Open Eng. 2015; 5:124–130 DE GRUYTER OPEN

Research Article Open Access

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Solitary wave and shock wave solutions of (1+1)-dimensional perturbed Klein-Gordon,(1+1)-dimensional Kaup-Keperschmidt and (2+1)-dimensional ZK-BBM equations

Abstract: In this paper, two different types of envelope solitons: solitary wave and shock wave have been obtained for the (1+1)-dimensional perturbed Klein-Gordon, (1+1)-dimensional Kaup-Keperschmidt and (2+1)-dimentional ZK-BBM equations using the solitary wave ansatz. The parameter regimes, for the existence of the solitons are identified during the derivation of the solution. Since, the nonlinear wave is one of the fundamental object of nature and a growing interest has been given to the propagation of nonlinear wave in dynamical system.

Keywords: Solitary wave solution, shock wave solution, (1+1)-dimensional perturbed Klein-Gordon equation, (1+1)-dimensional Kaup-Keperschmidt equation, (2+1)-dimentional ZK-BBM equation, solitary wave ansatz

DOI 10.1515/eng-2015-0014
Received July 27, 2014; accepted January 19, 2015

1 Introduction

Nonlinear evolution equations (NLEEs) have become attention for the scientists and researchers due to their variety of applications in electrochemistry, electromagnetic, fluid dynamics, acoustics, cosmology, astrophysics and plasma physics [1–4].

A challenging task is to look for solutions of these NLEEs. There are various types of solutions that are available for these equations. Some of them are soliton solutions, solitary wave solutions, cnoidal and snoidal waves, periodic solutions, shock wave solutions as well as various other types [5–9]. In recent times, a lot of research has been done in this area [20]-[39]

In this article, two different types of envelope solitons, solitary and shock wave, have been constructed for (1+1)-dimensional perturbed Klein-Gordon, (1+1)-dimensional Kaup-Keperschmidt and (2+1)- dimensional ZK-BBM equations. These solitons can propagate in nonlinear dispersive media. Compared with the solitary soliton which is a pulse on a zero-intensity background, the shock soliton appears as an intensity dip in an infinitely extended constant background. Remarkably, the interest in shock and solitary solitons has been growing steadily in recent years.

In recent time, Biswas *et al.* [10] and Sassaman and Biswas [11] investigated the (1+1)-dimensional perturbed Klein Gordon equation (KGE), which is

$$u_{tt} - u_{xx} + f(u) - \epsilon(\alpha u + r u_t + s u_x + \beta u_{xt} + \gamma u_{tt}) = 0, \quad (1)$$

and obtained the exact 1-soliton solution. Where $f(u) = au - bu^2$ and $a, b, \alpha, \beta, \gamma, r, s$ are constants while ϵ is perturbation parameter.

With the choice of $r = s = 0$, the above equation (1) reduces to (1+1)-dimensional perturbed Klein–Gordon equation, without local inductance and dissipation effect, in the following form

$$u_{tt} - u_{xx} + f(u) - \epsilon(\alpha u + \beta u_{xt} + \gamma u_{tt}) = 0. \quad (2)$$

These perturbation terms typically arise in the study of long Josephson junction in the context of the sine-Gordon equation (SGE). Since the SGE can be approximated by the KGE, an exact solution of the perturbed KGE will make sense in the context of the study of the SGE. For the perturbation terms, α represents losses across the junction, r accounts for dissipative losses in Josephson junction theory due to tunneling of normal electrons across the dielectric barrier, s is generated by a small inhomogeneous part

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of the local inductance, β represents diffusion, and γ is the capacity inhomogeneity. For $\epsilon = 0$, the equation (1) reduces to the KGE with quadratic nonlinearity:

$$u_{tt} - u_{xx} + au - bu^2 = 0. \tag{3}$$

If

$$f(u) = au - bu^3. \tag{4}$$

The equation (1) becomes the KGE with cubic nonlinearity:

$$u_{tt} - u_{xx} + au - bu^3 = 0. \tag{5}$$

It appears in theoretical physics in the context of relativistic quantum mechanics.

The main aim of this paper is to construct the solitray wave and shock wave solutions of perturbed KGE (1) with quadratic nonlinearity in the (1+1)-dimensional and also with local inductance and dissipation effect. In this paper, we will use the ansatz method to construct the solitary wave and shock wave solutions to the equations (6), (7) and (8).

$$\begin{aligned} &u_{tt} - u_{xx} + au - bu^2 \\ &- \epsilon(\alpha u + ru_t + su_x + \beta u_{xt} + \gamma u_{tt}) = 0. \end{aligned} \tag{6}$$

The Kaup-Kupershmidt equation is the nonlinear fifth-order partial differential equation. It is the first equation in a hierarchy of integrable equations with Lax operator. It has properties similar (but not identical) to those of the better-known KdV hierarchy. Fifth-order KdV type equations arise naturally in modeling many different wave phenomena such as gravity-capillary waves, the propagation of shallow water waves over a flat surface and magneto-sound propagation in plasmas [12]. Although, the KK equation is completely integrable [13] and has bilinear representations [14, 15], and is read as

$$u_t + u_{xxxxx} + 10uu_{xxx} + 25u_xu_{xx} + 20u^2u_x = 0. \tag{7}$$

The generalised form of the (2+1)-dimensional ZK-BBM equation is given as:

$$u_t + u_x - a(u)_x^2 + (bu_{xt} - ku_{yt})_x = 0, \tag{8}$$

where a, b and k are arbitrary constants. It arises as a description of gravity water waves in the long-wave regime [16, 17]. A variety of exact solutions for the (2+1)-dimensional ZK-BBM equation [18, 19] are developed by means of the tanh and the sine-cosine methods.

2 Solitary Waves Solitons

In this section, the solitary wave solution or non-topological solution for the nonlinear (1+1)-dimensional perturbed Klein-Gordon equation (6), (1+1)-dimensional Kaup-Keperschmidt equation (7) and (2+1)-dimensional ZK-BBM equation (8) have been found, using the solitary wave ansatz, in the following manner.

$$u(x,t) = \frac{A}{\cosh^p \xi} \quad \text{and} \quad \xi = B(x - vt). \tag{9}$$

Where A is the amplitude of the soliton, B is the inverse width of the soliton and v is the velocity of the solitary wave. For the (2+1)-dimensional equation we have

$$u(x,y,t) = \frac{A}{\cosh^p \xi} \quad \text{and} \quad \xi = Bx + Cy - vt. \tag{10}$$

Where A is the amplitude of the soliton, B and C are the inverse width of the soliton and v is the velocity of the solitary wave.

2.1 The (1+1)-dimensional perturbed Klein-Gordon equation

From the equation (9), it can be followed

$$u_{tt} = \frac{AB^2v^2p^2 \tanh\xi}{\cosh^p \xi} - \frac{AB^2v^2p(p+1)}{\cosh^{p+2}\xi} \tag{11}$$

$$u_{xx} = \frac{AB^2p^2}{\cosh^p \xi} - \frac{AB^2p(p+1)}{\cosh^{p+2}\xi} \tag{12}$$

$$u = \frac{A}{\cosh^p \xi} \tag{13}$$

$$u^2 = \frac{A^2}{\cosh^{2p}\xi} \tag{14}$$

$$u_t = \frac{ABvp \tanh\xi}{\cosh^p \xi} \tag{15}$$

$$u_x = \frac{-ABp \tanh\xi}{\cosh^p \xi} \tag{16}$$

$$u_{xt} = \frac{-AB^2vp^2}{\cosh^p \xi} + \frac{AB^2vp(p+1)}{\cosh^{p+2}\xi} \tag{17}$$

After substituting equations (11) - (17) into equation (6), the following equation is obtained

$$\begin{aligned} &\frac{AB^2v^2p^2 \tanh\xi}{\cosh^p \xi} - \frac{AB^2v^2p(p+1)}{\cosh^{p+2}\xi} - \frac{AB^2p^2}{\cosh^p \xi} \\ &+ \frac{AB^2p(p+1)}{\cosh^{p+2}\xi} + \frac{aA}{\cosh^p \xi} - \frac{bA^2}{\cosh^{2p}\xi} \\ &- \epsilon\left\{\frac{\alpha A}{\cosh^p \xi} + \frac{rABvp \tanh\xi}{\cosh^p \xi} - \frac{sABp \tanh\xi}{\cosh^p \xi} - \frac{\beta AB^2vp^2}{\cosh^p \xi}\right. \\ &+ \frac{\beta AB^2vp(p+1)}{\cosh^{p+2}\xi} + \frac{\gamma AB^2v^2p^2 \tanh\xi}{\cosh^p \xi} \\ &\left.- \frac{\gamma AB^2v^2p(p+1)}{\cosh^{p+2}\xi}\right\} = 0. \end{aligned} \tag{18}$$

It may be noted that $p = 2$ is being calculated when exponents $2p$ and $p + 2$ are equated equal to each other. Furthermore, set the coefficients of the linearly independent terms to zero. Thus, we can write

$$\begin{aligned}&-AB^2v^2p(p+1)+AB^2p(p+1)-bA^2-\epsilon\beta AB^2vp(p+1)\\&+\epsilon\gamma AB^2v^2p(p+1)=0,\\&AB^2v^2p^2-AB^2p^2+aA-\epsilon\alpha A-\epsilon rABvp\\&+\epsilon sABp+\epsilon\beta AB^2vp^2-\epsilon\gamma AB^2v^2p^2=0.\end{aligned}$$

Solving the above system of equations and also set $p = 2$, then it can be written

$$\begin{aligned}&A=\frac{6B^2(-v^2+1-\epsilon\beta v+\epsilon\gamma v^2)}{b},\quad B=B,\\&v=\frac{1}{4(-1+\gamma)B}\Big(2\epsilon\beta B-\epsilon r\\&\pm\Big(\epsilon(4\epsilon\beta^2B^2-4\epsilon\beta Br+\epsilon r^2+4\alpha-8sB-4\gamma\alpha+8\gamma sB)\\&+16B^2-4a-16\gamma B^2+4\gamma a\Big)^{\frac{1}{2}}\Big)\end{aligned}$$

Hence, the solitary wave solution of the (1+1)-dimensional perturbed Klein-Gordon equation is given by

$$u(x,t)=\frac{A}{\cosh^2\{B(x-vt)\}}. \tag{19}$$

2.2 The (1+1)-dimensional Kaup-Keperschmidt equation

It can, thus, be written from the equation (9) as follow

$$u_t=\frac{ABvp\tanh\xi}{\cosh^p\xi} \tag{20}$$

$$\begin{aligned}u_{xxxxx}=&\frac{-AB^5p^5\tanh\xi}{\cosh^p\xi}\\&+\frac{AB^5p(p+1)(p+2)(2p^2+4p+4)\tanh\xi}{\cosh^{p+2}\xi}\\&-\frac{AB^5p(p+1)(p+2)(p+3)(p+4)\tanh\xi}{\cosh^{p+4}\xi}\end{aligned} \tag{21}$$

$$uu_{xxx}=\frac{-A^2B^3p^3\tanh\xi}{\cosh^{2p}\xi}+\frac{A^2B^3p(p+1)(p+2)\tanh\xi}{\cosh^{2p+2}\xi} \tag{22}$$

$$u_xu_{xx}=\frac{-A^2B^3p^3\tanh\xi}{\cosh^{2p}\xi}+\frac{A^2B^3p^2(p+1)\tanh\xi}{\cosh^{2p+2}\xi}$$

$$u^2u_x=\frac{-A^3Bp\tanh\xi}{\cosh^{3p}\xi} \tag{23}$$

After substituting equations (20)-(23) into equation (7), the following equation is obtained

$$\begin{aligned}&\frac{ABvp\tanh\xi}{\cosh^p\xi}-\frac{AB^5p^5\tanh\xi}{\cosh^p\xi}\\&+\frac{AB^5p(p+1)(p+2)(2p^2+4p+4)\tanh\xi}{\cosh^{p+2}\xi}\\&-\frac{AB^5p(p+1)(p+2)(p+3)(p+4)\tanh\xi}{\cosh^{p+4}\xi}\\&-\frac{10A^2B^3p^3\tanh\xi}{\cosh^{2p}\xi}\\&+\frac{10A^2B^3p(p+1)(p+2)\tanh\xi}{\cosh^{2p+2}\xi}-\frac{25A^2B^3p^3\tanh\xi}{\cosh^{2p}\xi}\\&+\frac{25A^2B^3p^2(p+1)\tanh\xi}{\cosh^{2p+2}\xi}-\frac{20A^3Bp\tanh\xi}{\cosh^{3p}\xi}=0.\end{aligned} \tag{24}$$

It may be noted that $p = 2$ is being calculated when exponents $2p + 2$ and $3p$ are equated equal to each other. Furthermore, set the coefficients of the linearly independent terms to zero. Thus, we can write

$$\begin{aligned}&-AB^5p(p+1)(p+2)(p+3)(p+4)+10A^2B^3p(p+1)(p+2)\\&+25A^2B^3p^2(p+1)-20A^3Bp=0,\\&ABvp-AB^5p^5=0,\\&AB^5p(p+1)(p+2)(2p^2+4p+4)-10A^2B^3p^3\\&-25A^2B^3p^3=0.\end{aligned}$$

Solving the above system of equations and also set $p = 2$, then it can be written

$$A=\frac{12B^2}{7},\quad B=B,\quad v=B^4.$$

Hence, the solitary wave solution of the (1+1)-dimensional Kaup-Keperschmidt equation is given by

$$u(x,t)=\frac{A}{\cosh^2\{B(x-vt)\}} \tag{25}$$

2.3 The (2+1)-dimensional ZK-BBM equation

From the equation (10), it can be followed

$$u_t=\frac{Avp\tanh\xi}{\cosh^p\xi} \tag{26}$$

$$u_x=\frac{-ABp\tanh\xi}{\cosh^p\xi} \tag{27}$$

$$u_x^2=\frac{-A^2Bp\tanh\xi}{\cosh^{2p}\xi} \tag{28}$$

$$
\begin{aligned}
(bu_{xt} - ku_{yt})_x =& \frac{bAB^2vp^3\tanh\xi}{\cosh^p\xi} \\
&- \frac{bAB^2vp(p+1)(p+2)\tanh\xi}{\cosh^{p+2}\xi} \\
&- \frac{kABCvp^3}{\cosh^p\xi} \\
&+ \frac{kABCvp(p+1)(p+2)\tanh\xi}{\cosh^{p+2}\xi}
\end{aligned} \tag{29}
$$

After substituting equations (26)-(29) into equation (8), the following equation is obtained

$$
\begin{aligned}
&\frac{Avp\tanh\xi}{\cosh^p\xi} - \frac{ABp\tanh\xi}{\cosh^p\xi} + \frac{aA^2Bp\tanh\xi}{\cosh^{2p}\xi} \\
&+ \frac{bAB^2vp^3\tanh\xi}{\cosh^p\xi} - \frac{bAB^2vp(p+1)(p+2)\tanh\xi}{\cosh^{p+2}\xi} \\
&- \frac{kABCvp^3}{\cosh^p\xi} + \frac{kABCvp(p+1)(p+2)\tanh\xi}{\cosh^{p+2}\xi} = 0
\end{aligned} \tag{30}
$$

It may be noted that $p = 1$ is being calculated when exponents $2p$ and $p+2$ are equated equal to each other. Furthermore, set the coefficients of the linearly independent terms to zero. Thus, we can write

$$
\begin{aligned}
&aA^2Bp - bAB^2vp(p+1)(p+2) \\
&+ kABCvp(p+1)(p+2) = 0, \\
&Avp - ABp + bAB^2vp^3 - kABCvp^3 = 0.
\end{aligned}
$$

Solving the above system of equations and also set $p = 1$, then it can be written

$$
\begin{aligned}
&A = \frac{12B(bB - kC)}{a(1 + 4bB^2 - 4kBC)}, \\
&B = B, \quad C = C, \quad v = \frac{B}{1 + 4bB^2 - 4kBC}
\end{aligned}
$$

Hence, the solitary wave solution of the (2+1)-dimensional ZK-BBM equation is given by

$$
u(x, y, t) = \frac{A}{\cosh(Bx + Cy - vt)}. \tag{31}
$$

In the following section, the shock wave solutions have been found.

3 Shock wave solutions

In this section, the Shock wave solution or topological solution to the nonlinear (1+1)-dimensional perturbed Klein-Goron equation (6), (1+1)-dimensional Kaup-Keperschmidt equation (7) and (2+1)-dimensional ZK-BBM equation (8) have been found using the following solitary wave ansatz. For this, we may have

$$
u(x, t) = A\tanh^p\xi \quad \text{and} \quad \xi = B(x - vt), \tag{32}
$$

where A and B are free parameters and are the amplitude and inverse width of the soliton, while v is the velocity of the soliton. The value of the exponent p is determined later. For (2+1)-dimensional equation we have

$$
u(x, y, t) = A\tanh^p\xi \quad \text{and} \quad \xi = Bx + Cy - vt. \tag{33}
$$

Where A , B and C are free parameters. The value of the exponent p is determined later.

3.1 The (1+1)-dimensional perturbed Klein-Gordon equation

From the equation (32), it can be followed

$$
\begin{aligned}
u_{tt} =& AB^2v^2p(p-1)\tanh^{p-2}\xi \\
&- 2AB^2v^2p^2\tanh^p\xi + AB^2v^2p(p+1)\tanh^{p+2}
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
u_{xx} =& AB^2p(p-1)\tanh^{p-2}\xi \\
&- 2AB^2p^2\tanh^p\xi + AB^2p(p+1)\tanh^{p+2}\xi
\end{aligned} \tag{35}
$$

$$
u = A\tanh^p\xi \tag{36}
$$

$$
u^2 = A^2\tanh^{2p}\xi \tag{37}
$$

$$
u_t = -ABvp\tanh^{p-1}\xi + ABvp\tanh^{p+1}\xi \tag{38}
$$

$$
u_x = ABp\tanh^{p-1}\xi - ABp\tanh^{p+1}\xi \tag{39}
$$

$$
\begin{aligned}
u_{xt} =& -AB^2vp(p-1)\tanh^{p-2}\xi \\
&+ 2AB^2vp^2\tanh^p\xi - AB^2vp(p+1)\tanh^{p+2}\xi
\end{aligned} \tag{40}
$$

After substituting equations (34) - (40) into (6), the following equation is obtained

$$
\begin{aligned}
&AB^2v^2p(p-1)\tanh^{p-2}\xi - 2AB^2v^2p^2\tanh^p\xi \\
&+ AB^2v^2p(p+1)\tanh^{p+2} \\
&- AB^2p(p-1)\tanh^{p-2}\xi + 2AB^2p^2\tanh^p\xi \\
&- AB^2p(p+1)\tanh^{p+2}\xi + aA\tanh^p\xi \\
&- bA^2\tanh^{2p}\xi - \epsilon\alpha A\tanh^p\xi \\
&+ \epsilon rABpv\tanh^{p-1}\xi - \epsilon rABpv\tanh^{p+1}\xi \\
&- \epsilon sABp\tanh^{p-1}\xi + \epsilon sABp\tanh^{p+1}\xi \\
&+ \epsilon\beta AB^2vp(p-1)\tanh^{p-2}\xi \\
&- 2\epsilon\beta AB^2vp^2\tanh^p\xi + \epsilon\beta AB^2vp(p+1)\tanh^{p+2}\xi \\
&- \epsilon\gamma AB^2v^2p(p-1)\tanh^{p-2}\xi + 2\epsilon\gamma AB^2v^2p^2\tanh^p\xi \\
&- \epsilon\gamma AB^2v^2p(p+1)\tanh^{p+2} = 0
\end{aligned}
$$

It may be noted that $p = 2$ is being calculated when exponents $2p$ and $p+2$ are to be set equal to each other. Furthermore, set the coefficients of the linearly independent terms to zero. It can, thus, be written as

$$\begin{aligned}&AB^2v^2p(p+1) - AB^2p(p+1) - bA^2 + \epsilon\beta AB^2vp(p+1)\\&- \epsilon\gamma AB^2v^2p(p+1) = 0,\\&- \epsilon rABpv + \epsilon sABp = 0,\\&- 2AB^2v^2p^2 + 2AB^2p^2 + aA - \epsilon\alpha A\\&- 2\epsilon\beta AB^2vp^2 + 2\epsilon\gamma AB^2v^2p^2 = 0,\\&\epsilon rABpv - \epsilon sABp = 0,\\&AB^2v^2p(p-1) - AB^2p(p-1) + \epsilon\beta AB^2vp(p-1)\\&- \epsilon\gamma AB^2v^2p(p-1) = 0.\end{aligned}$$

Solving the above system of equations and also set $p = 2$, then it can be written

$$\begin{aligned}A &= \frac{-6B^2(-v^2+1-\epsilon\beta v+\epsilon\gamma v^2)}{b},\\v &= -\frac{\sqrt{(-1+\epsilon\gamma)(-1+\epsilon\beta)}}{(-1+\epsilon\gamma)}\\B &= \frac{1}{4}\frac{\pm\sqrt{-(-2v^2+2-2\varepsilon\beta v+2\epsilon\gamma v^2)(a-\epsilon\alpha)}}{(-v^2+1-\epsilon\beta v+\epsilon\gamma v^2)}\end{aligned}$$

Hence, the shock wave solution of the (1+1)-dimensional perturbed Klein-Gordon equation is given by

$$u(x,t) = A\tanh^2 B(x-vt). \tag{41}$$

3.2 The (1+1)-dimensional Kaup-Keperschmidt equation

From equation (32), it can be followed that

$$u_t = -ABvp\tanh^{p-1}\xi + ABvp\tanh^{p+1}\xi \tag{42}$$

$$\begin{aligned}u_{xxxxx} &= AB^5p(p-1)(p-2)(p-3)(p-4)\tanh^{p-5}\xi\\&- 5AB^5p(p^4-6p^3+15p^2-18p+8)\tanh^{p-3}\xi\\&+ AB^5p(10p^4-20p^3+50p^2-40p+16)\tanh^{p-1}\xi\\&- AB^5p(10p^4+20p^3+50p^2+40p+16)\tanh^{p+1}\xi\\&+ 5AB^5p(p^4+6p^3+15p^2+18p+8)\tanh^{p+3}\xi\\&- AB^5p(p+1)(p+2)(p+3)(p+4)\tanh^{p+5}\xi\end{aligned} \tag{43}$$

$$\begin{aligned}uu_{xxx} &= A^2B^3p(p-1)(p-2)\tanh^{2p-3}\xi\\&- A^2B^3p(3p^2-3p+2)\tanh^{2p-1}\xi\\&+ A^2B^3p(3p^2+3p+2)\tanh^{2p+1}\xi\\&- A^2B^3p(p+1)(p+2)\tanh^{2p+3}\xi\end{aligned} \tag{44}$$

$$\begin{aligned}u_xu_{xx} &= A^2B^3p^2(p-1)\tanh^{2p-3}\xi\\&- A^2B^3p^2(3p-1)\tanh^{2p-1}\xi\\&- A^2B^3vp^2(3p+1)\tanh^{2p+1}\xi\\&+ A^2B^3vp^2(p+1)\tanh^{2p+3}\xi\end{aligned} \tag{45}$$

$$u^2u_x = A^3Bp\{\tanh^{3p-1}\xi - \tanh^{3p+1}\xi\} \tag{46}$$

After substituting equations (42)-(46) into (7), the following equation is obtained

$$\begin{aligned}&- ABvp\tanh^{p-1}\xi + ABvp\tanh^{p+1}\xi\\&+ AB^5p(p-1)(p-2)(p-3)(p-4)\tanh^{p-5}\xi\\&- 5AB^5p(p^4-6p^3+15p^2-18p+8)\tanh^{p-3}\xi\\&+ AB^5p(10p^4-20p^3+50p^2-40p+16)\tanh^{p-1}\xi\\&- AB^5p(10p^4+20p^3+50p^2+40p+16)\tanh^{p+1}\xi\\&+ 5AB^5p(p^4+6p^3+15p^2+18p+8)\tanh^{p+3}\xi\\&- AB^5p(p+1)(p+2)(p+3)(p+4)\tanh^{p+5}\xi\\&+ 10A^2B^3p(p-1)(p-2)\tanh^{2p-3}\xi\\&- 10A^2B^3p(3p^2-3p+2)\tanh^{2p-1}\xi\\&+ 10A^2B^3p(3p^2+3p+2)\tanh^{2p+1}\xi\\&- 10A^2B^3p(p+1)(p+2)\tanh^{2p+3}\xi\\&+ 25A^2B^3p^2(p-1)\tanh^{2p-3}\xi\\&- 25A^2B^3p^2(3p-1)\tanh^{2p-1}\xi\\&- 25A^2B^3vp^2(3p+1)\tanh^{2p+1}\xi\\&+ A^2B^3vp^2(p+1)\tanh^{2p+3}\xi\\&+ 20A^3Bp\{\tanh^{3p-1}\xi - \tanh^{3p+1}\xi\} = 0.\end{aligned}$$

It may be noted that $p = 2$ is being calculated when exponents $2p+3$ and $3p+1$ are to be set equal to each other. Furthermore, set the coefficients of the linearly independent terms to zero. It can, thus, be written as

$$\begin{aligned}&- AB^5p(p+1)(p+2)(p+3)(p+4)\\&- 10A^2B^3p(p+1)(p+2)\\&+ A^2B^3vp^2(p+1) - 20A^3Bp = 0,\\&5AB^5p(p^4+6p^3+15p^2+18p+8)\\&+ 10A^2B^3p(3p^2+3p+2)\\&- 25A^2B^3vp^2(3p+1) + 20A^3Bp = 0,\\&ABvp - AB^5p(10p^4+20p^3+50p^2+40p+16)\\&- 10A^2B^3p(3p^2-3p+2) - 25A^2B^3p^2(3p-1) = 0,\\&- ABvp + AB^5p(10p^4-20p^3+50p^2-40p+16)\\&+ 10A^2B^3p(p-1)(p-2) = 0.\end{aligned}$$

Solving the above system of equations and also set $p = 2$, then it can be written

$$A = \frac{-68B^2}{55}, \quad v = 136B^4, \quad B = B.$$

Hence, the shock wave solution of the (1+1)-dimensional Kaup-Keperschmidt equation is given by

$$u(x,t) = A\tanh^2 B(x - vt). \tag{47}$$

3.3 The (2+1)-dimensional ZK-BBM equation

From equation (32), it can be written as

$$u_t = -Apv\tanh^{p-1}\xi + Apv\tanh^{p+1}\xi, \tag{48}$$

$$u_x = ABp\tanh^{p-1}\xi - ABp\tanh^{p+1}\xi, \tag{49}$$

$$u_x^2 = A^2Bp\tanh^{2p-1}\xi - A^2Bp\tanh^{2p+1}\xi, \tag{50}$$

$$\begin{aligned}(bu_{xt} - ku_{yt})_x &= -bABvp(p-1)\tanh^{p-2}\xi\\ &+ 2bABvp^2\tanh^p\xi - bABvp(p+1)\tanh^{p+2}\xi\\ &+ kACvp(p-1)\tanh^{p-2}\xi - 2kACvp^2\tanh^p\xi\\ &+ kACvp(p+1)\tanh^{p+2}\xi\end{aligned} \tag{51}$$

After substituting equations (48) - (51) into (8), the following equation is obtained

$$\begin{aligned}&-Apv\tanh^{p-1}\xi + Apv\tanh^{p+1}\xi + ABp\tanh^{p-1}\xi\\ &- ABp\tanh^{p+1}\xi - aA^2Bp\tanh^{2p-1}\xi\\ &+ aA^2Bp\tanh^{2p+1}\xi - bABvp(p-1)\tanh^{p-2}\xi\\ &+ 2bABvp^2\tanh^p\xi - bABvp(p+1)\tanh^{p+2}\xi\\ &+ kACvp(p-1)\tanh^{p-2}\xi - 2kACvp^2\tanh^p\xi\\ &+ kACvp(p+1)\tanh^{p+2}\xi = 0.\end{aligned}$$

It may be noted that $p = 1$ is being calculated when exponents $2p + 1$ and $p + 2$ are to be set equal to each other. Furthermore, set the coefficients of the linearly independent terms to zero. It can, thus, be written as

$$\begin{aligned}&aA^2Bp - bABvp(p+1) + kACvp(p+1) = 0,\\ &-Apv + ABp = 0,\\ &Apv - ABp = 0,\\ &-aA^2Bp + 2bABvp^2 - 2kACvp^2 = 0,\\ &-bABvp(p-1) + kACvp(p-1) = 0.\end{aligned}$$

Solving the above system of equations and also set $p = 1$, then it can be written

$$A = \frac{2v(bB - kC)}{aB}, \quad v = B \quad \text{and} \quad C = C.$$

Hence, the shock wave solution of the (2+1)-dimensional ZK-BBM equation is given by

$$u(x,y,t) = A\tanh(Bx + Cy - vt). \tag{52}$$

4 Conclusions

In this paper, the topological (shock wave) and non-topologial (solitary wave) solutions to the (1+1)-dimensional perturbed Klein-Gordon with quadratic nonlinearity, (1+1)-dimensional Kaup-Keperschmidt and (2+1)-dimensional ZK-BBM equations, were obtained. The solitary wave ansatz is rather heuristic and processes significant features that make it practical for the determination of soliton solutions for a wide class of nonlinear evolution equations. These results will be reported in future and we clearly see that the consistency, which has recently been applied successfully.

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