Research Article

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‘Mathematics Achievement in Crisis’: Modelling the Influence of Teacher Knowledge and Experience in Senior High Schools in Ghana

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Abstract: Mathematics achievement in senior high schools (SHS) in Ghana is not encouraging and this has become a concern for major stakeholders. Several studies have been conducted to provide information to understand the issue of poor mathematics achievement. Of all the studies, areas regarding the joint influence of experience, pedagogical content knowledge (PCK) and subject matter knowledge (SMK) have been less explored in Ghana. In this study, the interplay among these variables is examined in an attempt to explain the variances in students’ mathematics achievement. To achieve this purpose, 210 SHS teachers and 8,400 students in SHS across the country were surveyed and subsequently administered tasks on algebra taking into consideration the variables of interest. The study revealed that teacher knowledge and teaching experience significantly influenced students’ mathematics achievement. It was discovered that PCK had a significant indirect effect on students’ mathematics achievement through teachers’ SMK. The findings also showed that experience did not significantly moderate the relationship between teachers’ SMK and students’ mathematics achievement. The study, based on the outcome, made conclusions and recommendations for practice. Suggestions for the implementation of future studies were also highlighted.

Keywords: Pedagogical content knowledge; subject matter knowledge; senior high school; experience; mathematics achievement; algebra; mathematics teachers.

1 Introduction

The Sustainable Development Goal (SDG) 4 of the United Nations is geared towards attaining equitable and inclusive quality education as well as promoting long-life learning prospects for all. As a key indicator for the achievement of the Education Agenda 2030 (SDG 4), teaching quality is largely acknowledged by educational researchers, practitioners, and policymakers (United Nations Economic and Social Commission for Asia and the Pacific, 2015). The quality of the teaching process is a momentous element in the school environment and is known to significantly influence academic achievement (Gichuru & Ongus, 2016; Quansah & Ankoma-Sey, 2020). Fenster (2014) retains that a highly efficacious instructor guarantees an improvement in learners’ academic learning in both long-term and short-term. Thus, teacher quality in any school milieu is the most precarious constituent for improving student achievement and reducing students’ achievement gaps (Borisade, 2011). This presupposes that teacher quality has implications on students’ achievement and/or achievement gaps in mathematics in schools.

All over the world, mathematics is an important subject taught in schools and this is because of its significance to other subjects, particularly, in the development of science, humanities and technology (Ezenweani, 2006). The field of Mathematics is vital for its application in machine learning, management, national defence, technology, finance, industrial processes, among others. In light of this, students’ mathematics achievement have frequently been viewed by stakeholders as an indicator of the overall health of schools and the country’s general intellectual capability (Kolawole & Oluwatayo, 2005). Undeniably, teaching quality and other school factors have been found as the variables contributing to poor mathematics achievement in schools (Borisade, 2011). The gross prevalent underachievement in mathematics in public examinations could, thus, be attributed to teacher quality and other constraining school variables (Borisade, 2011).

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The West African Examination Council (WAEC), for several decades, has been the only body in Ghana for assessing the performance of students after the completion of Junior High School (JHS) and Senior High School (SHS). Some concerns have been expressed by a large number of citizens on SHS students' failure rate, particularly in mathematics, reported by WAEC every year. Due to this, many attempts have been made by governments in the past to improve the performance of mathematics in schools. Despite governments' efforts, mathematics achievement has not undergone much change; there are consistently low achievement levels in mathematics among SHS students (WAEC, 2014, 2015, 2016). It is regrettable, therefore, that in recent times, many SHS students struggle with mathematics and perform very poorly in their final examinations in most jurisdictions (Bosson-Amedenu, 2017; Fletcher, 2018). A 5-year mathematics performance data (2013-2017) of SHS students, obtained from WAEC (2018), revealed that more than half of students who sat within the period could not obtain grade C6¹ or better in mathematics which is needed to qualify them for university admissions. The pass rate was 36.68% in 2013, 32.45% in 2014, 24.09% in 2015, 33.12% in 2016, and in 2017 it was 41.66%. Although there is no definitive pattern in the performance data, it is important to emphasize that over 50% of the students who sat for the examination each year were not eligible to gain admission to any university in Ghana. This has become a disquiet for students, parents, and other stakeholders in education in the country.

Considering the national outcry in Ghana over the students' poor mathematics performance in national assessments, which prevents a bulk of SHS graduates from pursuing further studies, it is obvious that this problem is critical, and requires investigation to understand which variables seem to negatively impact on students' mathematics achievement and performance. Many studies have been conducted on the factors influencing mathematics performance at the SHS level (see Abreh, Owusu, & Amedahe, 2018; McCarthy, McCarthy, Gyan, Baah-Korang, 2015; Appiagyei, Joseph & Fentim, 2014; Enu, Osei, & Nkum, 2015; Fletcher, 2018). These studies highlight the role variables (such as students' attitude

towards mathematics, interest in the subject, inadequate teaching and learning materials, insufficient mathematical practice by students, home factors, and peer factors) play in students' mathematics performance. Other studies have investigated teacher variables and mathematics achievement of students (also see Akpo, 2012; Anney & Bulayi, 2020; Dial, 2008; Ewetan & Ewetan, 2015; Gichuru & Ongus, 2016; Kimani, Kara & Njagi, 2013; Kosgei, Kirwa, Odhiambo, & Ayugi, 2013; Yusuf & Dada, 2016). Although these previous studies focused on teacher variables, much emphasis was placed on specific teacher variables like qualification, age, preparedness, gender, communication skills, classroom management and teaching experience. It is only recently that Ansah, Mensah, and Wilmot (2020) examined the role of experience in the relationship between subject matter knowledge (SMK) and elective mathematics performance. Ansah et al. did not include pedagogical content knowledge (PCK) despite the role it plays in mathematics achievement. Thus, no instructor can teach without PCK and thus, it is only through PCK that SMK can be utilised in teaching (Lee, Bicer, Kwon, & Capraro, 2019). This study combines SMK, PCK, and teaching experience, and examines how they interact to influence mathematics performance.

Teacher knowledge (SMK and PCK) is a significant indicator of teacher competence (Ding, He, & Leung, 2014) and serves as the basis for teachers' classroom instructional practice (Lee, Capraro, & Capraro, 2018). Even (1993, p.94) defined SMK as "emphasizing knowledge and understanding of facts, concepts, and principles and how they are organized, as well as knowledge about the discipline". Shulman (1987), on the other hand, described PCK as the knowledge which extends SMK and embodies the aspects of content highly connected to its teachability. Several scholars have found the role of teachers' SMK and PCK in students' academic achievement to be important (see Barlow & Cates, 2006; Ding et al., 2014; Lee et al., 2018; Valanides, 2000). In particular, SMK and PCK have been found to influence the development of misinterpretations, misunderstandings, misconceptions, self-esteem, self-efficacy, self-concepts, attitudes, beliefs, and interest of students and teachers in mathematics (Barlow & Cates, 2006; Valanides, 2000; Quinn, 1997). Thus, SMK and PCK together are the starting point for the formation of such psychological variables which later affects the teaching and learning of any subject, especially in mathematics (Rizvi, 2004). Other researchers have also indicated that SMK and PCK contribute to teachers' choice and utilisation of TLMs (Teaching and Learning Materials), and classroom management practices (Ozden, 2008; Schmidt et al., 2009). In other words, teachers with rich SMK and

¹ The grading system and interpretation (by percentage score required to earn the grade) used by WAEC presented as A1 for excellent, for raw marks from 75% to 100%; B2 for very good, for raw score from 70% to 74%; B3 for good, for raw score from 65% to 69%; C4 for credit, for raw score from 60% to 64%; C5 for credit, for raw score from 55% to 59%, C6 for credit, for raw score from 50% to 54%, D7 for pass, for raw score from 45% to 49%; E8 for pass, for raw score from 40% to 45%; and F9 for failure, for raw score from failure 0% to 39%.

PCK, for example, would be able to improvise in the absence of adequate TLMs and have an excellent way of making difficult topics appear easy for all students. From the foregoing, it is clear that teacher knowledge is critical to teaching and learning as compared to other factors like TLMs availability, attitudes, interest, self-efficacy, among others. This backed our choice of introducing “teacher knowledge” as a possible contributor to poor achievements in Mathematics. We were also compelled to include years of teaching experience because literature has revealed that experience enhances SMK (Ewetan & Ewetan, 2015). Thus, teachers with several years of teaching experience have been found to develop robust SMK (Anney & Bulayi, 2020).

In this study, we modelled teachers’ SMK and PCK with students’ mathematics achievement and teaching experience was used as a moderating variable between SMK and mathematics achievement. In short, this research examined how teachers’ SMK, PCK and years of teaching interact to explain the variances in students’ mathematics achievement. In terms of approach, this study is among the few ones which applied Structural Equation Modelling (SEM) (e.g., Ansah et al., 2020). The rest of the paper is organized into seven sections. Following the introduction are the review of previous studies, hypotheses development and conceptual framework. The methods section follows where discussions regarding the participants, data collection instruments and approaches, as well as data analysis procedures were highlighted. The fourth and fifth sections present the results and discussion, and the conclusions, respectively. The significance of the study was also discussed. The paper ends with the recommendations for practice and suggestions for further studies.

2 Hypotheses Development and Conceptual Framework

Teachers’ SMK reflects teachers’ mastery over their area of specialisation (Adediwura & Tayo, 2007). It is indisputable that teachers should be knowledgeable in their field of endeavour since they need to impact and train students (Stevenson, 2020). Thus, if an instructor is not adequately knowledgeable in his/her subject area, then, any confidence in teaching efficiency goes right ‘outside-the-box’. Scholars like Fakeye (2012) have indicated that there is a significant relationship between what instructors know and what they teach. Therefore, for any instructor to effectively teach, the SMK of such an instructor is

paramount. Of course, instructors will be handicapped if they are unacquainted with issues in the content area (Lee et al., 2018). Supporting this view, Nixon, Toerien, and Luft (2018) also indicated that no instructor could explain what they do not know or understand.

Teacher PCK, on the other hand, centres on the style and approach adopted by the instructor in presenting their lessons in an exciting manner (Lee et al., 2019). If instructors can present their lessons to the learner in an appealing manner, then, they have excellent PCK (Tsafie, 2013). By this, an instructor with an excellent PCK in mathematics can simplify complex mathematical procedures, thereby, allowing all categories of students to understand what is being taught (Ball, 2003). Other scholars believe that teachers’ pedagogical skills are related to students’ performance, although they provided little evidence to support this claim (see Adedoyin, 2011; AbdulRaheem & Amali, 2014). It is important to note that teachers’ PCK, though strongly related to SMK, is not the same as SMK. Thus, instructors command and utilisation of appropriate teaching strategies in the classroom show their level of SMK (Fakeye, 2012). In other words, teachers need some level of PCK in addition to SMK to effectively impact students.

Furthermore, years of teaching have been found to significantly influence learners’ performance at all levels (see Akpo, 2012; Anney & Bulayi, 2020; Ewetan & Ewetan, 2015; Olaleye, 2011). Instructors who are experienced have a rich pool of knowledge to draw from and can offer ideas and insight into instruction and learning (Kosgei et al., 2013). Olaleye (2011) also revealed that learners taught by more experienced instructors attain high academic excellence since most of these instructors have mastery of the content (i.e. SMK) and have a great deal of classroom management skills to help all kind of students. Moreover, experienced instructors are deemed to use the most suitable instructional methods (i.e. PCK) to teach learners with different background and abilities (also see Harber et al., 2012; Lai, 2011; Rivers & Sanders, 2002; Stronge, Ward, Tucker, & Hindman, 2007). Based on the aforementioned discourse, the three hypotheses were developed:

H_{A1}: Teacher knowledge (ie. SMK and PCK) and years of teaching will positively influence students’ mathematics achievement.

H_{A2}: Teachers’ PCK will positively mediate the relationship between teachers’ SMK and students’ mathematics achievement.

H_{A3}: Teachers’ years of teaching will positively moderate the relationship between teachers’ SMK and students’ mathematics achievement.

Figure 1: Conceptual framework on the influence of teacher knowledge and experience on students' mathematics achievement.

3 Research Methods

3.1 Sample

The study comprised two categories of individuals: students and mathematics teachers in SHS in Ghana. Specifically, grade 11 students with their respective mathematics teachers were the focus of this study. Grade 11 students were employed because the grade 10 students had not extensively studied Algebra (which is the focus for achievement measures) and thus could not be in the position to respond to the tasks. The grade 12 students, on the other hands, were unavailable because they were writing their final examination. Through the cluster sampling technique, 14 single-sex male schools, 14 single-sex female schools and 14 mixed schools were selected. Five mathematics teachers were further sampled from each school making a total of 210 teachers (i.e., 42×5). For each of the 210 teachers, 40 students of theirs were randomly sampled. In all, 8,400 students (i.e., 40×210) participated in the study, with a response rate of 97.2% ($n=8,169$). The male students were 53.1% ($n=4,338$) and the females were 46.1% ($n=3,831$). Out of the 210 mathematics teachers, 162 (77.1%) were male teachers whereas 48 (22.9%) were female teachers.

3.2 Choice of Topic Area

The nature of the study required the choice of the topic area which the teacher knowledge will focus on. This narrowed the scope and decreased the influence of any extraneous variables which may explain the findings of the study. Thus, the SMK, PCK and students' academic achievement

were all restricted to one topical area in mathematics (i.e. Algebra). Algebra highlights mathematical symbols and its associated rules used to manipulate such symbols (Silver, 1997). Algebra is the foundation and merging point in virtually all aspects of mathematics (Olson, 1994). Topics under algebra include, but not limited to, algebraic expressions, real number system, solving equations, proportions and proportional reasoning, functions, linear inequalities, polynomials, and factoring.

It must be said that students with a strong foundation in algebra have higher chances of having a strong understanding of further topical areas in mathematics (Pascopella, 2000). Essentially, if teachers do not have mastery or even have poor pedagogy of the area, students' achievement in mathematics will continue to be poor. As a starting point, it is imperative to understand teachers' content knowledge in algebra, how they teach topics in this area, their experience in teaching, and how these variables influence students' achievement in algebra.

3.3 Measurement of Variables

This research comprised three major variables, namely, teacher knowledge (SMK and PCK), teaching experience, and students' achievement in algebra. The teaching experience was conceptualized as the number of years of teaching mathematics in SHS. The rest of the variables were measured using an instrument adapted from the Knowledge of Algebra for Teaching (KAT) project (2012), Michigan State University. The instrument contained standardised and psychometrically sound items to measure teachers' SMK and PCK in algebra (See Appendix). The reliability estimate for the instrument was .79 using the Kuder-Richardson 20 procedure. Ten tasks were provided under each construct (i.e. SMK and PCK) and teachers were required to respond to them. The same 10 items used to measure teachers' SMK were used to measure the achievement of students in algebra.

In adapting the instrument, most of the contexts and wording of the questions were changed to reflect Ghanaian contexts as part of the adaptations process. For instance, the prices of items were changed to reflect market values in Ghana as the dollar was replaced with the cedi equivalent. Also, a few words were changed to reflect words used in the Ghanaian contexts. A word like "pants" for example was replaced by "trousers" as it is commonly used in Ghana.

3.4 Instrument Administration Procedures

After obtaining ethical clearance from the University of Cape Coast review board, contacts were made with the regional offices of Ghana Education Service (GES), heads and/or the assistant heads of the senior high schools. Earlier contacts were also made with the mathematics teachers in the selected schools. At the meetings with the teachers/headteachers and reps from GES, the purpose of the study and the voluntary nature of participation were discussed; allowing them to decide, without any form of coercion, whether to participate or not to participate in the study. We had interaction with the students and assured them of confidentiality and anonymity as well as seeking their consent.

The tasks were administered before the start of the normal school hours so as not to disrupt classes and also for the participants to attempt the tasks with a sound mind. In each of the participating schools, students were assembled in the assembly hall where they were given 60 minutes to complete the tasks under the supervision of a trained research assistant. While students were responding to the tasks, the selected teachers were also assembled and carried out their tasks with a time limit of 120 minutes. Because the instrument contained tasks of ability, there was strict supervision just as in an examination situation. The data were taken with ten research assistants who were trained before the data collection. The data collection lasted for 39 weeks.

3.5 Data Analysis

Data were gleaned from 210 mathematics teachers and 8,169 students. For each of the teachers, the average achievement score for their students was computed and used as the overall achievement for the class the teacher handles. Thus, 210 average achievement scores were generated and matched with the data from the teachers for the analysis. A SEM approach was used to analyse the data using SPSS (version 25) via Analysis of Moment Structures (AMOS version 21) (Alavifar et al., 2012). The bootstrapping approach, specifically 5,000 bootstrap samples, was used as the parameter estimations procedure (Hayes, 2018), using a confidence interval of 95%. To interpret whether the result was significant or not using the confidence interval, the lower and upper limit values should not include zero (Hayes, 2018).

The use of SEM is advantageous over other approaches and statistical models like multivariate regression for several reasons. First, SEM allows for multidimensional,

Figure 2: Modelling teacher knowledge and experience on students' mathematics achievement.

complex and more accurate analysis of data by considering the distinct aspect of examined real and abstract theoretical concepts (Karimi & Meyer, 2014). Also, SEM can estimate for measurement errors through the utilisation of several indicator latent factors, and the testing of complex mediational mechanisms through the decomposition of effects (Tarka, 2018). It has been established that any linear model (e.g., regression) appears to perform worse relative to SEM, not only due to the exclusion of correction in the measurement errors but also because it is possible to ignore the indirect effects. Finally, SEM permits model fitting and theory development (Nunkoo & Ramkissoon, 2012).

4 Results

H₀: Teacher knowledge (ie. SMK and PCK) and years of teaching will not influence students' mathematics achievement.

As shown in Figure 2, SMK (with mean of 5.74 and variance of 3.41) predicted students' mathematics achievement by .21, PCK (with mean of 4.74 and variance of 4.42) predicted mathematics achievement by .19, Experience (with mean of 6.09. and variance of 4.09) predicted mathematics achievement by .18; all with error variance of .30 and intercept of 2.88. The covariance between SMK and PCK was .98, SMK and Experience was .73, and PCK and Experience was .46.

As shown in Table 1, teachers' SMK significantly and positively predicted students' mathematics achievement, $b=.205$, $BootCI(.156, .252)$. Also, teachers' PCK positively influenced students' mathematics achievement, $b=.194$, $BootCI(.154, .230)$. Further, the analysis revealed that

Table 1: Prediction of Teacher Knowledge on Students' Mathematics Achievement.

Parameter			Estimate	Boot SE	Std. Beta	Critical Ratio	BootLCI	BootUCI
Achievement	<---	SMK	.205	.024	.399	9.567	.156	.252
Achievement	<---	PCK	.194	.020	.428	10.401	.154	.230
Achievement	<---	Experience	.181	.020	.384	9.463	.140	.219
Intercept			2.875	.163	--	17.677	2.875	23.308
Bootstrap R ²			.670	--	--	--	.568	.757

Table 2: Direct, Total, and Indirect Effects of SMK on Mathematics Achievement.

	Effect	BootSE	BootLLCI	BootULCI
Direct effect of SMK on Achievement:	.287	.029	.180	.296
Total effect of SMK on Achievement	.300	.037	.222	.370
Indirect effect of SMK on Achievement	.059	.017	.026	.094
Effect sizes of the total & indirect effects				
Total	.582	.056	.460	.679
SMK → PCK → Achievement	.114	.031	.052	.172

Mediator: PCK

years of teaching mathematics (experience) positively influenced students' mathematics achievement, $b=.181$, $BootCI(.140, .219)$. Comparatively, teachers' PCK was found as the strongest predictor (standardized estimate=.428), followed by teachers' SMK (standardized estimate=.399) and the least predictor was experience (standardized estimate=.384). Overall, teachers' SMK, PCK and experience explained 67% of the variances in students' mathematics achievement ($R^2=.670$).

H₀2: Teachers' PCK will not mediate the relationship between teachers' SMK and students' mathematics achievement.

The path diagram displayed (Figure 3) reveals that SMK (with mean of 5.74 and variance of 3.41) predicted PCK by .29 (error of 4.41 and intercept of 3.09). Also, PCK and SMK predicted mathematics achievement by .20 and .24 respectively; all with error variance of .43 and intercept of 3.72

The results, as shown in Table 2, revealed that PCK had a significant indirect effect of teachers' SMK on students' mathematics achievement, $b=.059$, $BootCI(.026, .094)$ with an effect size of .114. Thus, the null hypothesis that

Figure 3: The Mediating Role of PCK in the Relationship between SMK and Students' Mathematics Achievement. **Figure 4:** Moderating Effect of Experience in the Relationship between Teachers' SMK and Students' Mathematics Achievement.

“teachers' PCK will not mediate the relationship between teachers' SMK and students' mathematics achievement” was rejected.

H₀3: Teachers' years of teaching will not moderate the relationship between teachers' SMK and students' mathematics achievement.

As shown in Figure 4, Experience (with mean of 6.09 and variance of 4.09) predicted students' mathematics achievement by .18, SMK (with mean of 5.74 and variance

Table 3: Moderating Role of Experience in the Relationship between Students' Mathematics Achievement.

Parameter			Estimate	Boot SE	Std. Beta	Critical Ratio	BootLCI	BootUCI
Achievement	<---	Experience	.181	.068	.385	2.671	-.042	.346
Achievement	<---	SMK	.247	.069	.479	3.592	.012	.408
Achievement	<---	SMK*EXP	.002	.012	.038	-.026	.037	.037
Intercept			3.478	.387	--	8.984	2.534	4.952

Moderator: SMK*EXP

of 3.41) predicted mathematics achievement by .25, Interaction term (moderator, SMK*EXP) (with mean of 35.65. and variance of 283.78) predicted mathematics achievement by .00; all with error variance of .45 and intercept of 3.48. The covariance between Experience and SMK was .73, SMK and Moderator was 22.67 and, Experience and Moderator was 226.33.

As presented in Table 3, years of teaching experience did not significantly moderate the relationship between teachers' SMK and students' mathematics achievement, $b=.002$, $BootCI(-.026, .037)$. Based on this result, we failed to reject the null hypothesis which states that "teachers' years of teaching will not moderate the relationship between teachers' SMK and students' mathematics achievement".

5 Discussion

The findings from our study revealed a significant influence of teachers' SMK, PCK and experience on students' mathematics achievement. Further analysis revealed that teachers' PCK significantly mediates the relationship between teachers' SMK and students' mathematics achievement. Of course, one cannot give what he/she does not have. That is to say, that teachers with limited content mastery and pedagogical content knowledge will produce students with limited mastery of what has been taught. It must be said that SMK is not enough for a teacher to greatly impact students with regards to mathematical abilities. Accordingly, teachers with the same level of SMK but with different levels of PCK will have a differential impact on their respective students' mathematics achievement. With the same level of SMK, the teacher with higher PCK will produce students with higher mathematical ability as compared to a teacher with lower PCK. These findings from this study corroborate the observations of Adediwura and Tayo (2007) who found out that there is some relationship between what

instructors knew and what they teach. Consistent with the findings, other scholars like Fakeye (2012) have discovered that instructors command and utilisation of appropriate teaching strategies in the classroom show their level of SMK and this reflects in the achievement of students.

The findings of this study further showed that experience did not significantly moderate the relationship between teachers' SMK and students' mathematics achievement. In other words, teachers' years of teaching failed to adapt the relationship between teachers' SMK and students' mathematics achievement. This suggests that with similar levels of SMK, teachers' with more years of experience and those with fewer years of experience will have a similar impact on students' mathematics achievement. Arguably, a teacher with strong content mastery does not need much experience to train students to be effective in mathematical computations. The understanding is that many years of experience may not have a significant improvement in the mastery the teacher has. Therefore, teachers after attaining some level of SMK do not require more years of teaching to be able to produce students with higher mathematical ability. In contrast to the findings of this study is the observations of Ansah et al. (2020), who indicated that years of teaching experience strengthens the link between teachers' SMK and students' mathematics achievement. Other studies like that of Olaleye (2011), who highlighted the essence of experience being a moderating variable, had contradictory findings relative to this study.

6 Conclusions

The outcome of this study has highlighted the essence of teachers knowledge, particularly PCK, and years of teaching experience in solving the crisis in mathematics achievement of students. Teachers are the major agent in the school setting and are required to gain adequate pedagogical knowledge on how to assist learners with

different abilities to gain mastery in mathematics. This outcome of the study provides a platform for the development and deployment of quality teachers having the pre-requisite PCK to assist students to acquire complex mathematical skills.

This study focused on only a specific topic area in mathematics, which is algebra. Therefore, variables in this study except for experience were tied to algebra. Consequently, teachers' PCK and SMK, as well as students' achievement were all centred on algebra. Although algebra is fundamental in almost all mathematical areas, the results should be understood and interpreted in the context of algebra. This is because the number of tasks administered to measure SMK and PCK is inadequate to conceptualise teachers' SMK and PCK in every aspect of mathematics. This is not to say, that the findings of this study do not apply to other topical areas in mathematics, however, such generalization should be done with caution.

7 Significance of the Study

The findings of this research provide empirical and comprehensive knowledge to relevant stakeholders on how teacher knowledge (i.e. SMK and PCK) significantly and directly determine students understanding in mathematics. This study provides a valuable tool of reference to teachers to undertake a self-evaluation in terms of their effectiveness and efficiency in teaching. This serves as a prompt to mathematics teachers on the need to improve their pedagogy and knowledge on the subject; by way of seminar and workshops or obtaining additional academic qualification(s) to positively impact their respective students. This shows the necessity of constantly improving the academic capabilities of the teacher as this will, in turn, impact the social and scientific base of the nation. The findings, most importantly, provide higher education administrators with relevant information to rethink and/or improve on the training of pre-service teachers on the education programme in the country.

8 Recommendations for Practice and Future Direction

The study recommends that all stakeholders; Ministry of Education, Ghana Education Service, education planners, and teacher unions should periodically provide

windows of opportunities like actualisation workshops on new pedagogical approaches covering specific topics in mathematics. Seminars and in-service training should also be organised for mathematics teachers to help them adopt appropriate pedagogical skills in teaching the subject. The results also have implications for the pre-service teacher education programme. The management of such institutions should incorporate the element of PCK as a core aspect of the training of pre-service teachers. For future implementation, further studies need to be conducted to cover other topics in mathematics excluding algebra.

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Appendix

Subject Matter Knowledge Of Teacher & Students

- At a storewide sale, shirts cost ₦80 each and a pair of trousers cost ₦120 each. If S is the number of shirts and P is the number of trousers bought, which of the following is a meaning for the expression $80S + 120P$?
 - The number of shirts and trousers bought
 - The cost of 80 shirts and 120 trousers
 - The cost of P shirts and S trousers
 - The cost of S shirts and P trousers

- If $f(x) = ax^3 + bx^2 + cx + d$, what is the slope of the line tangent to this curve at $x = 2$?
 - $8a + 4b + 2c$
 - $8a + 4b + 2c + d$
 - $12a + 4b + c$
 - $12a + 4b + c + d$

- Find the area under the curve below in square units

- How many real solutions does the equation $\sqrt{x-2} = \sqrt{1-x}$ have?
 - none
 - one
 - two
 - infinitely many

- In the figure below, ABC is a right-angled triangle. $ABDE$ is a square of area 200 square inches and $BCGF$ is a square of area 100 square inches. What is the length, in inches of AC ?

- Given the table to the right, determine $f(g(3))$

A.	4	x	$f(x)$	$g(x)$
B.	-1	-2	0	5
C.	1	-1	6	3
D.	5	0	4	2
		1	-1	1
		2	3	-1

- A manufacturing company makes closed metal containers each with a capacity of $2000\pi\text{cm}^3$. If the radius of the cylinder is r cm, find the minimum area of the metal sheet required.

- 1885.2cm^2
- 942.5cm^2
- 1000cm^2
- 642.3cm^2

- Mr. Ansah and Mr. Boateng work together. Mr. Ansah's salary is GH¢200.00 a year and he has an annual increment of GH¢20.00. Mr. Boateng is paid at first at the rate of GH¢80.00 a year and has an increment of GH¢8.00 every half-year. At the end of how many years will Mr. Boateng receive more salary than Mr. Ansah?

- 6 years
- 5 years
- 6.5 years
- 5.5 years

- Solve the inequality: $x^2 + x - 20 > 0$ in two essentially different ways.

- Let $A(x_1, y_1)$ be a point under a reflection in the line $y = mx + c$ and let $A^1(a, b)$ be the image of the point A . Show that:

- $mb + a = x_1 + my_1$
- $b - ma - 2c = mx_1 - y_1$

Pedagogical Content Knowledge Of Teacher

- Which of the following (taken by itself) would give substantial help to a student who wants to expand $(x + y + z)^2$?
 - See what happens in an example, such as $(3 + 4 + 5)^2$
 - Use $(x + y + z)^2 [(x + y) + z]^2$ and the expansion of $(a + b)^2$
 - Use the geometric model shown below:

	x	y	z
x	x^2	xy	xz
y	xy	y^2	yz
z	xz	yz	z^2

- A. iii only
B. i and ii only
C. ii and iii only
D. i, ii and iii
2. Which of the following can be represented by areas of rectangles?
- The equivalence of fractions and percentages, e.g. $\frac{3}{5} = 60\%$
 - The distributive property of multiplication over addition. For all real numbers **a**, **b** and **c** we have **$a(b + c) = ab + ac$**
 - The expansion of the square of a binomial **$(a + b)^2 = a^2 + 2ab + b^2$**
- A. ii only
B. i and ii only
C. ii and iii only
D. i, ii and iii
3. The statement ‘For all whole numbers, if to the product of two consecutive whole numbers we add the larger number, the result is equal to the square of the larger number’ can be expressed symbolically as: for all whole number n .
- A. $n^2 + 1 = n(n - 1) + n + 1$
B. $(n + 1)^2 = n^2 + 2n + 1$
C. $n^2 = n(n - 1) + n$
D. $(n + 1)n = n^2 + n$
4. Every number to the power zero is one, says a student, therefore $0^0 = 1$. Which of the following is true about this?
- A. $0^0 = 0$ and not equal to 1
B. The theorem does not include zero
C. 0^0 is indeterminate because it has multiple possible answers and not 1, the agreed upon answer
D. Zero cannot be an exponent
5. Which of the following is a valid conclusion to the statement “If a student is a high school band member, then the student is a good musician”?
- A. All good musicians are high school band members
B. A student is a high school band member
C. All students are good musicians
D. All high school band members are good musicians
6. In a first year elective mathematics class, which of the following is NOT an appropriate way to introduce the concept of slope of a line?
- A. Toss a ball in the air and use a motion detector to graph its trajectory
B. Apply the formula slope $\frac{\text{rise}}{\text{run}}$ to several points in the plane
C. Discuss the meaning of m in the graphs of several equations of the form $y = mx + b$
D. Talk about the rate of change of a graph of a line on an interval.
7. Consider the statement below:
For all a, b s, if $ab = 0$, then either $a = 0$ or $b = 0$
For which of the following sets is the above statement true?
- The set of real numbers
 - The set of complex numbers
 - The set of 2×2 matrices with real number entries
- A. i only
B. ii only
C. iii only
D. i and ii only
8. Which of the following situations can be modelled using an exponential function?
- The height h of a ball t seconds after it is thrown into the air
 - The amount of money A in a jar after w weeks, if each week \$ d are put in the jar
 - The value v of a car after t years if it depreciates $d\%$ per year
- A. i only
B. ii only
C. iii only
D. ii and iii only
9. Notice that $\frac{1}{2} = \frac{5}{10}$ and $\frac{1+2}{2} = \frac{3}{2}$ and $\frac{5+10}{10} = \frac{15}{10} = \frac{3}{2}$. Is it true in general that, if $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$ for all real numbers $a, b, c, d, \neq 0$? Justify your answer
10. A student asked his teacher, “Is the square root of 16 four and negative four?” The teacher answered, “You are right”. Explain whether you agree or disagree with the teacher.
[Write your solution]