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Vibrations Analysis of Rectangular Plates with Clamped Corners

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Abstract: This paper discusses the fundamental natural frequency of a thin elastic rectangular, isotropic and orthotropic, plates with clamped corners. Rayleigh's method was used to analytically calculate the plate lowest natural frequency. In this solution, the vibration mode shape was assumed in a form that certifies the displacement as well as the rotational boundary conditions of the current problem. Finally, this paper provides useful information for evaluating the natural frequency of a plate with fixed corners with different mass attachments configurations.

1 Introduction

Rectangular elastic plates [1, 2] are widely used in engineering applications; e.g. in printed circuit boards and solar collecting panels. The exact solutions of free vibration of elastic plates are possible only for a few cases, particularly for plate structures with well-defined boundary conditions. However, real-life vibration problems may pose some difficulties in satisfying boundary conditions, such as plates having fixed (or clamped) corners. In these cases, only approximate solutions, using Rayleigh methods, are available [3].

In engineering applications, plates with clamped corners frequently appear in the fatigue life analysis of electronic printed circuit boards (PCBs). For the PCBs, the clamped constraint can arise from the fact that the elastic plate is clamped at each corner using threaded screws, washers and nuts to attach the plate structure to a base support. This type of attachment prevents both displacement and rotation of the structure at the support locations. This kind of problem has been thoroughly investigated us-

ing the analytical solution of the superposition method invented by Gorman [5–10]. However, this analytical-type solution is very complicated and considered to be impractical.

A similar problem that has been investigated in literature is the vibration of elastic plate with pinned corners. The pinned-corner support boundary condition scheme, unlike the clamped corner scheme, only prevents displacements at the support locations, having no restriction on the rotation. Xiang-sheng [11] used Rayleigh method to analytically approximate the first resonant frequency of a plate having point-pinned supports at plate corners. Kim et al. [12, 13] used beam shape functions to solve for the deflections of plates with point supported at corners. Kerstens [14, 15] used a combination of Rayleigh-Ritz method and modal constraint method to obtain the free vibration problem of plates with such supports. As a powerful tool, the finite element analysis (FEA) has also been extensively used in the vibration analysis of plates with corner supports. Raju et al. [16] and Rao et al. [17] studied the natural frequencies of point-supported elastic plates using finite element method. Pitarresi and Kunz [18, 19] developed a simple technique using two-dimensional nonlinear least-squares fit, and finite element simulations, to study the natural frequency of plates having point supports. Recently, Gharaibeh et al. [20, 21] employed the assumed mode method to evaluate the fundamental natural frequency of corner-supported plates using an electronic package.

In addition, elastic rectangular plates having corner supports may exist in many renewable energy and power generation systems such as concentrating solar power (CSP) systems, including solar dishes, parabolic trough solar collectors, and solar central receiver systems. The analysis and characteristics of vibrations of rectangular plates has been widely and extensively investigated based on various techniques. For example, Torabi et al. [22] studied the vibration characteristics of a rectangular plate having a circular central hole with point support, based on the Rayleigh-Ritz method, which is widely used, based on orthogonal polynomials and beam functions. Similarly, Li [23] employed the same approach to characterize vi-

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brations of rectangular plates with general elastic boundary supports. Watkins et al. [24] computed normalized frequencies of elastically point-supported rectangular plates using Eigensensitivity analysis. Shahrjerdi et al. [25] used second-order shear deformation theory (SSDT) to study the vibration of solar-graded plates. Du et al. [26] employed Fourier series method for the in-plane vibration analysis of rectangular plates with elastically restrained edges.

The current work discusses the fundamental natural frequency of an elastic rectangular plate having fixed (or clamped) corners using the Rayleigh method. Both isotropic and orthotropic material systems were considered. In this paper, we first start by introducing the problem of the plate with clamped corners and resulting boundary conditions. This is followed by the use of the Rayleigh method in the system's first natural frequency analysis. Subsequently, the careful selection of the mode shape function that satisfies the displacement and rotational boundary conditions of the problem is also presented. Finally, the natural frequency results for several geometric and mass attachment configurations are discussed in detail.

2 Description of the Problem

The plate under consideration in the present work is shown in Figure 1. The supports on the plate corners prevent both displacements and rotations at the support locations, and the edges are completely free elsewhere. The plate width is (a) and length is (b) with thickness (h). The complete set of boundary conditions can be described as:

Displacement

$$w(0, 0) = w(a, 0) = w(a, b) = w(0, b) = 0 \quad (1)$$

Rotations

$$w_x(0, 0) = w_x(a, 0) = w_x(a, b) = w_x(0, b) = 0 \quad (2a)$$

$$w_y(0, 0) = w_y(a, 0) = w_y(a, b) = w_y(0, b) = 0 \quad (2b)$$

Figure 1: Plate with clamped corners schematic.

Moments

$$w_{xx}(0, y) + \nu w_{yy}(0, y) = 0 \quad (3a)$$

$$w_{xx}(a, y) + \nu w_{yy}(a, y) = 0 \quad (3b)$$

$$w_{yy}(x, 0) + \nu w_{xx}(x, 0) = 0 \quad (3c)$$

$$w_{yy}(x, b) + \nu w_{xx}(x, b) = 0 \quad (3d)$$

Effective shear forces

$$w_{xxx}(0, y) + (2 - \nu)w_{xyy}(0, y) = 0 \quad (4a)$$

$$w_{xxx}(a, y) + (2 - \nu)w_{xyy}(a, y) = 0 \quad (4b)$$

$$w_{yyy}(x, 0) + (2 - \nu)w_{xxy}(x, 0) = 0 \quad (4c)$$

$$w_{yyy}(x, b) + (2 - \nu)w_{xxy}(x, b) = 0 \quad (4d)$$

3 Natural Frequency Analysis by Rayleigh's Method

As previously mentioned, the present work aims to produce an approximate analytical solution for calculating the first natural frequency of a plate with clamped corners using Rayleigh's method. Therefore, it is essential to express the general energy of the system, which is composed

of the potential and kinetic energies of the plate structure. Considering an isotropic plate having a constant thickness, the plate potential energy (V) can be expressed as [27]:

$$V = \frac{D}{2} \iint_A \left[(w_{xx} + w_{yy})^2 - 2(1 - \nu) (w_{xx}w_{yy} - w_{xy}^2) \right] dx dy \quad (5)$$

where $D = \frac{Eh^3}{12(1-\nu)}$ is the plate flexural rigidity; h , E and ν are the plate thickness, plate isotropic material modulus of elasticity, and Poisson's ratio, respectively; w is the plate deflection in the transverse direction; $w_{xx} = \frac{\partial^2 w}{\partial x^2}$, $w_{yy} = \frac{\partial^2 w}{\partial y^2}$ and $w_{xy} = \frac{\partial^2 w}{\partial x \partial y}$.

The plate kinetic energy (T) is [27]:

$$T = \frac{\rho h}{2} \iint_A \left[(\dot{w})^2 \right] dx dy \quad (6)$$

where ρ is the mass per unit volume of the plate material; $\dot{w}$ is the plate velocity in the lateral direction. The double integrals shown in Eq. (5) and Eq. (6) are evaluated over the plate surface area A .

The deflection of the plate, whose motion is harmonic, is:

$$w(x, y, t) = W(x, y)e^{\hat{i}\omega t} \quad (7)$$

where $\hat{i}$ is the complex number $\hat{i} = \sqrt{-1}$, ω is the vibration circular frequency, and t is the time. The function $W(x, y)$ is the plate mode shape. The proper selection of this mode function will be discussed later.

To use Rayleigh's method, which is based on the energy conservation theorem, the maximum potential energy (V_{max}) of the system must be equal to the kinetic energy (T_{max}) of the system [8]:

$$V_{max} = T_{max} \quad (8)$$

where,

$$V_{max} =$$

$$\frac{D}{2} \iint_A \left[(W_{xx} + W_{yy})^2 - 2(1 - \nu) (W_{xx}W_{yy} - W_{xy}^2) \right] dx dy \quad (9)$$

$$T_{max} = \omega^2 \frac{\rho h}{2} \iint_A \left[(W)^2 \right] dx dy \quad (10)$$

Therefore, the first natural frequency of the plate is written as:

$$\omega = \sqrt{\frac{D}{\rho h} \frac{\iint_A \left[(W_{xx} + W_{yy})^2 - 2(1 - \nu) (W_{xx}W_{yy} - W_{xy}^2) \right] dx dy}{\iint_A \left[(W)^2 \right] dx dy}} \quad (11)$$

Considering orthotropic plate material system, the kinetic energy of the plate is still expressed as in Eq. (6) and Eq. (10) above. However, the potential energy is written as [28]:

$$V = \frac{1}{2} \iint_A \left[D_x W_{xx}^2 + D_y W_{yy}^2 + 2D_x \nu_y W_{xx} W_{yy} + 4D_k W_{xy}^2 \right] dx dy \quad (12)$$

where: $D_x = \frac{E_x h^3}{12(1 - \nu_x \nu_y)}$, $D_y = \frac{E_y h^3}{12(1 - \nu_x \nu_y)}$ and $D_k = \frac{G_{xy} h^3}{12}$. The material parameters E_x , E_y and G_{xy} are the modulus of elasticity in x -direction, the modulus of elasticity in y -direction, and the shear modulus in xy plane, respectively. Also, ν_x and ν_y are Poisson's ratio in xy and yx planes, respectively.

By applying Rayleigh's method as shown previously, the natural frequency of the orthotropic plate can be expressed as:

$$\omega = \sqrt{\frac{\left[D_x W_{xx}^2 + D_y W_{yy}^2 + 2D_x \nu_y W_{xx} W_{yy} + 4D_k W_{xy}^2 \right] dx dy}{\rho h \iint_A \left[(W)^2 \right] dx dy}} \quad (13)$$

The accuracy of the calculated natural frequency of Eq. (11) and Eq. (13) greatly depends on the engineered selection of the mode shape function, $W(x, y)$, to be discussed next.

4 Mode Shape Selection

As explained earlier, the proper selection of the mode shape in the Eq. (11) and Eq. (13) is very important. This mode function should be carefully selected in a mathematical form that satisfies the problem displacement as well as rotational boundary conditions. For the current analysis, the plate is fixed in all directions on the corners and free elsewhere. As formulated in Eq. (1) to Eq. (4), this type of

constraint will result in zero displacements and rotations at the corners. Whereas the plate deflections and rotations on the edges are different from zero. Therefore, the mode shape of this problem can be formulated as:

$$W(x, y) = C_1 \left(1 - \cos \left(\frac{2\pi x}{a} \right) \right) + C_2 \left(1 - \cos \left(\frac{2\pi y}{b} \right) \right) + C_3 \left(1 - \cos \left(\frac{2\pi x}{a} \right) \right) \left(1 - \cos \left(\frac{2\pi y}{b} \right) \right) \quad (14)$$

where a and b are the plate dimensions in x and y directions, respectively. Also, constants C_1 and C_2 are the deflections of the midpoints of the plate sides in x and y directions, respectively. The constant C_3 represents the displacement of the plate center ($x = \frac{a}{2}$, $y = \frac{b}{2}$). All mode shape constants are independent, as well as unknown, and therefore need to be determined.

Obviously, the mode shape formulated in Eq. (14) completely satisfies the displacement as well as the rotation boundary conditions of Eq. (1) and Eq. (2). However, the satisfaction of the bending moment and shear forces of Eq. (3) and Eq. (4), respectively, are not adequately achieved. Here, if we can satisfy the bending moment and/or effective shear forces boundary conditions, we might be able to greatly increase the calculated natural frequency of Eq. (11) and (13). Furthermore, this can decrease the number of the independent constants of the mode shape function, Eq. (14), therefore achieving much simpler solution. Consequently, we aim to satisfy the shear forces conditions of Eq. (4), having $y = 0$ and $x = 0$, thus obtaining:

$$C_1 = \beta_1 C_3 \quad (15)$$

$$C_2 = \beta_2 C_3 \quad (16)$$

where β_1 and β_2 are constants depends on the plate aspect ratio $\left(\frac{b}{a}\right)$, and Poisson's ratio is defined as:

$$\beta_1 = \left(\frac{a}{b}\right)^2 (2 - \nu) \quad (17)$$

$$\beta_2 = \left(\frac{b}{a}\right)^2 (2 - \nu) \quad (18)$$

By applying Eq. (15) to Eq. (18), and into Eq. (14), the mode shape function can be modified to:

$$W(x, y) = C_3 \left[\beta_1 \left(1 - \cos \left(\frac{2\pi x}{a} \right) \right) + \beta_2 \left(1 - \cos \left(\frac{2\pi y}{b} \right) \right) + \left(1 - \cos \left(\frac{2\pi x}{a} \right) \right) \left(1 - \cos \left(\frac{2\pi y}{b} \right) \right) \right] \quad (19)$$

As can be clearly seen in Eq. (19), the number of independent constants in the mode shape are reduced from three to one (C_3). This will result, as expected, in much simpler and accurate solution for the plate with clamped corners problem.

For the orthotropic plate analysis, the shear boundary conditions of the current plate problem become [28]:

$$w_{xxx}(0, y) + \left(2 \frac{D_{xy}}{D_x} - \nu_y \right) w_{xyy}(0, y) = 0 \quad (20a)$$

$$w_{xxx}(a, y) + \left(2 \frac{D_{xy}}{D_x} - \nu_y \right) w_{xyy}(a, y) = 0 \quad (20b)$$

$$w_{yyy}(x, 0) + \left(2 \frac{D_{xy}}{D_y} - \frac{D_x}{D_y} \nu_y \right) w_{xxy}(x, 0) = 0 \quad (20c)$$

$$w_{yyy}(x, b) + \left(2 \frac{D_{xy}}{D_y} - \frac{D_x}{D_y} \nu_y \right) w_{xxy}(x, b) = 0 \quad (20d)$$

where $D_{xy} = 2D_x \nu_y + 2D_k$.

Following the same procedure above, the mode shape constants of Eq. (19) are obtained as:

$$\beta_1 = \left(\frac{a}{b}\right)^2 \left(\frac{2D_{xy} - D_x \nu_y}{D_x} \right) \quad (21)$$

$$\beta_2 = \left(\frac{b}{a}\right)^2 \left(\frac{2D_{xy} - D_x \nu_y}{D_y} \right) \quad (22)$$

5 Results and Discussions

Plate with fixed corners with no mass attachments

To find an equation of the fixed-corners plate's first natural frequency, and considering an isotropic plate system, we substitute Eq. (19), combined with Eq. (17) and Eq. (18), into Eq. (11) to obtain:

$$\omega = \frac{\lambda^2}{a^2} \sqrt{\frac{D}{\rho h}} \quad (23)$$

where a is the plate shortest length, and λ^2 is the plate eigenvalue written as:

$$\lambda^2 = 4\pi^2 \sqrt{\frac{S_1 + 2\left(\frac{a}{b}\right)^2 + S_2\left(\frac{a}{b}\right)^4}{S}} \quad (24)$$

where,

$$S = 6(\beta_1^2 + \beta_2^2) + 12(\beta_1 + \beta_2) + 8\beta_1\beta_2 + 9 \quad (25a)$$

$$S_1 = 2\beta_1^2 + 4\beta_1 + 3 \quad (25b)$$

$$S_2 = 2\beta_2^2 + 4\beta_2 + 3 \quad (25c)$$

It important to mention that β_1 and β_2 of Eq. (25) are to be obtained from Eq. (17) and Eq. (18), respectively. As this eigenvalue is a function of β_1 and β_2 , then it is a function of plate aspect ratio b/a and Poisson's ratio ν only. Assuming that $\nu = 0.3$, then the values of β_1 , β_2 and λ^2 for several plate aspect ratios are summarized in Table 1.

To simplify the analysis, an empirical formula for calculating the plate eigenvalue as a function of the aspect ratio of the plate has been established. For this formula, the suggested general form is:

$$\lambda^2 = a_1 \left(\frac{1}{\frac{b}{a}} \right)^{a_2} \quad (26)$$

where a_1 and a_2 are unknown constants that need to be determined. To determine such constants, a non-linear regression analysis using MINITAB is performed. Thus:

$$\lambda^2 = 22.27 \left(\frac{1}{\frac{b}{a}} \right)^{1.66} \quad (27)$$

Eq. (27) was compared to the analytical solution data, as shown in Figure 2. From this comparison, it is found that the results of the curve-fitted equation are in agreement with the values of Table 1.

Figure 2: Plate eigenvalues for different plate geometries.

For an orthotropic plate, we substitute Eq. (19), combined with Eq. (21) and Eq. (22), into Eq. (13) to get:

$$\omega = \frac{4\pi^2}{a^2} \sqrt{\frac{D_{xy}}{\rho h}} \sqrt{\frac{S_x \frac{D_x}{D_{xy}} + 2\left(\frac{a}{b}\right)^2 + S_y \frac{D_y}{D_{xy}} \left(\frac{a}{b}\right)^4}{S_{xy}}} \quad (28)$$

where:

$$S_{xy} = 6(\beta_1^2 + \beta_2^2) + 12(\beta_1 + \beta_2) + 8\beta_1\beta_2 + 9 \quad (29a)$$

$$S_x = 2\beta_1^2 + 4\beta_1 + 3 \quad (29b)$$

$$S_y = 2\beta_2^2 + 4\beta_2 + 3 \quad (29c)$$

It important to mention that β_1 and β_2 of Eq. (29) are to be obtained from Eq. (21) and Eq. (22), respectively. This eigenvalue is a function of a/b , E_x , E_y , ν_x and ν_y .

Plate with fixed corners having mass attachments

After discussing the problem of a plate with fixed corners, we have a look at the problem of a plate with two mass attachment types:

- Case 1: Plate with fixed corners with a centrally placed mass.
- Case 2: Plate with fixed corners with edge-center-placed masses.

In the present analysis, only the isotropic plate material system was studied. The same procedure could be followed for orthotropic plate system.

For case 1, the concentrated mass (M) is mounted at the plate center point ($x = \frac{a}{2}, y = \frac{b}{2}$), as shown in Figure 3(a). For case 2, as depicted in Figure 3(b), four equal masses (the mass of each is M) are attached at the centers of the plate edges $\left\{ \left(x = \frac{a}{2}, y = 0 \right), \left(x = a, y = \frac{b}{2} \right), \left(x = \frac{a}{2}, y = b \right), \left(x = 0, y = \frac{b}{2} \right) \right\}$.

This mass addition will have no effect on the structure's potential energy. Generally, however, for a plate with attached mass (either for case 1 or case 2), the kinetic energy of the system could be expressed as:

$$T = \frac{\rho h}{2} \iint_A [\dot{w}(x, y)]^2 dx dy + \frac{M}{2} \sum_{i=1}^N \dot{w}^2(\zeta_i, \eta_i) \quad (30)$$

where $\dot{w}(\zeta_i, \eta_i)$ are the velocities of the plate at the mass fix-up locations, and N is the total number of attached masses.

Following the previously described natural frequency analysis procedure, the system's fundamental natural frequency is:

$$\omega = \frac{\lambda^2}{a^2} \sqrt{\frac{D}{\alpha \frac{M}{a^2} + \rho h}} \quad (31)$$

where α is the reduced coefficient of the mass, and it is function of plate aspect ratio b/a , and Poisson's ratio ν . Again, assuming that $\nu = 0.3$, then the values of α for several plate aspect ratios, for both cases, are summarized in Table 2.

It is important to mention at this point that this mass addition has no effect on the mode shape constants and

Table 1: Mode shape constants and isotropic plate eigenvalues for different plate aspect ratios ($\nu=0.3$).

b/a	β_1	β_2	λ^2
1.0	1.700	1.700	21.916
1.5	0.756	3.825	12.350
2.0	0.425	6.800	7.2033
2.5	0.272	10.63	4.6175
3.0	0.189	15.30	3.1940
3.5	0.139	20.80	2.3370
4.0	0.106	27.20	1.7831

Table 2: Reduced coefficient of the mass for several plate aspect ratios for plate with center mass $\nu=0.3$.

b/a	1.0	1.5	2.0	2.5	3.0	3.5	4.0
α Case 1	4.336	2.591	1.713	1.267	1.005	0.835	0.716
α Case 2	6.876	5.021	4.204	3.616	3.154	2.785	2.486

Figure 3: Corner-clamped plate with mass attachments; (a) case 1, (b) case 2.

plate eigenvalues, and their values are still equal to those listed in Table 1.

As done previously, a general equation to calculate the reduced coefficient of the mass, for both cases, was formulated. The suggested general form for this equation is:

$$\alpha = c_1 \left(\frac{1}{\frac{b}{a}} \right)^{c_2} \quad (32)$$

where c_1 and c_2 are coefficients determined using a non-linear regression analysis as $c_1 = 4.35$, $c_2 = 1.32$ for case 1, and $c_1 = 6.85$, $c_2 = 0.72$ for case 2. A comparison between the fitted formula and the actual (analytical) data of α is shown in Figure 4. For both cases, the fitted formulas are in excellent match with the analytically derived data.

As a special configuration, we shall consider the mass (M) of the added mass (or masses) to be a function of the

plate mass. Thus:

$$M = nbahp \quad (33)$$

where n is the plate mass fraction, and it can be any positive number: i.e., if $n = 0.5$ or $n = 2$, then the total added mass is half of the plate mass or double of it, respectively. Now, substitute Eq. (33) into Eq. (31), and after some simple mathematical manipulation we achieve:

$$\omega = \frac{\lambda^2}{a^2} \sqrt{\frac{1}{n\alpha \frac{b}{a} + 1}} \sqrt{\frac{D}{\rho h}} \quad (34)$$

or:

$$\omega = \frac{\Lambda^2}{a^2} \sqrt{\frac{D}{\rho h}} \quad (35)$$

where $\Lambda^2 = \lambda^2 \sqrt{\frac{1}{n\alpha \frac{b}{a} + 1}}$ is the massed plate eigenvalue.

From Eq. (35) above, Λ^2 is now function of b/a , ν and α , as well as n . Luckily, α is function of b/a (besides to ν), for both cases. Therefore, we can implicitly say that Λ^2 is a function of b/a , ν and n only. As before, assuming $\nu = 0.3$, Table 3 and Table 4 summarize the values of Λ^2 for different b/a and n combinations for case 1 and case 2, respectively.

To establish a formula to calculate Λ^2 as a function of the plate aspect ratio and of mass fraction factor, for both cases, a general non-linear form, which is a combination of Eq. (27) and Eq. (35), was selected as follows:

$$\Lambda^2 = a_1 \left(\frac{1}{\frac{b}{a}} \right)^{a_2} \sqrt{\frac{1}{d_1 n \left(\frac{1}{\frac{b}{a}} \right)^{d_2} + 1}} \quad (36)$$

Figure 4: The reduced coefficient of the mass vs plate aspect ratio for (a) case 1 and (b) case 2.

Figure 5: The massed plate eigenvalues (Λ^2) vs. the plate mass fraction (n) at different b/a for (a) case 1 and (b) case 2.

Here, a_1 and a_2 , for case 1 and case 2, are similar to those of Eq. (27). Also, d_1 and d_2 are two unknown coefficients. To obtain them, a multi-parametric fit was performed considering both case 1 and case 2. The resultant constant values are ($d_1 = 4.33$, $d_2 = 0.29$) and ($d_1 = 6.89$, $d_2 = -0.26$) for case 1 and case 2, respectively. Figure 5 shows a comparison between the analytically derived data and the fitted formulas for both mass attachment cases, for several plate aspect ratios. In conclusion, the Λ^2 values for case 1 and case 2, from both solutions, are in a great agreement.

6 Conclusions

This paper has discussed the free vibration problem of plates with clamped corners. Rayleigh's method was used to analytically approximate the fundamental natural frequency of the plate under study. From this approximation, the plate eigenvalue was obtained for several plate dimensions. In addition, two types of mass attachments, namely plate with centrally placed mass and plate with masses on the edges centers, were discussed. A special configuration considering the mass of the added masses as a function of the plate mass was also presented. Finally, this paper developed non-linear equations for calculating the plate eigenvalues for both massed and un-massed configurations.

Table 3: Values of Λ^2 at different b/a and n for case 1 ($\nu=0.3$).

b/a	n							
	0.25	0.50	0.75	1.0	1.25	1.50	1.75	2.0
1.0	15.1814	12.3131	10.6283	9.4875	8.6496	8.0005	7.4785	7.047
1.5	8.7954	7.1987	6.2418	5.5869	5.1026	4.7257	4.4216	4.1696
2.0	5.2867	4.3733	3.8127	3.4239	3.1341	2.9073	2.7235	2.5706
2.5	3.4495	2.8726	2.5132	2.2619	2.0734	1.9254	1.8052	1.7049
3.0	2.4119	2.017	1.7687	1.594	1.4626	1.3591	1.2749	1.2046
3.5	1.7765	1.4896	1.3081	1.18	1.0834	1.0072	0.9451	0.8932
4.0	1.3612	1.1434	1.005	0.9071	0.8332	0.7748	0.7272	0.6874

Table 4: Values of Λ^2 at different b/a and n for case 2 ($\nu=0.3$).

b/a	n							
	0.25	0.50	0.75	1.0	1.25	1.50	1.75	2.0
1.0	13.291	10.4032	8.8324	7.8092	7.0752	6.5156	6.0707	5.7061
1.5	7.2737	5.6572	4.7896	4.2282	3.8269	3.5218	3.2796	3.0814
2.0	4.0899	3.1576	2.665	2.3485	2.1232	1.9524	1.8171	1.7066
2.5	2.5574	1.9653	1.6555	1.4573	1.3166	1.2101	1.1259	1.0571
3.0	1.741	1.3342	1.1225	0.9875	0.8918	0.8194	0.7622	0.7156
3.5	1.2606	0.9643	0.8107	0.7129	0.6436	0.5913	0.5499	0.5162
4.0	0.955	0.7297	0.6131	0.5390	0.4866	0.4469	0.4157	0.3901

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