Review Article

Daniela Velichová*, Jana Gabková, and Peter Letavaj

Case study from Slovak University of Technology, Bratislava

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Abstract: Analysis of the experimental results presented in the following chapter reveals experiences with introduction of active learning methods in basic mathematics courses at the Faculty of Mechanical Engineering, Slovak University of Technology in Bratislava. Experiment was implemented in the academic year 2018/19, in basic courses Mathematics I and Mathematics II scheduled in the first year of bachelor study programs and in subject Basics of Statistical Analysis for bachelor students in the second year of their studies. The aim of the experiment was to find achieved level of knowledge acquisition and to compare abilities of students to solve mathematical problems individually or by collective work and collaboration in small groups. Anonymous questionnaire answered by students after completion of the experiment provided rich feedback and overview of their opinions, from which their attitudes towards different teaching methods applied in 3 compulsory subjects from their study programs could be deduced and summarised. Based on received data, research analysed also results that students achieved comparing their achievement from the secondary schools and in both maths courses at the university.

1 Active learning methods implementation at STU in Bratislava

Slovak University of Technology in Bratislava (STU) is the largest and best technical university in Slovakia with 7 faculties and the Institute of Management. STU is a public

university and offers education mainly in technical, technological, technical-economical, technical-information and technical-artistic fields of study using the modern methods of education, laboratories and practical training. It is aimed at the study branches with stable opportunities of students' employment at the labour market. The studies at STU and its faculties are performed at 3 levels: the bachelor degree, the engineer or master degree study program, and the doctorate degree study program.

STU offers education in foreign languages, primarily in the following areas: civil engineering, mechanical engineering, electrical engineering, chemistry and food technology, architecture. STU in Bratislava strives to be an internationally recognized and important, research-oriented technical university. It seeks to provide a high quality, internationally comparable education to a broad spectrum of students from the young generation in promising fields, based on independent and critical thinking, entrepreneurship and creativity, with a view to practical application and success in life, and taking into account the human aspects of education and technological progress.

The Faculty of Mechanical Engineering (FME), as one of the faculties of STU, delivers study through seven bachelor programs of 3 years duration (leading to the Bachelor of Science degree - BSc, and twelve 2-year graduate programs leading to the Master of Science degree - MSc or Engineer (Ing.)). Both programs operate on a winter and summer semester system. Each semester lasts 13 weeks followed by a six week examination period. Based on the European Community Courses Credit Transfer System (ECTS) the total for each year of study corresponds to 60 credits. Each BSc and MSc program consists of relatively fixed curricula which are taken during specific semesters and years. A program is composed of obligatory and elective subjects. The portion of electives is highest in the final year of study. The minimum requirement for a Bachelor of Science degree is to gain 180 credits, and 120 credits for Master of Science (Engineer) respectively. Degree programs at FME STU strive to prepare all graduates as professionals with a sound, technological training, analytical skills and valuable assets for the employment market.

It has been proved by the everyday evidence that implementation of active learning methods in teaching increases

*Corresponding Author: Daniela Velichová: Slovak University of Technology, Bratislava, Slovakia; Email: daniela.velichova@stuba.sk

Jana Gabková: Slovak University of Technology, Bratislava, Slovakia; Email: jana.gabkova@stuba.sk

Peter Letavaj: Slovak University of Technology, Bratislava, Slovakia; Email: peter.letavaj@stuba.sk

enthusiasm for both learners and facilitators. And even more, active learning also improves learners' perception and attitude towards studied subject, strengthens their motivation and raises their interest and involvement into the entire learning process and desire to acquire new knowledge. These are all critical attitudes in establishing an active learning environment. In order to improve continually teaching methods and learning scenarios, university teachers at FME STU are open to new ideas and implementation of active learning methods, in all subjects taught at the faculty. Institute of Mathematics and Physics, responsible for teaching all subjects related to mathematics, physics and their applications, actively cooperated in the DrIVE MATH project aimed to implementation of active learning methods in teaching mathematics to engineering students.

An analysis of the results of experiment carried out at the Faculty of Mechanical Engineering, Slovak University of Technology in Bratislava, within the DRIVE MATH project activities is presented in the following. Experiment was implemented in the academic year 2018/19, in basic courses Mathematics I and Mathematics II scheduled in the first year of bachelor study programs and in subject Basics of Statistics for bachelor students in the second year of their studies.

The main goals of the experiment were to find out:

- abilities of students to solve mathematical problems independently, and within a small stable group throughout the semester,
- attitudes and opinion of students on this form of teaching scenario,
- opinion of students on introduction of applied maths problems to the work in groups.

Several resources and tools were used to analyse the results of implemented empirical experiment, as study of research resources, natural didactic experiment, didactic tests and questionnaire that was non-standardised, cognitive and objectively scorable. Questionnaire was aimed to discover attitudes and opinions of students on this novel teaching scenario. Evaluation of experiment results and students' achievements is presented in statistical data supplemented by analysis of feedback obtained from students.

2 Mathematics I

The group of 99 students was randomly chosen from the pool of faculty freshmen in the academic year 2018/19. This large group scheduled to attend lectures presented in the lecture theatre was distributed to 5 working groups attend-

ing practical exercises, about 20 students in each, as some of them interrupted their studies during semester.

EduScrum method was implemented for the group work of students, as introduced in Andriotis (2018); Delhij *et al.* (2015); Sutherland (2014). In addition to 5 sprints of team work prepared and implemented in these 5 groups of students, there was applied also an alternative method of individual work realised by 4 worksheets prepared for students as individual tests after self-study period. For the team work, students in each of the 5 small groups were distributed into steady working teams of 5 students, who cooperated during the whole semester. Groups were formed by the own choice of individual students, almost randomly, as they did not know each other during the first week of semester.

The experiment was realised in 5 topics for team work:

- Linear algebra
- Functions with one real variable - basic properties
- Differential calculus
- Integral calculus I - indefinite integrals
- Integral calculus II - definite integrals

Each topic for team work consisted of 5 problems

- 1st - 4th problem were standard mathematical problem
- 5th problem was applied problem from the mechanical engineering field.

All teams solved the same problems in each from 5 working groups, while different problems were solved in each of the working groups. During the team work students cooperated together, thought each other, and each one was responsible for solution of one from the problems distributed by team leader. Always another student in the team was the scrum master in the role of team leader, responsible for the organization of the team work and final presentation of one of the problems in front of the class. It was not determined in advance which of the solved problems had to be presented by particular team, and its scrum master.

Major part, about 60 minutes of the practical exercise lasting 100 minutes, was spent on solving the problems, about 10 minutes were taken by administration, and remaining 30 minutes were scheduled for presentations of solutions. Students could achieve 2 points in each problem - max 10 points together, which was the score for all team members.

Individual work was applied also as an alternative method. It was realised by 4 worksheets prepared for students as individual tests after a self-study period, while the following topics were covered:

- Limits of functions and equations of asymptotes to function graphs
- Derivatives of functions
- Integration methods for indefinite integrals
- Determination of integration domains for definite integrals

Students had to solve about 8 easy problems in each topic in about 40 minutes. Score they could achieve in the 1st - 3rd topic was max 12 points, in the 4th topic it was max 14 points.

Experiment was completed with a questionnaire that was aimed to discover attitudes and opinions of students on both teaching scenarios and on inclusion of applied mathematics problems with technical engineering contents to the subject itself. Majority of students, 97 out of 99, wanted to answer posed questions (2 students were ill), while after the formal part there were free interviews with students. They openly reflected their opinions and were not reluctant to share their views with teachers.

Basic information about cohort of students

The group consisted of majority of male students, only 1/7 of them were female students. Almost 50% were grammar schools graduates, more than 30% graduated from technical secondary schools with mechanical specialisation and almost 20% graduated from other secondary schools.

Secondary schools graduates ratio - grammar : technical-mechanical : other = 5 : 4 : 2.

Almost half of the students reached matura from mathematics, which can be considered rather positively.

With respect to secondary school education, many students attended 5 and more lessons from mathematics per week due to their choice to attend seminar from mathematics aimed to prepare students to matura from mathematics and to take some topics at the university level (differentiation and integration of functions). Number of students, whose mark from mathematics at the secondary school was 1, 2 or 3, can be regarded as comparable, which was reflected in the overall average mark from mathematics at secondary school that is very good - 2,2 (median is 2). This average mark was influenced only insignificantly by number of students (8%) who received weak mark 4 from mathematics. Further on, it was found that almost one half of students attended Preparatory course of Mathematics which is realised each academic year for university newcomers before the start of the respective academic year winter semester. This course is aimed to students who need to supplement and strengthen their knowledge of secondary school mathematics. As much as 63% students from those, whose mark

from mathematics at the secondary school was 4, also took this course.

It can be concluded from the above that students from secondary schools have got very good "starting conditions" for their study at the FME STU. It was assumed that the better mark from mathematics at the secondary school means the better score achievement in the experiment.

Figure 1: Cohort of students

Figure 2: Mathematics attendance at the secondary schools

* – mark from maths in the last school year at the secondary school

** – number of maths lessons in the last school year at the secondary school

Results of team and individual work

Students' results achieved in respective topics for both work types are presented in Figure 3 – 5. Average point score achieved by students in the first two topics (Linear algebra, Functions with one real variable - basic properties) in team work was almost 66%. Both topics are known to students from the secondary schools.

Figure 3: Results of team work

* – Functions with one real variable - basic properties

** – Integral calculus I - indefinite integrals

*** – Integral calculus II - definite integrals

Figure 4: Results of individual work

* – Limits of functions and equations of asymptotes to function graphs

** – Integration methods for indefinite integrals

*** – Determination of integration domains for definite integrals

Figure 5: Results of team and individual work

The low score in topic Differential calculus, less than 50%, could be explained by the fact that it belongs to the curricula at university level. Just a small group of students was acquainted with this topic in the mathematical seminars at some grammar schools. Majority of students could learn about this topic only at the university. The main problem was to learn properly how to find derivative of function.

Average point score for team and individual work was about 50%. In team work, this score was influenced by a very small, almost zero number of points students achieved for solving applied mathematical problem in each topic. Score in individual work was influenced by problems included in the third topic focused on the first and second derivative of function.

Questionnaire

More than half of interviewed students (52,6%) regard mathematics as demanding subject ("very demanding" and "demanding"), while it is not demanding only for 5,2% of students.

Figure 6: Complexity of mathematics

Figure 7: Evaluation of mathematical knowledge and skills from the secondary school

One of the rather negative aspects is the fact that after 2 months of study at the faculty almost one third of students evaluated their knowledge and skills from the secondary school mathematics as weak and almost one tenth of them even as "very weak".

These facts could be particularly explained by the following

- students came to study at FME STU from different types of secondary schools with different approach adopted to teaching mathematics and different scope of the subject, which was reflected in different levels of their mathematical knowledge and skills
- students do not have sufficient working methods, habits and skills for study at the university (to work independently, systematically, to be able to apply knowledge in practical tasks, to acquire knowledge, which is steady and sustainable, ...)
- university freshmen in the 1st year must overcome the difficulties of the transition period changing their study approach adopted at the secondary school to the university style
- students who attended Preparatory course of Mathematics were "directed" to take it by their parents, or these students were the best ones, who expected to receive more information there, not only to repeat the knowledge they have already acquired.

Figure 8: Method to improve knowledge of mathematics

(You can also tick off more answers.)

- I consult the problem with my teacher directly during the tutorials or at the lecture
- I consul the problem with my classmates - about 74%
- I utilise organised consultations at the Institute of Mathematics and Physics - !!!

- I attend private lessons in mathematics
- I practise self-study approach - about 81%
- I do nothing

Figure 9: The knowledge assessment during the semester

About 92% of interviewed students prefer to have assessment of their knowledge 5-times per semester ("rather no than yes" and "no"). EduScrum is great!

Figure 10: Ways of knowledge assessment

Figure 11: Individually solving of problems in each team work

Some of the rather negative aspects could be deduced from the feedback received through answers that students gave on the questions related to ways of cooperation. The outcomes are presented in the following graphs.

Figure 12: Classmate's help to me

Figure 13: My help to classmates

To summarize these facts we see that

- almost one third of students solved only "my" problem
- only 2% of students solved all problems
- only 6 students didn't need their classmates' help to solve problems
- about 87% of students had to help their classmates with their problems

Students had to choose one from the 4 given possibilities:

- I have solved some problems in advance - more than one third
- I have studied the topic in advance
- I have studied the topic and solved some problems in advance - nearly 50%

- I did nothing and I relied on the help from my classmates

Figure 14: Preparation for solving problems in the team work

Self-estimation and awareness of own weak and strong abilities was tested in the following question.

- 45% students answered positively ("yes" and "rather yes than no")
- 45% students don't know how to compare their success with their classmates.

Figure 15: Comparison with classmates

Answers:

- I agree with inclusion of theoretical question into each from problems - 45%
- I agree with inclusion of theoretical question into each from problems provided it will be a part of the theoretical examination as it is in case of problem solutions - 38%
- I do not agree with inclusion of theoretical question into problems for semestral activities at all
- it is all the same to me

e) I do not know

Figure 16: Opinion on the test of mathematical theory during the semester

(You can also tick off more answers.)

- a) it has taught me how to work in team - 54%
- b) it has taught me to be responsible for my solution - 50%
- c) it has taught me to be responsible for the whole team
- d) it has taught me to discuss the problem in a constructive way - 45%
- e) in discussion about the problem I could understand the topic better - 46%
- f) I have acquired better knowledge about solution of the problem
- g) I was motivated to study the topic in advance
- h) I was motivated to solve in advance some problems related to the topic
- i) I was motivated to improve my knowledge from mathematics - about 30%
- j) I have used consultations from mathematics
- k) in a no way

Figure 17: The influence of team work on the student

As mentioned before, average score in team work was influenced by applications of mathematics included in each from the topics. These problems caused difficulties to 75% of students ("very difficult" and "difficult").

Figure 18: Complexity of application of mathematics

Figure 19: Need for mathematics at technical university

Interviews with students revealed also that:

- applications of mathematics demonstrate how mathematics can be used continually in special technical subjects,
- these problems need to be introduced into teaching mathematics from the first year of study,
- students "started" to work with different variables (in mathematics variables are denoted usually only as x , y), and also with constants denoted generally (e.g. constant g - gravitational acceleration usually understood as value $9,81 \text{ m/s}^2$),
- students became aware of the parallel in the conceptual apparatus of mathematics and technical subjects, for instance mathematical concept of stationary

points of function is related to concept of the equilibrium position of the state in technical disciplines.

Remark: All used applications of mathematics were selected from the lecture notes for specialised subjects taught at the faculty. Some of these are introduced in the lecture notes as examples of solved problems. The advantage of these applications is that once in the further study students will "remember them".

Figure 20: Team work on solving mathematical problems

Team work helped more than to 75% of students to understand learned content better. As students pointed out in the interview - this taught them also to work in team, to be responsible for their solutions and for the team as a whole, to lead constructive discussions about problem, and to gain new knowledge on problem solving.

Figure 21: Opinion on presentation of problem solutions

Answers:

- presentation by students suits me well - 1/4 of students
- presentation by students does not suit me
- presentation by teacher suits me best - nearly 50%
- it is all the same to me; important is the fact that I learn the solution

Figure 22: The influence of individual work on the student

(You can also tick off more answers.)

- it motivated me to study the topic in advance
- it motivated me to solve problems related to the topic in advance - nearly 60%
- it motivated me to improve my previous knowledge of mathematics - about 46%
- I have utilised consultations from mathematics
- I did nothing

Figure 23: Individual work on solving mathematical problems

Individual work did not only help to 1/7 of students to practise learned material better, but up to 85% of students grasped the learned topics considerably better, also due to their self-study efforts. Majority of students consider individual tests as a good assessment method, because it stimulates students to work on their own. They were aware of the fact that they could not rely on the help from their colleagues, which was sometimes the case of weaker students in the team work knowledge assessment. Better students, and especially the excellent ones, appreciated this test because they could work for themselves in their own speed, and receive as many points as they managed to earn in the respective time period. They could also somehow test their own abilities in order to improve individual study efforts, if necessary.

Figure 24: Ways of assessing knowledge during semester

Interview results showed that up to 82,5% of students would prefer to assess knowledge in mathematics during semester not only by individual work but in combination with the newly implemented eduScrum method.

Applications of mathematics

As mentioned before average score in team work was influenced by applications of mathematics included in each from the topics. These problems caused difficulties to 75% of students, who regarded it as "very difficult" or "difficult".

Positive aspects of eduScrum method

- Continuous assessment of students' knowledge during whole semester, from separate parts of curricula consequently
- method eduScrum taught students
 - to work in the team
 - to be responsible for their solutions

- to be responsible for the whole team as a unit
- to lead a constructive discussion about problem
- to understand better the problem through discussion
- to acquire new knowledge on possible problem solutions
- to learn how and why to improve knowledge from mathematics

Figure 25: Complexity of application of mathematics - groups / teams

Figure 26: Complexity of application of mathematics - student

Negative aspects of eduScrum method

- Random distribution of students to working teams
 - based on their friendship after the arrival to university study, not on base of knowledge level from mathematics, from which the consequences can be formulated as
- weaker students
 - * learned how to "grasp" problem, how to start to solve it

- * appreciated help from better students with solutions of their own problems
- * acquired more points in their "score" from particular topics thanks to better students in their team
- better students were "constrained" by weaker students
 - * they explained to weaker students how to solve their problems in order to receive as much points as possible for their own score they solved entirely also problems of other students, with the same points for everybody
 - * they often received less points on their "score" for not correct solutions of problems solved by weak students, as they could receive provided they had solved the problems themselves and correctly, or in another constitution of the working team.
- Better students were willing to work, later during the semester, in smaller groups (2 - 3 members) and they agreed on finding solutions of all 5 problems in the given time limit.

Discussions with students at the end of course Mathematics I revealed that students would like to work in this learning scenario also in the course Mathematics II, but in other distribution of working teams as it was in the case of Mathematics I.

Traditional ways of assessing knowledge of students at the secondary schools, and also at universities (up to now), are based on individual work of students in form of written tests. EduScrum method introduced to students a new way of knowledge assessment, which students rated quite positively.

Example of team worksheet with solutions for subject Mathematics I, topic Differential calculus I is presented in the following.

CALCULUS I - 5 problems with theoretical questions

PROBLEM 1. Determine domain of definition of function f and sketch function graph. Find range of function f :

- a) $f : y = 1 - (x - 3)^2$
- b) $f : y = \log_{\frac{1}{3}}(2x + 4)$
- c) $f : y = 3 \cdot \cos(x + \pi)$
- d) $f : y = -\cot(\pi - x)$

Solution:

- a) $D(f) = \mathbb{R}, \quad H(f) = (-\infty, 1]$
- b) $D(f) : x > -2, \quad H(f) = (-\infty, \infty)$
- c) $D(f) = \mathbb{R}, \quad H(f) = \langle -3, 3 \rangle$
 $f : y = -\cot(\pi - x) = \cot(x)$
- d) $D(f) = \mathbb{R} \setminus \{2k\pi\}, \quad H(f) = \mathbb{R}$

THEORY: Let the composite function be given $F : y = \sqrt{4 - x^2}$.

- a) Write the inner and the main (outer) part of this function.
- b) Determine domains of definition of both parts of function F .

Solution:

- a) $u(x) = 4 - x^2, \quad F(u) = \sqrt{u}$
- b) $D(u) = \mathbb{R}, \quad D(f) = \langle 0, \infty \rangle \Rightarrow x \in \langle -2, 2 \rangle$

PROBLEM 2. Determine domain of definition of function f and sketch function graph. Find range of function f :

- a) $f : y = \sqrt{2 - 3x} + \frac{1}{2}$
- b) $f : y = (x - 2)^3 - 1$
- c) $f : y = e^{4x-2} - 3$
- d) $f : y = -\tan\left(x - \frac{\pi}{2}\right)$

Solution:

- a) $D(f) = \left(-\infty, \frac{2}{3}\right], \quad H(f) = \left[\frac{1}{2}, \infty\right)$
- b) $D(f) = \mathbb{R}, \quad H(f) = \langle -1, \infty \rangle$
- c) $D(f) = \mathbb{R}, \quad H(f) = (-3, \infty)$
- d) $D(f) = \mathbb{R} \setminus \{2k\pi\}, \quad H(f) = \mathbb{R}$

THEORY: Sketch graph of cyclometric function $f(x)$, which is decreasing and bounded on its domain of definition $D(f)$.

Solution:

$$f(x) = \arccos(x), \quad D(f) = \langle -1, 1 \rangle$$

PROBLEM 3. For functions f that are 1-to-1 on their domains of definition find inverse functions f^{-1} and determine range of these inverse functions.

- a) $f : y = \sqrt{2 - 3x} + \frac{1}{2}$
- b) $f : y = \log_{\frac{1}{3}}(2x + 4)$
- c) $f : y = e^{4x-2} - 3$
- d) $f : y = (x - 2)^3 - 1$

Solution:

- a) $f^{-1} : y = \frac{2 - (x - \frac{1}{2})^2}{3}$
- b) $f^{-1} : y = \frac{1}{3^{x-2}} - 2$
- c) $f^{-1} : y = \frac{\ln(x+3)}{4} + \frac{1}{2}$
- d) $f^{-1} : y = 2 + \sqrt[3]{x+1}$

THEORY: Let function $f(x)$ be an even function, complete its graph accordingly.

PROBLEM 4. Prismatic rod of length l , which is firmly clamped at one end and free at the other one, is loaded by axial forces F and R . (Force of the rod own weight will be neglected.) Dependence of the strain energy of the accumulation A in this rod on the force R , under assumption that force F is constant, is determined as

$$A = 12 \left[\left(\frac{R}{F} \right)^2 - \frac{4}{3} \left(\frac{R}{F} \right) + \frac{2}{3} \right]$$

Draw graph of the strain energy of accumulation A for force $R \in [-F, 2F]$.

$$A = 12 \left[\left(\frac{R}{F} \right)^2 - \frac{4}{3} \left(\frac{R}{F} \right) + \frac{2}{3} \right] = 12 \left[\left(\frac{R}{F} - \frac{2}{3} \right)^2 + \frac{2}{9} \right]$$

Solution:

$$A = 12 \left(\frac{R}{F} - \frac{2}{3} \right)^2 + \frac{8}{3}$$

$$V = \left[\frac{2}{3}F, \frac{8}{3} \right]$$

THEORY: Sketch graph of function that is 1-to-1 on its $D(f)$ and it is not simply monotone.

Solution:

Individual examples

PROBLEM 5. Given function $f : y = \frac{\pi}{2} + \arccos(x - 1)$

- a) Determine domain $D(f)$, and range $H(f)$.
- b) Find inverse function f^{-1} to f if it exists.
- c) Determine domain $D(f^{-1})$ and range $H(f^{-1})$ of the inverse.
- d) Sketch graphs of both f and f^{-1} .

Solution:

$$D(f) = [0, 2]$$

$$H(f) = \left[\frac{\pi}{2}, \frac{3\pi}{2} \right]$$

Function is decreasing (one-to-one)

$$f^{-1} : y = 1 + \cos\left(x - \frac{\pi}{2}\right)$$

$$D(f^{-1}) = \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$$

$$H(f^{-1}) = [0, 2]$$

THEORY: What relation holds between continuity of function $f(x)$ at point a and function $f(x)$ limit at the point a ?

Solution:

Function $f(x)$ is continuous at the point a if and only if the following holds: $\lim_{x \rightarrow a} f(x) = f(a)$

3 Mathematics II

On the base of good responses from the questionnaire and personal interviews with students participating in the experiment carried in the first semester in subject Mathematics I, similar teaching scenario was adopted for the same students in the second semester of the academic year 2018/19. Group of 77 students participated in the following experiment, those who fulfilled the requirements for continuing in study at the FME STU in the summer semester 2020. Two forms of work were adopted, team work and individual work for assessments. Students worked in the same groups and teams as in the course Mathematics I. Main goal of the experiment continuation was to recognise possible improvements in abilities of students to solve mathematical problems and applied maths problems independently, and within a small stable group throughout the semester.

Team work

All 77 students were distributed to 4 working groups, about 19-20 students in each. For team work, small teams of 4-5 students were formed, voluntarily, mostly based on their experience from the first semester. Team work assessment was realized in 3 topics from Mathematics II curricula:

- Coordinate geometry
- Extremes of function with two real variables
- Multiple integrals

Sprint prepared for each topic consisted of 5 problems

- 1st - 4th problem was standard mathematical problem
- 5th problem was applied problem from the mechanical engineering field.

All teams in one working group solved the same problems, different problems were solved in each from 4 working group. During the team work students cooperated together, though each one was responsible for solution of at least one from the problems distributed by team leader. This position changed so that always another volunteering student in the team was the scrum master responsible for organization and presentation of the team work.

Students could achieve together max 55 points, whereas they could earn max 10 points in the sprint on topic Coordinate geometry, max 20 points for solving problems in the sprint Extremes of function with two real variables $f(x, y)$ and max 25 points for correct solutions of problems in the topic Multiple integrals.

In the same way as for the subject Mathematics I, major part, about 60 minutes of the practical exercise lasting 100 minutes, was spent on solving the problems. Each group had to present one of the problems in front of the class for the last 30 minutes.

Individual work

This approach was applied again as an alternative method. It was realised by 3 worksheets prepared for students as individual tests after self-study period. Students had to solve about 5 problems in each topic in about 40 minutes, while assessments covered topics

- Differential equations
- Partial derivatives of function, quadratic surfaces
- Multiple integrals - Elementary regions.

Students could achieve together - max 35 points, whereas they could earn max 14 points in topic Differential equations, max 10 points solving the sprint concerning Partial derivatives of $f(x, y)$ and Quadratic surfaces, and

max 11 points in topic Multiple integrals - Elementary regions. Comparison statistics of results in team work and individual work was analysed, and students' achievements in respective topics in both work types are presented in Figure 27–30.

Figure 27: Results of team work

Figure 28: Results of individual work

Figure 29: Results of team and individual work

Figure 30: Comparative results of team and individual work in subjects

Average point score achieved by students in the first two topics (Coordinate geometry and Function) in team work was more than 66%. Positive achievements can be seen in average score received in topic Coordinate geometry. This topic is well familiar to students from the grammar schools and some secondary vocational schools. Other students acquired the knowledge at the university.

The low score in topic Multiple integrals, less than 60%, could be explained by the fact that it belongs to the difficult part of the curricula at university level. Students acquired skills in calculation of indefinite and definite integrals (mostly easy problems) in the basic course Mathematics I. They encountered difficulties with solutions in

- description and sketch of solid bounded by quadratic surfaces
- (consequently) evaluations of double and triple integrals.

Achieved average score in team work, more than 60%, can be seen as positive.

Students achieved rather low scores in separate sprints for individual work. These low average scores consequently influenced average score from individual work as whole, which is deeply under the bound of 50%.

In the presented graph we can see that the lowest average score was achieved by students in the topic Differential equations. This topic was selected for individual work on purpose. Differential equations form unseparable part of solutions in practical problems from professional technical subjects. It is very important to realise that every student should acquire mathematical understanding and skills necessary to solve these problems. This means for instance the knowledge how to solve equations of movement, which are simple differential equations - engine location and work, oscilation of mechanical systems, kinematic excitation of

movements of car on non-linear road, bending vibration of beams for seismic calculations of buildings, etc.....

In the basic course Mathematics I students learned how to differentiate, i. e. find the derivatives $f'(x)$, $f''(x)$ of function $f(x)$. Problem appeared

- in derivation of partial derivatives of function
- in acquiring knowledge on sketches of basic quadratic surfaces.

The highest average score was achieved in topic Elementary regions, which was almost 50%. Also this score is influenced by problems students face in sketching sets $M \subset E^2$, $M \subset E^3$ and in their transformations to polar and cylindrical coordinates. (Set $M \subset E^2$ is bounded by graphs of basic elementary functions and conic sections, and $M \subset E^3$ is bounded by parts of quadratic surfaces.)

Average point score for team work was about 60% and for individual work about 35%. Average score in individual work represents about 57% comparing to team work score. We assess negatively the fact that average score in team and individual work does not even reach 50% (only 38,67%) of the maximal number of points in both types of work. This can be explained on the base of our observations of students during the team work and individual work. We have found out that in team work better students not only helped to solve but often entirely solved problems attached for solution to weaker students. Better students wanted to get as many points as possible, as these points achieved by team were given to each individual student in the team. Problems of weaker students in solving some of the tasks became apparent in individual work.

Mathematics I and Mathematics II comparison

Results of team and individual work in basic Mathematics I and Mathematics II courses for the first year engineering students at the bachelor study programs revealed the following:

- The average scores in team work are almost equal.
- In individual work there was recorded a decline in the average score in course Mathematics II.

Average score in individual work in Mathematics II formed only 75% of the average score in individual work in Mathematics I.

The decrease was mainly caused by new topics in the Mathematics II curricula, which

- are not known to students from the secondary school
- it is explicitly the more advanced subject matter at the university level,

- built on the knowledge of students from the course Mathematics I and at the same time on the knowledge acquired at the secondary school.

Average score in both work types in Mathematics II formed 71% of the average score in both work types in Mathematics I.

In conclusions we would like to summarize some of the positive and negative aspects of implemented method eduScrum at FME STU in Bratislava, Slovakia (basic Mathematics I and II courses), which were mentioned and commented.

Positive aspects

- continuous assessment of students' knowledge gains increasing performance
- learning how and why to improve knowledge from mathematics
- working in the team
- responsibility - individual and for the whole team as a unit
- leading constructive discussion about problems
- understanding better the problem through discussions
- acquiring new knowledge on possible problem solutions

Negative aspects

- weaker students could benefit on help from better students with solutions of their own problems and acquired more points in their "score" from particular topics thanks to better students in their team
- weaker students often restrained from work and relied on better students
- better students were "constrained" by weaker students, had to explain them how to solve their problems having thus less time for their own problems
- better students had to work more hard without any extra benefits
- better students received less points on their "score" for not correct solutions of problems solved by weaker students, as they could receive provided they had solved the problems themselves and correctly, or in another constitution of the working team

Better students wanted to work, later during the semester, in smaller groups (2 - 3 members) and they agreed on finding solutions of all 5 problems in the given time limit.

In connection to general recommendation for future broad general implementation of eduScrum method in teach-

ing mathematics to the first year engineering students at the bachelor study programs we suggest the following

- to recognise in each course of mathematics at least 2-3 teams of excellent students and "train" them as excellent teams to solve practical problems in their respective technical professional subjects and follow their careers
- to support healthy competitiveness between these teams of excellent students
- to develop common worksheets for team work with problems that could be solved by teams of students at technical universities in some of the European countries, analyse the overall results consecutively, and contribute thus to better mathematical education of future European engineers.

Example of team worksheet with solutions for subject Mathematics II, topic Integral calculus II is presented in the following.

MULTIPLE INTEGRALS II - teamwork

PROBLEM 1. Describe and sketch planar region bounded by graphs of functions $y = \sin x$, $y = \frac{2x}{\pi}$ and calculate its area by means of double integral.

Solution:

$$M = M_1 \cup M_2,$$

$$M_1 = \{[x, y] \in E^2; -\frac{\pi}{2} \leq x \leq 0, \sin x \leq y \leq \frac{2x}{\pi}\},$$

$$M_2 = \{[x, y] \in E^2; 0 \leq x \leq \frac{\pi}{2}, \frac{2x}{\pi} \leq y \leq \sin x\}, P_{M_1} = P_{M_2},$$

$$P_M = 2P_{M_2}$$

$$P_M = 2 \int_0^{\frac{\pi}{2}} \int_{\frac{2x}{\pi}}^{\sin x} dy dx = 2 \int_0^{\frac{\pi}{2}} [y]_{\frac{2x}{\pi}}^{\sin x} dx =$$

$$= 2 \int_0^{\frac{\pi}{2}} \left(\sin x - \frac{2x}{\pi} \right) dx = 2 \left[-\cos x - \frac{x^2}{\pi} \right]_0^{\frac{\pi}{2}} =$$

$$= 2 \left\{ 0 - \frac{\pi}{4} - (-1 - 0) \right\} = 2 - \frac{\pi}{2}$$

THEORY: Explain possible physical and geometric interpretation of obtained result.

Solution:

Area of the bounded planar region, mass of a homogenous lamina in the form of the described region with material specific density equal to 1.

PROBLEM 2. Calculate mass and coordinates of centre of gravity of non-homogeneous lamina in the form of a planar region determined by inequalities $1 \leq x^2 + y^2 \leq 4$, $y \geq |x|$, if specific density of lamina material is defined by function $\mu(x, y) = x^2 + y^2$. Describe and sketch the region with the calculated centre of gravity.

Solution:

$$M = \{[x, y] \in E^2 : 1 \leq x^2 + y^2 \leq 4, y \geq |x|\}$$

Polar transformation

$$x = \rho \cos \varphi$$

$$y = \rho \sin \varphi$$

$$J(\rho, \varphi) = \rho$$

$$M^* = \{(\rho, \varphi) \in R^2 : 1 \leq \rho \leq 2, \frac{\pi}{4} \leq \varphi \leq \frac{3\pi}{4}\}$$

$$m_M = \iint_M \mu(x, y) dx dy = m_{M^*} =$$

$$= \iint_{M^*} \mu(\rho \cos \varphi, \rho \sin \varphi) J(\rho, \varphi) d\rho d\varphi$$

$$= \iint_{M^*} (\rho^2 \cos^2 \varphi + \rho^2 \sin^2 \varphi) \rho d\rho d\varphi =$$

$$= \int_1^2 \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \rho^3 d\varphi d\rho = \int_1^2 \rho^3 d\rho \cdot \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} d\varphi$$

$$= \left[\frac{\rho^4}{4} \right]_1^2 \left[\varphi \right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = \frac{\pi}{2} \left(4 - \frac{1}{4} \right) = \frac{15\pi}{8}$$

$$T = [x_T, y_T]$$

$$\begin{aligned}
 x_T &= \frac{1}{m_M} \iint_M x \mu(x, y) dx dy \\
 &= \frac{1}{m_{M^*}} \iint_{M^*} \varrho \cos \varphi \varrho^2 d\varrho d\varphi \\
 &= \frac{8}{15\pi} \int_1^2 \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \varrho^4 \cos \varphi d\varphi d\varrho \\
 &= \frac{8}{15\pi} \int_1^2 \varrho^4 d\varrho \cdot \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \cos \varphi d\varphi \\
 &= \frac{8}{15\pi} \left[\frac{\varrho^5}{5} \right]_1^2 \cdot [\sin \varphi]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \\
 &= \frac{8}{15\pi} \left(\frac{32}{5} - \frac{1}{5} \right) \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \right) = 0
 \end{aligned}$$

$$\begin{aligned}
 y_T &= \frac{1}{m_M} \iint_M y \mu(x, y) dx dy = \frac{1}{m_{M^*}} \iint_{M^*} \varrho \sin \varphi \varrho^2 d\varrho d\varphi \\
 &= \frac{8}{15\pi} \int_1^2 \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \varrho^4 \sin \varphi d\varphi d\varrho \\
 &= \frac{8}{15\pi} \int_1^2 \varrho^4 d\varrho \cdot \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin \varphi d\varphi = \frac{8}{15\pi} \left[\frac{\varrho^5}{5} \right]_1^2 \cdot [-\cos \varphi]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \\
 &= \frac{8}{15\pi} \left(\frac{32}{5} - \frac{1}{5} \right) \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right) = \frac{248\sqrt{2}}{75\pi} \\
 T &= \left[0, \frac{248\sqrt{2}}{75\pi} \right]
 \end{aligned}$$

THEORY: Explain properties of double integrals and illustrate on simple examples.

Solution:

Individual examples

PROBLEM 3. Calculate volume of solid bounded by graphs of functions $f(x, y) = 4xy - y^3$ and $g(x, y) = -y$ defined over the region M in the coordinate plane xy , with boundaries in graphs of functions $y = x^3$, $y = \sqrt{x}$. Sketch and describe region M and solid T as set of points.

Solution:

$$x^3 = \sqrt{x}$$

$$x^6 = x \Rightarrow x^6 - x = 0 \Rightarrow x(x^5 - 1) = 0 \Rightarrow x = 0 \vee x^5 = 1 \Rightarrow x = 0 \vee x = 1$$

$$\begin{aligned}
 M &= \{[x, y] \in E^2 : 0 \leq x \leq 1, x^3 \leq y \leq \sqrt{x}\}, \\
 T &= \{[x, y, z] \in E^3 : [x, y] \in E^2, -y \leq z \leq 4xy - y^3\}
 \end{aligned}$$

$$\begin{aligned}
 V &= \iiint_T 1 dx dy dz = \int_0^1 \int_{x^3}^{\sqrt{x}} \int_{-y}^{4xy-y^3} 1 dz dy dx \\
 &= \int_0^1 \int_{x^3}^{\sqrt{x}} [z]_{-y}^{4xy-y^3} dy dx = \int_0^1 \int_{x^3}^{\sqrt{x}} (4xy - y^3 + y) dy dx \\
 &= \int_0^1 \left[2xy^2 - \frac{y^4}{4} + \frac{y^2}{2} \right]_{x^3}^{\sqrt{x}} dx \\
 &= \int_0^1 \left(2x^2 - \frac{x^2}{4} + \frac{x}{2} - 2x^7 + \frac{x^{12}}{4} - \frac{x^6}{2} \right) dx \\
 &= \int_0^1 \left(\frac{7x^2}{4} + \frac{x}{2} - 2x^7 + \frac{x^{12}}{4} - \frac{x^6}{2} \right) dx \\
 &= \left[\frac{7x^3}{3 \cdot 4} + \frac{x^2}{4} - \frac{x^8}{4} + \frac{x^{13}}{4 \cdot 13} - \frac{x^7}{2 \cdot 7} \right]_0^1 \\
 &= \frac{7}{3 \cdot 4} + \frac{1}{4} - \frac{1}{4} + \frac{1}{4 \cdot 13} - \frac{1}{2 \cdot 7} \\
 &= \frac{49 \cdot 13 + 21 - 6 \cdot 13}{3 \cdot 4 \cdot 7 \cdot 13} = \frac{4313 + 21}{7 \cdot 12 \cdot 13} = \frac{580}{1092} = \frac{145}{273}
 \end{aligned}$$

THEORY: Write and explain Fubini theorem for triple integrals.

Solution:

Individual text

PROBLEM 4. Calculate mass of non-homogeneous solid bounded by cylindrical surface with equation $x^2 + y^2 = 4$ and planes $z = 0$, $y = 2 - 2z$, if specific density of solid material is $\mu(x, y, z) = x + 2$. Sketch and describe solid T .

Solution:

$$T = \left\{ [x, y, z] \in E^3; x^2 + y^2 \leq 4, 0 \leq z \leq \frac{2-y}{2} \right\}$$

Cylindrical transformation

$$x = \rho \cos \varphi$$

$$y = \rho \sin \varphi$$

$$z = z$$

$$J(\rho, \varphi, z) = \rho$$

$$T^* = \left\{ [\rho, \varphi, z] \in E^3 : 0 \leq \rho \leq 2 \right. \\ \left. 0 \leq \varphi \leq 2\pi \right. \\ \left. 0 \leq z \leq 1 - \frac{\rho \sin \varphi}{2} \right\}$$

$$m = \iiint_T \mu(x, y, z) dx dy dz \\ = \iiint_{T^*} \mu(\rho \cos \varphi, \rho \sin \varphi, z) J(\rho, \varphi, z) d\rho d\varphi dz \\ = \int_0^2 \int_0^{2\pi} \int_0^{1 - \frac{\rho \sin \varphi}{2}} (\rho \cos \varphi + 2) \rho dz d\varphi d\rho$$

$$= \int_0^2 \int_0^{2\pi} \rho(\rho \cos \varphi + 2) \left(1 - \frac{\rho \sin \varphi}{2} \right) d\varphi d\rho \\ = \int_0^2 \int_0^{2\pi} \left(\rho^2 \cos \varphi + 2\rho - \frac{\rho^3 \sin \varphi \cos \varphi}{2} - \rho^2 \sin \varphi \right) d\varphi d\rho \\ = \int_0^2 \int_0^{2\pi} \left(\rho^2 (\cos \varphi - \sin \varphi) + 2\rho - \frac{\rho^3}{2} \cdot \frac{\sin 2\varphi}{2} \right) d\varphi d\rho \\ = \int_0^2 \left(\rho^2 [\sin \varphi + \cos \varphi]_0^{2\pi} + 2\rho [\varphi]_0^{2\pi} \right. \\ \left. + \frac{\rho^3}{2} \cdot \left[\frac{\cos 2\varphi}{2 \cdot 2} \right]_0^{2\pi} \right) d\rho \\ = \int_0^2 4\pi \rho d\rho = 4\pi \left[\frac{\rho^2}{2} \right]_0^2 = 8\pi$$

THEORY: List and explain basic properties of triple integrals.

Solution:

Individual text

PROBLEM 5. Using multiple integration prove validity of the solution of problem posed and solved by Greek mathematician Archimedes (about 287 B. C. - 212 A. D.) who used the exhaustion method (division of solid to small general prisms): Volume of rotational paraboloid inscribed into rotational cylinder equals one half of the volume of cylinder.

Solution guide: (Fig.)

By means of triple integrals express

- volume of paraboloid V_P (with height h and radius r of its base circle) determined by equation $z = h \left(1 - \frac{x^2 + y^2}{r^2} \right)$
- volume of cylinder V_V (with height h and radius r of its base circle) determined by equation $z = h$

Solution:

$$T_P = \left\{ [x, y, z] \in E^3; x^2 + y^2 \leq r^2, 0 \leq z \leq h \left(1 - \frac{x^2 + y^2}{r^2} \right) \right\}$$

$$T_V = \left\{ [x, y, z] \in E^3; x^2 + y^2 \leq r^2, 0 \leq z \leq h \right\}$$

Cylindrical transformation

$$x = \varrho \cos \varphi$$

$$y = \varrho \sin \varphi$$

$$z = z$$

$$J(\varrho, \varphi, z) = \varrho$$

$$T_P^* =$$

$$\left\{ [\varrho, \varphi, z] \in E^3 : 0 \leq \varrho \leq r, 0 \leq \varphi \leq 2\pi, 0 \leq z \leq h \left(1 - \frac{\varrho^2}{r^2} \right) \right\}$$

$$T_V^* = \left\{ [\varrho, \varphi, z] \in E^3 : 0 \leq \varrho \leq r, 0 \leq \varphi \leq 2\pi, 0 \leq z \leq h \right\}$$

$$\begin{aligned} V_P &= \iiint_{T_P} 1 dx dy dz = \int_0^r \int_0^{2\pi} \int_0^{h \left(1 - \frac{x^2 + y^2}{r^2} \right)} h \left(1 - \frac{x^2 + y^2}{r^2} \right) dx dy dz \\ &= \iiint_{T_P^*} J(\varrho, \varphi, z) d\varrho d\varphi dz \\ &= \int_0^r \int_0^{2\pi} h \left(1 - \frac{\varrho^2}{r^2} \right) \varrho d\varphi d\varrho = h \int_0^r \left(\varrho - \frac{\varrho^3}{r^2} \right) d\varrho \cdot \int_0^{2\pi} d\varphi \\ &= h \cdot \left[\frac{\varrho^2}{2} - \frac{\varrho^4}{4r^2} \right]_0^r \cdot [\varphi]_0^{2\pi} = h \cdot \left(\frac{r^2}{2} - \frac{r^4}{4r^2} \right) \cdot 2\pi = \frac{\pi h r^2}{2} \end{aligned}$$

$$\begin{aligned} V_V &= \iiint_{T_V} 1 dx dy dz = \int_0^r \int_0^{2\pi} \int_0^h h dx dy dz \\ &= \iiint_{T_V^*} J(\varrho, \varphi, z) d\varrho d\varphi dz \\ &= \int_0^r \int_0^{2\pi} h \varrho d\varphi d\varrho = h \int_0^r \varrho d\varrho \cdot \int_0^{2\pi} d\varphi = h \cdot \left[\frac{\varrho^2}{2} \right]_0^r \cdot [\varphi]_0^{2\pi} \\ &= h \cdot \left(\frac{r^2}{2} - 0 \right) \cdot 2\pi = \pi h r^2 \end{aligned}$$

$$\left. \begin{aligned} V_P &= \frac{\pi h r^2}{2} \\ V_K &= \pi h r^2 \end{aligned} \right\} \frac{V_P}{V_V} = \frac{\frac{\pi h r^2}{2}}{\pi h r^2} = \frac{1}{2} \Rightarrow V_P = \frac{1}{2} V_V$$

THEORY: Write equations of integral transformations of triple integrals to spherical coordinates. Illustrate on example of calculation of volume of a solid formed by parts of concentric spheres with centres in origin of coordinate system and radii 1 and 3 and half-planes $x \geq 0, y \geq 0, z \geq 0$.

4 Basics of Statistical Analysis

In the academic year 2019/2020 we continued with the experiment in the subject Basics of statistical analysis for the second year engineering students at the bachelor study programs. Participating students were aware of this innovative learning scenario from the first year of their study, and many of them were looking forward to follow the same schedule.

Our aim was to find out whether these matured students will adopt different approach to their study duties and benefit more from the team work or the individual one. 36 students participated in the experiment, while 28 students chose this subject from the list of selective subjects, and for 8 students at the new study program "Professional bachelor" the subject was appointed as compulsory.

The curriculum of BSA consists of two parts with different topics:

- Probability (Basic definition of probability, Random variables, Multivariate random variables)
- Statistics (Creation of random sample and descriptive statistics, Point and interval estimation, Statistical intervals and sample size at a given point, Estimate accuracy, Tests of hypotheses for a single sample, Statistical inference for two samples)

Students worked in team (in part Probability) and individually (in part Statistics), while individual work was applied as an alternative method to team work, in order to compare achievements and satisfaction of students with these two learning scenarios. The main goals of the experiment were to find out:

- abilities of students to solve problems of basic BSA course independently and within a small stable group throughout the semester attitudes
- attitudes and opinion of students on this form of teaching scenario
- performance and engagement due to this

Team work

36 students were distributed to 3 groups:

- 1st group = 8 students at the program "Professional bachelor",
- 2nd and 3rd group = 28 students at other study programs (14 students in each)

Students in each group worked on 5 sprints in teams, small working groups, while distribution to 2-3 teams of 4-5 students, i. e. 1 team = 1 small group, was left for them.

Sprints for team work covered the following topics from probability

- General probability concept
- Discrete random variable X
- Continuous random variable X
- Two discrete random variables X, Y
- Two continuous random variables X, Y

Each topic for team work consisted of 4 problems; all teams solved the same problems in all working groups. During the team work students cooperated together, though each one was responsible for solution of one from the problems distributed by team leader, and always another one of the team was scrum master in the role of team leader.

Students could achieve 4 points in each topic, so maximal achievement from all team work sprints was 20 points. Major part, about 70 minutes of the practical exercises lasting 100 minutes, was spent solving the problems; the remaining time was left for presentations of solutions, while each team had to present one of the problems in front of the class.

Individual work

This scenario was applied as an alternative method in the part Statistics, where it was realised on 4 topics

- Point and interval estimation
- Tests of hypotheses for a single sample
- Statistical inference for two samples
- Project

Students could achieve in 1st - 3rd topics max 18 points, in 4th topic max 32 points. They had to solve individually

2 basic problems in each topic in about 90 minutes. Each problem consisted of 2-4 partial problems. Students solved these problems using environment of statistical software Statgraphics available in the computer laboratory.

Questionnaire

The experiment was completed with a questionnaire for students. It was aimed to discover attitudes and opinions of students on this teaching scenario. 28 out of 36 students answered the questionnaire, and it was followed by free interviews with students.

Results of team and individual work

Students' results achieved in the respective topics for both work types are presented in Figure 31–33. Students achieved a minimal average score in the part Probability. This topic is known to students from the secondary schools but it is a rather demanding unit from the curriculum. The score achieved in team work can be seen positively, as it was almost 70%.

We can see in the presented graph that "Statistics" is quite demanding topic for students. Average score in all sprints for individual work is relatively low - about 30%. The only exception is the last topic Project, where the average score is just above the level of 50%, but with regard to the really practical contents of the solved problem it can be seen as positive and foreseen, because students were motivated by concrete applications.

These low average scores consequently influenced average score from individual work as whole, which is under the bound of 50%.

Figure 31: Results of team work

Figure 32: Results of individual work

Average point score for team work was about 70% and for individual work it was about 45%. This finding that the average score in team work was almost 1.5 of the average score in individual work was not surprising. Weaker students working individually could not reach higher numbers of points as it was possible in team work, where they benefited on the support from better students. We negatively evaluate the fact that summative average score in both team work and individual work is very closely above 50% (52.15%) from the maximal number of points in both working scenarios. Considering the fact that students asked to be taught in this active learning attitude, we conclude that they counted on the team work and better results they could achieve here. We were quite disappointed by their lower study discipline and very weak interest in gaining new knowledge.

Figure 33: Results of team and individual work

This failure can be explained based on the ways of students' work during tutorials:

- in team work all problems were solved by team - better and weaker students together,

- in individual work all problems were solved by individual students - better or weaker ones.

This was evidently presented on the achieved results.

Questionnaire

At the end of semester students were asked to answer similar questions as in the first semester, but related to subject Basics of statistical analysis - BSA. Questions were related to complexity of subject topics. About 60% of interviewed students regarded BSA as demanding subject ("very demanding" and "demanding"), while BSA is not demanding only for 3.6% of students. This assumption is proved also by results which students achieved in the individual work (Figure 32).

Figure 34: Complexity of statistic

Subject BSA consists of two part - "Probability" and "Statistics." Assessment of your knowledge was realized 10 times per semester (5 times in each part). Would you prefer to have this assessment rather 2-times per semester - after the part "Probability"?

Figure 35: The knowledge assessment during the semester

Only 32% of interviewed students does not prefer assessment of their knowledge during semester after each from the subject main topics. ("certainly yes" and "rather yes than no").

Figure 36: Ways of knowledge assessment

Almost 2/3 of students preferred team work prior to individual work to test their knowledge. The question arises: Is eduScrum really so great?

Figure 37: Individually solving of problems in each team work

One of the positive aspects is the fact that almost 43% of students managed to solve more than half of the problems and about 1/5 of students (18%) managed to solve almost all problems.

Figure 38: Classmate's help to me

One of the negative aspects - as many as 46% (close 50%) students need a classmate's help to solve with more than half of the problems.

Figure 39: My help to classmates

50% of students must help their classmates with more than half of the problems.

Figure 40: Preparation for solving problems in the team work

- a) I have solved some problems in advance
- b) I have studied the topic in advance
- c) I have studied the topic and solved some problems in advance - less than one third !!!
- d) I did nothing and I relied on the help from my classmates

Figure 41: Comparison with classmates

57% of students answered positive ("yes" and "rather yes than no"), and almost one third of students (28.6%) does not know to compare their success with their classmates.

Figure 42: The influence of team work on the student

(You can also tick off more answers.)

- a) it has taught me how to work in team - 85.7%
- b) it has taught me how to work with literature
- c) it has taught me to be responsible for my solution - 40%
- d) it has taught me to be responsible for the whole team
- e) it has taught me to discuss the problem in a constructive way - 64.3%
- f) in discussion about the problem I could understand the topic better - 64.3%
- g) I have acquired better knowledge about solution of the problem - almost 43%
- h) I was motivated to study the topic in advance
- i) I was motivated to solve in advance some problems related to the topic
- j) I was motivated to improve my knowledge from Basic of statistical analysis - only 14.3%

Figure 43: Team work on solving statistical problems

k) I have used consultations from Basic of statistical analysis - !!!

l) in a no way

Team work helped more than to 82% of students to understand learned content better.

Figure 44: The influence of individual work on the student

(You can also tick off more answers.)

- a) it motivated me to study the topic in advance - 46%
- b) it motivated me to solve problems related to the topic in advance - 39%
- c) it motivated me to improve my previous knowledge of Basic of statistical analysis - 36%
- d) I have utilised consultations from Basic of statistical analysis - !!!
- e) I did nothing - 18% (almost 1/5)

Figure 45: Individual work on solving mathematical problems.

Individual work did not help to about 2/3 of students to practise learned material better, but up to about 1/3 of students grasped the learned topics considerably better.

Interview results showed that up to 64.3% of students would prefer to assess knowledge in BSA during semester only by team work with the newly implemented eduScrum method and only one third of students would prefer to assess knowledge both work.

Figure 46: Ways of assessing knowledge during semester.

Figure 47: Project in the subject Basics of statistical analysis.

Almost 60% of students agree and 1/4 of students do not agree with inclusion of Project to the educational process.

Figure 48: Advice (to choose as compulsory) subject BSA to other students

We can see in the presented graph that 57% of students would advice and 43% of students would not advice (to choose as compulsory) subject BSA also to other students.

Course Basics of statistical analysis belongs to a list of subject that are selectively optional. We were interested to know, why students had chosen just this one from the list of seven other subjects. From interviews we could conclude that students realize the importance of Statistics in technical professional subjects and in practise, in particular. With respect to this, they articulated their opinion on the need to classify this subject as a compulsory one.

Students were interested also in implementation of eduScrum method into the subject BSA. They have learned about this didactic method from their classmates in the course Mathematics I and II and they were eager to "try it", too. That was the reason why we have adopted this method also in this particular subject.

The aim of the teachers of this subject was to prepare students for practice as good as possible. After graduation from the study at the technical university, our students will work in companies in the mechanical engineering industry. Statistical data processing will become a daily necessity in their work.

It is very important that students already during their studies

- acquire basic knowledge from statistics - mainly terms as normal population, recognition the difference between population and random sample, numeric and graphical methods of descriptive statistics, null and alternative hypothesis (be able to formulate them), the test statistic, the critical value, $1000(1 - \alpha)\%$ confidence interval, type I / II error probability, the level of significance, P - value, make a conclusion - reject / fail to reject null hypothesis at,
 - learn to work in some statistical software.
- During tutorials students worked with software Statgraphics. This software is very similar to software Minitab, which is widely used by companies in mechanical engineering industry. (Teachers' plans are to use software Minitab for teaching this subject in the next academic year.)
- verified the statistical data processing with data acquired from real practical situations - the specified Project was an assessment of the knowledge they obtained (each student received his own project).
- 2 weeks' time was delimited for elaboration of the project. They could benefit from personal consultations or lecture notes published for the subject, where are presented also solved problems.

It was found from the Questionnaire that almost 2/3 of students prefer team work (method eduScrum) prior to individual work to test their knowledge.

– What makes eduScrum great in BSA?

- As the students said in the interview and in the questionnaire - this taught them
 - * to lead constructive discussions about problem
 - * to understand the topic better in discussion about the problem
 - * how to "grasp" problem, how to start to solve it
 - * to gain new and better knowledge on problem solving
 - * to improve knowledge from BSA
- As one of the active-learning methods it helps students to further develop their soft skills (to work in team, leadership, responsibility, creativity, empathic behaviour, ...)

– What does not make eduScrum great?

Negative aspects of this method become apparent when it became a part of the examination. Exams from subject BSA proved this hypothesis. Subject BSA consisted of two parts, therefore also the exam was realised by two parts - Probability and Statistics. Students could take the exam also during the semester. In each part of the exam student gained points for his work during semester ("Probability" - team work, "Statistics" - individual work) and points from the final complex test from each part. (Complex test from "Statistics" was realised as Project.) Requirements for the exam completion during semester were known to students from the beginning of the academic year.

- In team work weaker students often relied on the help of better students with solutions of their own problems and thanks to better students they acquired more points in their "score" from particular topics. (Points achieved by the team were awarded to each of the students, better and weaker equally.)
- This fact was proved not only by achievements of students in the individual work (Figure 32) but also by their results in the final test from "Probability" (max points score per student = 30, average point score per student = 13,45 (45%).).

Examples of sprints for team and individual work and tests used for assessment of students' theoretical and practical knowledge gains in subject Basics of Statistical Analysis are presented.

4.0.0.1 PROBABILITY - one of the team works:

Continuous random variables

PROBLEM 1. The weight of sophisticated running shoe is normally distributed with a mean of 12 ounces and a standard deviation of 0.5 ounce.

- a) What is the probability that a shoe weighs more than 13 ounces?
Present the solution on graphs too.
- b) What must be the standard deviation of weight in order for company to state that 99% of its shoes are less than 13 ounces?
- c) If the standard deviation remains at 0.5 ounce, what must be the mean weight in order for the company to state that 99.9% of its shoes are less than 13 ounces?

Solution:

X - the weight of sophisticated running shoe

$$X \sim N(12; 0.5^2)$$

$$\begin{aligned} \text{a) } P(X > 13) &= 1 - P(X \leq 13) = 1 - F(13) = \\ &= P\left(Z > \frac{13-12}{0.5}\right) = 1 - P\left(Z \leq \frac{13-12}{0.5}\right) = 1 - P(Z < 2) = \\ &= 1 - \Phi(2) = 1 - 0.97725 = 0.02275 \approx 2.23\% \end{aligned}$$

$$\begin{aligned} \text{b) } P(X < 13) &= 0.99 \\ P\left(Z < \frac{13-12}{\sigma}\right) &= 0.99 \\ \Phi\left(\frac{1}{\sigma}\right) &= 0.99 \\ \frac{1}{\sigma} &= 2.33 \Rightarrow \sigma = \frac{1}{2.33} \\ \sigma &\approx 0.43 \\ \text{c) } P(X < 13) &= 0.99 \\ P\left(Z < \frac{13-\mu}{0.5}\right) &= 0.999 \\ \Phi\left(\frac{13-\mu}{0.5}\right) &= 0.999 \\ \frac{13-\mu}{0.5} &= 3.1 \Rightarrow \mu = 13 - 0.5 \cdot 3.1 \\ \mu &= 11.45 \end{aligned}$$

THEORY: What numerical characteristic is given by the relationship $\int_{-\infty}^{\infty} xf(x)dx$?

PROBLEM 2. The compressive strength of samples of cement can be modeled by a normal distribution with a mean

of 6000 kilograms per square centimeter and a standard deviation of 100 kilograms per square centimeter.

- What is the probability that a sample's strength is less than 6250 Kg/cm^2 ? Present the solution on graphs $f(x)$, $F(x)$, $\varphi(z)$, $\Phi(z)$ too.
- What is the probability that a sample's strength is between 5800 and 5900 Kg/cm^2 ?
- What strength is exceeded by 95% of the samples?

Solution:

X – the compressive strength of samples of cement

$X \sim N(6000; 100^2)$

- $P(X < 6250) = P\left(Z < \frac{6250-6000}{100}\right) = P(Z < 2.5) = \Phi(2.5) = 0.9938 = 99.38\%$
- $P(5800 < X < 5900) = F(5900) - F(5800) = P(-2 < Z < -1) = \Phi(-1) - \Phi(-2) = \Phi(2) - \Phi(1) = 0.97723 - 0.84134 = 0.1359 = 13.59\%$
- $P(X > x) = 0.95$
 $P(X \leq x) = 0.05$
 $P\left(Z \leq \frac{x-6000}{100}\right) = 0.05$
 $\Phi\left(\frac{6000-x}{100}\right) = 0.95 \Rightarrow \frac{6000-x}{100} = 1.65 \Rightarrow x = 5835$

THEORY: What distribution have the random variables $X \sim N(\mu, \sigma^2)$, $Z \sim N(0, 1)$? Write the relationship to transformation the random variable $X \sim N(\mu, \sigma^2)$ into the random variable $Z \sim N(0, 1)$.

PROBLEM 3. The reaction time of a driver to visual stimulus is normally distributed with a mean of 0.4 seconds and a standard deviation of 0.05 seconds.

- What is the probability that a reaction requires more than 0.5 seconds?
- What is the reaction time that is exceeded 90% of the time?
- 4500 reaction times of drivers from 5000 must correspond the norm. Determine interval whom bounds are symmetrical around the mean and it contains these the reaction times of drivers.

Solution:

X – the reaction time of a driver to visual stimulus

$X \sim N(0.4; 0.05^2)$

- $P(X > 0.5) = 1 - P(X \leq 0.5) = 1 - P\left(Z \leq \frac{0.5-0.4}{0.05}\right) = 1 - P(Z < 2) = 1 - \Phi(2) = 0.0228 = 2.28\%$
- $P(X > x) = 0.9$
 $P(X \leq x) = 0.1$
 $P\left(Z \leq \frac{0.4-x}{0.05}\right) = 0.1$
 $1 - \Phi\left(\frac{0.4-x}{0.05}\right) = 0.1$
 $\Phi\left(\frac{0.4-x}{0.05}\right) = 0.9 \Rightarrow \frac{0.4-x}{0.05} = 1.28 \Rightarrow x = 0.336$

$$\begin{aligned} \text{c) } P(0.4 - x \leq X \leq 0.4 + x) &= \frac{4500}{5000} \\ P\left(\frac{(0.4-x)-0.4}{0.05} \leq Z \leq \frac{(0.4+x)-0.4}{0.05}\right) &= 0.9 \\ P\left(\frac{-x}{0.05} \leq Z \leq \frac{x}{0.05}\right) &= 0.9 \\ \Phi\left(\frac{x}{0.05}\right) - \left(1 - \Phi\left(\frac{x}{0.05}\right)\right) &= 0.9 \\ \Phi\left(\frac{x}{0.05}\right) &= 0.95 \Rightarrow \frac{x}{0.05} = 1.65 \Rightarrow x = 0.0825 \\ [0.4 - x; 0.4 + x] &= [0.3175; 0.4825] \end{aligned}$$

THEORY: Write 2 properties which satisfies the probability function of continuous random variable X .

PROBLEM 4. Assume is normally distributed with a mean of 10 and a standard deviation of 2. Determine the following:

- $P(X < 13)$
- $P(X > 9)$
- $P(-2 < X < 8)$. Present the solution on graphs $f(x)$, $F(x)$, $\varphi(z)$, $\Phi(z)$ too.

Solution:

$X \sim N(10; 2^2)$

- $P(X < 13) = F(13) = P\left(Z < \frac{13-10}{2}\right) = P(Z < 1.5) = \Phi(1.5) = 0.9332 = 93.32\%$
- $P(X > 9) = 1 - P(X \leq 9) = 1 - P\left(Z \leq \frac{9-10}{2}\right) = 1 - \Phi(-0.5) = \Phi(0.5) = 0.6915 = 69.15\%$
- $P(-2 < X < 8) = F(8) - F(-2) = P\left(\frac{-2-10}{2} < Z < \frac{8-10}{2}\right) = P(-6 < Z < -1) = \Phi(6) - \Phi(1) = 0.1587 = 15.87\%$

THEORY: Determine the range of the cumulative distribution function of a continuous random variable X .

Team Work

Individual Work

5 Last remarks and summary

The aim of the experiment was to find out the level of knowledge acquisition and to compare abilities of students to solve mathematical problems independently, or as a collective work in the small groups. Anonymous questionnaire answered by students after completion of the experiment gave researchers a good feedback and overview of their opinions, from which we could compare their attitudes towards different teaching methods applied in 3 subjects from their study programs, Mathematics I and Mathematics II courses in the first year, and Basics of Statistical Analysis in the second year. Based on received data, research analysed results that students achieved at exams, comparing to their achievement from the secondary schools, and in both maths courses at the university.

EduScrum method showed students a new way of work during tutorials, a teamwork, which they rated very positively. It has to be stated that students with weaker knowledge benefited more from the team work as practised in the randomly formed small group. Very good students helped their classmates to receive better marks and took a greater part of the work on their shoulders. On the other hand, team represented one unit as all members were supposed to be equally responsible for their common achievements. This was often a strong motivation and driving force encouraging both - good students and not that good ones to do their best and contribute to the team final success. It is up to the teachers how they will introduce this method into their teaching. Team of maths teachers at the FME STU was enthusiastic and keen to implement new active learning scenarios in their teaching practise. They worked with enjoyment and this pleasurable attitude was naturally

transferred also to our students. Preparatory work and development of necessary didactic materials is a quite time consuming activity. It requires a real devotion of teachers who should plan carefully all activities in advance, support students with enough learning instructional materials and to be ready to help them with advice in mostly unforeseen situations that might occur during the activities.

In connection to general recommendation for future broad general implementation of eduScrum method in teaching mathematics to the engineering students at the bachelor or master study programs, we suggest the following few ideas in order to support development of the best students

- to recognise in each course of mathematics at least 2-3 teams of excellent students and "train" them as excellent teams to solve practical problems in their respective technical professional subjects
- to support healthy competitiveness between these teams of excellent students
- to build common worksheets for team work with problems that could be solved by these excellent teams of students, analyse the overall results consecutively, and contribute thus to better education of future European engineers in mathematics
- to organize at least one competitive teamwork session between excellent student teams
- to appreciate achieved results of all teams, regardless their composition
- to support and encourage students with weaker results
- to monitor and carefully record results of all students.

We believe that, given the current state of knowledge of students, their basic working and methodical skills necessary for the study work at universities and the new roles of teachers in the educational process, the goal of teachers will continue to be focused on the quality and not the quantity of students who complete the subject.

Gained experience and general findings of the project research team at the Slovak University of Technology in Bratislava were presented during the life of the project at international conferences related to the teaching of mathematics in engineering study programs at technical universities. Articles related to results achieved after introduction of active learning methods in several basic subjects during the course of our experiment were published in conference proceedings, see Gabková (2020); Gabková and Omachelová (2020); Gabková *et al.* (2019); Velichová (2018) and Velichová and Gabková (2019).

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