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AFFINE MOTION IN THE SYMMETRIC SAK_n^* (I)

1. Preliminaries

Throughout this paper the Latin indices have the range $1, 2, \dots, n$ and the Greek indices the range $3, 4, \dots, n$.

In [2] Wong generalized a notion of K^* -spaces, namely

D e f i n i t i o n 1. An n -dimensional analytic space A_n with a (symmetric) connexion Γ_{jk}^i will be called an AK_n^* -space, or simply AK_n^* , if its curvature tensor

$$(1) \quad B_{jkl}^i = \Gamma_{jl,k}^i - \Gamma_{jk,l}^i + \Gamma_{jl}^h \Gamma_{hk}^i - \Gamma_{jk}^h \Gamma_{hl}^i$$

satisfies the relations

$$(2) \quad B_{jkl|m}^i = B_{jkl}^i k_m$$

or else the relations

$$(3) \quad B_{jkl|m}^i = 0, \quad B_{jkl}^i k_m + B_{jlm}^i k_k + B_{jmk}^i k_l = 0,$$

where k_m is a non-zero vector and a solidus | denote a covariant differentiation. An AK_n^* is said to be recurrent (in the sense of Ruse) or symmetric (in the sense of Cartan) according as (2) or (3) are satisfied.

The main purpose of Wong was to show the existence of non-Riemannian AK_n^* . With this in view he has introduced a special AK_n^* , designed SAK_n^* :

D e f i n i t i o n 2. A non-flat AK_n^* will be called an SAK_n^* if there exists a system of coordinates x^i such that the components of the connexion

$$\Gamma_{11}^\alpha, \Gamma_{1\beta}^2 = \Gamma_{\beta 1}^2, \Gamma_{11}^2 \text{ are independent of } x^2,$$

and all the other components are zero.

Further, Wong has shown (Theorem 2.1 in [2]) that there exist five types of SAK_n^* (for $n \geq 3$), which will be denoted by I, II, III, IV and V, and has given the formulas for non-vanishing coordinates of the connexion and the conditions for the existence each of that types of SAK_n^* . In the same theorem he has proved that the space of the type I, II, III is recurrent and of the type IV, V is symmetric. Also he has proved (Theorem 4.2 in [2]) that a non-Riemannian SAK_n^* of the type II or IV has $n-1$ fields of parallel contravariant vectors and $n-1$ fields of parallel covariant vectors. A Riemannian SAK_n^* of the type II or IV has $n-2$ fields of parallel vectors.

It is known that the infinitesimal transformation

$$(4) \quad \bar{x}^i = x^i + v^i(x)dt$$

is an affine motion if

$$(5) \quad L_{\underset{v}{\nabla}}^i \Gamma_{jk}^i = 0 = v_{|jk}^i + B_{jkl}^i v^l,$$

where $v(x) = [v^1(x), \dots, v^n(x)]$ and $x = (x^1, \dots, x^n)$. Such motion may be considered in SAK_n^* also. K. Takano [1] has given a necessary and sufficient condition for the existence of the affine motion. In this paper we give effectively all affine motions v in SAK_n^* of the type IV. In the part (II) will be given one in SAK_n^* of the type V.

2. Affine motions in SAK_n^* of the type IV

D e f i n i t i o n 3. SAK_n^* is of the type IV if there exists a coordinate system in which the coordinates of the affine connexion have the form

$$(6) \quad \Gamma_{11}^{\alpha} = b^{\alpha} z, \quad \Gamma_{1\beta}^2 = a e_{\beta} z = \Gamma_{\beta 1}^2, \quad \Gamma_{11}^2 = c z,$$

where b^{α} , a , c are constants not vanishing simultaneously, and - similarly - e_{β} also are some constants with at least one different from zero, and $z = e_{\gamma} x^{\gamma}$.

Theorem 1 (see Theorem 3.2 in [2]). SAK_n^* of the type IV is a Riemannian space iff $a \neq 0$ and for some α there is $b^{\alpha} \neq 0$.

From this we immediately see that among all constants defining the Riemannian SAK_n^* of the type IV the constant c and some of b^{γ} may be vanishing.

The determination of affine motion (4) is reduced to the determination of all coordinates v^i . Then system (5) must be satisfied for each $x = (x^1, \dots, x^n)$, i.e., for each point of SAK_n^* . The solution of (5) is a vector field v ; we will calculate each of its coordinates $v^1, \dots, v^n$. With this purpose we write (5) in another form, namely ([3], p.41):

$$(8) \quad \mathcal{L}_v \Gamma_{jk}^i = a_{jk} v^i + v^s a_s \Gamma_{jk}^i - a_a v^i \Gamma_{jk}^a + a_j v^a \Gamma_{ak}^i + a_k v^a \Gamma_{ja}^i = 0.$$

Using (6) in (8) we obtain the system:

$$(9) \quad \begin{cases} (a) & \mathcal{L}_v \Gamma_{11}^1 = a_{11} v^1 - a_2 v^1 c z - a_{\alpha} v^1 b^{\alpha} z = 0 \\ (b) & \mathcal{L}_v \Gamma_{12}^1 = a_{12} v^1 = 0 \\ (c) & \mathcal{L}_v \Gamma_{1\beta}^1 = a_{1\beta} v^1 - a_2 v^1 a e_{\beta} z = 0 \\ (d) & \mathcal{L}_v \Gamma_{AB}^1 = a_{AB} v^1 = 0 \end{cases} \quad (A, B \in \{2, 3, \dots, n\}).$$

$$(10) \left\{ \begin{array}{l} (a) \quad L_{\nu}^2 = \partial_{11} v^2 + v^\alpha e_\alpha c - \partial_2 v^2 c z - \partial_\alpha v^2 b^\alpha z + 2\partial_1 v^1 c z + 2\partial_1 v^\alpha a e_\alpha z = 0 \\ (b) \quad L_{\nu}^2 = \partial_{12} v^2 + \partial_2 v^\alpha a e_\alpha z + \partial_2 v^1 c z = 0 \\ (c) \quad L_{\nu}^2 = \partial_{1\beta} v^2 + v^\alpha e_\alpha e_\beta a - \partial_2 v^2 a e_\beta z + \partial_1 v^1 a e_\beta z + \partial_\beta v^1 c z + \partial_\beta v^\alpha a e_\alpha z = 0 \\ (d) \quad L_{\nu}^2 = \partial_{22} v^2 = 0 \\ (e) \quad L_{\nu}^2 = \partial_{2\beta} v^2 + \partial_2 v^1 a e_\beta z = 0 \\ (f) \quad L_{\nu}^2 = \partial_{\alpha\beta} v^2 + \partial_\alpha v^1 a e_\beta z + \partial_\beta v^1 a e_\alpha z = 0, \end{array} \right.$$

$$(11) \left\{ \begin{array}{l} (a) \quad L_{\nu}^\mu = \partial_{11} v^\mu + v^\alpha b^\mu e_\alpha - \partial_2 v^\mu c z - \partial_\alpha v^\mu b^\alpha z + 2\partial_1 v^1 b^\mu z = 0 \\ (b) \quad L_{\nu}^\mu = \partial_{12} v^\mu + \partial_2 v^1 b^\mu z = 0 \\ (c) \quad L_{\nu}^\mu = \partial_{1\beta} v^\mu - \partial_2 v^\mu a e_\beta z + \partial_\beta v^1 b^\mu z = 0 \\ (d) \quad L_{\nu}^\mu = \partial_{AB} v^\mu = 0 \quad (A, B \in \{2, 3, \dots, n\}) \end{array} \right.$$

determining the vector field v in SAK_n^* of the type IV.

In our consideration we shall often use the evident

L e m m a . Let a_i, b_j, c_i, d_j be arbitrary numbers $(i, j \in \{1, \dots, n\})$. If the equation

$$a_i b_j = c_i d_j$$

holds for each i, j and $a_i \neq 0$ for some i , then

$$b_j = t d_j,$$

where t is some constant.

For affine motions we have

T h e o r e m 2. Each affine motion $v = [v^1, \dots, v^n]$ in the SAK_n^* of the type IV has the form

$$\begin{aligned} (i) \quad v^1 &= p_0^1 + p_1^1 x^1 + p_\alpha^1 x^\alpha, \\ (ii) \quad v^2 &= p_0^2 + p_i^2 x^i - \frac{1}{2} c e_\alpha [p_0^\alpha (x^1)^2 + \frac{1}{3} p_1^\alpha (x^1)^3] + c a^2 (m_0 e^{Qx^1} + m_1 e^{-Qx^1}) + \\ &+ a [a^3 (m_0 e^{Qx^1} - m_1 e^{-Qx^1}) - e_\alpha (p_0^\alpha x^1 + \frac{1}{2} p_1^\alpha (x^1)^2)] z - \frac{1}{3} a S z^3, \end{aligned}$$

$$(iii) \quad v^\mu = p_0^\mu + p_1^\mu x^1 + b^\mu Q^2 (m_0 e^{Qx^1} + m_1 e^{-Qx^1}) - \frac{1}{2} b^\mu e_\alpha [p_0^\alpha (x^1)^2 + \frac{1}{3} p_1^\alpha (x^1)^3],$$

where $Q = \sqrt{-e_\alpha b^\alpha}$ and $p_0^i, p_1^i, S, m_0, m_1$ are constants satisfying the conditions:

$$(12) \quad p_\alpha^1 b^\alpha = 0,$$

$$(13) \quad p_\beta^1 b^\mu = S b^\mu e_\beta,$$

$$(14) \quad p_\beta^1 a = a S e_\beta,$$

$$(15) \quad p_\beta^1 c = a(p_2^2 - p_1^1 - 2T) e_\beta,$$

$$(16) \quad a p_2^\mu = S b^\mu,$$

$$(17) \quad c p_2^\mu + p_\alpha^\mu b^\alpha = (2p_1^1 + T) b^\mu,$$

$$(18) \quad e_\alpha p_2^\alpha = 0,$$

$$(19) \quad e_\alpha p_\beta^\alpha = T e_\beta,$$

$$(20) \quad p_2^2 b^\alpha - a e_\alpha p_1^\alpha = c(T + 2p_1^1 - p_2^2),$$

$$(21) \quad e_\alpha p_0^\alpha = 0, \quad e_\alpha p_1^\alpha = 0 \quad \text{if } Q \neq 0.$$

P r o o f . The system (9) implies that

$$(22) \quad v^1 = p_0^1 + p_1^1 x^1,$$

where constants p_0^1, p_1^1 satisfy two conditions

$$(23) \quad a p_2^1 = 0,$$

$$(24) \quad c p_2^1 + p_\alpha^1 b^\alpha = 0.$$

Now the system (11) implies (iii), where $Q = \sqrt{-e_\alpha b^\alpha}$ and $p_0^\mu, p_1^\mu, m_0, m_1$ are constants satisfying conditions (21), (17) and

$$(25) \quad b^\mu p_2^1 = 0,$$

$$(26) \quad ap_2^\mu = Sb^\mu, \quad p_\beta^1 b^\mu = Sb^\mu e_\beta \quad (S \text{ is a constant}),$$

$$(27) \quad b^\mu e_\alpha p_2^\alpha = 0,$$

$$(28) \quad b^\mu e_\alpha p_\gamma^\alpha = Tb^\mu e_\gamma \quad (T \text{ is a constant}).$$

For receiving (iii) we have used (22) and the Lemma.

Now we consider (10). From (10d) we get

$$(29) \quad v^2 = g_2^2(x^1, x^\nu) x^2 + g_0^2(x^1, x^\nu),$$

where g_2^2, g_0^2 is a function independent of x^2 . Using this and (23) we get $g_2^2(x^1, x^\nu) = h_2^2(x^1)$ from (10c). Then from (10f) we obtain $\partial_{\alpha\beta} g_0^2(x^1, x^\nu) = -a(p_\alpha^1 e_\beta + p_\beta^1 e_\alpha)z$. This implies that

$$(30) \quad ap_\alpha^1 e_\gamma = ap_\gamma^1 e_\alpha$$

for each α, γ , because $\partial_{\alpha\beta\gamma} g_0^2(x^1, x^\nu) = \partial_{\gamma\beta\alpha} g_0^2(x^1, x^\nu)$. The condition (30) is satisfied iff (14) for each γ , where S is the same constant as in (26). Using (29), (10f), (10b), (10c), (10a) and the lemma we obtain (ii), where $p_0^2, p_1^2, p_2^2, p_\alpha^2$ are arbitrary constants satisfying the condition (20) and

$$(31) \quad ce_\alpha p_\gamma^\alpha = cTe_\gamma:$$

T is the same constant as in (28).

Moreover, this gives the further conditions for the constants $p_2^1, p_\alpha^1, p_2^\mu, p_\alpha^\mu$, namely (15) and

$$(32) \quad ap_2^\alpha e_\alpha = 0,$$

$$(33) \quad ae_\alpha p_\gamma^\alpha = aTe_\gamma,$$

$$(34) \quad cp_2^1 = 0,$$

$$(35) \quad ce_\alpha p_2^\alpha = 0.$$

From (23), (25), (34) we get $p_2^1 = 0$, from (28), (33), (31), in view of Lemma, we get (19) and (27), (32), (35) implies (18), because a, b^μ, c are not all zero. Using $p_2^1 = 0$ in (24) we obtain (12).

C o r o l l a r y . Each affine motion $v = [v^1, \dots, v^n]$ in the Riemannian SAK_n^* of the type IV has the form

$$(i) \quad v^1 = p_0^1 + p_1^1 x^1 + S z,$$

$$(ii) \quad v^2 = p_0^2 + p_1^2 x^1 - \frac{1}{2} c e_\alpha \left[p_0^\alpha (x^1)^2 + \frac{1}{3} p_1^\alpha (x^1)^3 \right] + c Q^2 (m_0 e^{Qx^1} + m_1 e^{-Qx^1}) + \\ + a \left[Q^3 (m_0 e^{Qx^1} - m_1 e^{-Qx^1}) - e_\alpha (p_0^\alpha x^1 + \frac{1}{2} p_1^\alpha (x^1)^2) \right] z - \frac{1}{3} a S z^3,$$

$$(iii) \quad v^\mu = p_0^\mu + p_1^\mu x^1 + \frac{S}{a} b^\mu x^2 + p_\alpha^\mu x^\alpha + b^\mu Q^2 (m_0 e^{Qx^1} + m_1 e^{-Qx^1}) - \\ - \frac{1}{2} b^\mu e_\alpha \left[p_0^\alpha (x^1)^2 + \frac{1}{3} p_1^\alpha (x^1)^3 \right],$$

where constants $a \neq 0, b^\alpha \neq 0, e_\beta \neq 0, z = e_\beta x^\beta, Q = \sqrt{-e_\alpha b^\alpha}$ and constants $p_0^i, p_1^i, S, m_0, m_1$ satisfy the conditions

$$(12) \quad p_\alpha^1 b^\alpha = 0,$$

$$(14') \quad S e_\alpha b^\alpha = 0,$$

$$(17') \quad p_\alpha^\mu b^\alpha = \frac{1}{2} (p_2^2 + 3p_1^1 - 3c \frac{S}{a}) b^\mu,$$

$$(19') \quad e_\alpha p_\beta^\alpha = \frac{1}{2} (p_2^2 - p_1^1 - c \frac{S}{a}) e_\beta,$$

$$(20') \quad p_\alpha^2 b^\alpha - a e_\alpha p_1^\alpha = \frac{1}{2} c (3p_1^1 - p_2^2 - c \frac{S}{a}),$$

$$(21) \quad e_\alpha p_0^\alpha = 0, \quad e_\alpha p_1^\alpha = 0 \quad \text{if} \quad Q^2 \neq 0.$$

P r o o f . This is a simple consequence of Theorems 1 and 2.

3. The number of parameters of the group of the affine motion in SAK_n^* of the type IV

SAK_n^* of the type IV is determined by constants a, b^α, c , not all zero, appearing in (6). These constants may be generally chosen in the seven various manners that not all of them are equal to zero, e.g. $a = 0, b^\alpha = 0, c \neq 0$ etc. For each such choice of these constants we get the corresponding case of SAK_n^* of the type IV and the corresponding group of affine motions in the mentioned SAK_n^* . We shall consider such groups with respect to the number of their parameters.

Theorem 3. The group of the affine motions in SAK_n^* ($n \geq 3$) of the type IV is reducible by a specification of constants a, b^α, c , with $e_\beta \neq 0$ determining this SAK_n^* , to one of the following seven forms:

A) if $a \neq 0, b^\alpha \neq 0, c$ arbitrary and $e_\alpha b^\alpha \neq 0$, then

$$\begin{cases} v^1 = p_0^1 + p_1^1 x^1, \\ v^2 = p_0^2 + p_1^2 x^1 + c Q^2 (m_0 e^{Qx^1} + m_1 e^{-Qx^1}) + a Q^3 (m_0 e^{Qx^1} - m_1 e^{-Qx^1}) z, \\ v^\mu = p_0^\mu + p_1^\mu x^1 + p_\alpha^\mu x^\alpha + b^\mu Q^2 (m_0 e^{Qx^1} + m_1 e^{-Qx^1}), \end{cases}$$

where p_0^i, p_j^i satisfy (21) and

$$(17A) \quad p_\alpha^\mu b^\alpha = \frac{1}{2} (3p_1^1 + p_2^2) b^\mu,$$

$$(19A) \quad e_\alpha p_\beta^\alpha = \frac{1}{2} (p_2^2 - p_1^1) e_\beta,$$

$$(20A) \quad p_\alpha^2 b^\alpha = \frac{1}{2} c (3p_1^1 - p_2^2);$$

B) if $a \neq 0, b^\alpha \neq 0, c$ arbitrary and $e_\alpha b^\alpha = 0$, then

$$\begin{cases} v^1 = p_0^1 + p_1^1 x^1 + Sz, \\ v^2 = p_0^2 + p_1^2 x^1 - \frac{1}{2} c e_\alpha \left[p_0^\alpha (x_1)^2 + \frac{1}{3} p_1^\alpha (x^1)^3 \right] - a e_\alpha \left[p_0^\alpha x^1 + \frac{1}{2} p_1^\alpha (x^1)^2 \right] z - \frac{1}{3} a S z^3, \\ v^\mu = p_0^\mu + p_1^\mu x^1 + b^\mu \frac{S}{a} x^2 + p_\alpha^\mu x^\alpha - \frac{1}{2} b^\mu e_\alpha \left[p_0^\alpha (x^1)^2 + \frac{1}{3} p_1^\alpha (x^1)^3 \right], \end{cases}$$

where p_0^1, p_j^1, S satisfy

$$(19B) \quad e_\alpha p_\beta^\alpha = \frac{1}{2} (p_2^2 - p_1^1 - c \frac{S}{a}) e_\beta,$$

$$(20B) \quad p_\alpha^2 b^\alpha - a e_\alpha p_1^\alpha = \frac{1}{2} c (3p_1^1 - p_2^2 - c \frac{S}{a});$$

C) if $a \neq 0, b^\alpha = 0, c$ arbitrary, then

$$\begin{cases} v^1 = p_0^1 + p_1^1 x^1 + Sz \\ v^2 = p_0^2 + p_1^2 x^1 - \frac{1}{2} c e_\alpha \left[p_0^\alpha (x^1)^2 + \frac{1}{3} p_1^\alpha (x^1)^3 \right] - a e_\alpha \left[p_0^\alpha x^1 + \frac{1}{2} p_1^\alpha (x^1)^2 \right] z - \frac{1}{5} a S z^3, \end{cases}$$

where p_0^1, p_j^1, S satisfy (19B) and

$$(20C) \quad a e_\alpha p_1^\alpha = \frac{1}{2} c (p_2^2 - 3p_1^1 + c \frac{S}{a});$$

D) if $a = 0, b^\alpha \neq 0, c \neq 0$ and $e_\alpha b^\alpha \neq 0$, then

$$\begin{cases} v^1 = p_0^1, \\ v^2 = p_0^2 + p_1^2 x^1 + c q^2 (m_0 e^{qx^1} + m_1 e^{-qx^1}), \\ v^\mu = p_0^\mu + p_1^\mu x^1 + b^\mu q^2 (m_0 e^{qx^1} + m_1 e^{-qx^1}) + \frac{1}{c} (T b^\mu - p_\alpha^\mu b^\alpha) x^2 + p_\alpha^\mu x^\alpha, \end{cases}$$

where p_0^1, p_j^1, T satisfy (19), (21) and

$$(20D) \quad b^\alpha p_\alpha^2 = c(T - p_2^2);$$

E) if $a = 0, b^\alpha \neq 0, c$ arbitrary and $e_\alpha b^\alpha = 0$, then

$$\begin{cases} v^1 = p_0^1 + p_1^1 x^1, \\ v^2 = p_0^2 + p_1^2 x^1 - \frac{1}{2} c e_\alpha \left[p_0^\alpha (x^1)^2 + \frac{1}{3} p_1^\alpha (x^1)^3 \right], \\ v^\mu = p_0^\mu + p_1^\mu x^1 - \frac{1}{2} b^\mu e_\alpha \left[p_0^\alpha (x^1)^2 + \frac{1}{3} p_1^\alpha (x^1)^3 \right], \end{cases}$$

where p_0^1, p_j^1 satisfy (17), (18), (19) and

$$(20E) \quad b^\alpha p_\alpha^2 = c(T + 2p_1^1 - p_2^2) \quad (T \text{ is a constant});$$

F) if $a = 0$, $b^\alpha \neq 0$, $c = 0$ and $e_\alpha b^\alpha \neq 0$, then

$$\begin{cases} v^1 = p_0^1, \\ v^2 = p_0^2 + p_1^2 x^1, \\ v^\mu = p_0^\mu + p_1^\mu x^1 + b^\mu Q^2 (m_0 e^{Qx^1} + m_1 e^{-Qx^1}), \end{cases}$$

where p_0^1 , p_j^1 , T satisfy (18), (19), (21) and

$$(17F) \quad p_\alpha^\mu b^\alpha = T b^\mu,$$

$$(20F) \quad b^\alpha p_\alpha^2 = 0; \quad (T \text{ is a constant});$$

G) if $a = 0$, $b^\alpha = 0$, $c \neq 0$, then

$$\begin{cases} v^1 = p_0^1 + p_1^1 x^1, \\ v^2 = p_0^2 + p_1^2 x^1 - \frac{1}{2} c e_\alpha \left[p_0^\alpha (x^1)^2 + \frac{1}{3} p_1^\alpha (x^1)^3 \right], \\ v^\mu = p_0^\mu + p_1^\mu x^1 + p_\alpha^\mu x^\alpha, \end{cases}$$

where p_0^1 , p_j^1 satisfy

$$(19G) \quad e_\alpha p_\beta^\alpha = (p_2^2 - 2p_1^1) e_\beta.$$

Moreover, for every case considered, the number of parameters of the group is given in the following table:

case	ab^α	c	$e_\alpha b^\alpha$	the number of parameters
A	$\neq 0$	arbitrary	$\neq 0$	$n^2 - 3n + 7$
B	$\neq 0$	arbitrary	0	$n^2 - 3n + 8$
C, E, G	0	arbitrary	0	$n^2 - 2n + 5$
D	0	$\neq 0$	$\neq 0$	$n^2 - 2n + 4$
F	0	0	$\neq 0$	$n^2 - 2n + 3$

P r o o f . To prove this theorem it is sufficient to use Theorem 2 and to calculate the difference between the number of used parameters in v^1 and the number of independent conditions bounded them. It is necessary to take into account that in the system of equations of the form $e_\alpha p^\alpha_\beta = T e_\beta$, $p^\mu_\alpha b^\alpha = T b^\mu$ there are $2n-5$ independent ones only. Moreover, in the case B the condition (18) follows from (17) and (19) for $c \neq 0$. If $c = 0$, then in the system of equations (17) and (19) there are $2n-5$ independent ones only.

R e m a r k . Cases A and B of the group of the affine motions in SAK_n^* of the type IV, in view of Theorem 1, are Riemannian.

4. An affine motion and the fields of parallel vectors

A non-Riemannian SAK_n^* of the type IV ($ab^\alpha = 0$) has $n-1$ fields of parallel vectors and a Riemannian one ($a \neq 0$, $b^\alpha \neq 0$) has $n-2$ such fields (see [2], Theorem 4.2). In a Riemannian space, in a Riemannian SAK_n^* also, each field of parallel vectors is an affine motion. This follows from (5) and Ricci-identity $v^i_{|jk} - v^i_{|kj} = B^i_{ajk} v^a$. The other situation is in a non-Riemannian space. There exists a field of parallel vectors, which is no affine motion. The same holds in a non-Riemannian SAK_n^* also. For example, if $b^\alpha = 0$, then $[p^1, p^2, \dots, p^n]$, where p^i are constants and $p^1 \neq 0$, $cp^1 + ae_\alpha p^\alpha = 0$, is a field of parallel vectors but is no affine motion. For this reason we have

T h e o r e m 4. Each affine motion in SAK_n^* of the type IV, whose field of parallel vectors has the form $v^1 = 0$, $v^2 = p^2_0$, $v^\mu = p^\mu_0$, where p^2_0 is an arbitrary constant, and where the constants p^μ_0 satisfy the condition $e_\alpha p^\alpha = 0$. All such motions form $(n-2)$ -parameters subgroup generated by $n-2$ independent fields of parallel vectors.

P r o o f . To prove this theorem it is sufficient to use (6) and (i), (ii), (iii) given in Theorem 2 and to solve the system

$$v^j_{|i} \frac{df}{ds} = \partial_i v^j + \Gamma^j_{is} v^s = 0.$$

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