Research Article

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Poly-Universe Resource for Solving Geometric Tasks by Portuguese Basic Education Students

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Abstract: This article presents the analysis and discussion of solving geometry tasks by Portuguese Basic Education students, using material from the Poly-Universe in Teacher Training Education (PUNTE) project. The teaching experience developed was aimed at testing and disseminating new resources, and innovative and transdisciplinary methodologies in the areas of Science, Technology, Engineering, Art and Mathematics (STEAM). The adopted methodology involved two phases. The first phase was exploratory, followed by a second phase where the exploration of geometry concepts in the plane (namely, the perimeter and the similarity of triangles) was carried out using the Poly-Universe material. The results of the study show that the Poly-Universe materials constituted an important teaching resource in solving the proposed tasks. Its manipulation allowed the visualisation of the situations involved in the tasks, as well as contributed to an implication in the activities developed by the students.

Keywords: perimeter, PUNTE, similarity of triangles, manipulative material

1 Introduction

Images are so important in today’s life that the need for fundamental changes in education is obvious (Nyiri, 2013). Considering the current curriculum, guidelines for the teaching of geometry, it is essential that the teacher provides opportunities and time for students to manipulate, view, and construct, providing favourable contexts for the development of spatial reasoning. The class of triangles deserves special attention for the study of cases of congruence, classification, and construction in which the realisation of experiments using dynamic geometry environments, such as GeoGebra, favours the understanding of properties and relationships (Direção-Geral da Educação - Ministério da Educação e Ciência [DGE-MEC], 2018). In geometry, visualisation is an intuitive aid and plays an important role in geometric reasoning (Hanna, 2000). This is recognised and supported by Duval’s cognitive model of geometrical reasoning (Duval, 1998) as it involves:

- Perceptual apprehension – this is what is recognised at first glance; perhaps, for instance, sub-figures which are not necessarily relevant to the construction of the geometrical figure;

- Sequential apprehension – this is used when constructing a figure or when describing its construction. In this case, the figurative units depend not on perception but on mathematical and technical constraints (in the latter case, this could be rulers and compasses, or perhaps the primitives in computer software).

- Discursive apprehension – perceptual recognition depends on discursive statements because mathematical properties represented in a drawing cannot be determined solely through perceptual apprehension, some must first be given through speech.

- Operative apprehension – this involves operating on the figure, either mentally or physically, which can give insight into the solution to a problem (Jones, 1998).

In this sense, manipulative materials are characterised by providing the student the direct contact, allowing them to touch and move them. The insertion of these materials causes students to be motivated and interested in learning, offering them a relaxed environment for reflection, and working at the same time with logical relationships developed by the students (Berkseth, 2013). However, as described by Dienes (1970), some care must be taken when using manipulative materials; care must be taken to avoid a simple association, that is, there must be a concrete manipulation of objects, a representative
map of these manipulations and then a formalisation of these representations in structures—rules.

The pieces of the Poly-UNiverse in Teacher Training Education (PUNTE) project are manipulative materials with good characteristics for learning in different areas of education (Szász-Saxon & Stettner, 2019). It is a set of 72 pieces in which they all differ from each other. They are divided into three different shapes: the square shape, the triangular shape, and the circular shape, and they are the combination of four different colours and no colour is repeated in a piece: yellow, blue, green, and red (Figures 1–3).

The pieces allow working on geometric concepts and procedures, namely the composition and decomposition of figures on the plane (2D figures), that, according to Kuzniak, Richard, and Michael (2018) promote geometric skills.

Creative and critical thinking skills, innovation, collaborative work, communication, and entrepreneurial spirit are among the most emphasised and valued aptitudes/competences in today’s society. With these rising expecta-


tions, students must learn new ways to approach problems, acquire skills and competencies, and create and use tools in innovative ways. The PUNTE’s pieces can provide an innovative educational experience.

The STEAM (Science, Technology, Engineering, Art, Mathematics) approach has received special attention in several countries (Pinheiro & Santos, 2021; Quigley & Herro, 2019), as it is a way of involving all students in creative activities, involving the use of innovative and appealing resources (Neto, Santos & Rodrigues, 2021).

The project PUNTE follows this path since it develops, tests, and disseminates new innovative, deeply transdisciplinary pedagogical methods in teacher training of the fields of STEAM and beyond (Andić et al., 2022)

The research question of the present study is the following: What is the contribution of Poly-Universe resources in solving tasks involving the concept of perimeter and similarity of triangles?

2 Method

This study aims to investigate the contribution of using material from the PUNTE project during activities in the learning of geometric topics by fifth-grade Portuguese students.¹ To achieve this objective, one of the authors

¹ The Portuguese system includes 12-years school up to higher education, nine years of Basic Education (BE) followed by three of Secondary Education (SE). BE consists of three cycles: the first of four years (with a single teacher), the second with two years, and the third with three years. In SE, students are oriented to different areas, namely Science, Humanities, Arts or Technology, and the Mathematics curriculum varies depending on the chosen area.
conducted a teaching experience based on the implementation of activities, during four lessons (45 min each).

The study is part of a “fundamentally qualitative research” approach (Creswell, 2007, p. 38), adopting an “interpretive perspective” (Creswell, 2007, p. 39). The research was carried out during the second semester of the academic year 2021–2022 in a public Portuguese Basic Education Institution. The participants were 21 students aged between 10 and 11 years who attend the fifth grade, and no students were colourblind. Data were collected from the written records produced by the students in groups of two during the activities.

3 Teaching Experience

The teaching experience involved two distinct phases. An exploratory phase with the objective of getting to know the pieces through their free exploration and the creation of figures. A second phase, according to essential learnings (DGE-MEC, 2018), of solving geometry tasks involves the concept of perimeter and using the criteria of equality of triangles in their construction and in solving problems in mathematical and non-mathematical contexts.

3.1 First Phase

In this phase, the students carried out a free-form activity in which they created several figures, based on the three basic pieces of Poly-Universe, which they later reproduced on paper. From the analysis of the students’ productions, we can record the following:

1) Most students create objects or living beings with which they are familiar (Figures 4 and 5).

2) Other students focused on geometric construction, of which two figures are shown, it was a construction with the three base pieces of the Poly-Universe, in an inclined position (Figure 6). In the second one, which also used the base pieces, it is observed that there was a junction of the blue colour of the semi-circle of the circular piece with the blue part of the square piece (the diameter of the blue semicircle has the same measure as the side of the blue square) (Figure 7).

3) Only one student used one of the pieces and finished the drawing with a free drawing (Figure 8).

It is worth noting the great involvement of students in handling this resource and in carrying out this first phase of the experience.

Figure 4: Creative drawing of a fish.

Figure 5: Creative drawing of a cat.

Figure 6: Geometric drawing 1.
3.2 Second phase

The second phase involved two tasks. The first task consisted of building a prototype of the triangular piece, with the measurements they wanted, obeying the ratios between the measurements of the sides of the four triangles of the prototype and the corresponding internal angles must be geometrically equal (they are all similar triangles with different similarity ratios). After observing the prototype of the equilateral triangle, the students recorded that the measure of the side of the blue triangle (equilateral) is half the measure of the side of the initial triangle. In turn, the measure of the side of the red triangle (equilateral) is the fourth part of the measure of the side of the original triangle. Finally, the measure of the side of the green triangle (equilateral) is the eighth part of the measure of the side of the initial triangle (Figures 9 and 10). A resolution using the dynamic geometry system (DGS) can be analyzed in the Figures 9 and 10.

The 18 students construct the prototype of the triangular piece (three students did not perform this first task of the second phase). During this task, all the students have on the table the pieces and the possibility to manipulate them. The constructions carried out by the students generally (13 constructions) obeyed the ratios involved between the measurements of the sides of the various triangles. However, some constructions (five constructions) did not comply with the measurements of the prototype (Figure 11).

It was found that of the 13 students who built correctly, eight of them resorted only to the drawing material and eight resorted in some way to the part to build the prototype. Of these five, a student used the piece to
measure all the sides of all the triangles that compose the prototype (Figure 12). The remaining four students drew the basic triangle using the drawing material (compass and ruler) and, later, went through the piece to define the vertices of the internal triangles and, finally, to join them. This student records all the measurements of each side of the construction, as well as the ratios between the measurements of the sides of the four triangles.

According to Figure 11, it is possible to verify that some built according to the prototype, and others built the figure with rotation. At Figure 13, we can see, the student records all the measurements of each side of the construction, as well as the ratios between the measurements of the sides of the four triangles.

The second task involved calculating the perimeter of the various triangles of the prototype and calculating the perimeter of the hexagon formed inside the triangle. The 21 students were challenged to perform the task and 16 solved it correctly; only two did not fulfil the task completely, and three did not solve the task (Figures 14).
The ratio of the various triangles and the respective ratios between them were also mentioned. After the student’s solutions, it was discussed with them the symbolic language used in the resolution of the task. Namely, the perimeter of the figures, the meaning of the symbology \( \times 3 \) or \( \times 8 \) and being a situation involving the calculation of the perimeter of the triangle, the correct symbology is symbology \( \times 3 \). The students verified that the ratio between the perimeters coincided with the similarity ratio of the triangles (Figure 15).

### 4 Conclusions

The first phase, exploratory phase, was important to promote a first contact with the Poly-Universe pieces allowing the students to touch and move them.

The Poly-Universe pieces facilitated the composition and decomposition of plane figures, as we observe in phase one of the study, contributing to the development of geometric skills (Kuzniak et al., 2018). The fact that it is an easy-to-use manipulative resource motivates the involvement of students in the tasks and gives it a dynamic character tasks (make and remake figures easily, following or not characteristics of the pieces, e.g., Figures 4–8), involving operation on the figure, either mentally or physically, which can give insight into the solution of a problem. The real benefits of using the presented Poly-Universe pieces were in the visualisation of the pieces (colours and sizes) in the exploratory phase and then in the second phase the decomposition and composition of figures (triangle), in support of the construction of the prototype, in which 18 students who resorted to the pieces. In addition, the identification of the similarity ratio between the various figures that compose the triangular prototype by

<table>
<thead>
<tr>
<th>Perimeter</th>
<th>Original triangle ( \rightarrow 8 \times 3 = 24 , \text{cm} )</th>
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<tbody>
<tr>
<td>Perimeter</td>
<td>Bigger triangle ( \rightarrow 4 \times 3 = 12 , \text{cm} )</td>
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<tr>
<td>Perimeter</td>
<td>Medium triangle ( \rightarrow 2 \times 3 = 6 , \text{cm} )</td>
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<tr>
<td>Perimeter</td>
<td>Smaller triangle ( \rightarrow 1 \times 3 = 3 , \text{cm} )</td>
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<tr>
<td>Perimeter of the hexagon (base)</td>
<td>( 8 - (1 + 4) = 8 - 5 = 3 )</td>
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<td></td>
<td>( 8 - (2 + 4) = 8 - 6 = 2 )</td>
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<td>( 8 - (1 - 2) = 5 )</td>
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<td></td>
<td>( 2 + 1 + 3 + 4 + 2 + 5 = 17 , \text{cm} )</td>
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<td></td>
<td>( P = 17 , \text{cm} )</td>
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Figure 14: Student’s task (left figure, legend right figure).

Figure 15: This student solves the task using the similarity ratios between the measurements of the sides of the triangles (left figure, legend right figure).
measuring the length of the sides of the triangle was based on direct observation of the piece.

It was possible to build the triangle prototype with the help of a DGS and of the geometrical materials and work on the concept of the perimeter of the blue triangle (Figure 10). This construction is in accordance with essential learning (DGE-MEC, 2018), which asks for the investigation of the possibility of building triangles given the lengths of the three sides, using a DGS or manipulative material, to systematise the results from the discussion by the class, and promoting the ability to work as a team and stimulate the identification of existing relationships between sides and angles.

On the concept of perimeter, the pieces supported the exploration and observation of the prototype in the study of the ratios of sides of similar triangles (Figure 13). Also in the plane, the study of the area includes the triangle, parallelogram, and circle, which favours the understanding of properties and relationships (DGE-MEC, 2018). Finally, at the level of free exploration (first phase of exploration) of the pieces by the students, small artists were imagined and built their “works of art.”

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