Research Article

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Designing Combinations of Physical and Digital Manipulatives to Develop Students' Visualisation

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Abstract: This paper focuses on two main objectives. The first, to describe and characterise instrumental approach involving artefact and instrument differentiation, instrumental genesis and orchestration along with the instrument-mediated activity model. The second, to present and discuss one of five cube cross-section lessons designed and carried out in our mixed methods research project. Altogether, a paper workbook, dynamic applets and 3D prints form a five-set toolkit, each corresponding to one of the five designed lessons. The main research idea was to explore how digital and physical manipulatives could be integrated into solid geometry lessons to support the development of students' visualisation. In this paper, Lesson 2 will be described in detail, which, like the others, has been particularly influenced by the instrumental approach. Its individual attributes will be depicted according to the personally adapted activity model.

Keywords: visualisation; instrumental approach; cube cross-section; 3D prints; GeoGebra applets.

1 Introduction

Understanding three-dimensional objects from their digital or paper-and-pencil images can be challenging. It means not all students can easily understand 3D virtual or 3D graphic representations of geometric solids in order to see the relative positions among their shapes and figures. The use of various digital technologies has grown steadily in the past years due to the promotion of technologies at various levels of education. In this way, as noted by Lieban (2019), physical resources seemed to be replaced more by the digital ones rather than being complemented by them. In (Gutiérrez, 1996a) the close relationship between

visualisation and spatial geometry was pointed out. The creation of 3D mental images and their subsequent external (digital or paper-and-pencil) representations are imperatives to progress in solid geometry learning. As proposed by Camou (2012), designing and implementing a multi representational approach has a positive impact on exploring three-dimensional objects (in Sinclair et al., 2016). Considering this approach, our mixed methods research was carried out with the aim of supporting the development of students' visualisation by integrating multiple (physical and digital) manipulatives into cube cross-section lessons. We anticipate the visualisation development could lead to the correct perception of virtual and paper-and-pencil representations of solids, and, therefore to the correct paper-and-pencil cube-sectional drawings of cubes. Altogether, a paper workbook, 3D prints and dynamic applets form a five-set toolkit designed and implemented in this project, each corresponding to one of the five lessons. The aim of this paper is to present and discuss Lesson 2 in detail, which, along with the others, has been particularly influenced by the instrumental approach. The object of this lesson is to appropriately integrate multiple manipulatives into students' work to (a) support the creation of their mental images, the improvement of external representations of the created mental images, the development of visual abilities and the reinforcement of visual processes; (b) help them apply solid geometry knowledge, as well as personal experiences with manipulatives, in paper-and-pencil constructions of cube cross-sections.

2 Theoretical Framework

Mixed methods research (later MMR), which combines quantitative and qualitative research approaches, allows researchers to combine quantitative and qualitative data into a single research study (or series of research studies). This combination aims to provide a better understanding of given research questions compared to what a quantitative or qualitative approach would provide on its own

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Figure 1: Research design overview.

(Creswell & Plano [2011] in Cohen et. al, 2018). However, MMR not only relates to a combination of qualitative and quantitative methodologies or methods; but also has a much wider embrace. This combination, following from Johnson et al. (2007), can be referred to (a) *what* is mixing; (b) *where* and *when* the mixing takes place; (c) the *scope* and *breadth* of the mixing; (d) the *reasons* for the mixing, and (e) the *research orientation*.

In our research project, two different mixing combinations were involved (see Figure 1). On the one hand, two research methodologies were mixed in the research design process. On the other hand, with respect to the individual research phases resulted from the research design, quantitative and qualitative research methodologies and methods were mixed in the process of data collecting and analysing. This paper is focused on a detailed description of Lesson 2 (designed and planned in Phase 1), the main idea of which is to combine physical and digital manipulatives to support the development of students' spatial visualisation, and thus support the correct cube cross-sectional drawings of cubes in the paper-and-pencil environment. The inspiration comes mainly from the two following sources. The preparation phase of the research (Phase 0), which comprises the literature review as well as the survey of solid geometry lessons at the three Slovak grammar schools (Vágová, 2019); and Lieban (2019) who presented and discussed some perspectives of the physical and digital modelling in mathematics education. From our point of view, these representations complement each other and offer a more holistic perspective on the

spatial arrangement of solid shapes. The following sections introduce a theoretical background that has been respected and followed in Phase 1 of our research, which is discussed in detail in this paper.

2.1 Visualisation

Mental or visual imagery (e.g., Lean & Clements, 1981), as well as spatial thinking or spatial visualisation (e.g., Korakakis et al., 2004; Vallo et al., 2013), introduce different forms of 'visualisation' as used in many psychological and pedagogical researches. Although these and many other investigations focused on the same topic, the concept of visualisation was used with different meanings. It means visualisation can be considered as physical objects (Schnotz, 2002), mental objects (Sharma in Phillips et al., 2010), processes (Zimmermann & Cunningham, 1991) or abilities (Piburn et al., 2005). In this research project, we consider **visualisation** as "the set of types of images, processes, and abilities necessary for geometry students to produce, analyse, transform, and communicate visual information related to real objects, models, and geometric concepts" (Gutiérrez, 2006). In other words, Gutiérrez (1996a) defined the so-called model of visualisation, where the visualisation consists of four main elements: external and internal representations, processes and abilities of visualisation. Based on the number of definitions, detailed in Vágová (2018), we define the individual elements as follows:

- **External representation** is any type of graphical (paper-and-pencil or digital), physical/material or verbal representation of terms, objects or their properties. More specifically, the term *external spatial representation* is understood as an external representation of three-dimensional objects that provides information about their spatial arrangement.
- **Internal representation** is the mental representation of a particular object in the form of a scheme, drawing, sketch, etc. depicting visual or spatial information with the presence or absence of its external representation. According to the Presmeg's (1986) classification, *concrete images* (pictures in mind, mental images of real objects), *kinaesthetic images* (mental images associates with muscular activity), and *dynamic images* (mental images, the parts of objects of which are in motion without any muscular activity) were taken into consideration in our research project.
- **Ability of visualisation** is the ability to create, maintain, manipulate (rotate, invert), move and transform internal representations/mental images in our mental space. Following from (Gutiérrez, 1986a), *figure-ground perception* (identification of a specific figure by isolating it from a complex background), *mental rotation* (visualisation of a moving configuration and creation of dynamic internal representations), *perceptual constancy* (remaining mental "orientation" of the object perceived from different perspectives, recognition of the independence of some properties of the actual internal representation from colour, position or size), *perception of spatial relationships* (identification of the related properties of several images/objects/pictures among them or within themselves, such as, e.g. perpendicularity), *perception/recognition of spatial positions* (relation of a mental image/object/picture to oneself), *visual discrimination* (comparison of several mental images, pictures and/or objects to identify similarities and differences among them) were the abilities discussed and examined in our research.
- **Processes of visualisation** – as defined by Gutiérrez (1996a), is a "mental or physical action where mental images are involved." Regarding the research questions (see Section 3.4), the Kosslyn model was used in the project (in Gutiérrez, 1996a): 1) *mental image generation* (creation of a mental image from given information); 2) *mental image inspection* (observation of a position or a presence of individual parts of the mental image); 3) *mental image*

transformation (decomposition/rotation/translation of the mental image); 4) *mental image use* (use of the mental image to reach the activity object).

In our project, three types of external representations were designed and planned in Phase 1: 3D graphic representations by paper-and-pencil worksheets, 3D physical representations by 3D prints and 3D virtual representations by GeoGebra software. The manipulation 3D prints and GeoGebra applets have played the main role in carrying out the research for the purpose of developing the students' visualisation and correct cross-sectioning a cube in lessons. With this intention, we designed the five-set toolkit, each corresponding to one of the five planned lessons. The aim of this paper is to present and discuss Lesson 2 in detail, which, along with the others, has been particularly influenced by the instrumental approach.

3 Instrumental approach

The **instrumental approach**, presented mainly by Pierre Rabardel (1995/2005), is a French theoretical framework based on the Activity Theory as introduced by Vygotsky, Leontiev and Engeström (Rabardel, 2002; Drijvers & Trouche, 2008; Gueudet et al., 2014; Hardman, 2008; Laisney, & Chatoney, 2018). In this context, **activity** is defined as "the logical temporal and spatial organisation of the actions and operations that aim at reaching a conscious goal" (Laisney & Chatoney, 2018). The activity can be understood as a set of actions/behaviours (Rabardel, 2002). It is expressed through goals and means, determined by the associated environment in which the subjects (e.g. workers, students) act (Simondon, 2017). The activity has a cultural and social dimension, is mediated by tools called artefacts (Laisney & Chatoney, 2018) and considered instrument-mediated (Rabardel, 2005).

The instrumental approach has been primarily used to study the technology application such as Computer Algebra System (e.g. Artigue, 2002; Trouche, 2003; Drijvers, 2012) or Dynamic Geometry System (e.g. Laborde, 2001; López, 2012) in the learning and teaching of mathematics. Nevertheless, it has also been employed for exploring the application of more "traditional" resources, such as printed (work)books used in the paper-and-pencil environment, or more recent tools in a virtual learning environment (Gueudet et al., 2018). Drijvers and Trouche (2008) believe that the instrumental approach "allows for an analysis of the learning process in technological environments of increasing complexity and takes into account the non-trivial character of using

computerized environments.” The instrumental approach generally studies an object-directed activity of one subject or a group of subjects which is mediated by artefacts, central tools in this study. Following Rabardel’s theory of instrument, the concepts ‘artefact’ and ‘instrument’ must be understood differently.

3.1 Artefact and instrument

In scientific literature, the terms ‘artefact’ and ‘instrument’ are used with distinctive meanings. Rabardel and his colleagues (1995/2005) stress the difference between these terms and clarify the definition of each of them. An **artefact** is considered as a physical tool, while an **instrument** is a physical tool which is used as a mental construction resulting from the artefact’s use (see Figure 2).

According to Drijvers and Trouche (2008), the **artefact** can be *material* (e.g. a computer, a calculator, etc.) or *non-material* (e.g. an algebraic symbol, language, etc.). It is firstly designed by a person or a group of people with the aim to meet one or more precise activity objectives. The artefact is one of the two instrumental components. That is, compared to the artefact, an instrument is not designed and “given” to the subject. The instrument is built from an artefact when the subject manipulates and uses it. Specifically, an artefact “converts” to the instrument if the subject associates this artefact with the act which makes it effective. “The instrument can be enriched according to the way it is used, in the specificity of the situations encountered by the subject in its activities” (Rabardel, 1995) in (Laisney & Chatoney, 2018).

Expanding on this, Rabardel (2002) defines an **instrument** as a mixed entity “incorporating an artefact (or a fraction of an artefact) and one or more utilisation schemes” which the subject associates with the artefact (see Figure 3). Every instrument can be understood as a subject’s knowledge.

Trouche (2004) introduces the instrument as a mixed entity consisting of:

- a *given material component* – it is an artefact, a part of an artefact or a set of artefacts.
- a *psychological component* – it is one or more schemes organising the subject’s activity.

In literature, the notion ‘scheme’ and its related terms such as script, scenario, etc. appear in a number of theoretical frameworks. For Piaget (1936a), the **scheme** is “an active organization of actual experience which integrates the past” in (Rabardel & Beguin, 2005). It can be understood as a structure with a past, which changes as it adapts to

Figure 2: From artefact to instrument.

Figure 3: Instrument as a mixed entity.

more diversified data and situations. A subject applies the scheme to the heterogeneity of the outside environment and generalises it with respect to the situations to which it applies. Subsequently, Piaget and his successors (e.g. Vergnaud, 1996/1998) developed the term ‘action scheme,’ which is the basis for the mentioned utilisation scheme introduced by Rabardel. **Action scheme** is defined as “the structured generalizable of the action that allows the same action to be repeated or applied to new contents” Piaget (1936) in (Rabardel, 2002).

As we indicated above, in the context of the instrumental approach, Rabardel (1995) introduces the notion of ‘**utilisation scheme**’ of an artefact. He defines this new concept as “a scheme organizing the activity with an artefact associated with the realization of a given task” in (Trouche, 2004). In the instrumental theory (see Figure 4), there are two plus one levels of utilization schemes (Trouche, 2003; Trouche, 2004; Rabardel, 2002; Gueudet et al., 2014):

1. *Usage schemes* – are considered as elementary schemes, directly related to the artefacts themselves. They are building blocks for later instrumented action schemes. When a subject examines an artefact, discovers it, and manipulates it in order to learn how to use it, we say that the subject creates a usage scheme.
2. *Instrumented action schemes or instrument-mediated action schemes* – are considered as higher-order, coherent and meaningful mental schemes, built up from elementary usage schemes, related to the object of the subject’s activity. When a subject manipulates an instrument with the aim to refer to the activity’s object, we say that the subject creates the instrumented action schemes. The created schemes are used for specific types of activities. These schemes are constituted by two components, between which there is a dialectical relationship:

Figure 4: Levels of utilisation schemes.

- a. Gestures – the subject’s elementary behaviour which can be seen/observed/detected. They make it possible to carry out the task.
- b. Operative invariants – the subject’s cognitive constructs which are developed within the actions for similar goals. They are understood as implicit knowledge: *theorems-in-actions* or *concept-in-actions*. It means the propositions or concepts implicitly are held as true. The operative invariants guide the gestures and simultaneously, the operative invariants are constructed through the activity.
3. *Instrumented collective activity schemes* or *collective instrument-mediated activity schemes* – the instruments are often used in the context of collective activity. The same artefact, a part of an artefact or a set of artefacts can be used jointly/simultaneously by subjects to carry out a shared/common activity. These schemes concern both the coordination of individual actions and the integration of these individual results as a contribution to the achievement of a common specific object.

Regarding the individual and collective aspect, Rabardel (2002) categorizes the utilization schemes according to their dimensions:

- *Private dimension* – private utilisation schemes are included here. These schemes are specific to each individual.
- *Social dimension* – social utilisation schemes are included here. These schemes are relative to collective activities developed among the subjects.

In addition to the different levels and dimensions, the utilisation schemes share the common characteristics.

These schemes are multifunctional in that they carry out the following functions (Rabardel, 2002; Trouche, 2004):

- *Epistemic function* – it allows the subject to understand the situation, what he/she is doing.
- *Heuristic function* – it allows the subject to orient and to control the activity, to anticipate and plan actions.
- *Pragmatic function* – it allows the subject to do something, to transform the situation and obtain results.

In our project, the artefacts are represented by physical (3D prints) and digital (GeoGebra applets) resources that become instruments for the students when manipulating them. The idea of such multiple resources is to support the development of students’ visualisation (graphic external representations in the paper-and-pencil environment, mental images, visual abilities and processes), which could lead to a better understanding of solid representations in plane and cube cross-sections. Following Lieban’s (2019) study, there is a terminological non-convergence using physical and digital resources in education. Some researchers (e.g. HersHKovitz, 2016) consider physical and virtual resources to be manipulative, while others do not include the digital ones (e.g. Faggiano et al., 2018). In our research, both physical and digital resources are considered to be manipulatives as stated by Lieban (2019). For physical manipulatives, the term ‘manipulative’ refers to the artefacts that can be manipulated. For the digital ones, it refers to the interaction of the user with the resource and not the manipulation of the physical device itself. In summary, the utilisation schemes together with the artefacts form the subject’s or subjects’ instruments developing through the object-directed activity. The process in which the instrument is developed is called

Figure 5: Personal adaptation of Trouche's (2004) instrumental genesis scheme.

instrumental genesis process. Its detailed description is given in the following section.

3.2 Instrumental genesis: when an artefact becomes an instrument

Guin and Trouche (2002) define the **instrumental genesis** as “a complex process, needing time, linked to the artefact's characteristics (its potentialities and its constraints) and to the subject's activity, his/her knowledge and former working habits.” It is a process, a building process, when the subject develops his/her instrument. An artefact becomes an instrument.

The process of instrumental genesis, “birth” of the instrument, refers to the interaction of both the subject and the artefact (Rabardel, 2002). It means the instrumental genesis is a bi-directional process (see Figure 5) involving a combination of instrumentation (a movement from the artefact towards the subject) and instrumentalisation (a movement from the subject towards the artefact). Both processes can be characterised as follows (Rabardel, 2002; Drijvers and Trouche, 2008; Gueudet et al., 2014; Laisney & Chatoney, 2018):

- *Instrumentation process* (artefact → subject) is related to the subject himself/herself. In this process, every artefact has its own potentialities and constraints which influence the subject's activity and the development of his/her knowledge. These potentialities/possibilities and constraints shape the

techniques and the conceptual understanding of the subject. The artefact shapes the thinking of the user.

- *Instrumentalisation process* (subject → artefact) is directed to the artefact. In this process, every subject has his/her specific knowledge, conceptions and preferences before engaging in the interaction with the artefact. These personal characteristics shape and change the ways in which he/she uses the artefact and may lead to changing or customizing it. In other words, this process is grounded in the artefact's intrinsic (internal) characteristics and properties. The subject shapes an artefact. Given these points, different subjects, although starting from the same artefact, can develop different instruments. As defined by Rabardel (2002), the instrumentalisation is “the process in which the subject enriches the artefact's properties” and it can go through different stages:
 1. A stage of *discovery* and *selection* of the relevant artefact's functions or keys
 2. A stage of *personalisation* of the artefact
 3. A stage of *transformation* of the artefact (either planned or unplanned by the designer). There are two levels of the artefact's transformation:
 - On the first level, the artefact is momentarily instrumentalised – the instrumentalisation is local, linked to a specific action of the subject's activity and the circumstances of its occurrence.
 - On the second level, the artefact is permanently instrumentalised – the acquired key or function is

permanently preserved as a property of the artefact in relation to the class of actions, situations and objects of the activity.

In both levels, there is no physical transformation of the artefact itself. Hence, the artefact has been enriched only with new extrinsic (external) properties acquired for a moment or permanently. For example, the subject uses a book as a ruler.

To summarise, the instrumental genesis is a process in which the subject transforms an artefact into an instrument by manipulating and using it. It refers to the bi-directional interaction between this artefact and the subject. That is, the instrumentation process directed towards the subject (artefact $\rightarrow$ subject) and the instrumentalisation process directed towards to the artefact (subject $\rightarrow$ artefact). Both processes are closely connected, and it is not possible to clearly distinguish between them. As follows from Guin and Trouche's (2002) definition, the instrumental genesis depends on the artefact's potentialities and constraints as well as the subject's knowledge. However, there is one more important component influencing the subject(s)'s instrumental genesis: instrumental orchestration. Its detailed characteristic is given in the following section.

3.3 Instrumental orchestration as a way to assist instrumental genesis

The utilisation schemes of an artefact, which the subject constructs in the process of instrumental genesis, always have individual and social aspects (Rabardel, 2002). They are shared in practice groups or communities and can lead to appropriation by the subjects. In this sense, Trouche (2004) indicates the two activities that influence the subject's instrumental genesis: *the subject's individual activity* (depending on the artefact's potentialities and constraints as well as the subject's knowledge) and *the teacher's organised activity*. The importance of the choice and the way of the activity promotion by the teacher are stressed by Dreyfus (1993). This means that the constitution of an effective learning instrument (or a system of instruments) from an artefact (or a set of artefacts) is influenced by and depends on a specific class organisation: instrumental orchestration. Laborde et al. (2005) point out the following, "We do not take the learner as an isolated individual facing the world, but take the learner as deeply embedded in his/her environment which is highly structured the ways the individual is learning" (in Drijvers & Trouche, 2008). **Instrumental orchestration** is defined as "the intentional and systematic organisation of

Figure 6: Illustration of a didactic configuration from our research project.

the various artefacts available in a learning environment by the teacher for a given mathematical situation, in order to guide students' instrumental genesis" (Drijvers and Trouche, 2008). It comprises how the teacher organises both the artefacts and the activity with the aim to support and guide the subjects' instrumental genesis. Following from the study results of Guin & Trouche (2002), Trouche (2003/2005), Drijvers & Trouche (2008), Drijvers et al. (2010) and Drijvers (2012), instrumental orchestration (a) is defined by five components; (b) is distinguished by a style; and (c) exists in several types. Subsequently, the individual attributes will be characterised one by one.

The components of instrumental orchestration can be defined as follows:

- A set of *subjects* – it is a group of people participating in the research (researcher(s), student(s), teacher(s), etc.).
- A set of *objects* – it is a complex of activity objects related either to the adaptation of the working environment or to carrying out the activities of a specific type.
- A *didactic configuration* – it is a general structure of the activity. It is a layout of the artefacts available in the environment with a specific layout for each action of the activity. Drijvers et al. (2010) define a didactic configuration as "an arrangement of artefacts in the environment, or, in other words, a configuration of the teaching setting and the artefacts involved in it." For example, a box with 3D prints is placed in the centre of the subjects' desk, a computer is placed on the teacher's desk, a screen or blackboard are placed in front of the subjects (see Figure 6).

- *Exploitation modes* – they are linked to the possible organisation of the configuration, how the teacher decides to exploit it for the benefit of his/her didactical intentions. It means each didactic configuration has a specific set of exploitation modes. It is a way the teacher decides to use the didactic configuration to come to the activity's objects. It includes decisions about how the activity is introduced and processed, about the possible roles of the artefacts to be used, and about the schemes and techniques that subjects have to develop and establish. They include decisions about which, when and how artefacts should be used to come to the objects of the activity. For example, the overhead projector is switched off as well as the workbooks are closed and subjects manipulate 3D prints.
- *A didactical performance* – it involves the “ad hoc” decisions made during the teaching process on how to actually act in the chosen didactic configuration and exploitation modes of that concrete configuration. For example, the teacher explains what happens on the screen and has to react to the subjects' behaviour and answer their questions.

The components, a set of ‘subjects’ and ‘objects’ together with a ‘didactical configuration’ and ‘exploitation modes’ were introduced in (Guin & Trouche, 2002; Trouche, 2003/2005 and Drijvers & Trouche, 2008). Designing and establishing these four components has a strong preparatory aspect. Although the exploitation modes are more flexible, the didactical configuration cannot be easily changed in lessons. For this reason, Drijvers and his colleagues (2010) needed to add the last component, a didactical performance, having a strong “ad hoc” aspect. The teacher has to deal with unexpected aspects of the activity, subjects' behaviour or complications connected with the use of the artefacts. In respect to the teacher's preferences, they distinguish between student-centred or teacher-centred orchestration styles. *Student-centred orchestration* is based on the teacher's discussion and interaction with subjects, while *teacher-centred orchestration* focuses on showing how to use artefacts, explaining how the task should be solved, and so on. Furthermore, the teacher can fulfil the function of either *orchestra conductor*, who combines various roles such as planner, conductor, technical assistant, explainer, facilitator, task setter, counsellor, collaborator, evaluator, manager, etc. (Zbiek & Hollebrands, 2008), or a *one-person band*, where subjects have opportunities to react, have more input, and there is more interaction between teacher and subjects/students (Drijvers et al., 2010).

The last attribute that plays an important role in our research is the type of instrumental orchestration. Drijvers and his colleagues (2010/2012) identified six basic and one specific orchestration types differing from each other by a didactical configuration and a set of exploitation modes in principal. Subsequently, we will present key characteristics of these orchestrations simultaneously adapted to our research project.

Technical-demo orchestration – it is based on the teacher's demonstration of an artefact (technical or/and non-technical) and it does not go beyond its use instructions in practice. This orchestration can be seen as teacher-centred and its component examples are as follows:

- a *didactical configuration*: an overhead projector is projected on the wall (screen) in front of the subjects that allows the subjects to follow the teacher's demonstration.
- a *set of exploitation modes*: a teacher demonstrates and shows artefacts themselves and their use in a new situation or task.

Based on the different types of artefacts that were designed and used in our project, we use the term ‘**Toolkit-demo orchestration**’ instead of Technical-demo orchestration. In this orchestration, the teacher demonstrates the workbook, 3D prints and applets, and explains how and when to manipulate them in practice.

Explain-the-screen orchestration – it is based on the teacher's explanation guided by what happens on the computer screen. The demonstration and explanation go beyond the artefact instructions, they involve mathematical content, too. This orchestration can be seen as teacher-centred and its component examples are as follows:

- *didactical configuration*: an overhead projector is projected on the wall (screen) in front of the subjects that allows the subjects to follow the teacher's demonstration (it can be similar to the first one).
- a *set of exploitation modes*: a teacher explains what happens on the computer screen. He/she carries out the task by using an artefact.

Following from our project, we distinguish several types of orchestration in this group:

1. ‘**Explain-the-3D prints orchestration**’ – the teacher explains a task while stressing and/or manipulating 3D prints with hands. We say she works in the physical environment.
2. ‘**Explain-the-applet orchestration**’ – the teacher explains a task while stressing and/or manipulating the applet projected on the screen (wall). We say she works in the digital environment.

3. **‘Explain-the-worksheet orchestration’** – the teacher explains a task while demonstrating the worksheet projected on the screen or held in hands. We say she works in the paper-and-pencil environment.
4. **‘Explain-the-board orchestration’** – the teacher explains a task while writing and drawing on the blackboard. We say she works in the collective paper-and-pencil environment.

Discuss-the-screen orchestration – it is based on the whole-class discussion about what happens on the screen. The aim is to support social/collective instrumental genesis. This orchestration can be seen as student-centred and its component examples are as follows:

- *didactical configuration*: an overhead projector is projected on the screen in front of the subjects that allows the subjects to follow the teacher’s demonstration (it can be similar to the first one). A classroom setting needs to be favourable for discussion.
- *a set of exploitation modes*: a teacher can either take the students’ work or a new task as a starting point for the discussion.

Following from our project, we distinguish several types of orchestration which are similar to those from the explain-the-screen orchestration group:

1. **‘Discuss-the-3D prints orchestration’** – the teacher and students discuss a task while stressing and/or manipulating 3D prints with hands. We say they work in the physical environment.
2. **‘Discuss-the-applet orchestration’** – the teacher and students discuss a task while stressing and/or manipulating the applet projected on the screen. We say they work in the digital environment.
3. **‘Discuss-the-worksheet orchestration’** – the teacher and students discuss a task while demonstrating the worksheet projected on the screen or held in hands. We say they work in the paper-and-pencil environment.
4. **‘Discuss-the-board orchestration’** – the teacher and students discuss a task while writing and drawing on the blackboard. We say they work in the collective paper-and-pencil environment.

Link-screen-board orchestration is based on the teacher’s stressing the relationship between what happens on the screen and how it is presented in a book, paper and blackboard. The teacher stresses the relationship between what happens in the digital and paper-and-pencil

environment. This orchestration can be seen as teacher-centred and its component examples are as follows:

- *didactical configuration*: a classroom setting needs to be favourable for both teacher and students. The screen and blackboard must be visible to everyone.
- *a set of exploitation modes*: the teacher may take either student work or start a new task as a starting point for stressing the desired relation.

Following from our project, we distinguish several types of orchestration in this group:

1. **‘Link-3D prints-3D prints orchestration’** – the teacher stresses the relationship among different categories of 3D prints designed in the project. We say they work in the physical environment.
2. **‘Link-3D prints-applet orchestration’** – the teacher stresses the relationship between what happens in the physical and digital environment. We say they work simultaneously in the physical and digital environment.
3. **‘Link-3D prints-worksheet orchestration’** – the teacher stresses the relationship between what happens in the physical and paper-and-pencil environment. We say they work simultaneously in the physical and paper-and-pencil environment.
4. **‘Link-applet-worksheet orchestration’** – the teacher stresses the relationship between what happens in the digital and paper-and-pencil environment. We say they work simultaneously in the digital and paper-and-pencil environment.
5. **‘Link-applet-board orchestration’** – the teacher stresses the relationship between what happens in the digital and collective paper-and-pencil environment. We say they work simultaneously in the digital and collective paper-and-pencil environment.

In our project, the link orchestrations is seen as student-centred orchestrations. A collective discussion is present here, too.

Spot-and-show orchestration – it is based on the identification of interesting student work during lesson preparation and its deliberate use in class discussion. In other words, the teacher spots a student solution while preparing the lesson and brings it to the fore in the next lesson. This orchestration can be seen as student-centred and its component examples are as follows:

- *didactical configuration*: besides previously mentioned configurations, it includes access to the desired artefacts to spot the student’s work while preparing for the next lesson.

- a *set of exploitation modes*: the teacher may invite the student to explain his/her reasoning and/or ask other students for reactions or feedback.

Following from our project, the term ‘Spot-and-show orchestration’ is used without any changes.

Sherpa-at-work orchestration – term coined by Tourche (2004), it is based on a so-called Sherpa-student¹ who manipulates an artefact. He/she uses the artefact to present his/her work or to carry out the actions guided by the teacher. This orchestration can be seen as student-centred and its component examples are as follows:

- *didactical configuration*: a classroom setting needs to be favourable for the Sherpa-students who manipulate an artefact, the teacher who controls the Sherpa-student and other students who follow the actions of both the Sherpa-student and the teacher.
- a *set of exploitation modes*: the teacher may invite the Sherpa-student to present and explain his/her work or ask him/her to carry out the specific actions in that concrete environment.

In our project, we use the term ‘**Student-at-lead orchestration**’ instead of Sherpa-at-work orchestration. In this orchestration, one student manipulates the artefact and the other students discuss with him/her, guide his/her work or only follow his/her actions. Students work in groups without a teacher, but if necessary, the teacher helps them and guides their work. The leadership position is experienced by all students from the group.

Work-and-walk-by orchestration – it is a particular orchestration resulted from a case study described by Drijvers (2012) in detail. In our research project, this orchestration was highly dominant. The teacher walks by and discusses with students personally. It can be seen as student-centred and its component examples are as follows:

- *didactical configuration*: students sit at their desks and have the required artefacts at their disposal. In addition, the screen and the blackboard must be visible to everyone, in case the teacher needs to write down additional explanations.
- a *set of exploitation modes*: students manipulate artefacts and the teacher walks around the classroom, checking students’ work. If necessary, the teacher comes to the student with the aim to help, advise, point out the error.

¹ The word sherpa refers to the person who guides and who carries the load during expeditions in the Himalaya as well as to diplomats who prepare international conferences (Drijvers & Trouche, 2008).

Figure 7: Vygotsky's Instrumental Act model.

Altogether, the instrumental orchestration can be understood as a concrete teacher's organisation of activities with the aim to assist subjects' instrumental genesis in lessons. Each orchestration is characterised by its specific components, styles, levels and types that are planned and/or unplanned by the teacher in advance. It is a way how the teacher can support the students' instrumental genesis that is happening in carrying out an instrument-mediated activity. In this sense, Rabardel (2002) put forward the instrument-mediated activity model as a tool to analysis the subject(s)'s activities. A detailed description of this model, which will be used as a key to explore the students' instrumental genesis in our project, is given in the following section.

3.4 Instrument-mediated activity model

To get back to the beginning, the instrumental approach is based on the Activity Theory that is interested in the subject's development impacted by socio-cultural factors. As stated by Hardman (2008), "The strength of this approach to studying teacher/student interactions in classrooms is found in its ability to situate general developmental principles in time and place." This framework had been evolving over three generations, each proposing an activity model enabling the student work analysis. Pierre Rabardel (1995/2005), as one of the most influential researchers in the instrumental field, put forward his instrument-mediated activity model focused on the three models from Activity Theory generations. In our project, Rabardel's activity model (Rabardel & Bourmaud, 2003) has been used as a tool for the task and activity analysis to explore the subjects' instrumental genesis with the aim to support their visualisation in solid geometry. However, before its detailed characterisation, the three generations will be shortly described for a better

Figure 8: Leontiev's three-level model (a) and Engeström's Activity System model (b).

understanding of the key elements (Rabardel, 2002; Drijvers & Trouche, 2008; Hardman, 2008):

- *Vygotsky's Activity Theory* – is referred to the 1st generation representing the subject's interaction with objects mediated by artefact. This generation centers on the concept 'mediation' as well as 'instrumental act.' **Mediation** is understood as "the process through which the social and the individual mutually shape each other" (Daniels, 2015). It is the main fact of psychological activity; it transforms psychological functions. The central presumption of the mediation is that the subject can accomplish more with assistance than he/she can on his/her own. Considering the mention, a triadic model for the so-called **instrumental act** was introduced by Vygotsky (see Figure 7). It is understood as a complex involving a problem/task/activity which needs to be solved, and the subject's mental processes and the artefacts that are used to coordinate as well as carry out these processes. In summary, a subject uses artefacts, also called *mediational means* or *mediators*, in order to act on the activity's object. The theory points out a way to understand the subject's learning.
- *Leontiev's Activity Theory* – is referred to the 2nd generation representing the distinction between object-oriented **activity** and goal-oriented **actions**. Considering the mention, Leontiev's three-level model was introduced (see Figure 8. a). In this model, activities, which are spread over a significant period of time, have to be distinguished from the shorter processes that compose them. Activities can be broken down into different steps or phases including actions or chains of actions, which in turn, consist of operations. The activities are relatively oriented towards objectives, mostly collective or even institutional, while the actions are more generally individual or cooperative and each has a defined and immediate goal. Finally, operations are well-defined routines, used unconsciously in response to

conditions encountered during the execution of an action. In Leontiev's model, activities, actions and operations are nested subsystems within each other. Only in this complex nesting do they take on their full meaning: the same activity can be carried out by different actions; conversely, an action can serve different activities.

- *Engeström's Activity Theory* – is referred to the 3rd generation representing an Activity System model that expands on Vygotsky's model and Leontiev's work (see figure 8. b). In this model, a subject or a group of subjects acts on the object using mediating artefacts (instruments) in order to come to the specific outcomes. In turn, the subject(s)'s position is influenced by the system of rules, the community and division of labour.

Overall, regardless of generations, Activity Theory studies the activity of a subject or group of subjects involved in object-directed activity that is mediated by artefacts/instruments (Gueudet et al., 2014). Rabardel & Bourmaud (2003) define the **instrument-mediated activity** as "an analysis unit that retains the characteristic properties of individuals, cultural tools and contexts" and enables interdisciplinary research on subject(s)'s use of artefacts. It is the unit in which the *instrumental interaction* between the subject(s) and the artefact(s) takes place. The term 'instrument-mediated activity' is also known as the 'instrument-mediated activity situation' or the 'instrumented activity'.

As we mentioned above, our research project is based on Rabardel's instrument-mediated activity model (see Figure 9) representing three types of mediations or orientations of mediations which can be observed in the instrumented activity (Rabardel & Bourmaud, 2003; Laisney & Chatoney, 2018):

1. *Objects mediations* – mediations towards the object of the activity, which are epistemic and pragmatic in nature:

- a. Epistemic mediations to the object – they are aiming at getting to know the activity object.
 - b. Pragmatic mediations to the object – they are aiming at the subject's actions.
 2. *Interpersonal mediations* – mediations towards other subjects. This type of mediation is obvious in both collective and individual activities. They also can be epistemic or pragmatic:
 - a. Epistemic mediations to other subjects – the subject gets to know the others.
 - b. Pragmatic mediations to other subjects – the subject acts upon the others.
- These mediations can also be defined as collaborative mediation (in the context of a collective work), social mediation, etc.
3. *Reflexive mediations* – mediations towards the subject himself/herself. These mediations concern the subject's relation to himself/herself that is mediated by the artefact/instrument.

Therefore, when the subject uses the artefact/instrument, it mediates the relation of the subject with the object, other subjects and/or himself/herself. Although the three types of mediations can be observed in the subject's activity, some of them can be either more dominant than others or absent completely. To underline, in the model, the instrument is perceived in two ways:

- *Instrument as a mixed entity* (see Section 3. 1)
- *Instrument as a mediator* – a medium world between the subject (student, a user of the instrument, operator, worker, etc.) and the goal of action and/or more generally, the object of activity.

Altogether, the instrument-mediated activity model can be understood as a tool enabling the teacher to analyse his/her students' work. The instrumental genesis thus takes place while subjects are carrying out the activity with the instrumental orchestration background. In our project, using this model or otherwise, by analysing subject(s)'s solutions, we will explore the student(s)'s instrumental genesis in order to answer the research questions: Does the interplay of physical and digital manipulatives promote the development of students' spatial abilities in solid geometry lessons? How does the interplay of physical and digital manipulatives affect the development of students' visualisation in solid geometry lessons? How should the combined use of physical and digital manipulatives be integrated into solid geometry lessons to develop students' visualisation?

Figure 9: Rabardel's model of instrument-mediated activity. The dashed (line) arrows represent mediations in instrumented activity (non-mediated relations).

Figure 10: Lesson Mind Map.

Regarding the instrumental approach, we put forward a Lesson Mind Map (see Figure 10) that was used as a starting point for design activities as well as analysis of subject(s)'s work. Three different categories of components are distinguished here:

1. **Pre-defined and invariable components** – attributes framed in blue. When designing lessons, these components were firmly defined.
2. **Pre-defined and variable components** – attributes framed in orange. When designing lessons, these components were defined but may change while carrying out the activity.

Figure 11: Five-set toolkit.

3. **Pre-assumed** and **variable components** – attributes framed in red. When designing lessons, these components were strategically planned in advance. Since these components are depending on the subject(s)'s actions, the activity has been designed assuming the presence of a particular type of component in the particular action. In data analysis (Phase 3) these components were observed and explored.

The up-coming section is focused on a detailed description of Lesson 2. Its individual attributes will be depicted according to the introduced Lesson Mind Map.

4 Cube Cross-Section Material: Lesson 2

Lesson 2 is one of the five lessons, which has been designed and planned in Phase 1 within our project. The whole material/five-set toolkit was named as 'Cube Cross-Section. Connecting the digital and material world' and includes three different external representations of geometric solids that students manipulated in cube cross-section lessons. These include: 3D paper-and-pencil representations in the form of paper worksheets, 3D physical representations in the form of 3D prints and 3D digital representations in the form of GeoGebra applets, with the latter two being considered artefacts in the sense of the instrumental approach. Figure 11 shows a five-set toolkit as was planned and designed for follow-up cube cross-section lessons. Each set is specific with a different combination and sequence of assigned representations. Every representation has its own symbol and the students' work with that concrete representation/in that concrete environment (paper-and-pencil, physical or digital) is specifically indicated in worksheets.

In Lesson 2, as well as in the others, students solved one series of six cube cross-section tasks (activities).

Although no task was duplicated, all tasks were of the same type and had the same objective. That is, to construct the cross-section of a cube by a plane passing through three given points. Subsequently, Lesson 2 will be described in detail as we designed and planned according to the individual attributes shown in Lesson Mind Map.

4.1 Object

The object of Lesson 2 was as follows: To appropriately integrate multiple manipulatives into students' work to:

- a) support the development of their visualisation (i. e. creation of mental images, improvement of graphic representations of the created mental images, development of visual abilities and reinforcement of visual processes);
- b) help them apply solid geometry knowledge, as well as personal experiences with manipulatives, in paper-and-pencil constructions of cube cross-sections.

4.2 Subjects

In total, the research team consisted of one researcher (teacher/observer/we) and four grammar school teachers. The teachers were working at three Slovak schools admitting both male and female students of all spatial ability levels (results and findings obtained in Phase 0). However, because of the current pandemic situation, only one school could participate in Phase 3 of our research project. That means one researcher, one grammar school teacher and two classes (47 students) participated in the study. The students were in the 11th grade and worked in groups of 4/5 members. All participants had previous experience with GeoGebra software but not in solid geometry lessons. The teacher volunteered and allowed us to carry out the research directly in cube cross-section lessons with a total of 6 teaching units (5 designed lessons and 1 lesson of the final exam).

4.3 Toolkit

A paper worksheet, 3D prints and GeoGebra applets forming a toolkit was designed and planned for this lesson. In other words, paper-and-pencil (PaPE), physical (PhyE) and digital (DigE) environments are those contexts in which students solved tasks *Cube C07 – Cube C12* in Lesson 2. The key and most important resource was the printed worksheet because

Figure 12: Illustration of worksheet W2 for Lesson 2.

Figure 13: Illustration of 3D prints for Lesson 2.

of its manual/guide character referring to the transition among the three different environments. It means the PaPE is the starting point from which students move to the other environment(s) and always return to it. This property is also shown in Figure 11, where the symbol of PaPE is centred in a single line. The main idea is, through using multiple manipulatives, to support students in a better understanding of different representations of three-dimensional objects. Since physical and digital representations are considered artefacts/ manipulatives, we will now focus on the description of worksheet W2, and physical and digital manipulatives will be discussed in the up-coming section.

As we mentioned above, the worksheet is the key starting resource, and 3D prints and GeoGebra applets are considered complementary. To make it easier for students to orient themselves on the worksheets, the same layout was followed in the design (see Figure 12). At first, there are symbols and pictures of working environments in which students solve each task in the C07-C12 series. To emphasise, a “gateway” from this PaPE to the DigE is also present here. Scanning a QR code or using a web page link, students move from the “real world” to the digital one. Afterwards, one exemplary well-solved task (see Figure 12) and six unresolved tasks follow the introductory part of worksheet W2. The aim of the exemplary solution

Figure 14: Illustration of toolkit for Lesson 2 in the GeoGebra online book.

is to demonstrate how and when to move/work/orient in that concrete environment. Students initially manipulate the applet and look for the corresponding 3D prints; then cross-section a cube and write down the construction steps based on personal experiences with artefacts, as well as their solid geometry knowledge.

4.4 Artefacts

In Lesson 2, two types of artefacts were manipulated by students: 3D prints and GeoGebra applet. The artefacts not only complement the workbook, but they also complement each other. That is, the identified constraints of 3D prints are identified potentialities of the GeoGebra applet and vice versa. With regards to the 3D prints, five categories were proposed in the project, the entire classification of which is given in (Vágová et al., 2020). The discussed lesson includes the following:

- **Category A** – white transparent cubes demonstrating the cube cross-section points. The section points are painted red and every print is labelled. In this category, 3D prints are named A10-A16.
- **Category C** – grey transparent cubes demonstrating the cube cross-section construction. The section points are painted red and every print is labelled. In this category, 3D prints are named C10-C16.

The 3D prints were named strategically. The letter (A or C) represents the class of the category. The first number (1) represents the transparent character of the 3D prints, while the second number (1-6) was randomly added to the 3D print regardless of tasks C07-C012. As can be inferred from Figure 13, each pair of 3D prints demonstrates the cube cross-section steps from beginning to end. The

idea is to allow students to become physically “familiar” with mathematics and to manipulate it. The following potentialities and constraints have been identified for physical manipulatives (artefacts):

- *Potentialities* – students can touch, take, play, manipulate, observe and/or discover “mathematics” by hands. Mathematics thus, comes alive and becomes real and physical for students. It can be looked from different perspectives.
- *Constraints* – students can touch, take, play, manipulate, observe and/or discover two stages of cube cross-section construction. Only the initial and final stages are in the disposal. They can see the “finish”, but not a way to reach it. The cube transparency cannot be changed, either.

The second artefact we designed for Lesson 2 was the GeoGebra applet. Altogether, demonstrative (passive) and operative (active) applets were proposed in the project, the entire classification of which is given in (Vágová et al., 2020). The discussed lesson includes the **demonstrative applet**, the idea of which is to provide a complex visualisation of how to cross-section a cube. This passive applet, as well as the others, is available in the GeoGebra online book published on the GeoGebra platform. By scanning the QR code or using the web page link shown in the worksheet, students move from the PaPE to the DigE. The structure and content of the online book is the same as the content of the paper workbook (it consists of five worksheets for Lesson 1 – Lesson 5). The online version is extra supplemented with PDF format of the paper worksheets, as well as photos of 3D prints (see Figure 14).

In Lesson 2, students manipulate only one applet that comprises the whole C07-C12 series (see Figure 15). On the left side, there is a so-called selection box, green

Figure 15: Illustration of the demonstrative applet for Lesson 2.

Figure 16: Solution steps in Lesson 2.

slider, colour-changeable slider, and play/pause button. On the right side, the cube cross-section construction is visualised. The applet does not require any skills with the software. When students manipulate this, they just choose a task from the selection box and a colour-changeable slider then appears (see blue slider in Figure 15). Afterwards, students move this slider and constructions steps are visualised one by one. The green (permanent) slider, which moves automatically, changes the transparency of the cube. If students click the play/pause button (located in the lower-left corner in Figure 15), the green slider stops and they can move the slider themselves. In addition, using a computer mouse, rotation of the cube is also possible.

The following potentialities and constraints have been identified for the digital manipulative (artefact):

- *Potentialities* – students can touch, take, play, manipulate, observe and/or discover all stages of cube cross-section construction. They can see a way how to cross-section a cube and not only the “result”

of the construction. It can be looked from different perspectives and the cube transparency can be changed, too. Thanks to the dynamic software, paper-and-pencil and physical representations comes alive for students.

- *Constraints* – students cannot freely manipulate and/or discover the cubes. The applet is passive, it is not possible to try another way of constructing the cross-section of cubes.

4.5 Actions of the Activity

We consider **students' actions** to be steps to solve a series of tasks in the paper-and-pencil environment using physical and digital manipulatives. The cycle of solution steps in Lesson 2, i.e. transition among the individual environments, can be described as follows (see Figure 16).

4.6 STEP 1

The first step represents the *transition* from the PaPE to the DigE. Based on the paper-and-pencil representation shown in the W2 worksheet, students choose the corresponding cube from the selection box in the applet and thus move the slider to visualise the construction steps one by one. The main idea is to present how the cross-section of a cube could be done. The goals of this step are to support:

- the creation of internal representations (*dynamic images*)
- the development of visual abilities (*figure-ground perception, mental rotation, perceptual constancy, perception of spatial relationships and perception/recognition of spatial positions*)
- the reinforcement of visual processes (*mental image generation, inspection and transformation*)

In carrying out the research, the students discussed with each other, thought about these steps, and compared the image shown in the worksheet with the image shown on the computer screen. The object mediations and reflexive mediations were pre-defined here.

4.7 STEP 2

The second step represents the *interconnection* of the DigE and the PhyE. Based on the digital representation shown in the applet, students are looking for 3D prints that match a dynamic image. The main idea is to “make mathematics tangible” and thus allow students to touch and manipulate it. Otherwise, to connect mathematics with physical reality. The goal of this step is to support:

- the creation of internal representations (*concrete, kinaesthetic* and dynamic images)
- the development of visual abilities (figure-ground perception, mental rotation, perceptual constancy, perception of spatial relationships and perception/recognition of spatial positions. and *visual comparison* in particular)
- the reinforcement of visual processes (mental image generation, inspection and transformation).

In carrying out the research, the students discussed with each other and compared the image on the computer screen with 3D prints. In this step, interpersonal and reflexive mediations were pre-defined.

4.8 STEP 3

The third step represents the *transition* from the DigE and PhyE interconnected to the PaPE. Based on the acquired practical experience and knowledge of solid geometry, students cross-section a cube in the W2 worksheet. The main idea is to transform both the interconnected physical and digital representation and the created mental image into one paper-and-pencil representation. The goal of this step is to support:

- the improvement of external representations (*paper-and-pencil*)
- the creation of internal representations (concrete images)
- the development of visual abilities (figure-ground perception, perceptual constancy, perception of spatial relationships and perception/recognition of spatial positions and visual comparison)
- the reinforcement of visual processes (*mental image use*).

Figure 17: Didactic configuration in Lesson 2.

In carrying out the research, the students discussed with each other and compared their paper-and-pencil representation with both 3D prints and image on the computer screen. The construction steps were also written down here. For this step, object, interpersonal and reflexive mediations were pre-defined.

4.9 Instrumental orchestrations

The last attribute that was designed and planned for Lesson 2, as well as for others, is the instrumental orchestrations.

Considering the didactic configuration (see Figure 17), the same toolkit arrangement was set up in all stages of the lesson (see below). While each group of students had one computer and one box of 3D prints in the disposal, the researcher disposed of a computer, projector and blackboard. The student computers were placed at the top of the table, the screen of which was visible only to members of the group. On the other, the main computer, i.e. researcher's computer, was placed on the teacher's desk, the screen of which was visible to everyone. Thanks to the overhead projector, the computer screen was projected on the “wall screen” placed in front of the students. The board was located on the side wall, also visible to everyone. All other components will be discussed below, taking into account the lesson's individual stages:

- *Stage 1: Homework check* – the ‘discuss/explain-the-worksheet’ and ‘discuss/explain-the-board’ orchestrations were dominant here. The researcher checked students' homework and, if necessary, solved the questionable task together. The orchestration was student-centred and we played the role of both orchestra conductor and one-person band.

- *Stage 2: Summary and repetition of Lesson 1* – the ‘discuss’ orchestrations were conducted here. The researcher asked students questions and, if necessary, discussed the questionable answer together. The orchestration was student-centred and we played the role of an orchestra conductor.
- *Stage 3: Solving a task series* – the researcher and students first examined the exemplary task in detail. ‘Toolkit-demo’ in STEP 1, ‘link’ in STEP 2 and ‘explain’ orchestration in STEPS 1, 2, 3 were conducted. The researcher explained when, how and which representations would be used and pointed out the interplay of physical and digital manipulatives. The orchestrations were teacher-centred and we played the role of an orchestra conductor. Secondly, students solved a series of six tasks/activities. In each STEP, ‘student-at-lead’, ‘work-and-walk-by’ and ‘discuss’ orchestrations were organised. The students worked in groups without the researcher but, if necessary, we helped them and guided their work. The orchestrations were student-centred and we played the role of one-person band.
- *Stage 4: final summary and repetition* – the ‘discuss’ orchestrations were conducted here. The researcher asked students questions and, if necessary, discussed the questionable answer together. The orchestration was student-centred and we played the role of an orchestra conductor.
- *Stage 5: homework assignment* – the ‘explain-the-worksheet’ and ‘explain-the-board’ orchestrations were dominant here. The researcher assigned homework to the students and, if necessary, we answered the students’ questions. The orchestration was teacher-centred and we played the role of an orchestra conductor.

In summary, Lesson 2 was designed, planned and carried out according to the visualisation model and instrumental theory. Each component was strategically intended to assist students in their instrumental genesis and thus support the development of their visualisation and paper-and-pencil constructions in solid geometry lessons.

5 Conclusion

The aim of this paper was twofold. On the one hand, an instrumental approach was introduced and discussed, as well as a visualisation model. We mainly focused on the differentiation of artefacts and instruments, instrumental

genesis and orchestration together with an instrument-mediated activity model. The key characteristics and components were described in detail and finally adapted to our research project. On the other hand, based on the theoretical background introduced in the first part of this paper, one of the five cube cross-section lessons was described in detail. These lessons were designed and planned in our mixed methods research project, the main idea of which was to explore how digital and physical manipulatives could be integrated into solid geometry lessons. Altogether, a paper worksheet, 3D prints and GeoGebra applets were designed for and used in carrying out the discussed lesson, i.e. Lesson 2. The object of this lesson was to appropriately integrate multiple manipulatives into students’ work to (a) support the creation of their mental images, the improvement of external representations of the created mental images, the development of visual abilities and the reinforcement of visual processes; (b) help them apply solid geometry knowledge, as well as personal experiences with manipulatives, in paper-and-pencil constructions of cube cross-sections. Currently, we are in the process of quantitative and qualitative data analysis. The following publications steps will lead to the remaining lessons and the findings and results obtained from the practice.

References

- Artigue, M. (2002). Learning Mathematics in a CAS Environment: The Genesis of a Reflection about Instrumentation and the Dialectics between Technical and Conceptual Work. *International Journal of Computers for Mathematical Learning*, 7(3), 245–264.
- Cohen, L., Manion, L., & Morrison, K. (2018). *Research Methods in Education (8th edition)*. London and New York: Routledge, Taylor & Francis Group.
- Daniels, H. (2015). Mediation: An expansion of the socio-cultural gaze. *History of the Human Sciences*, 28(2), 34–50. doi: 10.1177/0952695114559994
- Dreyfus, T. (1993). Didactic design of computer-based learning environments. In C. Keitel & K. Ruthven (eds.), *Learning from Computers: Mathematics Education and Technology*, 121 (pp. 101–130). Berlin, NATO ASI Series: Springer-Verlag.
- Drijvers, P. (2012). Teachers Transforming Resources into Orchestrations. In G. Gueudet, B. Pepin, & L. Trouche (eds.), *From Text to ‘Lived’ Resources* (pp. 265–281). Netherlands: Springer Science.
- Drijvers, P., & Trouche, L. (2008). From artifacts to instruments: a theoretical framework behind the orchestra metaphor. In K. Heid & G. Blume (eds.), *Research on Technology and the Teaching and Learning of Mathematics* (pp. 363–392). Charlotte, NC: Information Age.

- Drijvers, P., Doorman, M., Boon, P., Reed, H., & Gravemeijer, K. (2010). The teacher and the tool: instrumental orchestrations in the technology-rich mathematics classroom. In *Educ Stud Math*, 75, 213–234. <https://doi.org/10.1007/s10649-010-9254-5>
- Gueudet, G., Buteau, Ch., Mesa, V., & Misfeldt, M. (2014). Instrumental and documentational approaches: from technology use to documentation systems in university mathematics education. *Research in Mathematics Education*, 16(2), 139–155.
- Guin, D., & Trouche, L. (1999). The complex process of converting tools into mathematical instruments: the case of calculators. *International Journal of Computers for Mathematical Learning*, 3, 195–227.
- Guin, D., & Trouche, L. (2002). Mastering by the teacher of the instrumental genesis in CAS environments: necessity of instrumental orchestrations. *ZDM: the international journal on mathematics education*, 34(5), 204–211.
- Gutiérrez, A. (1996a). Visualization in 3-dimensional geometry: In search of a framework. In L. Puig, & A. Gutierrez (eds.), *Proceedings of the 20th PME International Conference*, 1, 3–19.
- Gutiérrez, A. (2006). La investigación sobre enseñanza y aprendizaje de la geometría. In P. Flores, F. Ruiz, & M. de la Fuente (eds.), *Geometría para el siglo XXI* (pp. 13–58). Badajoz: F.E.S.P.M. y Soc. Andaluza de Educación Matemática Thales.
- Hardman, J. (2008). Researching pedagogy: Activity Theory approach. *Journal of Education: Periodical of the Kenton Education Association*, 45, 65–96.
- Johnson, R. B., Onwuegbuzie, A. J., & Turner, L. (2007). Towards a definition of mixed methods research. *Journal of Mixed Methods Research*, 1(2), 112–33.
- Korakakis, G., Pavlatou, E. A., Palyvos, J. A., & Spyrellis, N. (2004). 3D visualization types in multimedia applications for science learning: A case study for 8th grade students in Greece. *Computers and Education*, 52, 390–401.
- Laborde, C. (2001). Integration of technology in the design of geometry tasks with Cabri-geometry. *International Journal of Computers for Mathematical Learning*, 6, 283–317.
- Laisney, P., & Chatoney, M. (2018). Instrumented activity and theory of instrument of Pierre Rabardel. *Philosophy of technology for technology education to Sense/Brill*. <hal-01903109>
- Lean, G. A., & Clements, K. (1981). Spatial ability, visual imagery, and mathematical performance. *Educational Studies in Mathematics*, 12(3), 267–299.
- Lieban, D. (2019). *Exploring opportunities for connecting physical and digital resources for mathematics teaching and learning* (Doctoral dissertation). Johannes Kepler Universität Linz, Austria.
- López, N. R. (2012). *Análisis del desarrollo de competencias geométricas y didácticas mediante el software de geometría didáctica GeoGebra en la formación inicial del profesorado de primaria* (Doctoral dissertation). Universidad Autónoma de Madrid, Spain.
- Phillips, L. M., Norris, S. P., & Macnab, J. S. (2010). *Visualization in Mathematics, Reading and Science Education*. London New York: Springer Dordrecht Heidelberg.
- Presmeg, N. C. (1986). Visualisation in high school mathematics. In *For the Learning of Mathematics*, 6(3), 42–46.
- Rabardel, P. (2001). Instrument Mediated Activity in Situations. In A. Blandford et al. (eds.), *People and Computers XV – Interaction without Frontiers* (pp. 17–30). Springer, London. https://doi.org/10.1007/978-1-4471-0353-0_2
- Rabardel, P. (2002). *People and technology: a cognitive approach to contemporary instruments*. Université Paris 8, pp.188. <hal-01020705f>
- Rabardel, P., & Beguin, P. (2005). Instrument mediated activity: from subject development to anthropocentric design. *Theoretical Issues in Ergonomics Science*, 6(5), 429–461.
- Rabardel, P., & Bourmaud, G. (2003). From computer to instrument system: a developmental perspective. *Interacting with Computers*, 15(5), 665–691.
- Schnotz, W. (2002). Towards an integrated view of learning from text and visual displays. *Educational Psychology Review*, 4(1), 101–120.
- Simondon, G. (2017). *On the Mode of Existence of Technical Objects*. (Translated by Cecile Malaspina and John Rogove). Washington: Univocal Publishing.
- Sinclair, N., Bartolini-Bussi, M., Villiers, M. D., Jones, K., Koretenkamp, U., Leung, A., & Owens, K. (2016). Recent research on geometry education: An ICME-13 survey team report. *ZDM – International Journal on Mathematics Education*, 48(5), 691–719. <https://doi.org/10.1007/s11858-016-0796-6>
- Trouche, L. (2003). From artifact to instrument: mathematics teaching mediated by symbolic calculators. *Interacting with Computers*, 15, 783–800.
- Trouche, L. (2004). Managing Complexity of Human/Machine Interaction in Computerized Learning Environments: Guiding Student's Command Process Through Instrumental Orchestrations. *The International Journal of Computers for Mathematical Learning*, 9(281). <https://doi.org/10.1007/s10758-004-3468-5>
- Trouche L. (2005) Instrumental Genesis, Individual and Social Aspects. In D. Guin, K. Ruthven, & L. Trouche (eds.), *The Didactical Challenge of Symbolic Calculators. Mathematics Education Library*, 36 (pp. 197–230). Springer, Boston, MA. https://doi.org/10.1007/0-387-23435-7_9
- Vágová, R. (2018). *Priestorová predstavivosť a vizualizácia objektov priestorovej geometrie v riešení problémov* (Unpublished doctoral thesis). Constantine the Philosopher University in Nitra, Slovakia.
- Vágová, R. (2019). Plane, physical or dynamic representations of 3D objects. Which do you prefer when teaching solid geometry? In *Aplimat 2019: Proceedings of the 18th conference on applied mathematics*, 1237–1251.
- Vágová, R., Kmetová, M., & Lieban, D. (2020). The interplay of physical and digital resources to explore cube cross-section. In *Aplimat 2020: Proceedings of the 19th conference on applied mathematics*, 1067–1081.
- Vallo, D., Ďuriš, V., Záhorská, J. (2013). *Specific method how to solve selected stereometry tasks in educational process*. In *Procedia - Social and Behavioral Sciences: proceedings from 5th World Conference on Educational Sciences – WCES 2013*, 116(2), 2957–2961. doi: 10.1016/j.sbspro.2014.01.687
- Verillon, P., & Rabardel, P. (1995). Cognition and Artifacts: A contribution to the Study of Thought in Relation to Instrumented Activity. *European Journal of Psychology of Education*, X(1), 77–101.
- Zbiek, R. M., & Hollebrands, K. (2008). A research-informed view of the process of incorporating mathematics technology into classroom practice by in-service and prospective teachers. In

M. K. Heid & G. W. Blume (eds.), *Research on technology and the teaching and learning of mathematics: Volume 1. Research syntheses* (pp. 287–344). Charlotte, NC: Information Age Publishing.

Zimmermann, W., & Cunningham, S. (1991). *Visualization in Teaching and Learning Mathematics*. Washington, DC, USA: Mathematical Association of America.