Research Article Open Access

Tunde Bello-Ochende*

Multi-scale pin fins: Scale analysis and mathematical optimization of micro-pin fins arranged in rows

DOI 10.1515/eng-2015-0023

Received October 09, 2014; accepted February 26, 2015

Abstract: This paper shows the performance of a cylindrical micro-pin fins with multiples-arrays structures for maximum heat transfer. The structures has a varying geometric sizes (diameter, height and spacing). The effects of Reynolds number and thermal conductivity ratio on the optimized geometric configurations and the maximum heat transfer rate is documented. Two design configuration were considered. Scales and computational fluid dynamics analysis shows that the benefits of varying fin height is minimal. Results show that performace is increased when three rows of micro pin fin heat sinks with a reduced degree of freedom (fixed height) when compared to two rows of micro pin fins heat sink for the same amount of material. The optimized diameters of the fins seems to have greatest effect on perfomance of the heat sink.

Keywords: thermal conductivity; heat transfer rate; gradient-based; geometry; micro-pin fin; constructal design

Nomenclature

C_p	Specific heat transfer, [J/kg]
D	Pin diameter, [m]
h	heat transfer coefficient, [W/m^{2}K]
H	Pin height, [m]
k	Thermal conductivity, [W/m.K]
L	Length, [m]
L_1	Width, [m]
n	numbers of rows [-]
P	Pressure, [Pa]
q	Rate of heat transfer, [W/m^2]
Q	Dimensionless heat transfer rate, [-]
R_e	Reynolds number base on axial length, L[-]
R_{th}	Thermal Resistance, [K/W]
s	Interfin spacing, [m]
T	Temperature, [K]
U	Velocity, [m/s]
u, v, w	Velocities in the x, y, z directions, [m/s]
V	Volume, [m^3]
x, y, z	Cartesian coordinates, [m]
Special characters	
Δ	Difference, [-]
μ	Dynamic viscosity, [kg/m.s]
ρ	Density, [kg/m3]
γ	Thermal conductivity ratio, [-]
λ	Lagrange multiplier, [-]
Subscripts	
1	First fin row
2	Second fin row
3	Third fin row
f	Fluid
$inlet$	Inlet
max	Maximum
opt	Optimum
r	Ratio
s	Solid
w	Wall
0	Free stream

1 Introduction

In the last decade there has been a considerable increase in power and ability of various devices and machinery. New concepts and ideas pushed for more versatile, efficient and powerful devices to be used to address rising requirements in industry. More power and ability also introduces the problem of an increase of heat of the various components of these devices. Primarily, these components would be the transistors and various other electronic components in electronic devices; heat exchangers found in industrial applications and other plant and equipment such turbines. Each of these components generates a considerable amount of heat due to the nature of their use, the fluids which they handle and their designs.

***Corresponding Author: Tunde Bello-Ochende:** Department of Mechanical Engineering, University of Cape Town, Private Bag X3, Rondebosch, 7701, South Africa, E-mail: tunde.bello-ochende@uct.ac.za

Micro pin-fin heat sinks are the more dominant in the micro heat sink category as they prove to yield increased heat dissipation characteristics under severe space and acoustic restraints [1]. Nevertheless, design considerations which include material selection, size and compactness greatly influences the heat dissipation rates that can be achieved by these heat sinks. A new evolutional design methodology known as constructal design handles all this design consideration in a simple manner [2–4].

[5] investigated the convective heat transfer and pressure drop phenomenon across a pin-fin micro heat sink by comparing its thermal resistance to that of a micro channel heat sink. They discovered that the thermo-hydraulic performance of a cylindrical micro pin-fin heat sink is superior to that of a micro channel heat sink as very high heat fluxes can be dissipated with low wall temperature rises across the heat sink. Their results showed that for fin diameters larger than 50 μm, the thermal resistance is less sensitive to changes in the fin diameter and for increased efficiency short pins should be used.

[1] optimized a fin heat sink by finding optimal geometric design parameters that minimize the entropy generation rate for both an in-line and staggered configuration. In-line arrangements gave lower entropy generation rates for both low and high thermal conductivity heat sink cases.

In a further study by [6] , the effects of geometric factors on the optimal design performance of pin-fin heat sinks were examined by using the entropy generation minimization scheme. They found that the thermal resistance of these heat sinks increases with an increase in the side and top clearance ratios resulting in a decrease in the entropy generation rate. They also documented that the pin height has an effect on the optimal entropy generation rate of heat sinks. [7], conducted a comparative study into the heat transfer performance of various fin geometries. The study consisted of fins having round, elliptical and plate cross sections both for in-line and staggered configurations. They found that round geometries out-performed sharp-edged fin shapes with the circular fin shape yielding the highest Nusselt number and that of the parallel plate having the lowest Nusselt number for the $110 \leq Re \leq 1320$ range considered. It was also found that at lower pressure drops, elliptical fins provide the best heat transfer performance with the circular fins taking over at higher pressure drops. Parallel plates however offered the best performance in terms of pressure drop and pumping power requirement.

[8] developed steady-state correlations predicting heat transfer performances of in-line and staggered pin-fins from which they found optimal designs. They revealed a dimensionless optimal pin-fin pitch in the span-wise and stream-wise direction of 0.135 and 0.173 respectively for the in-line arrangement. For the staggered arrangement, this dimensionless parameter was found to be 0.19 and 0.1 respectively. [9] developed a response surface methodology to find the optimal design parameters of a pin-fin heat sink. They documented that the fin height and fin diameter are the main factors that affect the thermal resistance of the heat sink while the pitch influences its pressure drop requirements. The conclusion that the most important design parameters affecting the thermal performance of pin-fin heat sink are the fin-diameter and height was also supported by [10]. Their work entailed an optimal design of pin-fin heat sink using a grey- fuzzy logic based on orthogonal arrays.

This work seeks to determine the optimal geometric configuration of a multi-scale micro-pin fin heat sink which will result in the maximal heat transfer rate. The resulting heat transfer across the cylindrical micro-pin fins is by laminar forced convection of uniform, isothermal free stream. The process is carried out using a gradient based mathematical optimization under total fixed volume and manufacturing constraints. It's important to note that the work covered in this paper follows that of [11]. The difference lies in the fact that the constructal design methodology has been combined with mathematical optimisation and applied to a known micro-pin fins heat sink with two and three rows.

2 Model

Consider a cylindrical micro pin fins heat sink consisting of multiple rows of fins as shown in Figure 1a. The flow assembly consist of rows of micro pin fins with diameter $D_1, D_2, D_3, D_4, D_5....D_n$, and heights $H_1, H_2, H_3, H_4, H_5, ...H_n$, spaced at distance $s_1, s_2, s_3, s_4, ...s_n$, from each other with the aims to enhance the extractions of heat at the base of the thermal conductive materials. The swept length is L, and it is fixed. The flow assembly is bathed by a free stream that is uniform and isothermal with temperature T_0 and velocity U_0, because of symmetry we select an elemental volume comprising of two, three cylindrical micro pin fins on the swept length L and width L_1 as shown in Figure 1b. A heat sink with dimensions of 1 mm×0.6 mm×1 mm is used for the numerical computation. The base of the heat sink is supplied with heat at a uniform temperature T_w. The respective continuity, momentum and energy equations governing the fluid flow and heat transfer for the cooling fluid within the heat sink are:

$$\nabla(\rho U) = 0 \quad (1)$$

$$\rho(U \cdot \nabla U) + \Delta P - \mu \nabla^2 U = 0 \quad (2)$$

$$\rho C_p(U \cdot \nabla T) - k_f \nabla^2 T = 0. \quad (3)$$

For the solid material, the momentum and energy governing equations are:

$$U = 0, \qquad k_s \nabla^2 T = 0, \quad (4)$$

where U is the velocity vector, C_p, the specific heat of the fluids, μ the dynamic viscosity and k_f and k_s the thermal conductivities of the fluid and solid. These conservation equations are solved over the fully discretized domain. A one-dimensional uniform velocity with constant temperature is assumed at the inlet:

$$u(x, y, 0) = v(x, y, 0) = 0;$$

$$w(x, y, z, 0) = U_0;$$

$$T(x, y, 0) = T_0. \quad (5)$$

Zero normal stress at the outlet. No-slip, no-penetration boundary condition is enforced on the fin and wall surfaces. Symmetry boundary condition is applied to the domain to reasonably represent the physical and geometric characteristics of flow through pin-fin arrays. A constant wall temperature T_w is imposed at the bottom of the micro pin-fin. Uniform isothermal free stream (air) is used as the working fluid. Other flow-related assumptions implemented include steady flow, laminar flow, incompressibility and constant fluid and material properties.

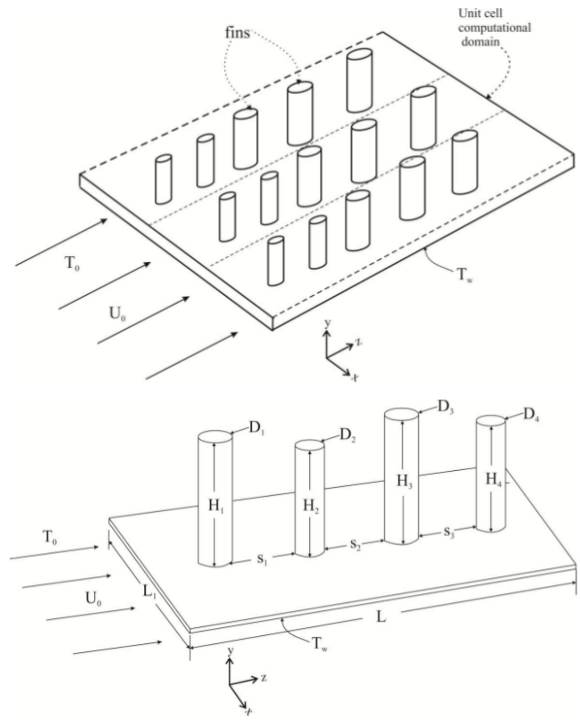
The objective function or the measure of goodness of the design is the total heat transfer rate. The dimensionless rate of heat transfer from the hot solid to the cold fluid is expressed as

$$Q = \frac{q/L}{k_f\,(T_w - T_0)}, \quad (6)$$

where q is the overall rate of heat transfer, T_w and T_0 are the wall and free-stream temperatures respectively.

3 Numerical formulation and optimization

In this section the mass, momentum and energy conservation Equations (1)-(5) are solved over the discretized domain shown using the finite volume software[1] coupled with the above-mentioned boundary conditions. A second-order upwind scheme was used in discretizing the momentum equation while a SIMPLE algorithm was used for the pressure velocity coupling. Convergence criteria were set to less than 1×10^{-4} for continuity and momentum residuals while the residual of energy was set to less than 1×10^{-7}.



(a)

(b)

Figure 1: (a) Physical model of micro finned heat sink. (b) Computational model of micro finned heat sink.

In order to verify the numerical model developed, grid independence tests were carried out on the pin-fin heat sink. For the two row configuration, the test was carried out on the pin-fin heat sink, whose dimensions are given in Table 1. The analysis was conducted for various control volume mesh sizes until the deviation in dimensionless heat transfer rate Q was negligible with the finest mesh consisting of 615 000 cells. The maximum average difference of Q encountered when using a mesh having greater than 159 768 cells was 2.2%, giving the confidence that the simulations carried out based on a 178 488-celled mesh provide satisfactory numerical accuracy.

The validation of the CFD[1] code used was carried out by comparing the numerical results obtained using this code with the analytical results obtained from the investigation carried out by [12] who analysed the performance of a cylindrical pin fin heat sink in laminar forced convection using thermal resistance network. The solution trends were in agreement with previous work [12] with a relative

1 Fluent Inc., Fluent Version 6 Manuals, Centerra Resource Park, 10 Cavendish Court, Lebanon, New Hampshire, USA, 2001.

difference of between 5% to 21% as shown in Table 2a and 15% to 18% in Table 2b.

For the three row configuration, three mesh sizes of 147 546 cells, 182 358 cells and 605 300 cells respectively were used for the verification procedure, it was found that the maximum difference in the dimensionless rate of heat transfer Q between the three mesh sizes is <1%. This gives confidence that a mesh with 182 358 cells will give satisfactory accuracy in the prediction of the heat transfer across the fin array.

3.1 Optimization problem

The objective of the optimization problem is to find the best geometric configuration of pin diameter, fin height (for case one) and inter fin spacing that will maximize the rate of heat transfer from the solid to the fluid. The automated optimization problem was carried out in MATLAB[2] coupled with the DYNAMIC-Q algorithm [13, 14], the mesh generation code GAMBIT[1] and a finite volume code[1]. Detailed discussion of the optimization procedure is outlined in reference [10, 15, 16].

3.2 Constraints

Total Fin Volume Constraint: In heat sink design, weight and material cost of fins are limiting factors. Therefore, the total volume of the cylindrical fins is fixed to a constant value.

$$\begin{aligned} V &= V_1 + V_2 + \ldots + V_n = \text{constant} \\ \therefore \sum V_i &= \text{Constant} \\ \sum \frac{\pi D_i^2}{4} H_i &= C \\ \sum D_i^2 H_i &= \frac{4C}{\pi} \\ \text{for} \quad & i = 1, 2 \text{ and } 3 \ldots n, \end{aligned} \tag{7}$$

where V is the volume of the fins and n the numbers of rows of fins, and C a constant associated with the volume. Manufacturing Restraint: Pin-fin manufacturing and size constraint allows for typical aspect ratios in the range of 0.5 and 4 [17, 18]. Considering fabrication techniques, inter fin spacing is limited to 50 microns [19, 20].

2 The MathWorks, Inc., MATLAB & Simulink Release Notes for R2008a, 3 Apple Hill Drive, Natick, MA, 2008.

4 Scale analysis

Consider Figure 1a, let z be the fin positions along the plate and h be the heat transfer coefficient, the question how should rows of sequential fins be sized? We need to make the shape of each fin so that each fin is of minimum weight that is maximum heat transfer rate density. Using scale analysis, at the point of optimal configuration the scale of convective heat transfer scales as that of conduction heat transfer from the fin, using the procedure outline for two pin fins [2] for multi-scale fins with five row, we have mathematically,

$$k_s D^2 \frac{\Delta T}{H} \sim hDH\Delta T. \tag{8}$$

Next, if h depends on the fin position z along the plate such that $h1(z1) \neq h2(z2) \neq h3(z3) \neq h4(z4) \neq h5(z5)$. The total heat transfer from the fins scale as

$$\begin{aligned} \Big(h_1 D_1 H_1 \Delta T_1 &+ h_2 D_2 H_2 \Delta T_2 + h_3 D_3 H_3 \Delta T_3 \\ &+ h_4 D_4 H_4 \Delta T_4 + h_5 D_5 H_5 \Delta T_5\Big) \\ &\sim k_s \Big(\frac{D_1^2}{H_1}\Delta T_1 + \frac{D_2^2}{H_2}\Delta T_2 + \frac{D_3^2}{H_3}\Delta T_3 \\ &+ \frac{D_4^2}{H_4}\Delta T_4 + \frac{D_5^2}{H_5}\Delta T_5\Big). \end{aligned} \tag{9}$$

Assuming that,

$$\Delta T_1 \sim \Delta T_2 \sim \Delta T_3 \sim \Delta T_4 \sim \Delta T_5 = \Delta T. \tag{10}$$

Where ΔT is the temperature between successive fins, Equation (9) is to be maximized subject to the total volume, scales as $D_1^2 H_1 + D_2^2 H_2 + D_3^2 H_3 + D_4^2 H_4 + D_5^2 H_5$. We seek the extremum of

$$\begin{aligned} \phi &= \frac{D_1^2}{H_1} + \frac{D_2^2}{H_2} + \frac{D_3^2}{H_3} + \frac{D_4^2}{H_4} + \frac{D_5^2}{H_5} \\ &+ \lambda\left(D_1^2 H_1 + D_2^2 H_2 + D_3^2 H_3 + D_4^2 H_4 + D_5^2 H_5\right), \end{aligned} \tag{11}$$

where λ is the Lagrange multiplier. From Equation (8) we find that the diameter of each fin must scale as

$$\frac{D}{H^2} \sim \frac{h}{k_s}. \tag{12}$$

From Equations (9) and (10) we have

$$\begin{aligned} \phi &= \\ &\left(D_1^{3/2} h_1^{1/2} + D_2^{3/2} h_2^{1/2} + D_3^{3/2} h_3^{1/2} + D_4^{3/2} h_4^{1/2} + D_5^{3/2} h_5^{1/2}\right) \\ &+\lambda\left(D_1^2 h_1^{-5/2} + D_2^2 h_2^{-5/2} + D_3^2 h_3^{-5/2} + D_4^2 h_4^{-5/2} + D_5^2 h_5^{-5s/2}\right). \end{aligned} \tag{13}$$

Table 1: Heat sink dimensions used for the code validation process

D_1 (mm)	D_2 (mm)	H_1 (mm)	H_2 (mm)	s (mm)	L_1 (mm)	L (mm)	H_T (mm)
0.15	0.25	0.3	0.6	0.2	0.6	1	1

Table 2: Validation of code for thermal resistance versus (a) pin diameter (b) pin height

(a)			
D (mm)	Present Study R_{th} (K/W)	[12] R_{th} (K/W)	Relative Difference
1.0	122	148	0.21
1.5	92	92	0.00
2.0	66	66	0.00
2.5	51	48	0.05
(b)			
H (mm)	Present Study R_{th} (K/W)	[12]] R_{th} (K/W)	Relative Difference
6	127	104	0.18
8	96	80	0.17
10	78	66	0.15
12	66	56	0.15
14	59	50	0.15

Minimizing ϕ with respect to D_1, D_2, D_3, D_4, D_5 we have that

$$\begin{aligned} D_1 &= -\frac{3h_1}{5\lambda}, D_2 = -\frac{3h_2}{5\lambda}, D_3 = -\frac{3h_3}{5\lambda} \\ D_4 &= -\frac{3h_4}{5\lambda}, D_5 = -\frac{3h_5}{5\lambda} \\ \text{and } \frac{D_2}{D_1} &= \frac{h_2}{h_1}, \frac{D_3}{D_2} = \frac{h_3}{h_2}, \frac{D_4}{D_3} = \frac{h_4}{h_3}, \frac{D_5}{D_4} = \frac{h_5}{h_4}. \end{aligned} \tag{14}$$

From Eqaution (10) we have that

$$\begin{aligned} \frac{D_1}{H_1^2} &\sim \frac{h_1}{k_s}, \frac{D_2}{H_2^2} \sim \frac{h_2}{k_s}, \frac{D_3}{H_3^2} \sim \frac{h_3}{k_s} \\ \frac{D_4}{H_4^2} &\sim \frac{h_4}{k_s}, \frac{D_5}{H_5^2} \sim \frac{h_5}{k_s}. \end{aligned} \tag{15}$$

Combining Equations (12) and (13) we have that the scale of the height ratio is

$$\frac{H_2}{H_1} \sim \frac{H_3}{H_2} \sim \frac{H_4}{H_3} \sim \frac{H_5}{H_4} \sim 1. \tag{16}$$

Similarly [2], if we make the assumption that the convective coefficients is nearly the same (implying that the diameters are nearly the same) for all the fins locations and we maximizes Equation(9) subject to the volume constraints we have that,

$$\left(\begin{aligned} \frac{H_2}{H_1} &= \left(\frac{\Delta T_2}{\Delta T_1}\right)^{1/2} \le 1, \frac{H_3}{H_2} = \left(\frac{\Delta T_3}{\Delta T_2}\right)^{1/2} \le 1, \\ \frac{H_4}{H_3} &= \left(\frac{\Delta T_4}{\Delta T_3}\right)^{1/2} \le 1, \frac{H_5}{H_4} = \left(\frac{\Delta T_5}{\Delta T_4}\right)^{1/2} \le 1, \end{aligned} \right) \tag{17}$$

and,

$$\left(\begin{aligned} \frac{D_2}{D_1} &\sim \left(\frac{\Delta T_2}{\Delta T_1}\right) \le 1, \frac{D_3}{D_2} \sim \left(\frac{\Delta T_3}{\Delta T_2}\right) \le 1, \\ \frac{D_4}{D_3} &\sim \left(\frac{\Delta T_4}{\Delta T_3}\right) \le 1, \frac{D_5}{D_4} \sim \left(\frac{\Delta T_5}{\Delta T_4}\right) \le 1, \end{aligned} \right) \tag{18}$$

for $\Delta T_1 \ge \Delta T_2 \ge \Delta T_3 \ge \Delta T_4 \ge \Delta T_5$.

The scale analysis result is a useful guide in mathematical optimization that would be discuss in the next section. For the numerical optimization the number of rows would be limited to three.

5 Numerical results

5.1 Double row configuration

For the case when n is equal to two (double row), the optimization procedure was conducted using the DYNAMIC-Q algorithm for Reynolds numbers ranging from 30 to 411 with the effect of Reynolds number Re on the pin-fin geometry. Geometric configuration and heat transfer capabilities are investigated for the heat sink shown in Figure 1b. Figure 2a shows that the optimal fin-height ratio is in general independent of Reynolds number. This is evident in the insignificant change of the optimal fin-height ratio over the Re. This implies that for maximum heat transfer, the pin-fins in the first row should be slightly higher than the fins in the next row, these results also agrees with the scale analysis prediction that $\frac{D_2}{D_1} \sim \left(\frac{H_2}{H_1}\right)^2 = \frac{\Delta T_2}{\Delta T_1} \leq 1$, we can see that the aspect ratio of all the geometric parameters are less than 1 as shown in Figures 2a and 3a respectively.

Results obtained from the optimization problem show that the optimal spacing sopt between the pin-fins remains unchanged regardless of the Reynolds number value applied across the length of the control volume. This constant value coincides with the allowable spacing due to manufacturing restraints.

Figure 2a also shows a small increase in the optimal fin-diameter ratio with Reynolds number. With an error of less than 1%, the results can be correlated as:

$$\left(\frac{D_2}{D_1}\right)_{opt} = 0.464Re^{0.0314}. \tag{19}$$

The results further imply that the non-uniformity of the diameters of fins in the various rows plays a vital role in the heat transfer rate of pin-fins heat sinks. Furthermore, the results show that at lower Re, the diameter of the pin-fins in the first row should be about twice the diameter of those in the second row in order to achieve the maximum heat transfer rate, while at higher Reynolds numbers it should be about 1.8 times the diameter of those in the second row.

Figure 2b establishes the fact that the optimal rate of heat transfer increases with an increase in Reynolds number. This relationship between the maximum (optimal) dimensionless rate of total heat transfer Q and Reynolds number can be given by the expression:

$$Q_{max} = \sigma Re^{0.323}, \tag{20}$$

where σ is a constant dependent on the thermal conductivity ratio γ and the micro pin geometry. The thermal conductivity ratio is simply the ratio of the solid's thermal conductivity to that of the fluid's ($\gamma = k_s/k_f$). The correlation given in Equation (20) correlates with an error of less than 1% to the CFD results produced and it is in agreement with the work published by [11]. For a fixed thermal conductivity ratio of 100 and for microscale applications, the constant σ was found to be 9.78.

The effect of the thermal conductivity ratio γ on the maximized rate of heat transfer and the geometrical ratio of the micro-pin fins was investigated. Figure 3a show that the thermal conductivity ratio has no effect on the pin-fin-diameter ratio and height ratio. Their aspect ratio is less than or equal to one as predicted in Equations (16), (17) and (18).

Figure 3b shows that the maximized rate of heat transfer increases as the thermal conductivity ratio increases. However, at higher thermal conductivities ($\gamma > 1000$), the rate of heat transfer approximately reaches a maximum and as the thermal conductivity ratio increases, the heat transfer rate is invariant with γ. This is due to the fact that convection rather than conduction is the more dominant medium thus rendering the thermal conductivity property of little importance.

5.2 Triple row configuration

In case two that is when n equal to three, the vertically arranged micro pin-fins form part of a three-row-finned array with row-specific diameters D_1, D_2 and D_3 respectively. The various rows are spaced by a distance s_1 and s_2 as depicted in Figure 1b. Uniform row height assumption was made (from results obtained from n = 2 and scale analysis). Therefore, $H_1 = H_2 = H_3$.

The optimized geometric parameters (Figure 4a) predict that pin-fins in the first row D_1 should be larger than the pin-fins in the next row with this decreasing diameter trend continuing to the third row. It further shows that the optimal trend of the respective pin diameters D_1, D_2 and D_3 changes slightly as the Reynolds number across the finned array increases. It indicates that as the fluid velocity is increased, the pin diameter of the fins in the first row decreases slightly while the diameter of the fins in the third row increases slowly. The pin diameters in the penultimate row show independence with regard to an increasing Re.

An optimal search of the geometric parameters of the heat sinks that maximizes the heat transfer rate showed that as the Reynolds number increases, the dimensionless heat transfer rate also increases. The results are shown in Figure 4b; it explains the fact that the convective heat transfer coefficient is a strong function of the fluid velocity. For a thermal conductivity ratio of 100, the relationship between Reynolds number and the maximal rate of heat transfer can be correlated within an error of 1% as:

$$Q_{max} = 8.45Re^{0.375}. \tag{21}$$

The effect of various materials on the maximised rate of heat transfer of the heat sink was also investigated.

From Figure 5a, results show that for a Reynolds number of 123, the pin diameters for each row stay constant with an increase in the conductivity ratio γ. This result implies that solid-fluid medium combination is insignificant with regard to the geometric design of such heat sinks. It shows that the design is robust with respect to the conductivity ratio. In addition, it is intuitive that the minimum allowable spacing due to manufacturing constraints of 50 μm is the optimal spacing separating the pin-fins in the various rows.

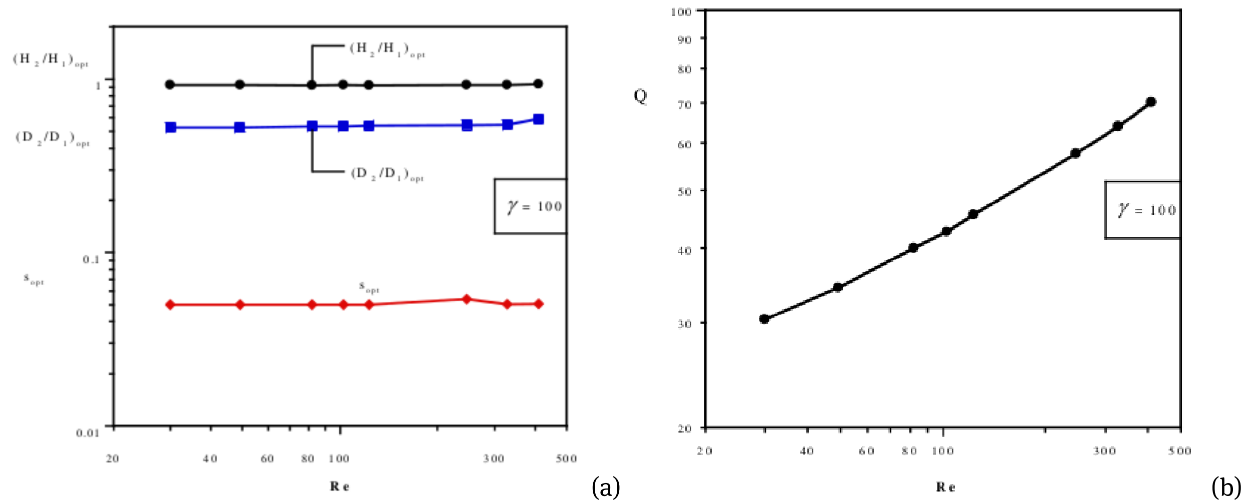


Figure 2: (a) The influence of Reynolds number on the optimized geometric configuration, for thermal conductivity ratio of 100 for double row finned heat sink. (b) The maximized rate of heat transfer as a function of Reynolds number with the conductivity ratio γ equal to 100 for double row finned heat sink.

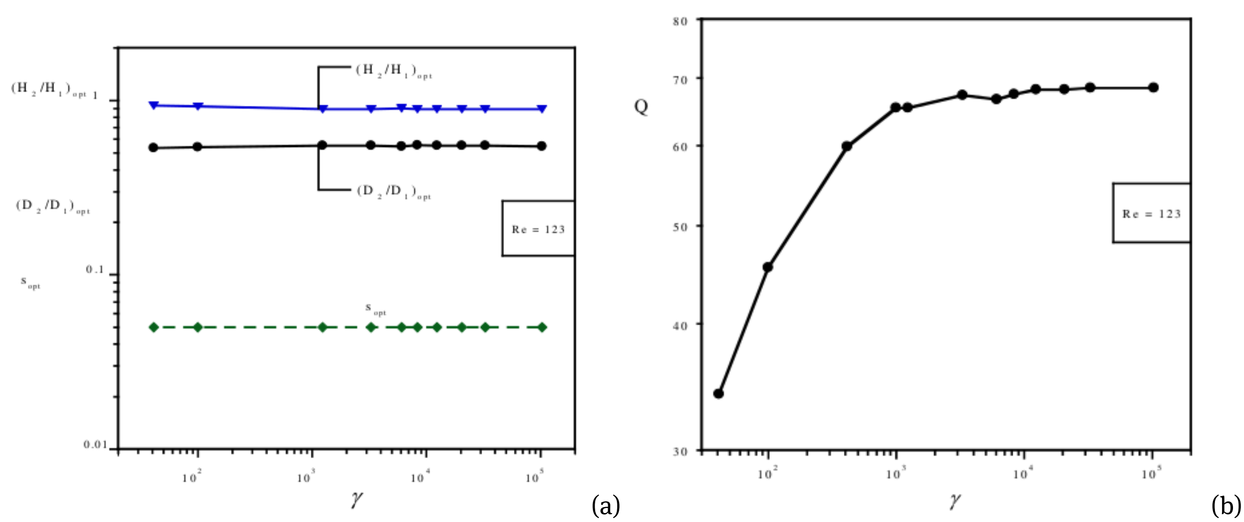


Figure 3: (a) The effect of the thermal conductivity ratio on the optimized geometric configuration of a double row finned heat sink at a Reynolds number of 123. (b) The effect of the thermal conductivity ratio on the maximized rate of heat transfer at a Reynolds number of 123.

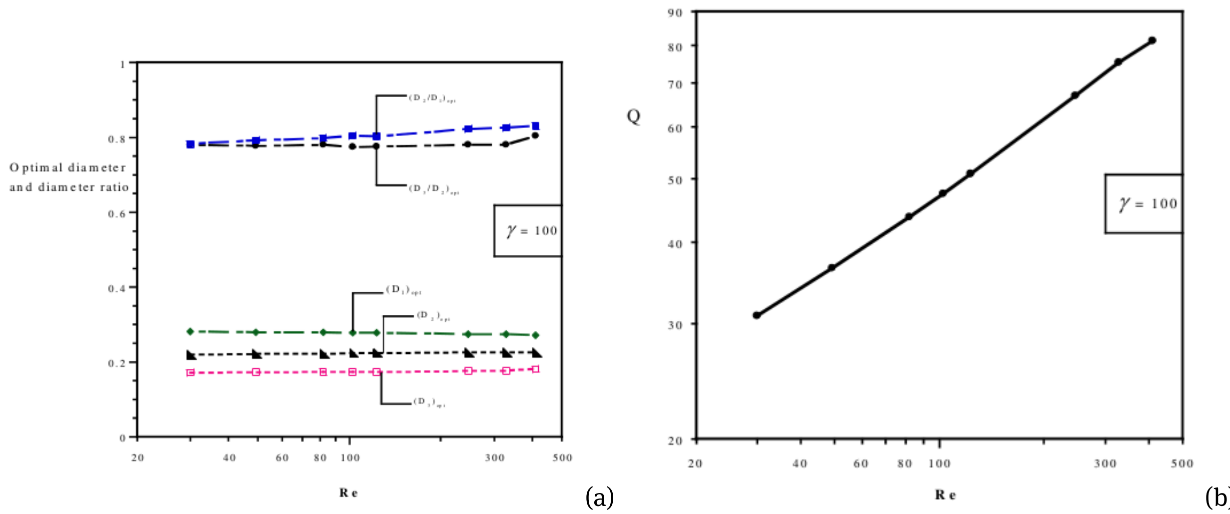


Figure 4: (a) The relationship between the optimal configurations for each fin row as a function of Reynolds number for triple rows for a thermal conductivity ratio of 100. (b) The relationship between the optimal dimensionless rate of heat transfer and Reynolds number for a triple row heat sink for a thermal conductivity ratio of 100.

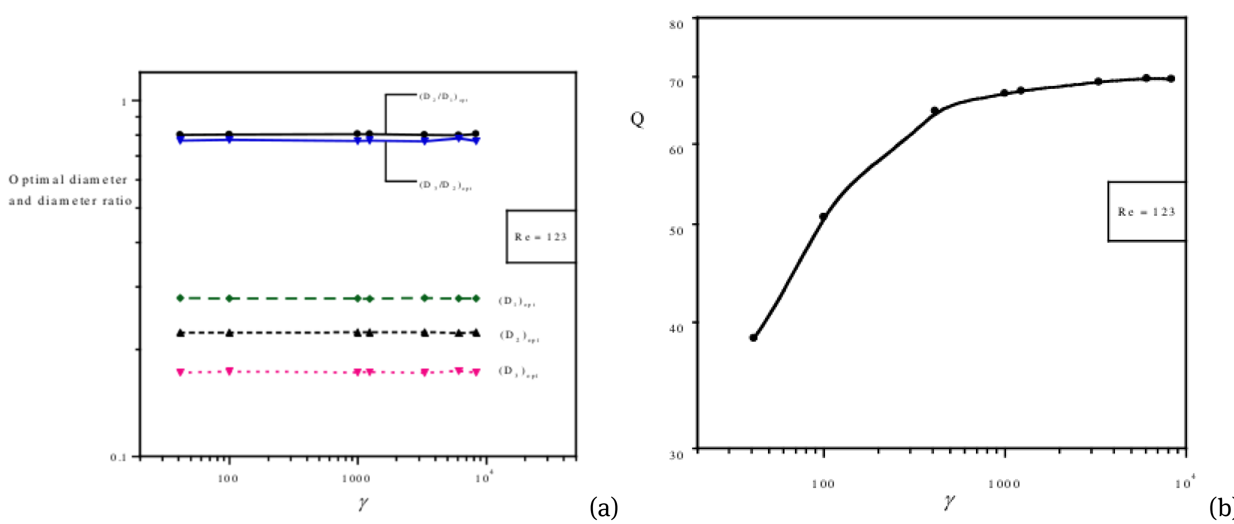


Figure 5: (a) The effect of the thermal conductivity ratio on the optimized geometric Configurations for a triple row finned heat sink at a Reynolds number of 123. (b) The effect of the conductivity ratio on the maximized heat transfer rate for a triple row micro heat sink for a Reynolds number of 123.

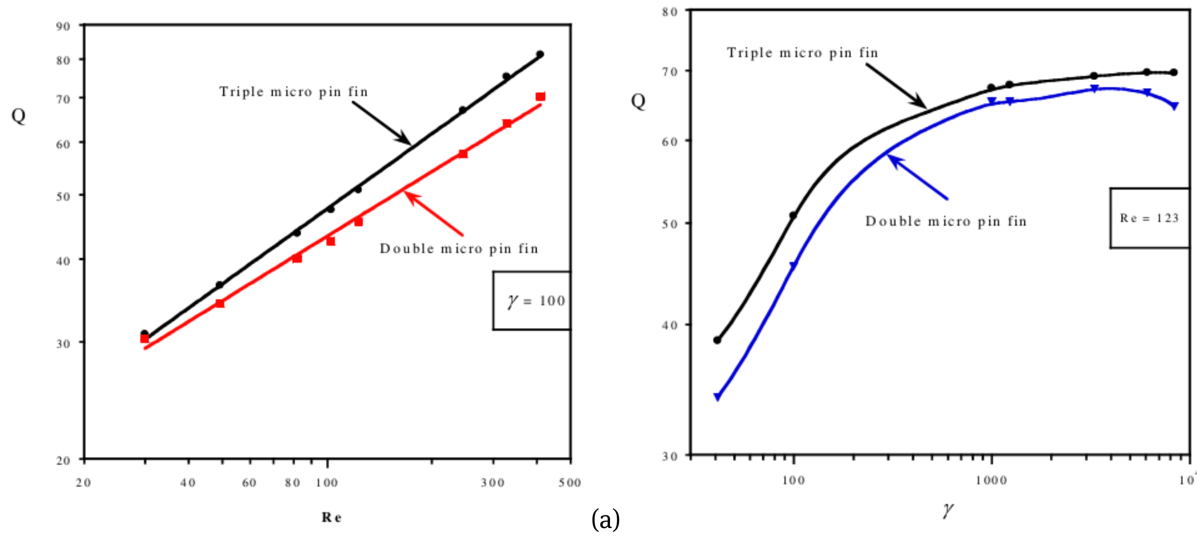


Figure 6: (a) The effect of Reynolds number on the maximum heat transfer for both the optimized double row micro-pin fins and triple row micro-pin fins heat sink. (b) he effect conductiviy ratio on the heat transfer for both the optimized double row micro-pin fins and triple row micro-pin finned heat sink.

Table 3: Optimal results for various computational widths.

L_1 (mm)	D_2/D_1	D_3/D_2	Q
0.4	0.799	0.792	52.3
0.5	0.779	0.782	50.9
0.6	0.802	0.775	50.8
0.8	0.786	0.797	52.7

Figure 5b further shows that an increase in the thermal conductivity ratio γ causes an increase in the maximal heat transfer rate. However, varying gradients of the dimensionless heat transfer rate as a function of γ are experienced with higher positive gradients experienced at lower γ (less than 500) and almost zero gradients for conductivity ratios greater than 1 000. The results suggest that a heat sink designed to operate within a medium where the conductivity ratio is about 400 will perform very well and increasing the conductivity ratio will not significantly increase the dimensionless heat transfer rate.

Table 3 shows the optimal parameters for various computational widths. The results show that optimal geometric design parameters are insensitive to the computational width.

5.3 Comparison between triple rows micro-pin fins to double row pin fins

Figures 6a and 6b shows the effect of Reynolds number and conductiviy ratio on the heat transfer for both the optimized double row fins and triple row micro pins, it is observed that as the Reynolds number increases the heat transfer rate also increases. The triple rows pin fins have a larger heat transfer rate when compared to the double rows micro pin fins for the same amount of material. The same behaviour is observed for Figure 6b, that the triples row micro pin fins gives a higher heat transfer as the thermal conductivity ratio increases. It was found that by adding a third row of pin-fins, the rate of heat transfer is enhanced with enhancement greater than 10% achievable for Re > 100. However, the enhancement rate decreases at higher thermal conductivity ratio γ.

6 Conclusion

This work has demonstrated the effective use of mathematics optimization coupled with geometric constraints for the design of optimum micro-pin fins heat sinks for maximum heat removal capabilities. The numerical optimization technique implemented shows that variable geometry in the various rows of a finned array will bring about maximized rate of heat transfer. However, it was discovered that the influence of non-uniform fin height to the optimal solution is quite negligible. The maximized heat transfer rate is a result of a balanced conduction-convection ratio. Results also proved that thermal conductivity ratio does not influence the optimal geometrical configuration of such heat sinks. It was found that by adding a third row of pin-fins with a reduced degree of freedom, the rate of heat transfer is enhanced with enhancement greater than 10% achievable for $Re > 100$. Future work will include inserting the micro-pin fins of different cross-sectional shape inside a micro-channel heat sink and relaxing the axial length. The optimal design of micro-pin fins heat sink is expected to facilitate the heat flow for maximum heat transfer in the design of heat exchangers for the cooling of electronic devices.

Acknowledgement: The author acknowledges Mr F.U. Ighalo, Profs J.P. Meyer and Adrian Bejan for their contribution to this work, and the South African National Research Foundation (NRF) for its financial support.

References

[1] Khan W.A., Culham J.R., Yovanovich M.M., Optimization of pin-fin heat sinks using entropy generation minimization, IEEE Trans. Compon. Packag. Technol., 2005, 28, 1–13.

[2] Bello-Ochende T., Meyer J.P., Bejan A., Constructal ducts with wrinkled entrances Int. J. Heat Mass Transfer, 2009, 53, 3628–3633.

[3] Bello-Ochende T., Meyer J.P., Ighalo F.U., Combined numerical optimization and constructal theory for the design of heat sinks, Numer. Heat Tran. A, 2010, 58, 1–18.

[4] Lorenzini G., Biserni C., Garcia F.L., Rocha L.A.O., Geometric optimization of a convective T-shaped cavity on the basis of constructal theory, Int. J. Heat Mass Transfer, 2012, 5(23-24), 6951–6958.

[5] Peles Y., Koşar A., Mishra C., Kuo C., Schneider B., Forced convective heat transfer across a pin fin micro heat sink, Int. J. Heat Mass Tran., 2005, 48, 3615–3627.

[6] Khan W.A., Yovanovich M.M., Optimization of pin-fin heat sinks in bypass flow using entropy generation minimization method, (Proceedings of IPACK 2007), Vancouver, Canada, 2007.

[7] Soodphakdee D., Behnia M., Copeland D.W., A comparison of fin geometries for heatsinks in laminar forced convection: Part I - round, elliptical, and plate fins in staggered and in-line configurations, Int. J. Microcir. Elect. Packag., 2001, 24, 68–76.

[8] Tahat M., Kodah Z.H., Jarrah B.A., Probert S.D., Heat transfers from pin-fin arrays experiencing forced convection, Appl. Energ., 2000, 67, 419–442.

[9] Chiang K., Chang F., Application of response surface methodology in the parametric optimization of a pin-fin type heat sink, Int. Comm. Heat Mass Tran., 2006, 33, 836–845.

[10] Chiang K., Chang F., Tsai T., Optimum design parameters of pin-fin heat sink using the grey- fuzzy logic based on the orthogonal arrays, Int. Comm. Heat Mass Tran., 2006, 33, 744–752.

[11] Bello-Ochende T., Meyer J.P., Bejan A., Constructal multi-scale pin-fins, Int. J. Heat Mass Transfer 2010, 53, 2773–2779.

[12] Khan W.A., Culham J.R., Yovanovich M.M., Performance of shrouded pin-fin heat sinks for electronic cooling, J. Thermophys. Heat Transfer, 2006, 20, 408–414.

[13] Snyman J.A., Practical Mathematical Optimization: An Introduction to Basic Optimization Theory and Classical and New Gradient-Based Algorithms, Springer, New York, 2005.

[14] Snyman J.A., Hay A.M., The DYNAMIC-Q Optimization Method: An Alternative to SQP? Comput. Math. Appl., 2002, 44, 1589–1598.

[15] Ighalo F.U., Optimization of Micro-channels and Micro-pin-fin Heat Sinks with Computational Fluid Dynamics in Combination with a Mathematical Optimization Algorithm, M.Eng. Thesis, University of Pretoria, South Africa, 2011.

[16] Ighalo, F.U., Bello-Ochende, T., Meyer, J.P., Geometric optimization of multiple-arrays of micropin-fins, (Proceeding of the 8th ASME/JSME Thermal Engineering Joint Conference, AJTEC 2011), Honolulu, Hawaii, United States of America, 2011.

[17] Achanta V. S., An experimental study of endwall heat transfer enhancement for flow past staggered non-conducting pin fin arrays, PhD Thesis, Department of Mechanical Engineering, Texas A & M University, U.S.A., 2003.

[18] Lyall M.E., Heat transfer from low aspect ratio pin fins, PhD Thesis, Department of Mechanical Engineering, Virginia Polytechnic Institute and State University, 2006.

[19] Husain A., Kim K., Shape optimisation of micro-channel heat sink for micro-electronic cooling, IEEE Trans. Compon. Packag. Technol., 2008, 31, 322–330.

[20] Li J., Peterson G.P., Geometric optimisation of a micro heat sink with liquid flow, IEEE Trans. Compon. Packag. Technol., 2006, 29, 145–154.