_i \ln(N_i - V) + V - R_i - v_i \ln(N_i - R_i - I_i)] \quad (13)$$

Integration of (13) yield an analytical solution for the set of equations (4)–(6):

$$F_i^*(V, N_i, I_i, R_i, v_i) = \alpha_i(t - t_i^*), \quad (14)$$

$$F_i^* = \int_{R_i+I_i}^V \frac{dU}{(N-U)[v_i \ln(N_i - U) + U - R_i - v_i \ln(N_i - R_i - I_i)]}. \quad (15)$$

Thus, for every set of parameters $N_i, I_i, R_i, v_i, \alpha_i, t_i^*$ and a fixed value of V , integral (15) can be calculated and a corresponding moment of time can be determined from (14). Then functions $I(t)$ and $R(t)$ can be easily calculated with the use of formulas (11) and:

$$S = N_i - V, R = V - I \quad (16)$$

The final number of victims (final accumulated number of cases in i -th epidemic wave) can be calculated from:

$$V_{i\infty} = N_i - S_{i\infty}. \quad (17)$$

To estimate the final day of an epidemic wave, we can use the condition:

$$I(t_{if})=1. \quad (18)$$

which means that at $t > t_{if}$ less than one person still spreads the infection.

2.5 Parameter identification procedures

In the case of a new epidemic, the values of its independent six parameters are unknown and must be identified with the use of limited data sets. For the first wave of an epidemic starting with one infected person, the number of unknown parameters is only four, since $I_1 = 1$ and $R_1 = 0$. The corresponding statistical approach was used in [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20] to estimate the values of four unknown parameters.

For the next epidemic waves ($i > 1$), the moments of time t_i^* corresponding to their beginning are known. Therefore the exact solution (14)–(16) depend only on five parameters $-N_i, I_i, R_i, v_i, \alpha_i$. Then the registered number of victims V_j corresponding to the moments of time t_j can be used in eq. (15) in order to calculate $F_{i,j} = F_i^*(V_j, N_i, v_i, I_i, R_i)$ for every fixed values of N_i, v_i, I_i, R_i and then to check how the registered points fit the straight line (14).

Eq. (14) can be rewritten as follows:

$$y \equiv F_i^*(V, N_i, I_i, R_i, v_i) = \gamma t + \beta, \quad \gamma = \alpha_i, \quad \beta = -\alpha_i t_i^* \quad (19)$$

We can estimate the values of parameters γ and β , by treating the values $y_j \equiv F_i^*(V_i, N_i, I_i, R_i, v_i)$ and corresponding time moments t_j as random variables. Then we can use the observations of the accumulated number of cases and the linear regression in order to calculate the coefficients $\hat{\gamma}$ and $\hat{\beta}$ of the regression line

$$\hat{y} = \hat{\gamma}t + \hat{\beta} \quad (20)$$

using the standard formulas from, e.g., [25]. Values $\hat{\gamma}$ and $\hat{\beta}$ can be treated as statistics-based estimations of parameters γ and β from relationships (19).

The reliability of the method can be checked by calculating the correlation coefficients r_i (see e.g., [25]) for every epidemic wave checking how close is its value to unity. We can use also the F-test for the null hypothesis that says that the proposed linear relationship (19) fits the data set. The experimental values of the Fisher function can be calculated for every epidemic wave with the use of the formula:

$$F_i = \frac{r_i^2(n_i - m)}{(1 - r_i^2)(m - 1)}, \quad (21)$$

where n_i is the number of observations for the i -th epidemic wave, $m = 2$ is the number of parameters in the regression equation. The corresponding experimental value F_i has to be compared with the critical value $F_C(k_1, k_2)$ of the Fisher function at a desired significance or confidence level ($k_1 = m - 1$, $k_2 = n_i - m$). When the values n_i and m are fixed, the maximum of the Fisher function coincides with the maximum of the correlation coefficient. Therefore, to find the optimal values of parameters N_i, v_i, I_i, R_i , we have to find the maximum of the correlation coefficient for the linear dependence (19). To compare the reliability of different predictions (with different values of n_i) it is useful to use the ratio $F_i/F_C(1, n_i - 2)$ at fixed significance level. We will use the level 0.001; corresponding values of $F_C(1, n_i - 2)$ can be taken from [26]. The most reliable prediction yields the highest $F_i/F_C(1, n_i - 2)$ ratio.

The exact solution (14)-(16) allows avoiding numerical solutions of differential equations (4)-(6) and significantly reduce the time spent on calculations. In the case of sequential calculation of epidemic waves $i = 1, 2, 3 \dots$, it is possible to avoid determining the four optimal unknown parameters N_i, v_i, I_i, R_i , thereby reducing the amount of calculations and difficulties in isolation a maximum of the correlation coefficient. For parameters I_i, R_i it is possible to use the numbers of I and R calculated for the previous wave of epidemic at the moment of time when the following wave began. Then we need to calculate values $F_i^*(V_i, N_i, v_i)$, linear regression coefficients in (20), correlation coefficient $r_i, F_i/F_C(1, n_i - 2)$ and to isolate the values of parameters N_i and v_i corresponding to the maximum of r_i . Knowing the optimal values of five parameters $N_i, I_i, R_i, v_i, \alpha_i$, the SIR curves and other characteristics of the corresponding epidemic wave can be calculated with the use of formulas (10)-(18). This approach has been successfully used in [20]. In particular, six waves of the Covid-19 epidemic in Ukraine and four pandemic waves in the world were calculated.

Segmentation of epidemic waves and their sequential SIR simulations need a lot of efforts. To avoid this, a new method of obtaining the optimal values of SIR parameters was proposed in [20]. First of all we can use the relationship

$$V_i = I_i + R_i \quad (22)$$

To estimate the value V_i , we can use the smoothed accumulated number of cases (e.g., formula (1)). Then

$$V_i \approx \frac{1}{7} \sum_{j=i-3}^{j=i+3} V_j \quad (23)$$

where i corresponds to the moment of time t_i^* . To obtain one more relationship, let us use (5)-(7). Then

$$I_i = \frac{1}{\alpha_i(N_i - V_i)} \frac{dV}{dt} \Big|_{t=t_i^*} \quad (24)$$

To estimate the average number of new cases dV/dt at the moment of time t_i^* , we can use (2). Thus we have only two independent parameters N_i and v_i . To calculate the value of parameter α_i , some iterations can be used (see details in [20]). In this study we will use both methods of identification of SIR model parameters.

3 Results and Discussion

3.1 Detection of the COVID-19 pandemic waves in Ukraine

Applications of formulae (1)-(3) for the pandemic dynamics in Ukraine are shown in Figs. 1 and 2. The accumulated numbers of cases ("circles") were smoothed with the use of eq. (1) and shown by lines. "Triangles" and "stars" represent the results of differentiation (2) and (3) respectively. To make the results more visible, the first derivative (2) is multiplied by 100, the second one (formula (3)) – by 1000.

Fig. 1 demonstrates that the second derivatives increase after epidemic outbreak, then become smaller and negative. Such behavior is typical for the first wave of the epidemics (before May 17, 2020). The jumps in the values of the second derivative indicate changes in the conditions of the pandemic (for example, the

Figure 1: Epidemic dynamics in Ukraine in spring and summer 2020. Accumulated number of cases (blue markers and line, eq. (1)). Red markers represent first derivative (eq. (2)), black one show the second derivative (eq. (3)).

weakening of quarantine) and the transitions to the next waves with other values of the parameters of mathematical models. These jumps occurred on May 16, May 29, June 8, July 3 and July 19-20 (see Fig. 1). Therefore these days can be treated as the beginning of the second, third etc. waves.

The second epidemic wave in Ukraine was caused by the cancelation of the national lockdown on May 10. After the incubation period (approximately after May 16), the number of new cases began to grow faster. Further waves of the epidemic in Ukraine are associated with further easing of quarantine and mass non-compliance with social distancing. The fifth epidemic wave in Ukraine can be explained by the consequences of the holiday season, which increased the number of trips and violations of social distancing.

The COVID-19 pandemic characteristics for Ukraine in autumn 2020 are shown in Fig. 2. Differentiation of the smoothed number of accumulated cases (eq. (1), line) with the use of formulas (2) (“triangles”) and (3) (stars) allow us to detect the changes in epidemic dynamics. It can be seen that after November 25 the average daily number of new cases (“triangles”) started to decrease. Similar short periods of the epidemic stabilization occurred in May, June and August, 2020 (see Fig. 1).

The values of the second derivative (3) allow detecting the changes in the epidemic characteristics and separating its different waves. The jump in d^2V/dt^2 values corresponding to September 11 (see “stars” in Fig. 2) can be explained by the beginning of classes in schools and universities (on September 1). Children and young people are often asymptomatic carriers of the infection and bring it to their families. For example, employees of two kindergartens and two schools in the Ukrainian city of Chmelnytskii were tested for antibodies to COVID-19, [27]. In total 292 people work in the surveyed institutions. Some of the staff had already fallen ill with COVID-19 or were hospitalized. Therefore, they were not tested accordingly. Of the 241 educators tested, antibodies were detected in 148, or 61.4%. These results indicate the important role of children in the spread of COVID-19 infection and the fact that in Ukraine those people who have become ill and have antibodies to coronavirus infection, obviously, are much more than the official statistics (presented in Tables 1-3) states.

The severe jumps in d^2V/dt^2 values occurred also in October and November, 2020 (see “stars” in Fig. 2). Probably, this is due to the local elections and a presidential poll, which were held throughout Ukraine on October 25, 2020 and involved hundreds of thousands of people to campaign and work in election commissions (their number was about 30 thousand). This obviously increased the number of contacts and the likelihood of additional infections. The corresponding seventh epidemic wave in Ukraine was considered in [20].

In November 2020, the Ukrainian government introduced a weekend lockdown. In the period from 00:00 on Saturday to 00:00 on Monday in Ukraine from November 14 to November 30 it was prohibited: the work of catering establishments (except for takeaway services), the work of shopping and entertainment centers,

Figure 2: Pandemic dynamics in Ukraine in autumn, 2020. Accumulated number of cases (V_j - “circles”, Table 2 and 3; smoothed values - line, eq. (1)). “Triangles” show the first derivative (eq. (2)) multiplied by 100, “stars” - the second derivative (eq. (3)) multiplied by 1000.

entertainment establishments, the activities of economic entities engaged in trade and consumer services, except for food trade in retail space, at least 60 percent of which intended for trade in food, fuel, medicines and medical devices, veterinary drugs, feed. It was prohibited to operate cultural institutions and hold cultural events. Gyms, fitness centers and swimming pools should also be closed, [28]. Probably, positive results of this lockdown are visible in Fig. 2. Between November 21 and December 10 (after some incubation period) we can see decreasing of the second derivative (“stars”).

The SIR simulations of corresponding epidemic waves in Ukraine will be presented in the next Section. In some cases the distance between jumps of the second derivative was too small to make statistical estimates of the parameters (e.g., in the second half of May 2020), so the individual waves of the epidemic during these periods were not isolated.

The COVID-19 pandemic characteristics for Ukraine in December 2020 and January 2021 are shown in Figs. 3 and 4. Red color corresponds to the national statistics (UNS) [2, 3]; black - to JHU data [4]. “Circles” show the corresponding accumulated numbers of cases; lines represent the smoothed number of accumulated cases (eq. (1)); “crosses” – the first derivative (eq. (2)) multiplied by 10; “dots” – second derivative (eq. (3)) multiplied by 1000. Fig. 3 illustrates the data sets presented in Table 3. It can be seen one day shift for the values of the second derivatives calculated with the use of different data sets. If the numbers of cases reported by JHU are attributed to the previous day, the differences in the values of the second derivatives become almost imperceptible (see Fig. 4), but the numbers of accumulated cases according to JHU are still much higher than for UNS (see Fig. 4).

The jump of the second derivatives occurred on January 10-12, 2021 (see “dots” in Fig. 4) can be explained the New Year and Christmas celebrations. The increase in the number of contacts caused the increase in the number of new cases after some incubation period (see “crosses” in Figs. 3 and 4). The national lockdown in the period January 8-24, 2021 allowed stopping this tendency.

3.2 Results of SIR simulations

The first epidemic wave in Ukraine were already simulated with the use of classical SIR model [14, 15, 16, 17, 18, 19, 20]. Usually the number of cases during the initial period of a new epidemic outbreak is not reliable, since many cases are not detected. That is why the first waves need re-estimations after obtaining fresh data.

Figure 3: Pandemic dynamics in Ukraine in December 2020 and January 2021 calculated with the use of data sets from Table 3: UNS [2, 3] (red) and data set reported by JHU [4] (black). Accumulated number of cases - “circles”; smoothed values - lines, eq. (1). “Crosses” show the first derivatives (eq. (2)) multiplied by 10, “dots” - the second derivative (eq. (3)) multiplied by 1000.

Figure 4: Pandemic dynamics in Ukraine in December 2020 and January 2021 with the numbers of cases reported by JHU attributed to the previous day: UNS [2, 3] (red) and data set reported by JHU [4] (black). Accumulated number of cases - “circles”; smoothed values - lines, eq. (1). “Crosses” show the first derivatives (eq. (2)) multiplied by 10, “dots” - the second derivative (eq. (3)) multiplied by 1000.

To illustrate the influence of data on the results of SIR simulations, different estimations of the first epidemic wave in Ukraine are presented in Table 4. It can be seen, that the use of more recent (and complete) data has changed the estimation for the pandemic beginning. Table 4 illustrates that prediction 21 calculated with the use of number of cases from the period May 3-16 (immediately before the start of the second wave) yields much longer hidden period of the epidemic outbreak in Ukraine in comparison with the previous prediction 8, [16]. Prediction 19, [19] yields even longer hidden period, but it was obtained with the use of the dataset for the period May 13-26, which corresponds the transition from first to the second wave. This prediction yielded the smallest values of F_i and $F_i/F_C(1, n_i - 2)$ in comparison with predictions 8 and 21 (see Table 4). The maximum corresponding values of these parameters demonstrate that the prediction 21 estimating the epidemic outbreak in Ukraine in the beginning of January, 2020 is probably the most reliable. Similar

Table 4: Characteristics of the first epidemic wave in Ukraine calculated with the use of different data sets. Optimal values of parameters, final sizes and days (last three rows).

Characteristics	Ukraine, first wave, prediction 8, [16] $i=1$	Ukraine, first wave, prediction 19, [19] $i=1$	Ukraine, first wave, prediction 21, [20] $i=1$
Period taken for calculations	April 11-24	May 13-26	May 3-16
I_i	1	1	1
R_i	0	0	0
N_i	21380.9241600000	92770.3987200000	59002.368
ν_i	12190.2413389748	76647.6354922344	43307.0168670536
α_i	1.45792603472e-05	3.766268949867e-06	4.62451395337e-06
t_i	27.72672361552338	-32.4672286400970	-16.5560950369874
ρ_i	0.177724702176649	0.288675609635095	0.200273903780639
$1/\rho_i$	5.62667984671	3.46409591466375	4.99316177056849
r_i	0.998863416866565	0.998856483326519	0.999453409358706
F_i , eq. (21)	5269.98203160936	5237.97374486281	10968.1371753815
$F_i / F_C(1, n_i - 2)$	283.332367290826	281.611491659291	589.684794375351
S_{ix} , eq. (12)	6108	62507	30676
V_{ix} , eq. (17)	15273	30263	28327
t_{if} , eq. (18)	186.5	273.6	264.9
Final day of epidemic	July 26, 2020	October 20, 2020	October 11, 2020

simulations of the global dynamics show that the COVID-19 pandemic probably started in the beginning of August 2019, [20].

The characteristics of the pandemic waves 2-4 for Ukraine are presented in Table 5. The results of SIR simulations of the next pandemic waves in Ukraine are shown in Tables 6 and 7 and in Figs. 5-7. It can be seen that optimal values of the model parameters are rather different for different pandemic waves. In particular, the final sizes and durations of the pandemic significantly differ. It is not surprising, since different time periods T_{ci} with different conditions were used for calculations. Value $n_i = 14$ and general SIR model were used for all the cases. The periods T_{ci} taken for calculations were selected according to the time periods of the corresponding epidemic waves (see Tables 5-7).

Fig. 5 illustrates first six epidemic waves in Ukraine. The accumulated number of cases $V=I+R$ (solid lines) increases for each next wave. Every new wave also increases the number of infected and spreading the infection persons I . The calculated dependences $10^4 I(t)$ are shown by dashed lines. The accuracy of simulations is rather good, especially for second and further waves of pandemic, since blue “stars” (showing the accumulated values of confirmed cases used only for control the calculations) are located very close to the corresponding solid lines showing the calculated $V=I+R$ values. There are some discrepancies for the early stages of first waves, when the number of registered cases is lower than the real one due to the problems with the identification of the infected persons.

Fig. 6 illustrates rather good accuracy of SIR simulations. In particular, the V_j values (“triangles”) started to deviate from the solid red line only after September 16. This fact can be explained by the beginning classes in schools and universities (as mentioned above). This curve was calculated with the use of data set from the period August 9-22. The increased number of contacts in schools and universities caused rapid increase of the number of cases. This increase in the number of diseases intensified in October and November 2020 through elections and a presidential poll. The very irregular nature of the epidemic dynamics (particularly the large values of the second derivative (see “stars” in Fig. 1)) led to the fact that $V=I+R$ curve for the seventh wave

Table 5: Characteristics of second, third and fourth epidemic waves in Ukraine. Optimal values of parameters, final sizes and days (last three rows).

Characteristics	Ukraine, second wave, $i=2$	Ukraine, third wave, $i=3$	Ukraine, fourth wave, $i=4$
Period taken for calculations, T_{ci}	May 17-30	June 9-22	July 3-16
I_i	2196	2490	3736
R_i	16420	25891	43941
N_i	239587.2	149444.16	182294.495539200
v_i	209382.9208704	104004.527376384	119723.766724183
α_i	7.728220925e-07	2.117564829222e-06	1.35741149806184e-06
ρ_i	0.161815747046830	0.220236329252125	0.162514417542680
$1/\rho_i$	6.17986826529682	4.54057694929708	6.15330021250195
r_i	0.998894780369877	0.999714332158213	0.999456710370030
F_i , eq. (21)	5419.78688520125	20994.4146122108	11034.8339181942
$F_i / F_C(1, n_i - 2)$	291.386391677487	1128.73196839843	593.270640763128
S_{iso} , eq. (12)	178641	78509	89552
V_{iso} , eq. (17)	60946	70935	92742
t_{if} , eq. (18)	531.9	338.5	421.7
Final day of epidemic	July 5, 2021	December 24, 2020	March 17, 2021

Table 6: Characteristics of fifth, sixth and seventh epidemic waves in Ukraine. Optimal values of parameters, final sizes and days (last three rows).

Characteristics	Ukraine, fifth wave, $i=5$, [20]	Ukraine, sixth wave, $i=6$, [20]	Ukraine, seventh wave, $i=7$, [20]
Period taken for calculations, T_{ci}	July 20-Aug 2	August 9-22	October 1-14
I_i	4570	7525	17605.4705407746
R_i	55596	73424	200051.672316368
N_i	208624.896	336210.688000000	566602.33216
v_i	117860.44860629	166678.304458998	271873.891638209
α_i	1.241234878e-06	6.47063569e-07	6.811436620341e-07
t_i^*	182	201	255
ρ_i	0.1462924996005	0.107851458558717	0.185185178161924
$1/\rho_i$	6.83562043666339	9.27201183334553	5.40000020479830
r_i	0.9997539353297	0.999591012582638	0.999637260478932
F_i , eq. (21)	24374.834072843	14661.3785892930	16531.7953887678
$F_i / F_C(1, n_i - 2)$	1310.47495015287	788.246160714677	888.806203697193
S_{iso} , eq. (12)	79668	91157	168285
V_{iso} , eq. (17)	128957	245054	398317
t_{if} , eq. (18)	381	443.5	432
Final day of epidemic	February 5, 2021	April 8, 2021	March 27, 2021

Figure 5: Six epidemic waves in Ukraine. SIR curves (lines) and accumulated number of cases (markers) versus time. Red, black, blue, magenta, green, and brown colors correspond to waves 1, 2, 3, 4, 5, and 6 respectively. Numbers of infected and spreading $I*10$ (dashed lines) and victims $V=I+R$ (solid lines). “Circles” represent the values V_j taken for calculations, “stars” show the values V_j taken for the control accuracy of simulations only.

Figure 6: Sixth (red), seventh (black) and eighth (blue) COVID-19 pandemic waves in Ukraine in autumn 2020. Solid lines - the numbers of victims $V=I+R$, dashed lines - numbers of persons spreading the infection I multiplied by 10, dotted lines - the theoretical estimations of the daily numbers of new cases dV/dt multiplied by 10. The accumulated number of cases V_j taken for calculations are shown by circles; “triangles” and “stars” correspond the cases taken only for comparisons and verifications of the predictions.

very quickly began to deviate from the recorded number of cases V_j (compare solid black line and “stars” in Fig. 6).

The absence of sharp jumps of the second derivative after November 21, 2020 (see “stars” in Fig. 2), allowed us to predict quite accurately the further epidemic dynamics (compare solid blue line and “stars” in Fig. 6). Blue dashed line in Fig. 6 show that the number of persons spreading the infection diminished in December 2020. It can be seen that after December 16, the number of reported cases slightly exceeds the theoretical estimates (compare the blue solid line and “stars” in Fig. 6). As calculations of the eighth epidemic

wave were carried out during the period of influence of the weekend lockdown, its cancellation probably led to increase in the number of cases.

The results of SIR simulations with the use of two data sets (UNS and JHU, presented in Table 3) are shown in Table 7 and Fig. 7. The optimal parameters of SIR model and other epidemic characteristics can be compared with the results obtained for the eighth wave (see Table 7). The number of observations taken for calculations n_i was 14 in all the cases.

It can be seen that two data sets yield rather different values of the optimal parameters for the ninth wave (especially for N_9 , $S_{9\infty}$, and ν_9), nevertheless the final sizes of this wave $V_{9\infty}$ and ρ_9 are rather close; the duration based on the national statistics is one month longer in comparison with the calculations based on JHU data set. Both simulations for the ninth epidemic wave in Ukraine yield slightly higher final sizes in comparison with the eighth wave (see Table 7).

Table 7: Calculated optimal values of SIR parameters for the eighth and ninth waves of the COVID-19 epidemic in Ukraine

Characteristics	Eighth wave, $i=8$, UNS	Ninth wave, $i=9$, JHU, V_{j2}	Ninth wave, $i=9$, UNS, V_{j1}
Period taken for calculations T_{ci}	Nov 21-Dec 4, 2020	December 11-24, 2020	December 11-24, 2020
I_i	74,07.6732607497	62,384.6901000672	59,686.0031221196
R_i	548,996.183882108	830,900.452757076	821,069.282592166
N_i	1,454,508.4416	1,524,200.0384	2,037,235.2
ν_i	667,978.033362204	580,121.592967068	1,141,207.02807484
α_i	2.138650373242e-07	2.91326081249e-07	1.55930616187218e-07
ρ_i	0.142857147036735	0.169004550327192	0.177949115084894
$1/\rho_i$	6.9999997952	5.91700044799982	5.6195840003078
r_i	0.999333327224955	0.998082602967150	0.998062592012312
F_i , eq. (21)	8990.91853799112	3120.24523044173	3087.92417596763
$F_i / F_C(1, n_i - 2)$	483.382717096297	167.755119916222	166.017428815464
$S_{j\infty}$, eq. (12)	376,90	347,782	810,539
$V_{j\infty}$, eq. (17)	1,078,218	1,176,418	1,226,696
Final day of the epidemic wave, eq. (18)	June 23, 2021	June 15, 2021	July 14, 2021

Fig. 7 illustrates the results of SIR simulations of the ninth epidemic wave in Ukraine obtained with the use of two data sets: Ukrainian national statistics [2, 3] (red) and data set reported by JHU [4] (black). Numbers of victims $V(t)=I(t)+R(t)$ are represented by solid lines. Numbers of infected and spreading the infection persons $I(t)$ are shown by dashed lines. Dotted lines represent the derivatives dV/dt calculated with the use of eq. (13). This derivative yields the estimation of the daily number of new cases. Markers show accumulated numbers of cases V_{j1} and V_{j2} from Table 3. “Circles” correspond to the accumulated numbers of cases taken for calculations (during period of time T_{ci}); “triangles” – numbers of cases before T_{ci} ; “stars” – number of cases after T_{ci} . It can be seen that the accuracy of simulations based on the national statistics is rather good (the deviations between red “stars” and red solid line are small). The use of JHU data sets yields worth accuracy. Nevertheless, the real numbers of cases already exceed the predicted saturations levels for both data sets and corresponding simulations. As of February 14, 2021 the national statistics yield the figure 1,271,143 of accumulated cases in Ukraine [2, 3]. Thus the epidemic observations during 52 days (after T_c) demonstrated only 3.5% exceeding of the saturation level in the case of UNS.

Figure 7: The ninth COVID-19 epidemic wave in Ukraine. SIR simulations (lines) with the use of optimal values of parameters from Table 7: national statistics [2, 3] (red) and data set reported by JHU [4] (black). Numbers of victims $V(t)=I(t)+R(t)$ – solid lines; numbers of infected and spreading $I(t)$ multiplied by 10 – dashed; derivatives dV/dt (eq. (13)) multiplied by 10 – dotted. Markers show accumulated numbers of cases V_{j1} and V_{j2} from Table 3. “Circles” correspond to the accumulated numbers of cases taken for calculations (during period of time T_{c9}); “triangles” – numbers of cases before T_{c9} ; “stars” – number of cases after T_{c9} .

4 Conclusions

Constant changes in the COVID-19 pandemic conditions (i.e., in the peculiarities of quarantine and its violation, in situations with testing and isolation of patients, in coronavirus activity due to its mutations etc.) lead to new pandemic waves and cause the changes in the values of parameters of the mathematical models. The dynamics of COVID-19 pandemic in different countries was simulated with the use of different mathematical models [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94]. The results of this paper show that the accuracy of long-time predictions is limited due to these changes. Separate pandemic waves must be detected and separated in order have good accuracy of SIR models or any other simulation.

The smoothed dependence of the accumulated number of cases and its differentiation can provide fairly accurate and useful information about the course of the epidemic, identify important changes in its dynamics and provide timely recommendations for quarantine measures or control of social distancing. To simulate different pandemic waves (periods with more or less constant values of its dynamics parameters), the generalized SIR model and presented procedures of its parameters identification can be applied to calculate different pandemic waves.

Incomplete data and different methods of cases recording can give quite different values of model parameters and forecasts, which was demonstrated by the example of two data sets for Ukraine. The forecast of the duration of the epidemic in Ukraine is not optimistic. New cases will appear at least until July 2021. Let us hope that vaccinations can change this sad predictions. The obtained results can be also useful for assessing the effectiveness of mass vaccination in Ukraine and other countries.

Very long duration of the pandemic requires correction of our behavior, we can not live as before it occurred. Decreased feelings of insecurity and non-compliance with social distancing may further increase the pandemic duration and the number of the coronavirus victims. Total closure of settlements or regions can be recommended only in the event of a sharp increase in the number of cases. There are many things that can be done without loss to the economy and our daily lives:

1. Minimize the number of contacts and trips, not visit crowded places. Work and study remotely where possible.

2. *Refrain from shaking hands and kisses during meetings. Use masks in transport and crowded areas.*
3. *If you (or others) have any suspicious symptoms, do your best to avoid the spread of the infection.*

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