Erratum to

On the dynamical and arithmetic degrees of rational self-maps of algebraic varieties

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Let $f : X \dashrightarrow X$ be a dominant rational map of a smooth projective variety defined over a characteristic 0 global field K , let δ_f be the dynamical degree of f , and let $h_X \geq 1$ be an ample Weil height on $X(\bar{K})$. In the article [2] the authors proved an inequality of the form

$$(1) \quad h_X \circ f^n \ll (\delta_f + \epsilon)^n h_X,$$

and they used (1) to prove an inequality

$$\bar{\alpha}_{f,P} \leq \delta_f$$

relating the upper arithmetic degree

$$\bar{\alpha}_{f,P} := \limsup h_X(f^n(P))^{\frac{1}{n}}$$

of the f -orbit of $P \in X(\bar{K})$ to the dynamical degree. Unfortunately, there is an error in the proof of inequality (1) due to the fact that the constants $C_1, C_2, \dots$ defined after [2, equation (5.5)] depend on m . In particular, the upper bound in inequality [2, equation (5.15)] involves the quantity $(C_8 r m^r)^{\frac{q}{m\ell}}$, and it is asserted that this quantity goes to 1 as $m \rightarrow \infty$, but this conclusion is not justified because C_8 depends on m in an unspecified way. This error was pointed out to the authors by Karou Sano. Subsequently, Yohsuke Matsuzawa [3] gave a corrected proof of the height inequality (1) using a more intricate argument. In particular, Matsuzawa proves that the statement of Theorem 24 (Theorem 1) in [2], and its consequences, are true. We also remark that in the case that $f : X \rightarrow X$ is a morphism, the authors had earlier [1] given an alternative proof of a strong height formula that implies in particular the arithmetic/dynamical degree estimate

$$\alpha_{f,P} \leq \delta_f$$

for morphisms.

References

- [1] *S. Kawaguchi and J. H. Silverman*, Dynamical canonical heights for Jordan blocks, arithmetic degrees of orbits, and nef canonical heights on abelian varieties, *Trans. Amer. Math. Soc.* **368** (2016), no. 7, 5009–5035.
- [2] *S. Kawaguchi and J. H. Silverman*, On the dynamical and arithmetic degrees of rational self-maps of algebraic varieties, *J. reine angew. Math.* **713** (2016), 21–48.
- [3] *Y. Matsuzawa*, On upper bounds of arithmetic degrees, preprint 2018, <https://arxiv.org/abs/1606.00598v3>; *Amer. J. Math.*, to appear.

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