DEMONSTRATIO MATHEMATICA
Vol. XXXIII No 4 2000

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ON A WEAK FORM OF ALMOST WEAKLY CONTINUOUS FUNCTIONS

Abstract. A weak form of almost weak continuity, called subalmost weak continuity, is introduced. It is shown that subalmost weak continuity is strictly weaker than both almost weak continuity and subweak continuity. Subalmost weak continuity is used to improve a result in the literature concerning the graph of an almost weakly continuous function. Additional properties of these functions are also investigated.

1. Introduction

Janković [2] introduced the notion of almost weak continuity as a weak form of both weak continuity developed by Levine [4] and almost continuity introduced by Husain [1]. Recently the almost continuity due to Husain has been referred to as precontinuity. Almost weakly continuous functions were developed further by Popa and Noiri [11]. They were investigated recently by Paul and Bhattacharyya [10] under the name of quasi-precontinuity. Rose [12] introduced the concept of subweak continuity and showed that this condition is strictly weaker than weak continuity. The purpose of this note is to introduce the concept of subalmost weak continuity. We show that this condition is strictly weaker than both almost weak continuity and subweak continuity. We also establish that the graph of a subalmost weakly continuous function with a Hausdorff codomain is preclosed. This extends the corresponding result for almost weakly continuous functions in Popa and Noiri [11]. Finally we establish additional properties of subalmost weak continuity. For example, we show that the restriction of a subalmost weakly continuous function to a semi-open set is subalmost weakly continuous.

1991 *Mathematics Subject Classification*: 54C10.

Key words and phrases: subalmost weak continuity, almost weak continuity, subweak continuity.

2. Preliminaries

The symbols X and Y denote topological spaces with no separation axioms assumed unless explicitly stated. All sets are considered to be subsets of topological spaces.The closure and interior of a set A are signified by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A set A is preopen (α-open, semi-open) provided that $A \subseteq \mathrm{Int}(\mathrm{Cl}(A))$ ($A \subseteq \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$, $A \subseteq \mathrm{Cl}(\mathrm{Int}(A))$). A set A is preclosed (α-closed, semi-closed) if its complement is preopen (α-open, semi-open). Obviously A is preclosed if and only if $\mathrm{Cl}(\mathrm{Int}(A)) \subseteq A$. The preclosure of A, denoted by $\mathrm{pCl}(A)$, is the intersection of all preclosed sets containing A and it can be shown that $\mathrm{pCl}(A) = A \cup \mathrm{Cl}(\mathrm{Int}(A))$. Finally, if an operator is used with respect to a proper subspace, then a subscript will be added to the operator. Otherwise it is assumed that the operator refers to the space X or Y.

Definition 1. Levine [4]. A function $f : X \to Y$ is said to be weakly continuous if, for every $x \in X$ and every neighborhood V of $f(x)$, there is a neighborhood U of x such that $f(U) \subseteq \mathrm{Cl}(V)$.

Definition 2. Janković [2]. A function $f : X \to Y$ is said to be almost weakly continuous if $f^{-1}(V) \subseteq \mathrm{Int}(\mathrm{Cl}(f^{-1}(\mathrm{Cl}(V))))$ for every open subset V of Y.

Definition 3. Rose [12]. A function $f : X \to Y$ is said to be subweakly continuous if there is an open base $\mathcal{B}$ for the topology on Y such that $\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(\mathrm{Cl}(V))$ for every $V \in \mathcal{B}$.

Definition 4. A function $f : X \to Y$ is said to be precontinuous (Mashhour et al. [6]) (semi-continuous (Levine [5]), α-continuous (Popa and Noiri [11])) if $f^{-1}(V)$ is preopen (semi-open, α-open) for every open subset V of Y.

3. Subalmost weakly continuous functions

Popa and Noiri [11] established the following characterization of almost weak continuity.

Theorem 1. Popa and Noiri [11]. *A function $f : X \to Y$ is almost weakly continuous if and only if* $\mathrm{pCl}(f^{-1}(V)) \subseteq f^{-1}(\mathrm{Cl}(V))$ *for every open subset V of Y.*

We define a function $f : X \to Y$ to be subalmost weakly continuous (or briefly s.a.w.c.) provided there is an open base $\mathcal{B}$ for the topology on Y for which $\mathrm{pCl}(f^{-1}(V)) \subseteq f^{-1}(\mathrm{Cl}(V))$ for every $V \in \mathcal{B}$. Obviously almost weak continuity implies s.a.w.c. The following example shows that the concepts are not equivalent.

EXAMPLE 1. Let X be a nondiscrete T_1-space and let $Y = X$ have the discrete topology. The identity mapping $f : X \to Y$ is s.a.w.c. with respect to the base $\mathcal{B}$ consisting of the singleton subsets of Y. However, since X is nondiscrete T_1, X has a subset V that is open but not closed. Then we see that $f^{-1}(\mathrm{Cl}(V)) = f^{-1}(V) = V$, but $\mathrm{pCl}(f^{-1}(V)) = \mathrm{Cl}(V) \not\subseteq V$ and hence f is not almost weakly continuous.

Since $\mathrm{pCl}(A) \subseteq \mathrm{Cl}(A)$ for every set A, obviously subweakly continuous implies s.a.w.c. The next example shows that the converse implication does not hold.

EXAMPLE 2. Let X be an indiscrete space with at least two elements and let $Y = X$ have the discrete topology. Since $\mathrm{pCl}(\{x\}) = \{x\}$ for every $x \in X$, the identity mapping $f : X \to Y$ is s.a.w.c. with respect to the base of all singleton subsets of Y. However, since every singleton set in X is dense, f is not subweakly continuous.

Since $\mathrm{pCl}(A) = A \cup \mathrm{Cl}(\mathrm{Int}(A))$ for every set A, we have the following characterization of s.a.w.c. functions.

THEOREM 2. *A function $f : X \to Y$ is s.a.w.c. if and only if there is an open base $\mathcal{B}$ for Y for which $\mathrm{Cl}(\mathrm{Int}(f^{-1}(V))) \subseteq f^{-1}(\mathrm{Cl}(V))$ for every $V \in \mathcal{B}$.*

Paul and Bhattacharyya [10] proved that an almost weakly continuous, semi-continuous function is weakly continuous. (They use the name quasi-precontinuous for almost weakly continuous.) In the following theorem we show that the analogous result holds for s.a.w.c. functions. The proof follows from that of Paul and Bhattacharyya.

THEOREM 3. *If $f : X \to Y$ is s.a.w.c. and semi-continuous, then f is subweakly continuous.*

Proof. Let $\mathcal{B}$ be an open base for Y such that $\mathrm{pCl}(f^{-1}(V)) \subseteq f^{-1}(\mathrm{Cl}(V))$ for every $V \in \mathcal{B}$. Then for $V \in \mathcal{B}$, it follows from Theorem 2 that $\mathrm{Cl}(\mathrm{Int}(f^{-1}(V))) \subseteq f^{-1}(\mathrm{Cl}(V))$. Since f is semi-continuous, $f^{-1}(V)$ is semi-open and hence $\mathrm{Cl}(f^{-1}(V)) = \mathrm{Cl}(\mathrm{Int}(f^{-1}(V)))$. Therefore $\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(\mathrm{Cl}(V))$ and hence f is subweakly continuous. □

4. Graph related properties

By the graph of a function $f : X \to Y$ we mean the set $G(f) = \{(x, y) : x \in X, y = f(x)\}$.

Popa and Noiri [11] proved that the graph of an almost weakly continuous function with a Hausdorff codomain is preclosed. We show that almost weak continuity can be replaced with s.a.w.c. in this result.

THEOREM 4. *If $f : X \to Y$ is s.a.w.c. and Y is Hausdorff, then the graph of f, $G(f)$, is preclosed in $X \times Y$.*

Proof. Let $(x,y) \in X \times Y - G(f)$. Then $y \neq f(x)$. Let $\mathcal{B}$ be an open base for the topology on Y for which $\mathrm{pCl}(f^{-1}(V)) \subseteq f^{-1}(\mathrm{Cl}(V))$ for every $V \in \mathcal{B}$. Since Y is Hausdorff, there exist disjoint open subsets V and W in Y with $y \in V$, $f(x) \in W$, and $V \in \mathcal{B}$. Then $f(x) \notin \mathrm{Cl}(V)$ and hence $x \notin f^{-1}(\mathrm{Cl}(V))$. Because f is s.a.w.c., $\mathrm{pCl}(f^{-1}(V)) \subseteq f^{-1}(\mathrm{Cl}(V))$ and hence $x \notin \mathrm{pCl}(f^{-1}(V))$. Then we see that $(x,y) \in (X - \mathrm{pCl}(f^{-1}(V))) \times V \subseteq X \times Y - G(f)$. Since $X - \mathrm{pCl}(f^{-1}(V))$ is preopen and V is open, it follows from Nasef and Noiri [8] (Lemma 3.1) that $(X - \mathrm{pCl}(f^{-1}(V))) \times V$ is preopen and hence $G(f)$ is preclosed. □

COROLLARY 1. Popa and Noiri [11]. *If $f : X \to Y$ is almost weakly continuous and Y is Hausdorff, then $G(f)$ is preclosed.*

For a function $f : X \to Y$, the graph function of f is the function $g : X \to X \times Y$ given by $g(x) = (x, f(x))$ for every $x \in X$.

THEOREM 5. *If $f : X \to Y$ is s.a.w.c., then the graph function $g : X \to X \times Y$ is s.a.w.c.*

Proof. Let $\mathcal{B}$ be an open base for Y for which $\mathrm{pCl}(f^{-1}(V)) \subseteq f^{-1}(\mathrm{Cl}(V))$ for every $V \in \mathcal{B}$. Then $\mathcal{C} = \{U \times V : U \subseteq X \text{ is open and } V \in \mathcal{B}\}$ is an open base for the product topology on $X \times Y$. For $U \times V \in \mathcal{C}$, we have $\mathrm{pCl}(g^{-1}(U \times V)) = \mathrm{pCl}(U \cap f^{-1}(V)) \subseteq \mathrm{pCl}(U) \cap \mathrm{pCl}(f^{-1}(V)) = \mathrm{Cl}(U) \cap \mathrm{pCl}(f^{-1}(V)) \subseteq \mathrm{Cl}(U) \cap f^{-1}(\mathrm{Cl}(V)) = g^{-1}(\mathrm{Cl}(U) \times \mathrm{Cl}(V)) = g^{-1}(\mathrm{Cl}(U \times V))$. Thus the graph function g is s.a.w.c. □

THEOREM 6. *Let $f : (X, \mathcal{T}) \to Y$ be a function and $g : X \to X \times Y$ its graph function. Let $\mathcal{B}$ be an open base for Y. If g is s.a.w.c. with respect to the base $\mathcal{T} \times \mathcal{B}$ for $X \times Y$, then f is s.a.w.c.*

Proof. For $V \in \mathcal{B}$, $\mathrm{pCl}(f^{-1}(V)) = \mathrm{pCl}(g^{-1}(X \times V)) \subseteq g^{-1}(\mathrm{Cl}(X \times V)) = g^{-1}(X \times \mathrm{Cl}(V)) = f^{-1}(\mathrm{Cl}(V))$ and hence f is s.a.w.c. □

5. Additional properties

The following generalizations of T_1- and T_2-spaces will be required.

DEFINITION 5. Kar and Bhattacharyya [3]. A space X is said to be pre-T_1 provided that for each pair of distinct points x, y there exists a pair of preopen sets, one containing x but not y and the other containing y but not x.

DEFINITION 6. Kar and Bhattacharyya [3]. A space X is said to be Pre-T_2 provided that for each pair of distinct points x, y there exists a pair of disjoint preopen sets, one containing x and the other containing y.

Paul and Bhattacharyya [10] (Theorem 5.1) proved that, if $f : X \to Y$ is almost weakly continuous (their term is quasi-precontinuous) and injective and Y is Urysohn, then X is pre-T_2. The following example shows that almost weak continuity cannot be replaced by s.a.w.c. in this result.

EXAMPLE 3. Let $X = E \cup \{x_1, x_2\}$, where E is an infinite set and x_1 and x_2 are distinct points not in E, have the topology $\mathcal{T}$ given by (*i*) $U \in \mathcal{T}$ if $U \subseteq E$ and (*ii*) $U \in \mathcal{T}$ if x_1 or $x_2 \in U$ and $X - U$ contains only a finite number of elements of E. Kar and Bhattacharyya [3] established that $\mathcal{T}$ is indeed a topology and that X is pre-T_1 but not pre-T_2. Let $Y = X$ have the indiscrete topology and let $f : X \to Y$ be the identity mapping. The space Y is obviously Urysohn. Since X is pre-T_1, the singleton subsets of X are preclosed (Kar and Bhattacharyya [3]). It then follows that f is s.a.w.c. with respect to the base for Y consisting of all singleton subsets. Therefore f is a s.a.w.c. injection with an Urysohn codomain and a non pre-T_2 domain.

We do, however, have the following weaker result for s.a.w.c. functions.

THEOREM 7. *If $f : X \to Y$ is s.a.w.c., injective, and Y is Hausdorff, then X is pre-T_1.*

Proof. Let x_1 and x_2 be distinct points in X. Since f is injective, $f(x_1) \neq f(x_2)$. Let $\mathcal{B}$ be an open base for Y for which $\mathrm{pCl}(f^{-1}(V)) \subseteq f^{-1}(\mathrm{Cl}(V))$ for every $V \in \mathcal{B}$. Since Y is Hausdorff, there exist disjoint open sets U and V such that $f(x_1) \in U$ and $f(x_2) \in V$ and $V \in \mathcal{B}$. Then $f(x_1) \notin \mathrm{Cl}(V)$ and hence $x_1 \notin f^{-1}(\mathrm{Cl}(V))$. Since f is s.a.w.c., it follows that $X - \mathrm{pCl}(f^{-1}(V))$ is a preopen set containing x_1 but not x_2. Therefore X is pre-T_1. $\square$

The function in Example 2 is s.a.w.c. and injective and has an indiscrete domain and a discrete codomain. Therefore the pre-T_1 condition in Theorem 7 cannot be replaced by T_1.

As we see in the following example, the restriction of a s.a.w.c. function may not be s.a.w.c.

EXAMPLE 4. Let $X = \{a, b, c, d\}$ have the topology $\mathcal{T} = \{X, \emptyset, \{a, b\}\}$ and let $Y = X$ have the discrete topology. Since the singleton subsets of X are preclosed (Kar and Bhattacharyya [3]), the identity mapping $f : X \to Y$ is s.a.w.c. with respect to the base for Y consisting of singleton sets. However, if $A = \{a, c\}$, then $f|_A : A \to Y$ fails to be s.a.w.c.

Noiri [9] showed that the restriction of an almost weakly continuous function to an open set is almost weakly continuous. Later Popa and Noiri

[11] extended this result to semi-open sets. In what follows we show that the restriction of a s.a.w.c. function to a semi-open set is s.a.w.c. The following lemmas are required.

LEMMA 1. Mashhour et al. [7]. *If U is preopen in X and A is semi-open in X, then $U \cap A$ is preopen in A.*

Combining Lemma 1 with the fact that, for every set A, $x \in \text{pCl}(A)$ if and only if every preopen set containing x intersects A nontrivially (Popa and Noiri [11]) yields an immediate proof of the next lemma.

LEMMA 2. *If $B \subseteq A \subseteq X$ and A is semi-open in X, then $\text{pCl}_A(B) \subseteq \text{pCl}(B)$.*

THEOREM 8. *If $f : X \to Y$ is s.a.w.c. and A is a semi-open subset of X, then $f|_A : A \to Y$ is s.a.w.c.*

Proof. Let $\mathcal{B}$ be an open base for Y for which $\text{pCl}(f^{-1}(V)) \subseteq f^{-1}(\text{Cl}(V))$ for every $V \in \mathcal{B}$. Then using Lemma 2 we have for $V \in \mathcal{B}$, $\text{pCl}_A(f|_A{}^{-1}(V)) \subseteq A \cap \text{pCl}(f|_A{}^{-1}(V)) = A \cap \text{pCl}(A \cap f^{-1}(V)) \subseteq A \cap (\text{pCl}(A) \cap \text{pCl}(f^{-1}(V))) = A \cap \text{pCl}(f^{-1}(V)) \subseteq A \cap f^{-1}(\text{Cl}(V)) = f|_A{}^{-1}(\text{Cl}(V))$. Therefore $f|_A : A \to Y$ is s.a.w.c. □

THEOREM 9. *If $f : X \to Y$ is s.a.w.c. and A is an open subset of Y with $f(X) \subseteq A$, then $f : X \to A$ is s.a.w.c.*

Proof. Let $\mathcal{B}$ be an open base for Y for which $\text{pCl}(f^{-1}(V)) \subseteq f^{-1}(\text{Cl}(V))$ for every $V \in \mathcal{B}$. Then $\mathcal{C} = \{V \cap A : V \in \mathcal{B}\}$ is an open base for the relative topology on A. For $V \cap A \in \mathcal{C}$, we have $\text{pCl}(f^{-1}(V \cap A)) = \text{pCl}(f^{-1}(V)) \subseteq f^{-1}(\text{Cl}(V) = f^{-1}(\text{Cl}(V) \cap A)$. In what follows we show that $\text{Cl}(V) \cap A \subseteq \text{Cl}_A(V \cap A)$.

Assume $y \in \text{Cl}(V) \cap A$. Let $W \subseteq A$ be open in the relative topology on A with $y \in W$. Since A is open in Y, W is open in Y. Because $y \in \text{Cl}(V)$, $V \cap W \neq \emptyset$. Then $W \cap (V \cap A) = W \cap V \neq \emptyset$ and hence $y \in \text{Cl}_A(V \cap A)$. Thus $\text{Cl}(V) \cap A \subseteq \text{Cl}_A(V \cap A)$ and it then follows that $\text{pCl}(f^{-1}(V \cap A)) \subseteq f^{-1}(\text{Cl}(V) \cap A) \subseteq f^{-1}(\text{Cl}_A(V \cap A))$ and hence $f : X \to A$ is s.a.w.c. □

Paul and Bhattacharyya [10] defined an almost weakly continuous retraction (their term is quasi-precontinuous retraction) to be an almost weakly continuous mapping $f : X \to A$, where $A \subseteq X$ and $f|_A$ is the identity mapping on A. It is then proved (Theorem 5.5) that, if $f : X \to A$ is an almost weakly continuous retraction and X is Hausdorff, then A is preclosed. We prove the following comparable result for s.a.w.c. functions.

THEOREM 10. *Let $f : X \to X$ be s.a.w.c. and let $A \subseteq X$ such that $f(X) \subseteq A$ and $f|_A$ is the identity on A. Then, if X is Hausdorff, A is preclosed.*

Proof. Assume A is not preclosed. Let $x \in \mathrm{pCl}(A) - A$. Let $\mathcal{B}$ be an open base for X for which $\mathrm{pCl}(f^{-1}(V)) \subseteq f^{-1}(\mathrm{Cl}(V))$ for every $V \in \mathcal{B}$. Since $x \notin A$, $f(x) \neq x$. Since X is Hausdorff, there exist disjoint open sets V and W with $x \in V$, $f(x) \in W$, and $V \in \mathcal{B}$. Let U be any preopen subset of X with $x \in U$. Then $x \in U \cap V$ which is preopen (Mashhour et al. [6]). Because $x \in \mathrm{pCl}(A)$, it follows from Popa and Noiri [11] that $(U \cap V) \cap A \neq \emptyset$. Let $y \in (U \cap V) \cap A$. Since $y \in A$, $f(y) = y \in V$ and hence $y \in f^{-1}(V)$. Thus $y \in U \cap f^{-1}(V)$ and therefore $U \cap f^{-1}(V) \neq \emptyset$ and we have that $x \in \mathrm{pCl}(f^{-1}(V))$. However, $f(x) \in W$ which is open and disjoint from V. So $f(x) \notin \mathrm{Cl}(V)$ or, that is, $x \notin f^{-1}(\mathrm{Cl}(V))$, which contradicts the assumption that f is s.a.w.c. Therefore A is preclosed. □

THEOREM 11. *If $f_1 : X \to Y$ is α-continuous, $f_2 : X \to Y$ is s.a.w.c., and Y is Hausdorff, then the set $A = \{x \in X : f_1(x) = f_2(x)\}$ is preclosed.*

Proof. Let $x \in X - A$. Then $f_1(x) \neq f_2(x)$. Let $\mathcal{B}$ be an open base for Y for which $\mathrm{pCl}({f_2}^{-1}(V)) \subseteq {f_2}^{-1}(\mathrm{Cl}(V))$ for every $V \in \mathcal{B}$. Since Y is Hausdorff, there exist disjoint open sets V and W in Y with $f_1(x) \in V$, $f_2(x) \in W$, and $V \in \mathcal{B}$. Then $f_2(x) \notin \mathrm{Cl}(V)$ and hence $x \notin {f_2}^{-1}(\mathrm{Cl}(V))$. Then, since f_2 is s.a.w.c., $x \in X - {f_2}^{-1}(\mathrm{Cl}(V)) \subseteq X - \mathrm{pCl}({f_2}^{-1}(V))$. Thus $x \in {f_1}^{-1}(V) \cap (X - \mathrm{pCl}({f_2}^{-1}(V))) \subseteq X - A$. Since ${f_1}^{-1}(V)$ is α-open and $X - \mathrm{pCl}({f_2}^{-1}(V))$ is preopen, the intersection is preopen (Popa and Noiri [11], Lemma 4.1). It then follows that A is preclosed. □

COROLLARY 2. *Assume that $f_1 : X \to Y$ is α-continuous, $f_2 : X \to Y$ is s.a.w.c., and Y is Hausdorff. If f_1 and f_2 agree on an open dense set, then $f_1 = f_2$.*

Proof. Let $A = \{x \in X : f_1(x) = f_2(x)\}$ and let D be an open dense subset of X on which f_1 and f_2 agree. Then, since $D \subseteq A$, we see that $X = \mathrm{Cl}(D) = \mathrm{pCl}(D) \subseteq \mathrm{pCl}(A) = A$. It follows that $f_1 = f_2$. □

THEOREM 12. *If $f_\alpha : X \to Y_\alpha$ is s.a.w.c. for every $\alpha \in \mathcal{A}$, then $f : X \to \prod_{\alpha \in \mathcal{A}} Y_\alpha$ given by $f(x) = (f_\alpha(x))$ is s.a.w.c.*

Proof. For each $\alpha \in \mathcal{A}$, let $\mathcal{B}_\alpha$ be an open base for Y_α for which $\mathrm{pCl}({f_\alpha}^{-1}(V_\alpha)) \subseteq {f_\alpha}^{-1}(\mathrm{Cl}(V_\alpha))$ for every $V_\alpha \in \mathcal{B}_\alpha$. Then $\mathcal{B} = \{\prod_{\alpha \in \mathcal{A}} V_\alpha :$ $V_\alpha = Y_\alpha$ for all but finitely many coordinates and, if $V_\alpha \neq Y_\alpha$, then $V_\alpha \in \mathcal{B}_\alpha\}$ is an open base for the product topology on $\prod_{\alpha \in \mathcal{A}} Y_\alpha$. For $\prod_{\alpha \in \mathcal{A}} V_\alpha \in \mathcal{B}$, $\mathrm{pCl}(f^{-1}(\prod_{\alpha \in \mathcal{A}} V_\alpha)) = \mathrm{pCl}(\cap_{\alpha \in \mathcal{A}} {f_\alpha}^{-1}(V_\alpha)) \subseteq \cap_{\alpha \in \mathcal{A}} \mathrm{pCl}({f_\alpha}^{-1}(V_\alpha)) \subseteq \cap_{\alpha \in \mathcal{A}} {f_\alpha}^{-1}(\mathrm{Cl}(V_\alpha)) = f^{-1}(\prod_{\alpha \in \mathcal{A}} \mathrm{Cl}(V_\alpha)) = f^{-1}(\mathrm{Cl}(\prod_{\alpha \in \mathcal{A}} V_\alpha))$. Thus f is s.a.w.c. □

The author gratefully acknowledges the assistance of the referee.

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Received February 16, 2000.