Abstract: The outbreak of coronavirus disease 2019 (COVID-19) has been declared a pandemic by the World Health Organization on March 11, 2020. Here, a nonlinear mathematical model is proposed and analyzed to study the spread of coronavirus disease in a human habitat. In modeling the dynamics, the total population is divided into five subclasses: susceptible population, asymptomatic infective population, symptomatic infective population, recovered population, and vaccinated population. It is assumed that the disease is transmitted directly from infectives. It is further assumed that due to the effect of media, susceptible individuals become aware about the disease and avoid contact with the infectives. The analysis of the model is performed using the stability theory of differential equations. Furthermore, conditions that influence the persistence of the system are obtained. We have also conducted numerical simulations to validate the analytical results. The model analysis shows that with an increase in media awareness, the spread of coronavirus disease decreases with a decrease in the number of infective populations.

Keywords: mathematical model, coronavirus, COVID-19, stability theory

MSC 2020: 34D20, 34D23

1 Introduction

The new coronavirus disease 2019 (COVID-19) is a widespread infectious disease at the present [8,13,14,32–34]. Severe acute respiratory syndrome coronavirus [6], middle east respiratory syndrome coronavirus [4,15,16,35], and severe acute respiratory syndrome coronavirus 2 [10] are responsible for the coronavirus diseases that have struck the world, including this year's epidemic. Coronavirus disease causes respiratory problems, such as fever, and a dry cough, muscle or body aches, headache, and fatigue, among other symptoms [10]. The majority of countries around the world are affected by this disease, and its intensity is increasing everyday. As a huge number of individuals move from one country to another, the disorder is spreading among different countries, notably by air travel [5,9,18,19]. To prevent the spread of disease, world health organization issued a
warning to all countries about the importance of screening people at both entry and exit points [17,24]. COVID-19 has been confirmed in a huge number of persons, and a large number of patients are quarantined and in the asymptomatic stage. Because a significant number of persons are infected daily, exposed individuals and those with no symptoms are particularly dangerous. Because the emerging stage has a 214-day incubation period, it is communicable at any moment [36]. The asymptomatic stage, on the other hand, is more harmful than any other stage since it lasts an average of 3 days [31], and persons with no symptoms do not develop symptoms of the disease due to what they come into contact with. COVID-19 spreads quickly among people because they are unconcerned about it. However, due to the large number of virus cases and limited medical competence in many countries, diagnostic testing for asymptomatic, exposed, and isolated COVID-19 infection is limited. This fact also boosts the number of coronavirus-affected patients [13].

COVID-19 poses such a significant risk that, as of May 11, 2020, about 42 lakh persons had been infected with the virus, with 2,87,131 of them dying [36]. Because the disease spreads through interaction and there is now no effective treatment, the only option to slow the development of the disease is to reduce community distance and improve communication between people. The Chinese government has developed a strategy of closure to preserve social isolation and is able to control the development of the disease [13,17]. With the exception of a few countries, every government uses this approach. Large number of individuals commute from one location to another because of work in densely populated nations such as India, Bangladesh, and others, and many of these people originated from extensively affected countries. As a result, the disease is more likely to spread in these countries. The Indian government implemented a 21-day shutdown policy in the first phase, which began on March 25, 2020, and was prolonged until May 17, 2020, to restrict and stop human migration. COVID-19 is a worldwide extremely contagious disease. However, there is no specific vaccination, medicine, or antibiotic treatment for this condition. As a result, the existing global issue guards against these diseases. Other preventive measures, such as public safety, wearing a mask, and washing hands with soap and water on a regular basis, can be used to protect people from infection in this situation. Maintaining community distance is critical in preventing infection throughout the COVID-19 studies. Keeping a safe location among people who are not their own homes implies staying away from the community [24]. This community’s average elevation should be at least 6 feet [24]. Researchers from all over the world have been working to develop some effective COVID-19-fighting techniques [1–3,12,20–23,25–30]. Vaccination is an effective tool for controlling and preventing diseases. Vaccination and aggressive adoption of non-pharmacological therapies are key to lowering the global incidence of COVID-19 infection. Various methodologies have been developed to examine the complex dynamics of a continuous epidemic. We included it in our model to investigate the impact of this significant element (social distance) on illness dynamics. The primary purpose of this research is to investigate the disease dynamics of COVID-19 using a deterministic compartmental model for an Indian context and to identify preventive approaches for COVID-19 outbreak control in India.

The organization of this article is as follows: in Section 2, we have formulated the mathematical model. In Section 3, we study the steady-state analysis of the disease-free-equilibrium (DFE) point and existence of endemic equilibrium point. We study the basic reproduction number in Section 4. Sensitivity analysis for basic reproduction number $R_0$ is carried out in Section 5. The stability analysis of equilibrium points is done in Section 6, and persistence of the model in Section 7. In Section 8, the numerical simulation of the model is carried out. Finally, the concluding remarks are given in Section 9.

2 Mathematical model

To understand the dynamics of the nonlinear model, we consider a model as illustrated in Figure 1. In this model, the entire human population is divided into five different sub-compartments, namely, the susceptible $X(t)$, asymptomatic infective $Y(t)$, symptomatic infective $Y\widetilde{(t)}$, recovered compartment $R(t)$, and vaccinated compartment $V(t)$. The total population is given by: $N(t) = X(t) + Y(t) + Y\widetilde{(t)} + R(t) + V(t)$. Also, let $M(t)$ be the cumulative density of awareness programs driven by the media in that region at time $t$. For simplicity, we use $X(t) = X$, $Y(t) = Y$, $Y\widetilde{(t)} = Y$, $R(t) = R$, $V(t) = V$, and $M(t) = M$. Keeping the aforementioned assumptions in view, a mathematical model is proposed here:
We assumed that there has been a constant immigration within susceptible population from external source. It is also assumed that the media \( M(t) \) plays an important role in spreading the information or messages related to coronavirus. In coronavirus disease, as the susceptible population come in contact with asymptomatic infected persons, they become infected. In modeling process, \( \beta \) is the transmission rate among susceptible and asymptomatic infected individuals, and it is assumed that \( \beta \) reduces by an exponential factor \( e^{-\alpha_1 M} \), where \( \alpha_1 \) measures the effectiveness of the information influences over the disease-related messages. The description of the parameters is given in Table 1.

Since \( N = X + Y + Y_1 + R + V \), the aforementioned model System (1) can be rewritten as follows:

\[
\begin{align*}
\frac{dN}{dt} &= A - dN - aY_i, \\
\frac{dY}{dt} &= \beta Y(N - Y - Y_i - R - V) e^{-\alpha_1 M} - (\delta_1 Y + \delta_2 Y_i - (\xi + d)R), \\
\frac{dY_i}{dt} &= \beta Y_i - (d + \delta_2 + a)Y_i, \\
\frac{dR}{dt} &= \delta_1 Y + \delta_2 Y_i - (\xi + d)R, \\
\frac{dM}{dt} &= \mu N - \mu_0 M, \\
\frac{dV}{dt} &= \theta(N - Y - Y_i - R - V) - (\theta_0 + d)V.
\end{align*}
\]
In the following lemma, we state the bounds of system variables, which is further used to prove the analytic results. Boundedness of the system may be interpreted as a natural restriction to indefinite growth of infective population because of various constraints either due to natural conditions or due to preventive habits acquired by the population to keep themselves safe from the disease. We will show that the solutions of system are bounded to ensure the biological validity of the model.

2.1 Boundedness of solutions

Continuity of right-hand side of System (2) and its derivative implies that the model is well posed for $N > 0$ [11]. The invariant region where solution exists is obtained as follows:

$$\frac{A}{d + a} \leq \lim \inf N(t) \leq \lim \sup N(t) \leq \frac{A}{d} \quad \text{(as } t \to \infty),$$

since $N(t) > 0$ for all $t \geq 0$. Therefore, from the first equation of model System (2), $N(t)$ cannot blow up to infinity in finite time, and consequently, the model system is dissipative (solutions are bounded). Hence, the solution exist globally for all $t > 0$ in the invariant and compact set, $\Omega = \{(N(t), Y(t), Y_i(t), R(t), M(t), V(t)) \in R^6, 0 \leq Y, Y_i, R, V \leq N \leq \frac{A}{d}, 0 \leq M \leq \frac{\mu A}{\mu d} = M_M\}$, which gives a region of attraction all solution initiating in the interior of the positive orthant.

3 Equilibrium analysis

In this section, the existence of equilibrium points of the model System (2) is being investigated by equating right-hand side of System (2) to zero. We obtain the following two non-negative equilibria:

1. The DFE point $E_0$ is $(\frac{A}{d}, 0, 0, 0, \frac{\mu A}{\mu d}, \frac{\theta d A}{d(\theta + d + a)})$. The existence of $E_0$ is obvious. The equilibrium point $E_0$ implies that in the absence of infective individuals and COVID-19 virus density deposited on surfaces, areas, etc., the disease will not persist.

2. The endemic equilibrium point $E^*(N^*, Y^*, Y_i^*, R^*, M^*, V^*)$ implies that in the presence of infective individuals and surfaces contaminated with COVID-19 virus, disease will always persist and the existence for it is given in the following.

### Table 1: Description of parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Recruitment rate of population in susceptible classes</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Transmission rate of the disease directly from the asymptomatic infectives to the susceptibles</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>Measures how active the virus-associated communications can affect the spread ratio</td>
</tr>
<tr>
<td>$d$</td>
<td>Natural death rate</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Dissemination rate of awareness programs among susceptibles</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>Vaccine decrement rate</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>Rate at which asymptomatic infectives become symptomatic</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>Recovery rates of asymptomatic infective classes</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>Recovery rates of symptomatic infective classes</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Disease-induced death rate</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Rate with which awareness programs are being executed</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>Rate that message becomes outdated</td>
</tr>
</tbody>
</table>
3.1 Existence of $E^*$ ($N^*$, $Y^*$, $Y_1^*$, $R^*$, $M^*$, $V^*$)

\[
A - dN - \alpha Y = 0, \quad (3)
\]
\[
\beta Y(N - Y - Y_1 - R - V)e^{-\mu M} - (d + \beta_1 + \delta_2)Y = 0, \quad (4)
\]
\[
\beta_1 Y - (d + \delta_2 + \alpha)Y = 0, \quad (5)
\]
\[
\delta_1 Y + \delta_2 Y_1 - (\xi + d)R = 0, \quad (6)
\]
\[
\mu N - \mu_0 M = 0, \quad (7)
\]
\[
\theta(N - Y - Y_1 - R - V) - (\theta_0 + d)V = 0. \quad (8)
\]

From equations (3), (5), (6), (7), and (8), respectively, we have

\[
Y_1 = \frac{A - dN}{a} = g_1(N), \quad (9)
\]
\[
Y = \frac{(d + \delta_2 + \alpha)(A - dN)}{\beta_1 a} = g_2(N), \quad (10)
\]
\[
R = \frac{(A - dN)[\delta_1(d + \delta_2 + \alpha) + \beta_1 \delta_2]}{a \beta_1 (\xi + d)} = g_3(N), \quad (11)
\]
\[
M = \frac{\mu N}{\mu_0}, \quad (12)
\]

and

\[
V = \frac{\theta(N - g_3(N) - g_4(N) - g_5(N))}{(\theta + \theta_0 + d)} = g_4(N). \quad (13)
\]

In the equilibrium $E^*$, $N = N^* > 0$ is the positive root of the following equation, which can be obtained from equation (4), after using $Y_1, Y, R, M,$ and $V$ from equations (9), (10), (11), (12), and (13), respectively, we obtain

\[
F(N) = \beta e^{-\frac{\mu N}{\mu_0}}[N - g_3(N) - g_4(N) - g_1(N) - (d + \beta_1 + \delta_1)]. \quad (14)
\]

It is clear from equation (14) that

\[
F\left(\frac{A}{d + a}\right) = -\frac{A(\theta_0 + d)\beta e^{-\frac{\mu N}{\mu_0}}}{\beta_1 (d + a)(\theta + \theta_0 + d)} \left[\frac{(d + \delta_2 + \alpha)(\xi + d + \delta_1) + \beta_1 \delta_2}{\xi + d}\right] - (d + \beta_1 + \delta_1) < 0 \quad (15)
\]
and

\[
F\left(\frac{A}{d}\right) = \frac{A(\theta_0 + d)\beta e^{-\frac{\mu N}{\mu_0}}}{d(\theta + \theta_0 + d)} - (d + \beta_1 + \delta_1) = (d + \beta_1 + \delta_1)[R_0 - 1] > 0. \quad (16)
\]

provided $R_0 > 1$, where $R_0 = \frac{\beta(\theta_0 + d)\beta e^{-\frac{\mu N}{\mu_0}}}{d(\theta + \theta_0 + d)(d + \beta_1 + \delta_1)}$.

It would be sufficient if we show that $F(N) = 0$ has one and only one root. From equation (14), we note that

\[
F\left(\frac{A}{d + a}\right) < 0 \quad \text{and} \quad F\left(\frac{A}{d}\right) > 0. \quad \text{This implies that there exists a root} \ N^* \ \text{of} \ F(N) = 0 \ \text{in} \ \frac{A}{d + a} < N < \frac{A}{d}.
\]

Also, $F'(N) > 0$, provided $N < [g_1(N) + g_2(N) + g_3(N) + g_4(N)]$ in $\frac{A}{d + a} < N < \frac{A}{d}$. Thus, there exists a unique root of $F(N) = 0$, (say $N^*$) in $\frac{A}{d + a} < N < \frac{A}{d}$. So the equilibrium point $E^*$ exists provided $R_0 > 1$. 

Influence of media campaigns efforts to control spread of COVID-19
4 Basic reproduction number

In order to obtain an expression for the basic reproduction number \( R_0 \) associated with System (2), we use the approach of the next-generation matrix method [7]. To this, we find the transmission (new infection) and transition terms in the infected subsystem of System (2) as follows:

\[
F = \begin{bmatrix}
\beta Y (N - Y - Y_1 - R - V) e^{-\omega_t M} \\
0
\end{bmatrix},
\]

\[
V = \begin{bmatrix}
(\delta_1 + \beta_1) Y & 0 \\
-\beta_1 Y & (\delta_2 + \alpha) Y
\end{bmatrix},
\]

Thus, the transmission (new infection) and transition matrices corresponding to the equilibrium \( E_0 \) are, respectively, obtained as:

\[
F = \begin{bmatrix}
\frac{\beta A}{d} - \frac{\theta A}{d(\theta + \theta_0 + d)} e^{-\omega_t M} & 0 \\
0 & 0
\end{bmatrix},
\]

\[
V = \begin{bmatrix}
(\delta_1 + \beta_1) & 0 \\
-\beta_1 & (\delta_2 + \alpha)
\end{bmatrix}.
\]

It has been well established that the basic reproduction number of an epidemic model is the spectral radius of the next-generation matrix \( FV^{-1} \), i.e., \( R_0 = \rho(FV^{-1}) \). Thus, for model System (2), we obtain

\[
R_0 = \rho(FV^{-1}) = \frac{\beta A(\theta + d)e^{(-\omega_t M)}}{d(\theta + \theta_0 + d)(\delta_1 + \beta_1 + \delta_1)}.
\]

5 Sensitivity analysis of basic reproduction number \( R_0 \)

Sensitivity indices measure how the basic reproduction number \( R_0 \) changes in response to the small shifts in the value of a parameter. The initial disease transmission is directly related to the \( R_0 \):

\[
R_0 = \frac{\beta A(\theta + d) e^{(-\omega_t M)}}{d(\theta + \theta_0 + d)(\delta_1 + \beta_1 + \delta_1)}.
\]

If \( R_0 < 1 \), then the disease cannot prevade the population and the infection will die out over a period of time. If \( R_0 > 1 \), then the infection can spread through the population.

The normalized forward sensitivity index of \( R_0 \), which depends differentially on a parameter \( p \), is defined by:

\[
\gamma_p^{R_0} = \frac{\partial R_0}{\partial p} \times \frac{p}{R_0}.
\]

Proceeding in a similar manner, we determine and measure the sensitivity indices of \( R_0 \) using the parameter values as given in Section 8. Table 2 shows the sensitivity indices of \( R_0 \) for the parameters. The parameters are ordered from most sensitive to least.

In the sensitivity indices of \( R_0 \), since \( \gamma_p^{R_0} = +1 \) that means, increasing (or decreasing) the transmission rate of the disease directly from the asymptomatic infectives to the susceptibles \( \beta \), by 10% increases (or decreases) the reproduction number \( R_0 \) by 10%. Similarly, increasing (or decreasing) the natural death rate \( d \) by 10% decreases (or increases) the reproduction number \( R_0 \) by 5.507%. Furthermore, increasing (or decreasing) the recruitment rate of population in susceptible classes \( A \) by 10% increases (or decreases) \( R_0 \) by 7.250% and increasing (or decreasing) the rate at which asymptomatic infectives become symptomatic \( \beta_1 \) by 10% decreases (or increases) \( R_0 \) by 5.208%. The increase (or decrease) in the rate that message becomes outdated \( \mu_0 \) by 10%
increases (or decreases) $R_0$ by 2.750%. In the same manner, increasing (or decreasing) the recovery rates of asymptomatic infective classes $\delta_1$ measures how active the virus-associated communications can affect the spread ratio $\alpha_1$, the rate at which susceptible individuals may send communication about disease during an outbreak of coronavirus disease $\mu$, and the rate that message becomes outdated $\mu_0$ by 10% decreases (or increases) $R_0$ by 4.167, 2.750, and 2.750%, respectively.

Now, the effect of various parameters is shown on reproduction number $R_0$ in Figures 2–4. In Figure 2, the effect of variation in $d$ and $A$ is depicted on $R_0$. It is noted that as the value of $d$ and $A$ increases, the value of $R_0$ also increases. The effect of other parameters $d$ and $\beta$ is shown in Figure 3, and the effect of other parameters $d$ and $\beta_1$ is shown in Figure 4. The sensitivity of $R_0$ to some parameters of system is presented in Figure 5.

6 Stability analysis

6.1 Theorem 1

The DFE $E_0$ is unstable if $R_0 > 1$ and stable if $R_0 < 1$.

Table 2: Sensitivity indices of $R_0$ evaluated at the parameter values given in Section 8

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sensitivity Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>+1</td>
</tr>
<tr>
<td>$A$</td>
<td>+0.7250</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>0.2750</td>
</tr>
<tr>
<td>$d$</td>
<td>-0.5507</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-0.5208</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>-0.4167</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>-0.2750</td>
</tr>
<tr>
<td>$\mu$</td>
<td>-0.2750</td>
</tr>
</tbody>
</table>

Figure 2: Surface plot showing the effect of $d$ and $A$ on $R_0$.

Figure 3: Surface plot showing the effect of $d$ and $\beta$ on $R_0$. 
Proof. The variational matrix $V_0$ of model System (2) corresponding to the equilibrium point $E_0 \left( \frac{A}{d}, 0, 0, 0, \frac{\mu A}{\mu_d} \right)$ is given by:

$$V_0 = \begin{bmatrix}
-d & 0 & -\alpha & 0 & 0 & 0 \\
0 & r & 0 & 0 & 0 & 0 \\
0 & \beta_1 & -(d + \delta_2 + \alpha) & 0 & 0 & 0 \\
0 & \delta_1 & \delta_2 & -(\xi + d) & 0 & 0 \\
\mu & 0 & 0 & 0 & -\mu_0 & 0 \\
\theta & -\theta & -\theta & -\theta & 0 & -(\theta + \theta_0 + d)
\end{bmatrix},$$

where $r = \frac{A(\beta_0 + d) + \delta}{d(\theta + \theta_0 + d)} - (d + \beta_1 + \delta_0) = (d + \beta_1 + \delta_0)[R_0 - 1]$.

The eigenvalues of the aforementioned matrix are as follows:

$$\lambda = -d, -(d + \delta_2 + \alpha), -(\xi + d), -(\theta + \theta_0 + d), -\mu_0, (d + \beta_1 + \delta_0)[R_0 - 1](<0 \text{ if } R_0 < 1 \text{ and } >0 \text{ if } R_0 > 1).$$

Hence, DFE point is stable because all eigenvalues of the aforementioned matrix $V_0$ are found to be negative if $R_0 < 1$ and unstable since one eigenvalue is found to be positive if $R_0 > 1$. □

6.2 Theorem 2

The endemic equilibrium point $E^*$ is locally asymptotically stable, provided that the following conditions are satisfied:
\[(\beta Y^* e^{-\alpha M^*} + \theta)^2 < \frac{1}{5}\beta Y^* e^{-\alpha M^*}(\theta + \theta_0 + d), \quad (17)\]
\[
\frac{4\beta Y^* e^{-\alpha M^*}}{d(d + a + \delta_2)} < \frac{\beta Y^* e^{-\alpha M^*}}{5}, \quad (18)\]
\[
\beta_1 \delta_2^2 < \frac{1}{3}\delta_1(\xi + d)(d + \delta_2 + a), \quad (19)\]
\[
\theta^2 < \max\left[\frac{1}{4}(d(\theta + \theta_0 + d)), \frac{1}{4}q(d + a + \delta_2), \frac{1}{3}(\xi + d)\right], \quad (20)\]

where \(q = \frac{\beta Y^* e^{-\alpha M^*}}{\beta_1} (\theta + \theta_0 + d)\).

**Proof.** To compute the local stability of endemic equilibrium point \(E^*\), we linearize the model System (2) using small perturbations \(n, y, y_i, r, m,\) and \(v\) about \(E^*\), defined as:

\[
N = n + N^*, Y = y + Y^*, Y_i = y_i^* + Y_i^*, R = r + R^* = m + M^*, \quad \text{and} \quad V = v + V^*.
\]

Consider the following positive definite function:

\[
U_i = \frac{1}{2}(m_i n^2 + m_i y^2 + m_i y_i^2 + m_i r^2 + m_i m^2 + m_i v^2),
\]

where, \(m_i \ (i = 0, 1, 2, 3, 4, 5)\) are positive constants to be chosen appropriately. Differentiating the aforementioned equation with respect to \(t\) and using the linearized system of Model (2) corresponding to \(E^*\), we obtain

\[
\frac{dU_i}{dt} = -m_i n d^2 - m_i \beta Y^* e^{-\alpha M^*} y^2 - m_i (d + \delta_2 + a) y_i^2 - m_i (\xi + d) r^2 - m_i \mu m^2
\]
\[
- m_2(\theta + \theta_0 + d) y^2 - m_2(\delta_2 + a) y_i^2 - m_2 a n y_i + m_2 \beta Y^* e^{-\alpha M^*} n y - (m_2 \beta Y^* e^{-\alpha M^*} - m_2 \beta_1) n y_i
\]
\[
- (m_2 \beta Y^* e^{-\alpha M^*} - m_2 \delta_1) r y - (m_2 a \beta Y^* (N^* - Y^* - Y_i^* + R^* - V^*) e^{-\alpha M^*}) n y
\]
\[
- [m_2 \beta Y^* e^{-\alpha M^*} + m_2 \theta_1 n y + (m_2 \delta_2) r y_i + m_2 \mu m n + m_2 \theta v - m_2 \theta_1 y_i - m_2 \theta v].
\]

After choosing \(m_0 = m_1 = m_2 = 1, \ m_2 = \frac{\beta Y^* e^{-\alpha M^*}}{\beta_1}, \ m_3 = \frac{\beta Y^* e^{-\alpha M^*}}{\beta_1}, \) and \(\frac{\beta Y^* e^{-\alpha M^*}}{\beta_1} p < m_4 < \frac{1}{\beta_1} \) where \(p = \frac{\beta Y^* (N^* - Y^* - Y_i^* + R^* - V^*) e^{-\alpha M^*}}{\beta_1}\),

We obtain \(\frac{dU_i}{dt}\) to be negative definite showing that \(U_i\) is a Lyapunov function, and hence, \(E^*\) is locally asymptotically stable, provided that Conditions (17)–(20) are satisfied.

\[\square\]

7 Persistence of the model

Here, in this section, we found persistence of the disease. The word “persistence” stands for the survival of each population in future time. We study the persistence of the system to determine the conditions under which a disease persists in the system:

\[
\frac{dN}{dt} = A - dN - aY
\]
\[
\geq A - dN - aY_{\text{max}}
\]
\[
\geq (A - aY_{\text{max}}) - dN.
\]

Using the boundedness and comparison principle, we have \(N_{\text{min}} = \frac{A - aY_{\text{max}}}{d}\). If \(A > aY_{\text{max}}\), \(N_{\text{min}}\) is always positive:

\[
\frac{dY}{dt} = \beta_1^Y - (d + \delta_2 + a) Y_i.
\]
Again, using the comparison principle, we obtain $Y_{\text{min}} = \frac{\beta Y_{\text{min}}}{d + \delta + \alpha}$ is always positive:

$$\frac{dR}{dt} \geq \delta_1 Y_{\text{min}} + \delta_2 Y_{\text{min}} - (\xi + d)R.$$  

Using the boundedness and comparison principle, we obtain $R_{\text{min}} = \frac{\beta Y_{\text{min}} + \delta Y_{\text{min}}}{\xi + d}$ is always positive:

$$\frac{dM}{dt} = \mu N - \mu_0 M \geq \mu N_{\text{min}} - \mu_0 M.$$  

Again, using the comparison principle, we obtain $M_{\text{min}} = \frac{\mu N_{\text{min}}}{\mu_0}$ is always positive:

$$\frac{dV}{dt} = \theta N - \theta Y - \theta Y_1 - \theta R - \theta V - (\theta_0 + d)V \\
\geq \theta(N_{\text{min}} - Y_{\text{max}} - Y_{\text{max}} - R_{\text{max}} - V_{\text{max}}) - (\theta_0 + d)V \\
\geq \theta(B - B_1) - (\theta_0 + d)V.$$  

Using the boundedness and comparison principle, we obtain

$$V_{\text{min}} = \frac{\theta(B - B_1)}{\theta_0 + d},$$  

where $B = N_{\text{min}}$ and $B_1 = (Y_{\text{max}} + Y_{\text{max}} + R_{\text{max}} + V_{\text{max}})$.

$V_{\text{min}}$ is positive if $(B > B_1)$:

$$\frac{dY}{dt} = (\beta Y - \beta Y^2 - \beta Y_1 - \beta R - \beta V) e^{-a M_{\text{max}} - (d + \beta_1 + \delta_1)Y} \\
\geq [(\beta N - \beta Y_1 - \beta R - \beta V) e^{-a M_{\text{max}}} - (d + \beta_1 + \delta_1)] Y - \beta e^{-a M_{\text{max}}} Y^2 \\
\geq T Y Y^2.$$  

Again, using the comparison principle, we obtain

$$Y_{\text{min}} = \frac{T}{T_1},$$  

where $T = T_2 - T_3, T_1 = \beta e^{-a M_{\text{max}}}, T_2 = (\beta N_{\text{min}} - \beta Y_{\text{max}} - \beta R_{\text{max}} - \beta V_{\text{max}}) e^{-a M_{\text{max}}}, T_3 = d + \beta + \delta_1$, and $Y_{\text{min}}$ is positive if $T_2 > T_3$.

### 8 Numerical simulation

In this section, the numerical simulation of the model System (2) using MATLAB is performed. The following set of parameter values are used in numerical simulation:

- $A = 10$, $d = 0.06$, $\alpha = 0.02$, $\beta = 0.08$, $\alpha_1 = 0.001$, $\beta_1 = 0.5$, $\delta_1 = 0.4$, $\delta_2 = 0.7$, $\xi = 0.9$, $\mu = 0.99$, $\mu_0 = 0.6$, $\theta = 0.023$, and $\theta_0 = 0.0057$.

The equilibrium values of endemic equilibrium point $E^*$ for the aforementioned set of parameter values are obtained as follows:

- $N^* = 155.2967$, $Y^* = 53.2113$, $Y_1^* = 34.1098$, $R^* = 47.0431$, $M^* = 256.2396$, and $V = 5.4278$.

The eigenvalues corresponding to the Jacobian matrix of endemic equilibrium $E_3$ are as follows: $-6.0748$, $-1.1338 \pm 0.3885i$, $-0.6001$, $-0.0613$, and $-0.0742$.

It is noted here that four eigenvalues are negative and two eigenvalues have a negative real part; therefore, for the aforementioned set of parameter values, the endemic equilibrium $E^*$ is locally asymptotically stable. The results of the model analysis are displayed graphically in Figures 6–12. In Figure 6, four different values of the total human population $N(t)$, asymptomatic infective population $Y(t)$, and symptomatic infective population $Y_1(t)$ are considered. It is seen from the figure that all trajectories starting from different initial values approach to the equilibrium point. This shows that the endemic equilibrium $E^*$ is nonlinearly asymptotically stable. The initial starts of all trajectories to reach the equilibrium point are given in the following:
Figures 7 and 8 represent a variation of asymptomatic infective population and symptomatic infective population, respectively, with time for different values of $\beta$, the rate of transfer of the disease directly from the asymptomatic infectives to the susceptibles. When an infected person comes in contact with susceptible
person, then that susceptible person catches the disease unknowingly and becomes asymptomatic infective, and later, if they do not take precaution, they become symptomatic infective. Thus, it is depicted from Figure 7 that the asymptomatic infective population increases with an increase in the value of $\beta$, which, in turn, increases the symptomatic infective population (Figure 8).

As the value of $\alpha_1$, i.e., measure of how active the virus-associated communications can affect the spread ratio, increases, the asymptomatic infective population decreases, which is shown in Figure 9. Thus, on increase in the media communication, the people become aware of the coronavirus disease and if they acquire
some symptoms of COVID, they isolate themselves and start taking treatment, which decreases the symptomatic infective population, as depicted in Figure 10. Also, from Figures 11 and 12, we have seen that both the population asymptomatic and symptomatic infectives decrease, respectively, because when there is an increase in the awareness programs due to media among susceptible population, people become aware about the coronavirus disease and they obtain themselves vaccinated.

9 Conclusion

In this article, we develop a nonlinear mathematical model and analyze it to investigate the impact of media campaigns aimed at containing the COVID-19 pandemic by vaccination. In the modeling process, the entire human population is separated into five subclasses: the susceptible population, the infected population without symptoms, the infected population with symptoms, the population that has recovered, and the population that has given vaccinations. The effect of some critical parameters on the spread of the disease is studied. The analysis of the model has performed using the stability theory and numerical simulation and some inferences have been drawn by establishing local and nonlinear stability results. The study of the model displayed graphically, showing the variation of symptomatic and asymptomatic infective population with respect to time for different values of parameters. The model study demonstrates that the increase in media campaigns and vaccinations helps in the control of the dangerous coronavirus epidemic in 2019.

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References


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