Research Article

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The Relationship Between Problem-Solving Skills and Planning Ability by Using Tower of London Test

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Abstract: Mathematical problem-solving skills depend on many variables and cognitive skills are one of the most important of them. The purpose of this study is to determine the routine and non-routine problem-solving skills in terms of cognitive aspects. This aims to reveal the relationship between the planning ability and mathematical problem-solving skills of secondary school children, ages 10 to 13. An empirical quantitative research was done for this purpose. For the measurement of problem-solving skills, a curriculum-based mathematics achievement test was used and for assessing the planning ability, a computerized version of the Tower of London test was used. It was applied to a total of 416 students from the fifth, sixth, and seventh grade. As a result of the statistical analysis, a positive significant relationship was found between planning ability and problem-solving skills but there were not any significant differences in terms of gender.

Keywords: Problem-solving; Planning Ability; Mathematics Achievement; Executive Function; Tower of London Test.

1 Introduction

Understanding math concepts requires the generation of abstract representations of quantitative and qualitative relations between variables (Primi et al., 2010). Therefore, problem-solving can be one of the best methods that can be used to teach mathematical concepts to school-age children (Donaldson, 2011).

Problem-solving in mathematics refers to invisible, non-routine problems that require high-level understanding and cognitive skills to solve the problem in a meaningful way (Jagals and Van der Walt, 2016). It can be said that this way of solving problems related to the very hierarchical nature of mathematics; because the understanding of new concepts depends upon what has been learned before, and so the learning of the new topic becomes intimately tied to the knowledge of previous topics (Siadat, 2011). Therefore, parts of a task need to be solved first and kept in mind so that the student can solve the overall task (Lipnevich et al., 2016). For instance, some students consciously pay attention to the given problem by solving it hierarchically, but there are also students who just carelessly answer the problem when facing the test and in this frame, it can be stated that each student has a different process of thinking or plan of thinking in solving the problem (Pratama et al., 2019).

A problem schema can be defined as a cognitive construct that allows problem solvers to recognize a problem as belonging to a specific category that requires particular moves for a solution (Sweller and Cooper, 1985). This definition includes problem states, problem-solving operators, and their relations. Although there are different problem-solving models such as linear (hierarchical), ideal or recursive as defined by different researchers (Montague et al., 2000; Polya, 1962; Verschaffel et al., 1999), most of these models were developed on Polya's linear problem-solving model. According to Polya (2004), there are four steps in problem-solving. These are understanding the problem, planning, implementing the plan, and controlling the process. Therefore, solving a problem requires a good plan, in which a child takes decisions, selects strategies, and uses them, to successfully solve the problem. (Kroesbergen et al., 2010).

All the steps in problem-solving need the cognitive process to solve a mathematics problem. Bull et al. (2008) have propounded, as a consequence of their research, that cognitive skills help children to gain a higher level of math proficiency, and children identified as having a limitation in some aspects of cognitive functioning can

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have mathematical problems presented in a format that reduces the reliance on weaker cognitive skills.

Domain-general cognitive abilities especially executive functions (EFs) may provide good explanations for variability in early math learning (Kroesbergen et al., 2007). EFs comprise the mental capacities necessary for formulating goals, planning how to achieve them, and carrying out the plans effectively (Lezak, 1982). EFs contain different mathematical functions in their different structures such as planning, mental set-shifting, updating and monitoring, and inhibition (Miyake et al., 2000). For instance, inhibition is likely to be especially important at younger ages to suppress less sophisticated strategies, e.g., counting on the first addend, to use more sophisticated strategies, such as counting on the larger addend (Cragg and Gilmore, 2014). Shifting skills may help to switch between operations, solution strategies, quantity ranges, and notations (e.g., between verbal digits, written Arabic symbols, and non-symbolic quantity representations) and between the steps of a complex multi-step problem (Bull and Lee, 2014). Planning requires all these skills to think of alternatives, to weigh and make choices, and to evolve a conceptual framework or structure which can serve to direct activity (Lezak, 1982).

Roebbers et al. (2012) assumed a single factor EF and directly affected mathematics achievement (MA) as assessed by curriculum-based tests, teacher grades, and classroom learning-related behaviors. Depending on the significant relationship between EFs and MA, a theoretical model of EF skills and components of mathematical knowledge (Fig. 1.) was developed by Cragg et al. (2017).

A complex phenomenon like executive function skills, specifically planning ability (PA), cannot be measured with imprecise and over-simplified operationalizations. Researchers demonstrate the performance on cognitive tests such as Tower of Hanoi (TOH) or Tower of London (TOL) use working memory processes to generate, execute, manipulate, and concatenate these move sequences (Humes et al., 1997).

Many different paradigms for measuring these skills have been developed, for instance, TOH or TOL illustrate the influence of individual structural problem parameters on planning performance (Borys et al., 1982; Klahr and Robinson 1981; Spitz et al., 1982). When the TOL test was used as a single-time task to assess EFs, the performance was shown to develop with age, in line with children's increasing ability to control thoughts and actions as they grow older (Flavell, 1971; Siegler, 1983). In another study, Boghi et al., (2006) when investigating gender differences between fMRI measurement of brain functions in the measurement of planning skills by using TOL test,

Figure 1: Dashed lines represent relationships that change over the course of development. (Cragg et al., 2017).

they suggested a different process between groups in the system of planning and visuospatial working memory. Regarding that study, males relied on a strategy involving visual imagery whereas females relied on EFs (Boghi et al., 2006). On the other hand, Culbertson and Zillmer (2001) demonstrated that the gender factor does not have a significant effect on any output of the TOL test.

In light of these studies, for demonstrating the cognitive side of the mathematical routine and non-routine problem-solving skills in a structure of PA in EFs, it was investigated whether there is a positive and significant relationship between PA and math problem-solving skills.

2 Materials and Participants

This study was conducted face-to-face just before the Covid-19 pandemic restrictions began in schools. To reduce the possible effects of the socio-economic structure in the variables, schools were selected from different socio-economic backgrounds. There were four public schools in low (141 students), middle (146 students), and high (129 students) socio-economic backgrounds.

Participants of this study were secondary school students ages 10 to 13 from the fifth, sixth, and seventh grade in Istanbul. Participation rates at each grade level are close to each other (36.5% in 5th grade ages 10 to 11, 26.4% in 6th grade ages 11 to 12, and 37.00% in 7th grade ages 12 to 13). The gender distribution of the participants is also close to each other in all grades (203 girls and 213 boys).

To assess the PA, TOL-F test was used and for the mathematical problem-solving skills a curriculum-based MA test was used.

2.1 Mathematics Achievement Test (MA)

The MA test was prepared by the authors and three Mathematics teachers. The questions in the MA test were based on the curriculum for each grade. A total of 20 routine mathematical problems were asked from the subjects of algebra, geometry, data analysis, and number to measure the mathematical problem-solving skills. The participants took the tests in their own classrooms and one lesson hour (40 minutes) was given to all of them for the MA test.

2.2 Tower of London Test Freiburg Version (TOL-F)

TOL test has been widely used as a single-time task to assess executive functioning abilities such as planning and problem-solving as well as implicit learning (Schiff and Vakil, 2015). It holds non-routine problems of increasing difficulty. Implementation of the TOL-F version is computerized in the form of a realistic three-dimensional representation of a wooden model of the tower configuration shown in Fig. 2.

The test aims to convert a starting state into a defined finishing state in as few moves as possible. Therefore, the respondent must convert the start state into the goal state. The minimum number of moves is shown on the left next to the start state. The TOL-F is worked using the computer mouse. For example, the left-hand rod can hold three balls, the center rod holds two balls, and the right-hand rod one ball. The ball colors are red, yellow, and blue.

The following rules have to be respected during the moves:

- Only one ball can be moved at a time
- The balls can only be moved onto the rods of the starting configuration shown in the lower part of the screen
- Balls that are blocked by balls lying on top of them cannot be selected.
- Balls cannot be placed on rods that are already full.

The remaining working time is continuously reported via a bar at the border of the screen. If the time limit is exceeded, the item that is being worked on is automatically terminated. If the time limit is exceeded for three successive items, the TOL-F terminates automatically. The three-move problems represent the familiarization phase that is usual in an ability test to avoid distortion of individual test results on account of any remaining difficulties in understanding the task. In the TOL-F the actual measurement of planning ability is based on the four- to six-move problems that are presented.

Figure 2: screenshot of the TOL-F test used in this study.

Before starting the test, it was asked whether the students had experience with the TOL-F test and then shown how to do the test with examples. After confirming the student understood the test, each student performed the TOL-F test individually. In this study, it is concluded that no student had known about this test before.

3 Main Results

Students' performance in the two tests was measured to analyze the relationship between routine, non-routine problem-solving skills, and PA.

In Fig. 3 and Fig. 4, the participants showed normal distribution and in Table 1, the descriptive statistics are given in MA ($M = 10.03$, $SD = 4.63$) and TOL-F test for PA ($M = 3.00$, $SD = 1.47$). Mean score differences are related to the number of questions. There are 20 routine questions in the MA test and 12 questions in the TOL-F test for measuring PA in terms of problem-solving skills. The differences between standard deviations revealed that students show a more homogeneous distribution in terms of PA than mathematical problem-solving skills.

Girls outperformed boys on the planning and attention scales of the cognitive assessment system and achievement in Naglieri and Rojahn's (2001) research and the lower planning scores earned by the boys provide a cognitive explanation for procedural arithmetic errors and they suggest that girls may perform this activity better because of better planning processing. However, their findings suggest that there are not any significant differences in terms of routine, non-routine problem-solving skills, and PA in terms of gender differences in this study (Table 2).

Table 1: Descriptive statistics of the tests results.

	N	Minimum	Maximum	Mean	Std. Deviation
MA	416	1.00	20.00	10.03	4.63
PA	416	0.00	8.00	3.00	1.47

Table 2: Descriptive variables of genders in MA and PA.

Descriptive Variables	Grade 5		Grade 6		Grade 7	
	Mean	SD	Mean	SD	Mean	SD
MA test	10.69	4.69	9.37	4.68	9.83	4.46
Boys	10.40	4.76	8.65	4.28	9.65	4.37
Girls	11.02	4.61	10.26	5.03	10.04	4.57
PA	2.76	1.31	2.99	1.38	3.25	1.62
Boys	2.80	1.16	2.96	1.34	3.28	1.59
Girls	2.70	1.47	3.02	1.45	3.22	1.67
CS	9.03	2.62	10.66	1.75	10.89	1.30
Male	9.02	2.78	10.40	2.10	10.85	1.22
Female	9.05	2.44	10.97	1.12	10.94	1.40

3.1 Relationship Between Planning Ability and Problem-Solving Performance

All correlational analyses were conducted by the use of Pearson Product Moment correlation coefficients. Statistical analyses revealed that there was a significant positive correlation between PA within the scope of executive functions, and problem-solving skills ($p < 0.05$).

This correlation is practically significant ($p < 0.05$), indicating that, when students plan the solution of a problem, they monitor their progress and reflect on their mathematical knowledge. Thus, planning of the problem will construct their correct solution. When the positive relationship was examined at each grade level, there was the same significant ($p = 0.05$) correlation at the 6th and 7th grade but it is not significant at the 5th-grade level.

In the TOL test, which consists of non-routine problems in all grades, the questions are measured for two variables; one is for PA and the other one is correct solutions (CS). There is a significant correlation between MA and these two variables in Table 3. The level of significance is 0.05 with PA and MA; 0.01 with CS and MA. On the other hand, a correlation of CS was found at a 0.01 significance level in terms of grades, except PA and MA.

The results obtained from measuring routine and non-routine problem-solving skills demonstrated that the relationship between these two different types of problem-

Figure 3: Distribution of the planning ability scores of the participants. **Figure 4:** Distribution of the mathematics achievement scores of the participants.

solving skills increases as the grade level increases. The percentage of success in MA obtained from routine problems is approximately 50% at each grade level (Table 4).

In the TOL-F test, questions which are measured for PA, the percentage of general success in all grades is 25% (Table 4). Answers without considering the number of steps for the solution are measured for the variable CS. The percentage of the CS in all grades is 73% (Table 4).

There are large differences (75%) in the mean values (Table 2) of the same questions in the output of the TOL-F test in terms of CS and PA variables. The main reason for the difference between CS and PA is that participants make their first move without planning. Since it is impossible

Table 3: Correlations between variables.

Correlations				
	MA	PA	CS	Grade
MA				
Sig. (2-tailed)		0.019*	0.028*	0.122
N	415	415	415	415
PA				
Sig. (2-tailed)	0.019*		0.000**	0.003**
N	415	416	416	416
CS				
Sig. (2-tailed)	0.028*	0.000**		0.000**
N	415	416	416	416
grade				
Sig. (2-tailed)	0.122	0.003*	0.000**	
N	415	416	416	416

*. Correlation is significant at the 0.05 level (2-tailed).

**. Correlation is significant at the 0.01 level (2-tailed).

for someone who makes the first and second moves wrong to reach the planned number of steps, the number of steps towards resolution is important the evaluation of PA. Even if the results of the questions made without planning are correct, they are not taken into account in terms of PA when the number of steps is higher than the specified number of steps.

Since the significance levels of the correlations in Table 3 between the variables for comparing the success percentages obtained from the data are different, the mean, standard deviation, and success percentages of the variables are given in Table 4.

Accordingly, there is a difference in terms of success percentages of the PA and CS variables obtained from the MA and TOL tests with moderate correlation. Likewise, success percentages of highly correlated PA and CS variables are quite different from each other. In this case, although there is a significant correlation in solving routine and non-routine problems, they differ considerably from each other in terms of success percentages. A similar situation exists between MA and PA. Although the percentage of solving the desired number of steps correctly by using the given time, which expresses the PA in the TOL test, increases by the grade level this percentage of solving the desired number of steps is very low. This situation shows that secondary school students (ages 10 to 13) are weak at the planning stage in solving non-routine problems in this study. It was determined that

Table 4: Group statistics in all variables.

Group Statistics					
	grade	N	Mean	Std. Deviation	Percentage of the mean in the all questions
MA	7.00	154	9.84	4.46	49%
	6.00	110	9.37	4.68	47%
	5.00	152	10.66	4.69	53%
Mean of all grades					50%
PA	7.00	154	3.26	1.63	27%
	6.00	110	3.00	1.38	25%
	5.00	152	2.76	1.32	23%
Mean of all grades					25%
CS	7.00	154	10.90	1.31	78%
	6.00	110	10.66	1.76	76%
	5.00	152	9.04	2.62	65%
Mean of all grades					73%

when they were asked to do the same problems without measuring their PA and determining the number of steps, their success rates increased considerably. It can be said that students at the secondary school level prefer trial and error instead of planning in advance in solving non-routine problems. In both tests, the students who tried to solve a problem without planning and thinking about the solution, could not reach the main result.

4 Conclusion

The TOL test is a measurement tool that is generally used in clinical and experimental neuropsychology as putative measure of frontal lobe function (Welsh et al., 1999). It measures EF and specifically PA with non-routine problem situations in a certain time period. This study was planned interdisciplinarily and the relationship between mathematics achievement and these cognitive skills was investigated.

The most important thing in problem-solving is understanding, reasoning, and methodology up to

producing the right solution (Nursyahidah et al., 2018). In this research, the total number of correct solutions in each test indicates the level of children's planning and problem-solving efforts. In a new problem, planning is the priority for the solution. Therefore, in the analyses, it was seen that competence in routine and non-routine problem-solving was related to PA.

One of the reasons why arithmetic-based operations are better understood or performed than word problems in learning mathematics is that students try to find results using only the numbers they see without doing the problem-solving, understanding, and planning stages (Soylu and Soylu, 2006). Therefore, the low rate of routine math problems can be attributed to this and it may be due to the inability or deficiencies of students in using their cognitive skills. Coordinated use of these cognitive skills and competence of these skills are very important elements underlying mathematical success (Welsh, et al, 2010). High scores show that children are capable of developing and maintaining mental planning patterns over time. The findings of this quantitative investigation suggest that students who solve mathematical problems reflect their PA on problem-solving processes and become aware of their strategy of knowledge.

In a comprehensive study, a large sample of typically 9 years old children was tested on an extensive battery of measures, and Szűcs et al. (2014) found that visuospatial short-term and working memory were significant predictors of MA. This study also has a similar result and it shows the significant relationship between one of the cognitive factors of PA and MA. Therefore, the cognitive sides of mathematics are important in terms of a better understanding of mathematics concepts, and more effective use of problem-solving skills from the foundation of mathematics learning.

5 Limitations and Future Directions

The study was limited to participants within a certain age range and data from a limited region. Since the studies in which the TOL-F test was performed by young students are very few (Albert et al., 2011, Gangopadhyay et al., 2018), the data in this age range can be increased.

By expanding the scope of this study, the effect of cognitive skills on mathematics education and problem-solving skills can be investigated in terms of other executive functions. Since the TOL-F test is a non-verbal test, comparisons can be made by applying the available data to students in the same age group in different

countries and it can be determined whether the socio-cultural structure has an effect on cognitive factors.

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