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ON $\gamma\alpha$ -CONTINUOUS FUNCTIONS

Abstract. The aim of this paper is to introduce and study $\gamma\alpha$ -continuous functions as a generalization of preirresoluteness, α -precontinuity, α -irresoluteness, γ -irresoluteness, irresoluteness and semi α -irresoluteness. Furthermore, basic characterizations, preservation theorems and several properties concerning $\gamma\alpha$ -continuous functions are investigated. The relationships between $\gamma\alpha$ -continuous functions and the other types of continuity are also discussed.

1. Introduction

Many authors studied several stronger and weaker forms of continuity. It is well known that continuity is one of the basic topics in General Topology and in Analysis. Functions and continuity stand among the most important and most researched points in the whole of the Mathematical Science. Many different forms of continuous functions have been introduced over the years. Some of them are preirresoluteness [15], α -precontinuity [3], α -irresoluteness [8], γ -irresoluteness, irresoluteness [4] and semi α -irresoluteness [2].

The purpose of this paper is to give a new weaker form of some types of continuity including preirresoluteness, α -precontinuity, α -irresoluteness, γ -irresoluteness, irresoluteness and semi α -irresoluteness. In this paper, $\gamma\alpha$ -continuity is introduced and studied. Moreover, basic properties and preservation theorems of $\gamma\alpha$ -continuous functions are investigated and relationships between $\gamma\alpha$ -continuous functions and the other types of continuity are investigated.

2. Preliminaries

In the present paper, spaces X and Y always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X . For a subset A of X , $cl(A)$ and $int(A)$ represent the closure of A and the interior of A , respectively.

A subset A of a space X is said to be regular open (resp. regular closed) if $A = \text{int}(cl(A))$ (resp. $A = cl(\text{int}(A))$) [16].

A subset A of a space X is said to be α -open [14] (resp. semi-open [7], preopen [10], b -open [1] or γ -open [6] or sp -open [5]) if $A \subset \text{int}(cl(\text{int}(A)))$ (resp. $A \subset cl(\text{int}(A))$, $A \subset \text{int}(cl(A))$, $A \subset cl(\text{int}(A)) \cup \text{int}(cl(A))$).

It is shown in [14] that $\tau^\alpha = \alpha O(X)$ is a topology for X .

The complement of an α -open set is said to be α -closed [11].

The complement of a γ -open set is said to be γ -closed [6]. The intersection of all γ -closed sets of X containing A is called the γ -closure [6] of A and is denoted by $\gamma cl(A)$. The union of all γ -open sets of X contained A is called γ -interior of A and is denoted by $\gamma\text{-int}(A)$. A subset U of X is called a γ -neighborhood of a point $x \in X$ if there exists a γ -open set V such that $x \in V \subset U$.

The family of all α -open (resp. α -closed, γ -open, γ -closed) sets of X is denoted by $\alpha O(X)$ (resp. $\alpha C(X)$, $\gamma O(X)$, $\gamma C(X)$).

Let (X, τ) be a topological space and (x_α) a net in X . It is said that the net (x_α) is γ -convergent (resp. α -convergent) to x if for each γ -open (resp. α -open) set G containing x in X , there exists an index $\alpha_0 \in I$ such that $x_\alpha \in G$ for each $\alpha \geq \alpha_0$. It is said that a filter base Ψ is γ -convergent (resp. α -convergent) to a point x if for any $U \in \gamma O(X)$ (resp. $U \in \alpha O(X)$) containing x , there exists a $B \in \Psi$ such that $B \subset U$.

DEFINITION 1. A function $f : X \rightarrow Y$ is said to be

- (1) γ -continuous [6] if $f^{-1}(V)$ is γ -open for each open subset V of Y ,
- (2) preirresolute [15] (resp. α -irresolute [8], γ -irresolute, irresolute [4], semi α -irresolute [2], α -precontinuous [3]) if $f^{-1}(V)$ is preopen (resp. α -open, γ -open, semiopen, semiopen, preopen) for each preopen (resp. α -open, γ -open, semiopen, α -open, α -open) subset V of Y .

3. $\gamma\alpha$ -continuous functions

DEFINITION 2. A function $f : X \rightarrow Y$ is called $\gamma\alpha$ -continuous at a point $x \in X$ if each α -open set V containing $f(x)$ in Y , then there exists a γ -open set U of X such that $x \in U$ and $f(U) \subset V$ and f is called $\gamma\alpha$ -continuous if it has this property at each point of X .

THEOREM 3. For a function $f : (X, \tau) \rightarrow (Y, \nu)$, the following statements are equivalent:

- (1) f is $\gamma\alpha$ -continuous,
- (2) $f^{-1}(V)$ is γ -open for every α -open set V in Y ,
- (3) $f^{-1}(V)$ is γ -closed for every α -closed set K in Y ,
- (4) $f : (X, \tau) \rightarrow (Y, \nu^\alpha)$ is γ -continuous.

Proof. (1) $\Rightarrow$ (2): Let V be an α -open set and $x \in f^{-1}(V)$. Since $f(x) \in V$ and f is $\gamma\alpha$ -continuous, there exists a γ -open set U in X containing x such that $f(U) \subset V$. It follows that $x \in U \subset f^{-1}(V)$. Hence $f^{-1}(V)$ is γ -open.

(2) $\Rightarrow$ (1): Let $x \in X$ and V be an α -open set containing $f(x)$. By (2), $f^{-1}(V)$ is a γ -open set containing x . Take $U = f^{-1}(V)$. Then $f(U) \subset V$. This shows that f is $\gamma\alpha$ -continuous.

(2) $\Rightarrow$ (3): Let K be any α -closed set of Y . Since $Y \setminus K$ is an α -open set, by (2), it follows that $f^{-1}(Y \setminus K) = X \setminus f^{-1}(K)$ is γ -open. This shows that $f^{-1}(K)$ is γ -closed in X .

(3) $\Rightarrow$ (2): Let V be an α -open set. Then $Y \setminus V$ is an α -closed set. By (3), $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is γ -closed and thus $f^{-1}(V)$ is γ -open.

(1) $\Leftrightarrow$ (4): Obvious. ■

REMARK 4. The following diagram holds:

None of these implications is reversible.

EXAMPLE 5. Let $X = \{a, b, c, d\}$ and $Y = \{p, q, r, s\}$. Let τ and σ be topologies on X and Y , respectively, given by $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, b, c\}\}$ and $\sigma = \{\emptyset, Y, \{p, r\}\}$. Define a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = p$, $f(b) = r$, $f(c) = q$ and $f(d) = s$. Then f is $\gamma\alpha$ -continuous but not γ -irresolute, since $\{q, r, s\}$ is γ -open but $f^{-1}(\{q, r, s\}) = \{b, c, d\}$ is not γ -open in (X, τ) .

EXAMPLE 6. Let $X = \{a, b, c\}$ and $Y = \{-2, -1, 0, 1, 2\}$. Let τ and σ be topologies on X and Y , respectively, given by $\tau = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$ and $\sigma = \{\emptyset, Y, \{-2, -1, 0, 1\}\}$. Define a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = 0$, $f(b) = 1$, $f(c) = 2$. Then f is $\gamma\alpha$ -continuous but not α -precontinuous and α -irresolute, since $\{-2, -1, 0, 1\}$ is open but $f^{-1}(\{-2, -1, 0, 1\}) = \{a, b\}$ is not preopen in (X, τ) .

EXAMPLE 7. Let X, Y, σ be as in the previous example and a topology $\tau = \{\emptyset, X, \{b, c\}\}$ on X . Let f be as in the previous example. Then f is $\gamma\alpha$ -continuous but not semi α -irresolute, since $f^{-1}(\{-2, -1, 0, 1\}) = \{a, b\}$ is not semiopen in (X, τ) .

LEMMA 8. Let A and X_0 be subsets of a space (X, τ) . If $A \in \gamma O(X)$ and $X_0 \in \alpha O(X)$, then $A \cap X_0 \in \gamma O(X_0)$ [1, 6].

THEOREM 9. *If $f : X \rightarrow Y$ is a $\gamma\alpha$ -continuous function and $A \in \alpha O(X)$, then the restriction $f|_A : A \rightarrow Y$ is $\gamma\alpha$ -continuous.*

Proof. Let $G \in \alpha O(Y)$. Since f is a $\gamma\alpha$ -continuous function, then $f^{-1}(G) \in \gamma O(X)$. From the above lemma, it is obtained that $f|_A^{-1}(G) = f^{-1}(G) \cap A \in \gamma O(A)$. Hence, $f|_A$ is a $\gamma\alpha$ -continuous function. ■

LEMMA 10. *Let $A \subset X_0 \subset X$. If $X_0 \in \alpha O(X)$ and $A \in \gamma O(X_0)$, then $A \in \gamma O(X)$ [6].*

THEOREM 11. *Let $f : X \rightarrow Y$ be a function and $\{U_i : i \in I\}$ be an α -open cover of X . If for each $i \in I$, $f|_{U_i}$ is $\gamma\alpha$ -continuous, then $f : X \rightarrow Y$ is $\gamma\alpha$ -continuous.*

Proof. Let G be an α -open set in Y . Since $f|_{U_i}$ is $\gamma\alpha$ -continuous for each $i \in I$, $(f|_{U_i})^{-1}(G) \in \gamma O(U_i)$. Since $U_i \in \alpha O(X)$, by the previous lemma, $(f|_{U_i})^{-1}(G) \in \gamma O(X)$ for each $i \in I$. Then $f^{-1}(G) = \bigcup_{i \in I} [(f|_{U_i})^{-1}(G)] \in \gamma O(X)$. This shows that f is a $\gamma\alpha$ -continuous function. ■

THEOREM 12. *Let $f : X \rightarrow Y$ be a function and $x \in X$. If there exists $U \in \alpha O(X)$ such that $x \in U$ and the restriction of f to U is a $\gamma\alpha$ -continuous function at x , then f is $\gamma\alpha$ -continuous at x .*

Proof. Suppose that $G \in \alpha O(Y)$ containing $f(x)$. Since $f|_U$ is $\gamma\alpha$ -continuous at x , there exists $V \in \gamma O(U)$ containing x such that $f(V) = (f|_U)(V) \subset G$. Since $U \in \alpha O(X)$ containing x , it follows from Lemma 10 that $V \in \gamma O(X)$ containing x . This shows clearly that f is $\gamma\alpha$ -continuous at x . ■

THEOREM 13. *Let (X, τ) and (Y, ν) be topological spaces. If $f : (X, \tau) \rightarrow (Y, \nu)$ is $\gamma\alpha$ -continuous, then for each $x \in X$ and each net (x_α) which is γ -convergent to x , the net $(f(x_\alpha))$ is α -convergent to $f(x)$.*

Proof. Let (x_α) be a net which is γ -convergent to x in X and G be an α -open set containing $f(x)$ in Y . By $\gamma\alpha$ -continuity of f , $f^{-1}(G)$ is a γ -open set containing x . Since (x_α) is γ -convergent to x , it follows that there exists an index $\alpha_0 \in I$ such that $x_\alpha \in f^{-1}(G)$ for each $\alpha \geq \alpha_0$. We obtain that $f(x_\alpha) \in G$ for each $\alpha \geq \alpha_0$. Thus, the net $(f(x_\alpha))$ is α -convergent to $f(x)$. ■

THEOREM 14. *If a function $f : X \rightarrow Y$ is $\gamma\alpha$ -continuous, then for each point $x \in X$ and each filter base Ψ in X which is γ -convergent to x , the filter base $f(\Psi)$ is α -convergent to $f(x)$.*

Proof. Let $x \in X$ and Ψ be any filter base in X which is γ -convergent to x . Since f is $\gamma\alpha$ -continuous, then for any $V \in \alpha O(Y)$ containing $f(x)$, there exists $U \in \gamma O(X)$ containing x such that $f(U) \subset V$. Since Ψ is γ -convergent to x , there exists a $B \in \Psi$ such that $B \subset U$. This means that $f(B) \subset V$ and therefore the filter base $f(\Psi)$ is α -convergent to $f(x)$. ■

THEOREM 15. Let $f : X \rightarrow Y$ be a function and $g : X \rightarrow X \times Y$ the graph function of f , defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is $\gamma\alpha$ -continuous, then f is $\gamma\alpha$ -continuous.

Proof. Let U be an α -open set in Y , then $X \times U$ is an α -open set in $X \times Y$. It follows from Theorem 3 that $f^{-1}(U) = g^{-1}(X \times U) \in \gamma O(X)$. Thus, f is $\gamma\alpha$ -continuous. ■

DEFINITION 16. A function $f : X \rightarrow Y$ is said to be;

1. γ -irresolute if for each $x \in X$ and each $V \in \gamma O(Y, f(x))$, there exists a γ -open set U in X containing x such that $f(U) \subset V$,
2. γ -open if the image of each γ -open set is γ -open.

THEOREM 17. Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be functions.

1. If f is γ -irresolute and g is a $\gamma\alpha$ -continuous function, then $g \circ f : X \rightarrow Z$ is $\gamma\alpha$ -continuous,
2. If f is a γ -open surjection and $g \circ f : X \rightarrow Z$ is $\gamma\alpha$ -continuous, then g is $\gamma\alpha$ -continuous.

DEFINITION 18. A space X is said to be γ -compact [6] (resp. α -compact [9]) if each γ -open (resp. α -open) cover of X has a finite subcover.

THEOREM 19. The surjective $\gamma\alpha$ -continuous image of a γ -compact space is α -compact.

Proof. Let $f : X \rightarrow Y$ be a $\gamma\alpha$ -continuous function from a γ -compact space X onto a space Y . Let $\{G_\alpha : \alpha \in I\}$ be any α -open cover of Y . Since f is $\gamma\alpha$ -continuous, $\{f^{-1}(G_\alpha) : \alpha \in I\}$ is a γ -open cover of X . Since X is γ -compact, then there exists a finite subcover $\{f^{-1}(G_i) : i = 1, 2, \dots, n\}$ of X . Then it follows that $\{G_i : i = 1, 2, \dots, n\}$ is a finite subfamily which cover Y . Hence, Y is α -compact. ■

DEFINITION 20. A space X is said to be γ -connected [6] (resp. α -connected) if X cannot be written as a disjoint union of two non-empty γ -open (resp. α -open) sets.

THEOREM 21. If $f : X \rightarrow Y$ is a $\gamma\alpha$ -continuous surjective function and X is γ -connected space, then Y is an α -connected space.

Proof. Suppose that Y is not an α -connected space. Then Y can be written as a disjoint union of two non-empty α -open sets U and V . Since f is $\gamma\alpha$ -continuous, then $f^{-1}(U)$ and $f^{-1}(V)$ are non-empty disjoint γ -open sets. Thus, we obtain that X is not a γ -connected space. This is a contradiction. ■

DEFINITION 22. A space X is said to be $\gamma-T_1$ if for each pair of distinct points x and y of X , there exist γ -open sets U and V containing x and y , respectively, such that $y \notin U$ and $x \notin V$.

DEFINITION 23. A space X is said to be α - T_1 [12] if for each pair of distinct points x and y of X , there exist α -open sets U and V containing x and y , respectively, such that $y \notin U$ and $x \notin V$.

THEOREM 24. If $f : X \rightarrow Y$ is a $\gamma\alpha$ -continuous injection and Y is α - T_1 , then X is γ - T_1 .

Proof. Suppose that Y is α - T_1 . For any distinct points x and y in X , there exist $V, W \in \alpha O(Y)$ such that $f(x) \in V$, $f(y) \notin V$, $f(x) \notin W$ and $f(y) \in W$. Since f is $\gamma\alpha$ -continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are γ -open subsets of X such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$, $x \notin f^{-1}(W)$ and $y \in f^{-1}(W)$. This shows that X is γ - T_1 . ■

DEFINITION 25. A space X is said to be γ - T_2 if for each pair of distinct points x and y in X , there exist disjoint γ -open sets U and V in X such that $x \in U$ and $y \in V$.

THEOREM 26. If $f : X \rightarrow Y$ is a $\gamma\alpha$ -continuous injection and Y is T_2 , then X is γ - T_2 .

Proof. For any pair of distinct points x and y in X , there exist disjoint open sets U and V in Y such that $f(x) \in U$ and $f(y) \in V$. Since f is $\gamma\alpha$ -continuous, $f^{-1}(U)$ and $f^{-1}(V)$ is γ -open in X containing x and y , respectively. We have $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. This shows that X is γ - T_2 . ■

LEMMA 27. The intersection of an open and a γ -open set is a γ -open set [1].

THEOREM 28. If $f : X \rightarrow Y$ is continuous, $g : X \rightarrow Y$ is $\gamma\alpha$ -continuous and Y is T_2 , then $E = \{x \in X : f(x) = g(x)\}$ is γ -closed in X .

Proof. If $x \in X \setminus E$, then it follows that $f(x) \neq g(x)$. Since Y is T_2 , there exist open sets V and W such that $f(x) \in V$, $g(x) \in W$ and $V \cap W = \emptyset$. Since f is continuous and g is $\gamma\alpha$ -continuous, $f^{-1}(V)$ is an open set and $g^{-1}(W)$ is a γ -open set, respectively. Take $U = f^{-1}(V)$ and $G = g^{-1}(W)$. It is clear that $x \in U$ and $x \in G$. Set $O = U \cap G$. By the previous lemma, O is γ -open in X . Therefore $f(O) \cap g(O) = \emptyset$ and it follows that $x \notin \gamma\text{-cl}(E)$. This shows that E is γ -closed in X . ■

DEFINITION 29. A function $f : X \rightarrow Y$ is said to be:

1. $\gamma\alpha$ -open if $f(U) \in \alpha O(Y)$ for each $U \in \gamma O(X)$,
2. $\gamma\alpha$ -closed if $f(K) \in \alpha C(Y)$ for each $K \in \gamma C(X)$.

DEFINITION 30. A space X is said to be strongly γ -regular if for each γ -closed set F and each point $x \notin F$, there exists disjoint γ -open sets U and V such that $x \in U$ and $F \subset V$.

DEFINITION 31. A space X is said to be strongly α -regular [13] if for each α -closed set F and each point $x \notin F$, there exists disjoint α -open sets U and V in X such that $x \in U$ and $F \subset V$.

THEOREM 32. If f is a $\gamma\alpha$ -continuous $\gamma\alpha$ -open injective function from a strongly γ -regular space X onto a space Y , then Y is a strongly α -regular space.

Proof. Let F be an α -closed subset in Y and let $y \notin F$. Since f is $\gamma\alpha$ -continuous, then $f^{-1}(F)$ is a γ -closed subset such that $f^{-1}(y) = x \notin f^{-1}(F)$. Take $G = f^{-1}(F)$. Since X is strongly γ -regular, then there exists disjoint γ -open sets U and V in X such that $x \in U$ and $G \subset V$. We obtain that $y = f(x) \in f(U)$ and $F = f(G) \subset f(V)$ such that $f(U)$ and $f(V)$ are disjoint α -open sets. This shows that Y is a strongly α -regular space. ■

DEFINITION 33. A space X is said to be strongly γ -normal if for each pair of disjoint γ -closed subsets F and K of X , there exists disjoint γ -open sets G and H such that $F \subset G$ and $K \subset H$.

DEFINITION 34. A space X is said to be strongly α -normal [13] if for each pair of disjoint α -closed subsets F and K of X , there exists disjoint α -open sets G and H such that $F \subset G$ and $K \subset H$.

THEOREM 35. If $f : X \rightarrow Y$ is a $\gamma\alpha$ -continuous $\gamma\alpha$ -open injective function from a strongly γ -normal space X onto a space Y , then Y is strongly α -normal.

Proof. Let F_1 and F_2 be disjoint α -closed subsets in Y . Since f is $\gamma\alpha$ -continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are γ -closed sets. Take $U = f^{-1}(F_1)$ and $V = f^{-1}(F_2)$. We have $U \cap V = \emptyset$. Since X is strongly γ -normal, there exist disjoint γ -open sets A and B such that $U \subset A$ and $V \subset B$. We obtain that $F_1 = f(U) \subset f(A)$ and $F_2 = f(V) \subset f(B)$ such that $f(A)$ and $f(B)$ are disjoint α -open sets. Thus, Y is strongly α -normal. ■

DEFINITION 36. The graph $G(f)$ of a function $f : X \rightarrow Y$ is said to be $\gamma\alpha$ -closed if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist a γ -open set U in X containing x and an α -open V in Y containing y such that $(U \times V) \cap G(f) = \emptyset$.

LEMMA 37. The following properties are equivalent for the graph $G(f)$ of a function $f : X \rightarrow Y$:

- (1) $G(f)$ is $\gamma\alpha$ -closed;
- (2) for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist a γ -open set U in X containing x and an α -open V in Y containing y such that $f(U) \cap V = \emptyset$.

THEOREM 38. If $f : X \rightarrow Y$ is $\gamma\alpha$ -continuous and Y is T_2 , $G(f)$ is $\gamma\alpha$ -closed in $X \times Y$.

Proof. Suppose that Y is T_2 . Let $(x, y) \in (X \times Y) \setminus G(f)$. It follows that $f(x) \neq y$. Since Y is T_2 , there exist open sets V and W such that $f(x) \in V$, $y \in W$ and $V \cap W = \emptyset$. Since f is $\gamma\alpha$ -continuous, there exists a γ -open set U in X containing x such that $f(U) \subset V$. Therefore, $f(U) \cap W = \emptyset$ and $G(f)$ is $\gamma\alpha$ -closed in $X \times Y$. ■

THEOREM 39. *Let $f : X \rightarrow Y$ have a $\gamma\alpha$ -closed graph. If f is injective, then X is γT_1 .*

Proof. Let x and y be any two distinct points of X . Then, we have $(x, f(y)) \in (X \times Y) \setminus G(f)$. By Lemma 37, there exist a γ -open set U in X containing x and an α -open V in Y containing $f(y)$ such that $f(U) \cap V = \emptyset$; hence $U \cap f^{-1}(V) = \emptyset$. Therefore, we have $y \notin U$. This implies that X is γT_1 . ■

THEOREM 40. *Let $f : X \rightarrow Y$ has a $\gamma\alpha$ -closed graph $G(f)$. If f is surjective $\gamma\alpha$ -open function, then Y is T_2 .*

Proof. Let y_1 and y_2 be any distinct points of Y . Since f is surjective $f(x) = y_1$ for some $x \in X$ and $(x, y_2) \in (X \times Y) \setminus G(f)$. By $\gamma\alpha$ -closedness of graph $G(f)$, there exist a γ -open set U of X and an α -open V in Y such that $(x, y_2) \in U \times V$ and $(U \times V) \cap G(f) = \emptyset$. Then, we have $f(U) \cap V = \emptyset$. Since f is $\gamma\alpha$ -open, then $f(U)$ is α -open such that $f(x) = y_1 \in f(U)$. Since $f(U)$ and V are disjoint α -open sets, we have $y_1 \in f(U) \subset \text{int}(\text{cl}(\text{int}(f(U))))$, $y_2 \in V \subset \text{int}(\text{cl}(\text{int}(V)))$ and $\text{int}(\text{cl}(\text{int}(f(U)))) \cap \text{int}(\text{cl}(\text{int}(V))) = \emptyset$. This shows that Y is T_2 . ■

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