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ON SOME APPLICATIONS OF INTEGRAL EQUATIONS
WITH FIXED SINGULARITIES
TO BOUNDARY-CONTACT-PROBLEMS IN THERMOELASTICITY

Singular integral equations with fixed singularities occur in different problems of mechanics and mathematical physics. Some applications can be found in the Teubner-Text of Duduchava [1]. The simplest integral equation with a fixed singularity is the Bueckner integral equation

$$Au(x) := c_0 u(x) + \frac{1}{\pi i} \int_0^1 \left[\frac{c_1}{y-x} + \frac{c_2}{y+x} \right] u(y) dy = f(x), \quad 0 \leq x \leq 1.$$

The part containing the fixed singularity has influence on the Fredholm property and the index of the operator A.

The boundary-contact-problems to be considered in this paper lead to systems of integral equations, which have in the scalar case the following form

$$\begin{aligned} Au(x) := & c_0 u(x) + \frac{c_1}{\pi i} \int_{-1}^{+1} \frac{u(y)}{y-x} dy + \\ & + \sum_{r=0}^n \frac{c_{r+2}}{\pi i} \int_{-1}^{+1} \frac{(1-x)^{m_r^+}}{(1-y)^{m_r^+ - r} [2-(x+y)]^{r+1}} u(y) dy + \end{aligned}$$

The results of this paper were presented during the 3rd Symposium on Integral Equations and Their Applications held at the Technical University of Warsaw, December 3-6, 1984.

$$+ \sum_{r=0}^n \frac{c_{r+2}^-}{\pi i} \int_{-1}^{+1} \frac{(1+x)^{\frac{m_r^-}{r}}}{(1+y)^{\frac{m_r^-}{r}-r} [2+x+y]^{r+1}} u(y) dy + Tu = f(x), \quad -1 \leq x \leq +1.$$

There are results of Duduchava [1], who considered these equations in the space $L_p([-1, +1], (1+x)^\alpha (1-x)^\gamma)$, where $0 \leq \operatorname{Re} \frac{m_r^\pm}{r} \leq r$, $1 < p < \infty$, $-1 < \alpha, \gamma < p-1$, and T being a compact operator from this space into itself. To the operator A there is assigned a symbol (or a symbolic determinant in the matrix case), which represents a closed orientated curve C_A in the complex plane.

It is known [1] that A is a Fredholm-operator if and only if $0 \notin C_A$. If this condition is fulfilled, then $\operatorname{Ind} A = -\operatorname{ind} C_A$.

We will use these results in the case $p = 2$, $\alpha = \gamma = 0$.

Now we formulate the boundary-contact-problems. We introduce:

the Lamé operator $\underset{(j)}{A} u = \mu_j \Delta u + (\lambda_j + \mu_j) \operatorname{grad} \operatorname{div} u$,

the displacement vector $u = u_1 i_1 + u_2 i_2$,

the stress operator $\underset{(j)}{T}(n_x) u(x) = 2\mu_j \frac{\partial u}{\partial n_x} + \lambda_j n_x \operatorname{div} u + \mu_j [n_x \operatorname{rot} u]$.

We shall consider a disk of two different materials occupying the domain $D = D^+ \cup D^-$ (fig.1). We shall also study boundary-contact-problems for the exterior domain (fig.2).

Fig.1

Fig.2

Problem I(D;w) or I(CD;w): Find a vectorfield u satisfying the differential equation

$$(1) \quad \underset{(1)}{A} u = 0 \text{ in } D^+ \text{ or } CD^+, \quad \underset{(0)}{A} u = 0 \text{ in } D^- \text{ or } CD^-,$$

the contact condition

$$(2) \quad \{u\}^+ = \{u\}^-, \quad \left\{ \underset{(1)}{T} (n_x) u(x) \right\}^+ = \left\{ \underset{(0)}{T} (n_x) u(x) \right\}^- \text{ on } S_0 \text{ or } CS_0,$$

and the boundary condition I

$$(3) \quad u = w \text{ on } S = S^+ \cup S^-.$$

Problem II(D;w) or II(CD;w): Instead of (3) we impose the boundary condition II

$$(4) \quad \underset{(1)}{T} (n) u = P \text{ on } S^+, \quad \underset{(0)}{T} (n) u = P \text{ on } S^-,$$

or

$$(4') \quad \underset{(1)}{T} (n') u = P \text{ on } S^+, \quad \underset{(0)}{T} (n') u = P \text{ on } S^-.$$

If there are thermal stresses and body forces, the problem can be reduced to one of these problems with the aid of particular solutions, which satisfy the inhomogeneous differential equation and inhomogeneous contact conditions. Such particular solutions can be expressed explicitly by the Green contact tensor for two half planes with different Lamé module ([2], Theorem 4.3).

In several papers, which will appear, the contact condition of sliding without friction ([3], [4], [5], [6]), and the fixed contact in normal direction ([4], [5], [6], [7]) are considered, too.

We look for a solution in the following proper class of regularity:

$$u|_{D^\pm} \in C^0(\overline{D_\varepsilon^+}) \cap C^1(\overline{D_\varepsilon^+}) \cap C^\infty(D_\varepsilon^+) \text{ or } CD_\varepsilon^+,$$

$$u \text{ bounded in } D^+ \cup D^- \text{ or } CD^+ \cup CD^-,$$

$$\frac{\partial u}{\partial x_j} = O(\varrho_x^{-\gamma}), \gamma < 1, \text{ in the neighbourhood of } \bar{S} \cap \bar{S}_0,$$

$$u = O(1), \frac{\partial u}{\partial x_j} = O(|x|^{-2}) \text{ for } |x| \rightarrow \infty,$$

where $D_\varepsilon^\pm = D^\pm \setminus K_\varepsilon$, K_ε is the union of the open circles with radius ε and centre $\bar{S} \cap \bar{S}_0$, ϱ_x is the shortest distance from x to S measured in direction of the x_1 -axis.

We suppose $S \subset C_n^{1,\alpha}$, this means that $S \subset C^{1,\alpha}$ and the angle between S and x_1 -axis is the right one. This assumption is essential for the proof of the Fredholmness of the integral operators. Some results in particular those for the single layer potential, remain true, if only $S^+ \subset C^{1,\alpha}$, $S^- \subset C^{1,\alpha}$ and the angles between S^+ , S^- and the x_1 -axis are different from zero.

U n i q u e n e s s t h e o r e m . In the class of regularity the problems $I(D;0)$, $I(CD;0)$ admit only the trivial solution, the problem $II(D;0)$ has the solution $\text{span}\{a_1, a_2, a_3\}$, the problem $II(CD;0)$ has the solution $\text{span}\{a_1, a_2\}$, where $a_1 = i_1 = (1,0)$, $a_2 = i_2 = (0,1)$ are the translations and $a_3 = -x_2 i_1 + x_1 i_2$ is the infinitesimal rotation.

Now we turn over to the existence of regular solutions. To this purpose we construct the contact tensor $G(x,y)$. The column k of $G(x,y)$ represents the displacement in x of two coupled halfplanes with different Lamé module, if a single force acts in y in x_k -direction. $G(x,y)$ is the sum of the Somigliana fundamental solution and a compensatrix $V(x,y)$, so that the columns of $G(x,y)$ satisfy the differential equation and contact condition [8]. $V(x,y)$ is represented by the adjoint potential functions

$$Z = \ln \sqrt{(x_1 - y_1)^2 + (|x_2| + |y_2|)^2}, \quad W = \arctan \frac{x_1 - y_1}{|x_2| + |y_2|}.$$

With the aid of $G(x,y)$ we consider potentials of single and double layers type

$$V(x; \varphi) = \frac{1}{\pi} \int_S G(x, y) \varphi(y) ds_y.$$

$$W(x; \varphi) = \frac{1}{\pi} \int_S (T(n_y) G(x, y)^T)^T \varphi(y) ds_y.$$

These potentials satisfy a priori the differential equation (1) and the contact condition (2).

Theorem 1 ([2], Theorems 2.1, 2.2, 2.3). If $S \subset C_n^{1, \alpha}$, $\varphi \in L_2(S) \cap C^{0, \beta}(S \setminus K_\varepsilon)$ for every $\varepsilon > 0$, $0 < \beta < \alpha \leq 1$, then the following relations hold true:

$$V(x; \varphi) \Big|_{D^\pm} \in C^{1, \beta}(D^\pm \setminus K_\varepsilon) \cap C^\infty(D^\pm),$$

$$V(x; \varphi) \Big|_{CD^\pm} \in C^{1, \beta}(CD^\pm \setminus K_\varepsilon) \cap C^\infty(CD^\pm),$$

$$V(x; \varphi) = O(1), \quad \frac{\partial V(x; \varphi)}{\partial x_j} = O(\varrho_x^{-1/2}) \quad \text{for } x \rightarrow \bar{S} \cap \bar{S}_0.$$

Moreover

$$V(x; \varphi) = O(|x|^{-1}), \quad \frac{\partial V(x; \varphi)}{\partial x_j} = O(|x|^{-2}) \quad \text{for } |x| \rightarrow \infty$$

if and only if $\int_S \varphi \cdot a_k ds = 0$, $k = 1, 2$.

Theorem 2 ([4], Theorem 4.1-4.4). If $S \subset C_n^{1, \alpha}$, $\varphi \in C^{1, \beta}(S)$, $l = 0, 1$, $0 < \beta < \alpha \leq 1$, then:

$$W(x; \varphi) \Big|_{D^\pm} \in C^{1, \beta}(D^\pm \setminus K_\varepsilon) \cap C^\infty(D^\pm),$$

$$W(x; \varphi) \Big|_{CD^\pm} \in C^{1, \beta}(CD^\pm \setminus K_\varepsilon) \cap C^\infty(D^\pm),$$

$$W(x; \varphi) = O(1), \quad \frac{\partial W(x; \varphi)}{\partial x_j} = O\left(\frac{1}{\varrho_x^{1-\beta}}\right) \quad \text{for } x \rightarrow \bar{S} \cap \bar{S}_0,$$

$$W(x; \varphi) = O(|x|^{-1}), \quad \frac{\partial W(x; \varphi)}{\partial x_j} = O(|x|^{-2}) \quad \text{for } |x| \rightarrow \infty.$$

If $l = 1$, then

$$\lim_{D \ni x \rightarrow x_0 \in S} T(n_x)W(x; \varphi) = \lim_{CD \ni x \rightarrow x_0 \in S} T(n_x)W(x; \varphi) \in L_2(S).$$

Now we use the jump properties of these potentials to get the integral operators associated with the boundary-contact-problems:

Problem	Potential	Integral equation ($x_0 \in S$)
I(D;w)	$W(x; \varphi)$	$A_I \varphi := -\varphi(x_0) + W(x_0; \varphi) = w(x_0)$
I(CD;w)	$W(x; \varphi)$	$A_I^\infty \varphi := +\varphi(x_0) + W(x_0; \varphi) = w(x_0)$
II(D;P)	$V(x; \varphi)$	$A_{II} \varphi := +\varphi(x_0) + \frac{1}{\pi} \int_S T(n_{x_0})G(x_0, y)\varphi(y)ds_y = P(x_0)$
II(CD;P)	$V(x; \varphi)$	$A_{II}^\infty \varphi := -\varphi(x_0) + \frac{1}{\pi} \int_S T(n_{x_0})G(x_0, y)\varphi(y)ds_y = -P(x_0).$

Theorem 3 ([2] Theorem 3.5, [4] Theorem 3.1).

The operators A_I , A_I^∞ , A_{II} , A_{II}^∞ are ϕ -operators in the space $L_2(S)$. Their indexes are equal to zero.

This follows from results of Duduchava [1]. The symbolic curve C_A lies in the halfplane $\text{Re } z > 0$, C_A is symmetric with respect to the real axis ([2], fig.2). If $\lambda_0 = \lambda_1$, $\mu_0 = \mu_1$, then C_A contracts to a point of the interval $(9/16, 1)$.

For the adjoint operators we have

$$A_I^* = A_{II}^\infty, A_I^{\infty*} = A_{II}, A_{II}^* = A_I^\infty, A_{II}^{\infty*} = A_I.$$

Theorem 4 ([2] Theorem 3.6, [4] Theorem 3.3).

The relations $N(A_I) = \{0\}$, $N(A_I^\infty) = \text{span}\{a_1, a_2, a_3\}$ hold true.

The proof of the fact that the equation $A_I \varphi = 0$ has only the trivial solution in $L_2(S)$ is given in [4]. Moreover $\text{span}\{a_1, a_2, a_3|_S\} \subseteq N(A_I^\infty)$ easily follows from Betti's integral theorem. The proof of $N(A_I^\infty) = \text{span}\{a_1, a_2, a_3|_S\}$ is contained in [2]. It uses the following

R e g u l a r i t y t h e o r e m : Let A be one of the operators $A_I, A_I^\infty, A_{II}, A_{II}^\infty$ and $\varphi \in L_2(S)$ a solution of $A\varphi = w$. If $w \in C^{0,\beta}(S \setminus K_\varepsilon)$, then $\varphi \in C^{0,\beta}(S \setminus K_\varepsilon)$.

Problem II(D;P) [2]. The equation $A_{II}\varphi = P$ has in $L_2(S)$ a solution if and only if

$$P \in N(A_{II}^*) = N(A_I^\infty) = \text{span}\{a_1, a_2, a_3\},$$

which means

$$(5) \quad (P, a_k) = \int_S P \cdot a_k \, ds = 0, \quad k = 1, 2, 3.$$

The relations (5) are the necessary and sufficient equilibrium conditions.

If in addition $P \in C^{0,\beta}(S \setminus K_\varepsilon)$, then also $\varphi \in C^{0,\beta}(S \setminus K_\varepsilon)$ and $u(x) = V(x; \varphi)$ is a regular solution of the problem.

Problem II(CD;P) [5]. We have $N(A_{II}^{\infty*}) = N(A_I) = \{0\}$. Therefore $A_{II}^\infty \varphi = -P$ has a unique solution $\varphi \in L_2(S)$ for every $P \in L_2(S)$. The potential $V(x; \varphi)$ is regular at infinity if and only if $\varphi \perp a_1, a_2$. Suppose now that $A_{II}^\infty \varphi = -P$. We prove that $\varphi \perp a_1, a_2$ if and only if $P \perp a_1, a_2$. In fact suppose $a \in N(A_I)$. Then from $A_I^\infty a - A_I a = 2a$ it follows that $A_I a = -2a$. Therefore,

$$-(P, a) = (A_{II}^\infty \varphi, a) = (\varphi, A_{II}^{\infty*} a) = (\varphi, A_I a) = -2(\varphi, a).$$

We get that $u(x) = V(x; \varphi)$ is a regular solution of the problem, when $P \in L_2(S) \cap C^{0,\beta}(S \setminus K_\varepsilon)$ and $P \perp a_1, a_2$. The condition $P \perp a_1, a_2$ is also necessary for the existence of a regular solution.

Problem I(D;w) [4].

Theorem 5. Assume $S \subset C_n^{1,\alpha}$, $w \in C^{1,\beta}(S)$, $0 < \beta < \alpha \leq 1$. Then the problem I(D;w) has a unique regular solution of the form

$$u(x) = -\frac{1}{2} W(x; w) + \frac{1}{2} V(x; \psi),$$

where $A_I^* \psi = f$, $f = \{T(n_x)W(x; w)\}^+$.

P r o o f . From Theorem 2 it follows that $f \in L_2(S) \cap C^{0,\beta}(S \setminus K_\varepsilon)$. Therefore $A_I^* \psi = f$ has a unique solution $\psi \in L_2(S) \cap C^{0,\beta}(S \setminus K_\varepsilon)$ and $u(x)$ is regularly in D . From the jump properties of the potentials it follows that

$$(6) \quad \{u\}^+ - \{u\}^- = w$$

and

$$(7) \quad \begin{aligned} \{T(n_x)u\}^- &= -\frac{1}{2} \{T(n_x)W(x;w)\}^- + \frac{1}{2} A_{II}^\infty \psi = \\ &= -\frac{1}{2} f + \frac{1}{2} A_I^* \psi = 0. \end{aligned}$$

Since $W(x;w)$ is regular in D , we have $\text{fl} a_1, a_2, a_3$. Let $\varphi \in \text{span}\{a_1, a_2, a_3\} = N(A_I^\infty)$, $A_I^* \psi = f$. From $A_{II} \psi - A_{II}^\infty \psi = 2\psi$ we obtain $A_{II} \psi - f = 2\psi$ and

$$2(\psi, \varphi) = (A_{II} \psi - f, \varphi) = (A_{II} \psi, \varphi) = (\psi, A_{II}^* \varphi) = (\psi, A_I^\infty \varphi) = 0.$$

So, $u(x)$ is regular at infinity and (7) yields $u(x) = 0$ for $x \in CD$. Therefore $\{u\}^- = 0$ and $\{u\}^+ = w$.

Problem I(CD;w) [5].

L e m m a [5]. There exists a base ψ_1, ψ_2, ψ_3 in $N(A_I^{\infty*})$ with the following properties:

$$V(x; \psi_1), V(x; \psi_2) \in \text{span}\{a_1, a_2\} \text{ for } x \in D,$$

$$V(x; \psi_3) = \alpha_1 a_1 + \alpha_2 a_2 + \alpha_3 a_3, \alpha_3 \neq 0 \text{ for } x \in D,$$

$$(\psi_1, a_k) = \delta_{1k}, i, k = 1, 2, (\psi_3, a_1) = \delta_{31}, 1 = 1, 2, 3.$$

T h e o r e m 6. Let there be $S \subset C_n^{1,\alpha}$, $w \in C^{1,\beta}(S)$, $0 < \beta < \alpha \leq 1$. Then the problem I(CD;w) has a unique solution of the form

$$u(x) = \frac{1}{2} W(x;w) - \frac{1}{2} V(x;\psi) + \eta_1 a_1 + \eta_2 a_2 + \eta_3 V(x;\psi_3)$$

with

$$A_{II} \psi = f, \quad f = \{T(n_x)W(x;w)\}^-, \quad (\psi, a_1) = 0, \quad (\psi, a_2) = 0.$$

P r o o f . $W(x;w)$ is regular in D , hence $\{T(n_x)W(x;w)\}^+ = f|a_1, a_2, a_3$. Therefore $A_{II}\psi = f$ has a solution $\hat{\psi}$ and $\psi = \hat{\psi} - (\hat{\psi}, a_1)\psi_1 - (\hat{\psi}, a_2)\psi_2$ is solution of $A_{II}\psi = f$ with $(\psi, a_1) = 0$, $(\psi, a_2) = 0$. This implies that $u(x)$ is regular in CD . Let us consider $u^*(x) = \frac{1}{2} W(x;w) - \frac{1}{2} V(x;\psi)$, then

$$(8) \quad \{u^*\}^- - \{u^*\}^+ = w$$

and

$$\{T(n_x)u^*(x)\}^+ = \frac{1}{2} \{T(n_x)W(x;w)\}^+ - \frac{1}{2} A_{II}\psi = \frac{1}{2} f - \frac{1}{2} A_{II}\psi = 0.$$

By the uniqueness theorem

$$u^*(x) = x_1 a_2 + x_2 a_2 + x_3 a_3 \text{ in } D,$$

and from (8) we get

$$\{u^*(x)\}^- = w + x_1 a_1 + x_2 a_2 + x_3 a_3.$$

Therefore

$$\{u(x)\}^- = w + x_1 a_1 + x_2 a_2 + x_3 a_3 + \eta_1 a_1 + \eta_2 a_2 + \eta_3 (\alpha_1 a_1 + \alpha_2 a_2 + \alpha_3 a_3) = w,$$

if η_1, η_2, η_3 are chosen such that

$$x_1 + \eta_1 + \eta_3 \alpha_1 = 0, \quad x_2 + \eta_2 + \eta_3 \alpha_2 = 0, \quad x_3 + \eta_3 \alpha_3 = 0.$$

Since $\alpha_3 \neq 0$, the constants η_1, η_2, η_3 are uniquely determined.

R e m a r k . If S is not vertical to the x_1 -axis, then the integrals with the fixed singularities have the form

$$\int_{-1}^{+1} \frac{(1+\tau)^n (2\tau+\tau\tau)^{2m-1-n}}{[-a(1+\tau)+b(1+\tau\tau)]^2 + (2\tau+\tau\tau)^2}^m u(\tau) d\tau, \quad n \leq 2m-1,$$

where a, b are the cotangens of the angles between S^+ , S^- and the x_1 -axis. Equations of this kind can be reduced to operators of the kind

$$Bu(x) = c_0 u(x) + \frac{c_1}{\pi i} \int_0^1 \frac{u(y)}{y-x} dy + \\ + \sum_{k=1}^m \sum_{n_k=0}^{2k-1} \frac{c_k^{(n_k)}}{\pi i} \int_0^1 \frac{x^{n_k} y^{2k-1-n_k}}{[(-ax+by)^2 + (x+y)^2]^k} u(y) dy.$$

The symbol of B as operator in $L_p([0,1], x^\alpha)$, $1 < p < \infty$,

$-1 < \alpha < p-1$, $\delta = \frac{1}{p}$, $\beta = \frac{1+\alpha}{p}$ is given by

$$A_{\beta, \delta}(y, \xi) = \\ = \begin{cases} c_0 + c_1 \coth \pi(i\beta + y) + \sum_{k=1}^m \sum_{n_k=0}^{2k-1} \frac{c_k^{(n_k)}}{\pi i} m_{k, \beta}^{(n_k)}(y) & \text{for } -\infty < y \leq +\infty, \\ & \xi = -\infty \\ c_0 - c_1 \coth \pi(i\delta + \xi) & \text{for } y = \infty, -\infty < \xi \leq +\infty \end{cases}$$

with

$$m_{k, \beta}^{(n_k)}(y) = \int_{-\infty}^{+\infty} e^{iyt} \frac{e^{-(\beta+n_k)t}}{[(1+e^{-t})^2 + (b-ae^{-t})^2]^k} dt = \\ = \sqrt{1+a^2}^{iy-(\beta+n_k)} \bar{\xi}^{(\beta+n_k-iy-k)-\zeta k} \frac{\Gamma(\beta+n_k-iy)\Gamma(2k-\beta-n_k+iy)}{\Gamma(2k)} \cdot \\ \cdot {}_2F_1(k, \beta+n_k-iy; 2k; 1-e^{\bar{\xi}-\zeta}),$$

$$D = \frac{1}{\sqrt{1+a^2}} [1-ab+|a+b|i], \quad D = e^{\bar{\xi}}, \quad 0 \leq \operatorname{Im} \zeta < \pi.$$

T h e o r e m [6]. B is a ϕ -operator in $L_p([0,1], x^\alpha)$ if and only if $A_{\beta,\delta}(y,\xi) \neq 0$ for all y, ξ . If this conditions is fulfilled, then $\text{Ind } B = -\text{ind } A_{\beta,\delta}$.

In the case $a = b = 0$ the results of Duduchava

$$m_{k,\beta}^{(n_k)}(y) = \binom{\beta+n_k-1}{2k-1} (-1)^{2k-1} \frac{1}{\sinh \pi(y+(\beta+n_k)i)}$$

follows.

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Received December 6, 1984.