Research Article

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Jordan centralizer maps on trivial extension algebras

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Abstract: The structure of Jordan centralizer maps is investigated on trivial extension algebras. One may obtain some conditions under which a Jordan centralizer map on a trivial extension algebra is a centralizer map. As an application, we characterize the Jordan centralizer map on a triangular algebra.

Keywords: Jordan centralizer maps, trivial extension algebra, triangular algebra

MSC 2010: 15A78, 16W25, 47B47

1 Introduction and preliminaries

In this article, $\mathcal{R}$ will denote a commutative ring with identity, A will be an unital $\mathcal{R}$ -algebra, M will be an unital A -bimodule, and the unit of all algebras will denote by 1. For $a \in A$ and $m \in M$, we use $a \circ m$ to denote the Jordan product $am + ma$ of a and m . Let $\phi: A \rightarrow M$ be a linear map. Recall that ϕ is said to be a right (left) centralizer map if $\phi(ab) = a\phi(b)$ ($\phi(ab) = \phi(a)b$) for all $a, b \in A$. It is called a centralizer map if ϕ is both a right centralizer map and a left centralizer map. We say that ϕ is a Jordan centralizer map if $\phi(a \circ b) = a \circ \phi(b)$ for all $a, b \in A$. It is obvious that ϕ is a centralizer map if and only if $\phi(a) = \phi(1)a = a\phi(1)$ for all $a \in A$.

Clearly, each centralizer map is a Jordan centralizer map. The converse is, in general, not true [1, Example 2.6]. It is natural and interesting to find some conditions under which a Jordan centralizer map is a centralizer map. Liu [2] has proved that a Jordan centralizer map on a triangular algebra under some conditions is a centralizer map. Zalar [3] has showed that any Jordan centralizer map on a 2-torsion free semiprime ring is a centralizer map. In this article, we mainly deal with the problem of describing the form of Jordan centralizer maps on trivial extension algebras. One of our main results (Theorem 2.11) shows that, where the algebra A and the A -bimodule M are 2-torsion free, if there exists a nontrivial idempotent e in A such that $eme' = m$ for all $m \in M$, where $e' = 1 - e$, and such that $eAe' = \{0\} = e'Ae$ and $e'r \cdot \text{Ann}_A(M)e' = \{0\} = e'l \cdot \text{Ann}_A(M)e$, then every Jordan centralizer map on $A \ltimes M$ is a centralizer map (some conditions on trivial extension algebras are applied in [4–6]). This result is applied to triangular algebras. To prove it, several preparatory results are given. We first characterize, in Section 2, the general form of Jordan centralizer maps and centralizer maps on trivial extension algebras (see Lemmas 2.1 and 2.2). Then, we characterize in terms of the form of their components when every Jordan centralizer map on a trivial extension algebra can be written as a centralizer map (see Theorem 2.3). This approach allows us to treat each component of a Jordan centralizer map on a trivial extension algebra separately. For details, readers are referred to Lemmas 2.4–2.10.

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For the readers' convenience, we briefly recall the constructions of trivial extension algebras and triangular algebras. Recall that the direct product $A \times M$ together with the pairwise addition, scalar product, and the algebra product defined by

$$(a, m)(b, n) = (ab, an + mb) \quad (a, b \in A, m, n \in M),$$

is an unital algebra, which is called a trivial extension of A by M and will be denoted by $A \ltimes M$. Trivial extensions are also known as a fertile source of (counter-)examples in various situations in the functional analysis. It should be remarked that in the functional analysis literature, this algebra was termed “module extension.”

The triangular algebras were first introduced in ref. [7] and then studied by many authors [2,8,9]. Let A and B be unital algebras and M is an unital (A, B) -module, which is faithful as a left A -module as well as a right B -module. The algebra

$$\text{Tri}(A, M, B) = \left\{ \begin{pmatrix} a & m \\ 0 & b \end{pmatrix} : a \in A, m \in M, b \in B \right\};$$

under the usual matrix addition and formal matrix multiplication will be called a triangular algebra. Every triangular algebra is a trivial extension algebra. Indeed, the triangular algebra $\text{Tri}(A, M, B)$ can be identified with the trivial extension algebra $(A \times B) \ltimes M$, in which M as an $(A \times B)$ -module is equipped with the module operations

$$(a, b)m = am \quad \text{and} \quad m(a, b) = mb, \quad (a \in A, b \in B, m \in M).$$

2 Jordan centralizer maps on $A \ltimes M$

Our aim is to study a Jordan centralizer map on a trivial extension algebra. We give conditions under which it is a centralizer map. Let us start with a general description of these types of mappings on a trivial extension algebra.

Every linear mapping $f: A \ltimes M \rightarrow A \ltimes M$ can be presented in the following form:

$$f(a, m) = (f_A(a) + h_1(m), f_M(a) + h_2(m)), \quad ((a, m) \in A \ltimes M), \quad (2.1)$$

where the linear mappings $f_A: A \rightarrow A$, $f_M: A \rightarrow M$, $h_1: M \rightarrow A$ and $h_2: M \rightarrow M$ are given by $f_A(a) = (\pi_A \circ f)(a, 0)$, $f_M(a) = (\pi_M \circ f)(a, 0)$, $h_1(m) = (\pi_A \circ f)(0, m)$ and $h_2(m) = (\pi_M \circ f)(0, m)$, respectively. Here, $\pi_A: A \ltimes M \rightarrow A$ and $\pi_M: A \ltimes M \rightarrow M$ are the natural projections given by $\pi_A(a, m) = a$ and $\pi_M(a, m) = m$, respectively. In the sequel, we suppose that f has a presentation given as in (2.1).

We commence with the following lemmas that need standard arguments.

Lemma 2.1. *A linear map f is a Jordan centralizer map if and only if the following conditions hold:*

1. f_A is a Jordan centralizer map;
2. f_M is a Jordan centralizer map;
3. $h_1(a \circ m) = a \circ h_1(m) = 0$ for all $a \in A$ and $m \in M$;
4. $m \circ f_A(a) = h_2(a \circ m) = a \circ h_2(m)$ for all $a \in A$ and $m \in M$;
5. $mh_1(n) + h_1(n)m = 0$ for all $m, n \in M$.

Lemma 2.2. *A linear map f is a centralizer map if and only if the following conditions hold:*

1. f_A is a centralizer map;
2. f_M is a centralizer map;

3. $ah_1(m) = h_1(am) = 0 = h_1(ma) = h_1(m)a$ for all $a \in A$ and $m \in M$;
4. $ah_2(m) = h_2(am) = f_A(a)m$ and $mf_A(a) = h_2(ma) = h_2(m)a$ for all $a \in A$ and $m \in M$;
5. $h_1(m)n = 0 = mh_1(n)$ for all $m, n \in M$.

Now we give the first fundamental result.

Theorem 2.3. *Every Jordan centralizer map on $A \ltimes M$ is a centralizer map if and only if the following conditions hold:*

1. *Every Jordan centralizer map $g: A \rightarrow M$ is a centralizer map;*
2. *Every linear map $h: M \rightarrow A$, such that for all $a \in A, m, n \in M$, $h(a \circ m) = 0 = a \circ h(m)$ and $mh(n) + h(n)m = 0$, satisfies $ah(m) = h(am) = 0 = h(ma) = h(m)a$ and $h(m)n = 0 = mh(n)$ for all $a \in A, m, n \in M$;*
3. *Every Jordan centralizer map f on $A \ltimes M$ of the form $f(a, m) = (f_A(a), h_2(m))$ (i.e., $h_1 = 0$ and $f_M = 0$ in the presentation (2.1)) is a centralizer map.*

Proof. $\Rightarrow$ We only need to prove (1) and (2).

- (1). Let g be a Jordan centralizer from A into M . Clearly, $(0, g)$ is a Jordan centralizer on $A \ltimes M$. Then, by hypothesis, there exists a centralizer map $(D_A + \mathcal{K}, D_M + \mathcal{L})$, such that for all $a \in A, m \in M$,

$$(0, g(a)) = (D_A(a) + \mathcal{K}(m), D_M(a) + \mathcal{L}(m))$$

Take $a = 0$, we obtain $\mathcal{L}(m) = 0$. Hence, $g = D_M$, we are done.

- (2). By hypotheses, $(h, 0)$ is a Jordan centralizer map on $A \ltimes M$. Then, by hypothesis, there exists a centralizer map $(D_A + \mathcal{K}, D_M + \mathcal{L})$, such that for all $a \in A, m \in M$,

$$(h(m), 0) = (D_A(a) + \mathcal{K}(m), D_M(a) + \mathcal{L}(m))$$

Take $m = 0$, then we obtain $D_A = 0$. Therefore, $h = \mathcal{K}$, as desired.

$\Leftarrow$ Let $f: A \ltimes M \rightarrow A \ltimes M$ be a Jordan centralizer map. It is obvious that h_1 has the conditions of (2). Also, by hypothesis, f_M is a centralizer map. Conversely, Lemma 2.1 shows that the linear map $(a, m) \mapsto (f_A(a), h_2(m))$ is a Jordan centralizer map on $A \ltimes M$. Then, by (3), it is a centralizer map. By using Lemma 2.2, $(a, m) \mapsto (h_1(m), f_M(a))$ is a centralizer map. Since $f(a, m) = (f_A(a), h_2(m)) + (h_1(m), f_M(a))$, then f is a centralizer map. $\square$

Now we turn to our second aim. We study Jordan centralizer maps on $A \ltimes M$ when there exists a nontrivial idempotent e in A that satisfies $eme' = m$ for all $m \in M$ (where $e' = 1 - e$). Triangular algebras are examples of this kind of algebras. To get the second main result, we need some lemmas.

Lemma 2.4. [10, Proposition 2.5] *Consider a nontrivial idempotent e of an algebra A and set $e' = 1 - e$. For an A -bimodule M , the following assertions are equivalent:*

1. *For every $m \in M$, $eme' = m$;*
2. *For every $m \in M$, $e'm = 0 = me$;*
3. *For every $m \in M$, $em = m = me'$;*
4. *For every $m \in M$ and $a \in A$, $am = eaem$ and $ma = me'ae'$.*

The following lemma shows that the first condition of Theorem 2.3 holds.

Lemma 2.5. *Let $g: A \rightarrow M$ be a Jordan centralizer map. If there exists a nontrivial idempotent e in A such that $eme' = m$ for all $m \in M$ (where $e' = 1 - e$), then g is a centralizer map.*

Proof. Define $f_1(a) = g(eae')$ and $f_2(a) = g(eae + e'ae + e'ae')$ for all $a \in A$. Clearly, $g = f_1 + f_2$. We prove that f_1 is a centralizer map. Let $a, b \in A$. We have

$$\begin{aligned} f_1(ab) &= g(eabe') \\ &= g(eaebe') + g(eae'be') \\ &= g((eae) \circ (ebe')) + g((eae') \circ (e'be')) \\ &= eae \circ g(ebe') + eae' \circ g(e'be') \\ &= ag(ebe') \\ &= af_1(b). \end{aligned}$$

Conversely, we have

$$\begin{aligned} f_1(ab) &= g(eabe') \\ &= g(eaebe') + g(eae'be') \\ &= g((eae) \circ (ebe')) + g((eae') \circ (e'be')) \\ &= g(eae) \circ ebe' + g(eae') \circ e'be' \\ &= g(eae')b \\ &= f_1(a)b. \end{aligned}$$

Also, we prove that f_2 is a centralizer map. Note that, for all $a \in A$,

$$\begin{aligned} 0 &= g((eae) \circ (e'ae')) \\ &= eae \circ g(e'ae') \\ &= ag(e'ae'). \end{aligned}$$

Hence, replacing a by $e'ae' + e$ in the previous equation, we obtain

$$g(e'ae') = 0.$$

We have

$$\begin{aligned} 0 &= g((e'ae') \circ (eae)) \\ &= e'ae' \circ g(eae) \\ &= g(eae)a. \end{aligned}$$

Replacing a by $eae + e'$ in this equation, we obtain

$$g(eae) = 0.$$

By using these relations, we get $f_2(a) = g(e'ae)$ for all $a \in A$. Now, for every $a, b \in A$,

$$\begin{aligned} f_2(ab) &= g(e'abe) \\ &= g((e'ae') \circ (e'be)) + g((e'ae) \circ (ebe)) \\ &= g(e'ae') \circ e'be + g(eae') \circ ebe \\ &= bf_2(a). \end{aligned}$$

Conversely, we have

$$\begin{aligned} f_2(ab) &= g(e'abe) \\ &= g((e'ae') \circ (e'be)) + g((e'ae) \circ (ebe)) \\ &= e'ae' \circ g(e'be) + e'ae \circ g(ebe) \\ &= f_2(b)a. \end{aligned}$$

Therefore, we have $f_2(ab) = bf_2(a)$ and $f_2(ab) = f_2(b)a$ for all $a, b \in A$. Take $a = 1$ in the first equation and take $b = 1$ in the second equation, we get $f_2(b) = bf_2(1)$ and $f_2(a) = f_2(1)a$. Hence, f_2 is a centralizer map. This completes the proof. $\square$

The following lemma shows that also the second condition of Theorem 2.3 holds.

Lemma 2.6. *Let $h: M \rightarrow A$ be a linear map such that $h(a \circ m) = a \circ h(m)$ for all $a \in A, m \in M$. If there exists a nontrivial idempotent e in A such that $eme' = m$ for all $m \in M$ and $eAe' = \{0\} = e'Ae$, where $e' = 1 - e$, then $ah(m) = h(am) = 0 = h(ma) = h(m)a$ and $h(m)n = 0 = mh(n)$ for all $a \in A, m, n \in M$.*

Proof. Let $m \in M$. By hypothesis, we have $h(m) = h(em) = h(e \circ m) = e \circ h(m) = eh(m) + h(m)e$. Then, $eh(m)e = 0$. Similarly, we get $e'h(m)e' = 0$. Therefore, $h(m) = eh(m)e' + e'h(m)e$. By hypothesis $h = 0$. This completes the proof. $\square$

Lemmas 2.5 and 2.6 show that to get the desired result, one should focus on the Jordan centralizer map on $A \ltimes M$ of the form $f(a, m) = (f_A(a), h_2(m))$ (i.e., $h_1 = 0$ and $f_M = 0$ in the presentation (2.1)). In the sequel, we will refer to such a particular kind of Jordan centralizer map as a Jordan centralizer map of type Δ . Recall that in this case, f_A is a Jordan centralizer map and h_2 satisfies $m \circ f_A(a) = h_2(a \circ m) = a \circ h_2(m)$ for all $a \in A$ and $m \in M$ (Lemma 2.1).

Lemma 2.7. *Assume that the algebra A is 2-torsion free and e is a nontrivial idempotent in A , where $e' = 1 - e$. Then, for a Jordan centralizer map f_A on A , the following assertions hold for all $a \in A$:*

1. $ef_A(e'ae')e = 0$;
2. $e'f_A(eae)e' = 0$;
3. $ef_A(eae')e = 0$;
4. $e'f_A(eae')e' = 0$;
5. $ef_A(e'ae)e = 0$;
6. $e'f_A(e'ae)e' = 0$.

Proof. We prove only (1), (3) and (4). The other assertions are proved similarly.

(1). We have $0 = f_A(e \circ (e'ae')) = e \circ f_A(e'ae')$. Then, $ef_A(e'ae')e = 0$.

(3). We have $f_A(eae') = f_A(e' \circ (eae')) = e' \circ f_A(eae')$. This implies that $ef_A(eae')e = 0$ and $e'f_A(eae')e' = 0$. $\square$

Thus, using Lemma 2.7, a Jordan centralizer map f of type Δ can be decomposed as follows:

$$f = I + D, \quad (2.2)$$

where for all $(a, m) \in A \ltimes M$,

$$I(a, m) = (ef_A(eae + e'ae + e'ae')e' + e'f_A(eae + eae' + e'ae')e, 0), \quad (2.3)$$

$$D(a, m) = (ef_A(eae)e + ef_A(eae')e' + e'f_A(e'ae)e + e'f_A(e'ae')e', h_2(m)). \quad (2.4)$$

It is obvious that if there exists a nontrivial idempotent e in A such that $eAe' = \{0\} = e'Ae$, where $e' = 1 - e$. Then, I (defined by (2.3)) is equal to zero.

Now we prove that D is a centralizer map. For this, we use the following two lemmas. The first one can be of independent interest. It presents some properties of a Jordan centralizer map of type Δ .

Recall that the left annihilator, $l\text{-Ann}_A(M)$, of M is the set of all elements r in A such that $rM = 0$. Similarly, the right annihilator, $r\text{-Ann}_A(M)$, of M is defined.

Lemma 2.8. *Assume that the A -bimodule M is 2-torsion free, and there exists a nontrivial idempotent e in A such that $eme' = m$ for all $m \in M$ (where $e' = 1 - e$). Let f be a Jordan centralizer map of type Δ . Then, the following assertions hold:*

1. $h_2(am) = f_A(a)m$ and $h_2(ma) = mf_A(a)$ for all $a \in A$ and $m \in M$;
2. $mf_A(e') = f_A(e)m$ for all $m \in M$;

3. $f_A(ab) - f_A(a)b \in r. \text{Ann}_A(M) \cap l. \text{Ann}_A(M)$;
 4. $f_A(ab) - af_A(b) \in r. \text{Ann}_A(M) \cap l. \text{Ann}_A(M)$.

Proof.

(1). We need only to prove the first equality. The second one follows immediately. We have

$$\begin{aligned} 2h_2(am) &= h_2((ae + ea)m) \\ &= h_2((ae + ea) \circ m) \\ &= m \circ f_A(a \circ e) \\ &= m(f_A(a) \circ e) + (f_A(a) \circ e)m \\ &= 2f_A(a)m. \end{aligned}$$

Therefore, the 2-torsion freeness of M implies that $h_2(am) = f_A(a)m$, as desired.

(2). For every $m \in M$, $mf_A(e') = h_2(me') = h_2(em) = f_A(e)m$, as desired.

(3) and (4). Let $a, b \in A$ and $m \in M$. We have $h_2(m) = f_A(e)m$, which means that $h_2(m) - f_A(e)m = 0$, and so

$$\begin{aligned} 0 &= h_2(am) - f_A(e)am \\ &= f_A(a)m - f_A(e)am \\ &= (f_A(a) - f_A(e)a)m. \end{aligned} \tag{2.5}$$

Then,

$$\begin{aligned} h_2(abm) &= f_A(a)bm \\ &= f_A(a)bm + a(f_A(b) - f_A(e)b)m. \end{aligned}$$

Conversely, $h_2(abm) = f_A(ab)m$. Then,

$$f_A(ab)m = (f_A(a)b + a(f_A(b) - f_A(e)b))m. \tag{2.6}$$

By using (2.5) and (2.6), we get

$$(f_A(ab) - f_A(a)b)m = 0. \tag{2.7}$$

Conversely, since $h_2(m) = mf_A(e')$, then $h_2(m) - mf_A(e') = 0$. So

$$\begin{aligned} 0 &= h_2(ma) - maf_A(e') \\ &= mf_A(a) - maf_A(e') \\ &= m(f_A(a) - af_A(e')). \end{aligned} \tag{2.8}$$

From (2.8), we have

$$\begin{aligned} h_2(mba) &= mbf_A(a) \\ &= mbf_A(a) + m(f_A(b) - bf_A(e')). \end{aligned}$$

Also, we have $h_2(mba) = mf_A(ba)$. Then,

$$mf_A(ba) = m(bf_A(a) + (f_A(b) - bf_A(e'))). \tag{2.9}$$

By using (2.8) and (2.9), we get

$$m(f_A(ba) - bf_A(a)) = 0. \tag{2.10}$$

Since f_A is a Jordan centralizer map,

$$f_A(ab) - f_A(a)b = -f_A(ba) + bf_A(a).$$

Therefore, by (6) and (9) and this identity, the proof ends. $\square$

Lemma 2.9. Assume that the A -bimodule M is 2-torsion free and there exists a nontrivial idempotent e in A such that $eme' = m$ for all $m \in M$ (where $e' = 1 - e$). Let f be a Jordan centralizer map of type Δ . If f_A is a centralizer map, then $h_2(ma) = h_2(m)a$ and $h_2(am) = ah_2(m)$ for all $a \in A$, $m \in M$.

Consequently, the Jordan centralizer map f is a centralizer map.

Proof. For all $a \in A$ and $m \in M$, we have

$$h_2(ma) - h_2(m)a = ah_2(m) - h_2(am).$$

Then, by using the hypothesis and Lemma 2.8 ((1) and (2)), we get

$$\begin{aligned} h_2(ma) - h_2(m)a &= a(f_A(e)m) - f_A(a)m \\ &= af_A(e)m - (f_A(e'a) + f_A(ea))m \\ &= af_A(e)m - (f_A(e')e'a + f_A(e)ea)m \\ &= af_A(e)m - f_A(e)am = 0. \end{aligned}$$

Then, $h_2(ma) = h_2(m)a$. Since $h_2(a \circ m) = a \circ h_2(m)$, we obtain $h_2(am) = ah_2(m)$. As desired. $\square$

Lemma 2.10. Assume that the A -bimodule M is 2-torsion free. Suppose there exists a nontrivial idempotent e in A such that $eme' = m$ for all $m \in M$, where $e' = 1 - e$. If $eAe' = \{0\} = e'Ae$ and $e'r.Ann_A(M)e' = \{0\} = el.Ann_A(M)e$, then D (defined by (2.4)) is a centralizer map.

Proof. First, we show that the map $f': A \rightarrow A$, defined by $f'(a) = ef_A(eae)e + e'f_A(e'ae')e'$ for all $a \in A$, is a centralizer map.

By the hypothesis and the assertion (3) of Lemma 2.8, we have

$$\begin{aligned} ef_A(eaebe)e &= ef_A(eae)ebe, \\ ef_A(eae'be)e &= ef_A(eae')e'be = 0, \\ e'f_A(e'ae'be')e' &= e'f_A(e'ae')e'be', \text{ and} \\ e'f_A(e'aebe')e' &= e'f_A(e'ae)ebe' = 0. \end{aligned} \tag{2.11}$$

By the hypothesis and (2.8), we have

$$\begin{aligned} f'(ab) &= ef_A(eabe)e + e'f_A(e'abe')e' \\ &= ef_A(eaebe)e + ef_A(eae'be)e + e'f_A(e'aebe')e' + e'f_A(e'ae'be')e' \\ &= ef_A(eae)ebe + e'f_A(e'ae')e'be'. \end{aligned}$$

On the other hand, by the hypothesis, we have

$$\begin{aligned} f'(a)b &= ef_A(eae)eb + e'f_A(e'ae')e'b \\ &= ef_A(eae)ebe + e'f_A(e'ae')e'be'. \end{aligned}$$

By the hypothesis and the assertion (4) of Lemma 2.8, we have

$$\begin{aligned} ef_A(eaebe)e &= eaef_A(ebe)e, \\ e'f_A(e'ae'be')e' &= e'ae'f_A(e'be')e'. \end{aligned} \tag{2.12}$$

By the hypothesis and (2.12), we have

$$\begin{aligned} f'(ab) &= ef_A(eabe)e + e'f_A(e'abe')e' \\ &= ef_A(eaebe)e + e'f_A(e'ae'be')e' \\ &= eaef_A(ebe)e + e'ae'f_A(e'be')e'. \end{aligned}$$

On the other hand, we have

$$\begin{aligned} af'(b) &= aef_A(ebe)e + ae'f_A(e'be')e' \\ &= eaef_A(ebe)e + e'ae'f_A(e'be')e'. \end{aligned}$$

Therefore, $f'(a)b = f'(ab) = af'(b)$ for all $a, b \in A$. That is, f' is a centralizer map. Then, f' is a Jordan centralizer map. We can show easily that D is a Jordan centralizer map of type Δ . Therefore, since f' is a centralizer map, Lemma 2.9 implies that D is a centralizer map. $\square$

Finally, combining the above results we obtain our second main result.

Theorem 2.11. Assume that the algebra A and the A -bimodule M are 2-torsion free. Suppose there exists a nontrivial idempotent e in A such that $eme' = m$ for all $m \in M$, where $e' = 1 - e$. If $eAe' = \{0\} = e'Ae$ and $e'r.Ann_A(M)e' = \{0\} = el.Ann_A(M)e$, then every Jordan centralizer map on $A \ltimes M$ is a centralizer map.

As a consequence of Theorem 2.11, we have the following results.

Corollary 2.12. Assume that the algebras A, B and the $(A \times B)$ -bimodule M are 2-torsion free. If $r.Ann_B(M) = \{0\}$ and $l.Ann_A(M) = \{0\}$, then every Jordan centralizer map on $Tri(A, M, B)$ is a centralizer map.

Proof. Clearly, $e = (1, 0)$ is a nontrivial idempotent in $A \times B$, where $e' = (0, 1)$. It is obvious that $e(A \times B)e' = \{0\} = e'(A \times B)e$ and $eme' = m$ for all $m \in M$. Since $l.Ann_A(M) = \{0\}$ and $r.Ann_B(M) = \{0\}$, then $e'r.Ann_{A \times B}(M)e' = \{0\} = el.Ann_{A \times B}(M)e$. This completes the proof. $\square$

Corollary 2.13. Assume that the algebra A and the A -bimodule M are 2-torsion free. Suppose there exists a nontrivial idempotent e in A such that $eme' = m$ for all $m \in M$, where $e' = 1 - e$. If $r.Ann_A(M) \cap l.Ann_A(M) = \{0\}$, then every Jordan centralizer map on $A \ltimes M$ is a centralizer map.

Proof. The hypothesis all of the sets eAe' , $e'Ae$, $e'r.Ann_A(M)e'$ and $el.Ann_A(M)e$ are in $r.Ann_A(M) \cap l.Ann_A(M)$. Therefore, the proof is end. $\square$

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