Corrigendum

Moosa Gabeleh*

Corrigendum to the paper “Equivalence of the existence of best proximity points and best proximity pairs for cyclic and noncyclic nonexpansive mappings”

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Abstract: The purpose of this short note is to present a correction of the proof of the main result given in the paper “*Equivalence of the existence of best proximity points and best proximity pairs for cyclic and noncyclic nonexpansive mappings*,” Demonstr. Math. **53** (2020), 38–43.

Keywords: best proximity (point) pair, uniformly convex Banach space, noncyclic (cyclic) contraction

MSC 2020: 47H09, 46B20

1 On the result in [1]

The following theorem is the main result of [1].

Theorem 1.1. (Theorem 2.1 of [1]) *Let (A, B) be a nonempty, weakly compact and convex pair in a strictly convex Banach space X . Then every cyclic nonexpansive mapping defined on $A \cup B$ has a best proximity point if and only if every noncyclic nonexpansive mapping defined on $A \cup B$ has a best proximity pair.*

We mention that the proof of Theorem 1.1 is based on the fact that the mapping $\mathcal{P} : A_0 \cup B_0 \rightarrow A_0 \cup B_0$ is an isometry (see condition (2) of Proposition 1.8).

We mention that the isometricness of the mapping $\mathcal{P}$ means that both $\mathcal{P}|_{A_0}$ and $\mathcal{P}|_{B_0}$ are isometry, where in Proposition 1.8, we wrongly stated that $\|\mathcal{P}x - \mathcal{P}y\| = \|x - y\|$ for all $(x, y) \in A_0 \times B_0$.

To correct this, we need to introduce the following geometric notion.

Definition 1.2. Let (A, B) be a nonempty pair of subsets of a metric space (X, d) such that A_0 is nonempty. We say that the pair (A, B) has the diagonal property provided that

$$\begin{cases} d(x_1, y_1) = \text{dist}(A, B), \\ d(x_2, y_2) = \text{dist}(A, B), \end{cases} \Rightarrow d(x_1, y_2) = d(x_2, y_1),$$

for any $x_1, x_2 \in A_0$ and $y_1, y_2 \in B_0$.

For instance, if (A, B) is a pair of nonempty subsets of a metric space (X, d) such that $\text{dist}(A, B) = 0$, then (A, B) has the diagonal property.

Also, every two parallel segments in the Euclidian plan $\mathbb{R}^2$ have the diagonal property. In more general, it can be shown that every nonempty, closed and convex pair in Hilbert spaces has the diagonal property.

* **Corresponding author: Moosa Gabeleh**, Department of Mathematics, Ayatollah Boroujerdi University, Boroujerd, Iran, e-mail: Gabeleh@abru.ac.ir, gab.moo@gmail.com

We now have the following result regarding the projection mapping $\mathcal{P}$.

Proposition 1.3. *Let (A, B) be a nonempty, bounded, closed and convex pair in a reflexive and strictly convex Banach space X such that (A, B) has the diagonal property. Define $\mathcal{P} : A_0 \cup B_0 \rightarrow A_0 \cup B_0$ as*

$$\mathcal{P}(x) = \begin{cases} \mathcal{P}_{A_0}(x) & \text{if } x \in B_0, \\ \mathcal{P}_{B_0}(x) & \text{if } x \in A_0. \end{cases} \quad (1)$$

Then $\mathcal{P}$ is a relatively isometry, that is, $\|\mathcal{P}x - \mathcal{P}y\| = \|x - y\|$ for all $(x, y) \in A_0 \times B_0$.

Proof. It follows from Proposition 1.8 of [1] that

$$\|x - \mathcal{P}x\| = \|\mathcal{P}y - y\| = \text{dist}(A, B), \quad \forall (x, y) \in A_0 \times B_0.$$

Since (A, B) has the diagonal property, we obtain

$$\|\mathcal{P}x - \mathcal{P}y\| = \|x - y\|,$$

that is, $\mathcal{P}$ is a relatively isometry on $A_0 \cup B_0$. □

Thus, we need the condition diagonal property on the considered pair (A, B) in Theorems 2.1 and 2.3 of [1].

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Reference

- [1] M. Gabeleh and H. P. Künzi, *Equivalence of the existence of best proximity points and best proximity pairs for cyclic and noncyclic nonexpansive mappings*, Demonstr. Math. **53** (2020), 38–43.