Research Article

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Optimisation of stress distribution in a highly loaded radial-axial gas microturbine using FEM

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Abstract: The article discusses the stress optimisation process of the highly loaded disc of a high-speed radial-axial microturbine. At the design stage, the strength optimisation is vitally important for these types of devices because they must withstand very high temperatures (600°C in this case) and be capable of operating at high rotational speeds (96,000 rpm in this case). Calculations were made using a three dimensional FE numerical model. The optimisation process is strictly connected with the choice of materials – which in this case are Inconel 738 (nickel-cobalt super alloy) and silicon nitride. Several stress reduction methods were developed, which took into account the mass of the disc, the rotational speed of the rotor and the complex shape of the rear part of the disc. Numerical computations helped to choose the best optimisation method, which decreased maximum reduced von Mises stresses by about 45% (from 1,288 MPa to 705 MPa). The methods proposed in this article are universal and can be implemented in the design process of various high-speed radial-axial microturbines. This article could be of interest to scientists and engineers who deal with highly loaded microturbines, which are increasingly used in many industrial sectors.

Keywords: gas turbine; high-speed turbine; design optimisation; FEM analysis; stress analysis

Notation

$\dot{m}$	mass flow [kg/s]
$\epsilon_{rr}, \epsilon_{\theta\theta}$	radial/tangential strain [m/m]
$\sigma_{rr}, \sigma_{\theta\theta}$	radial/tangential stress [N/m ²]
η	efficiency[-]

κ	adiabatic index [-]
ω	angular speed [rad/s]
ρ	density[kg/m ³]
ν	Poisson ratio [-]
c_p	specific heat [J/kg]
E	Young's modulus [Pa]
F	rotational loading [N]
i	enthalpy [J/kg]
l_t, l_c	specific work of the turbine/compressor [J/kg]
q	heat per mass unit [J/kg]
r	radius [m]
s	entropy [J/K]
T	temperature [K]
t	temperature [°C]
u_r	displacement [m]

1 Introduction

People with basic technical knowledge are aware of the existence of fluid-flow machines such as gas turbines. However, not everybody knows that gas turbines play a remarkably significant role in distributed power engineering and CHP (Combined Heat and Power) systems, which have become increasingly popular in recent years [1, 2]. It is well known that gas turbines and other types of gas engines are widely used to power electric generators or turbochargers in planes, cars and ships, and are also used to power compressors of heat pumps [3]. It is obvious that they differ in terms of their design, materials and size. However, their operating principles are almost identical – turbines convert heat energy into kinetic energy [4].

While reflecting on the distributed power engineering mentioned, it is essential to take a closer look at microturbines. They are fairly small but high-speed devices that could easily be implemented in electricity-producing machines. The growth in popularity of these machines could have a positive impact on the development of distributed power engineering (directly) and on the environment (indirectly) [5]. Especially in Central and Eastern Europe, where most of the households are dependent on coal, as shown in

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Figure 1. The combined production of heat and electricity by burning gas and the use of a microturbine is much more environmentally friendly than the combustion of coal for heat production.

Figure 1: Brown coal consumption in Europe [6]

What is more, gas microturbines could work not only as individual energy sources but also in cooperation with other commonly known systems such as hybrid systems with photovoltaic panels [7–11].

Gas turbines can be used in the engines of jet aircraft and UAVs (unmanned aerial vehicles) as well as in power plants, where axial-flow turbines are the most commonly used [12]. Radial-flow turbines are most often used as gas microturbines; currently, they are widely used in turbochargers (also in automotive turbochargers). Most producers of combustion engines (especially Diesel engines) increase their efficiency and power by utilising waste heat and adding to these engines the turbocharger mentioned [13]. Excellent examples of the use of axial- and radial-flow microturbines can be found in the book by J. Kiciński and G. Żywica [14]. There are described, among other things, radial-flow microturbines with power capacities of about 3 kW, which operate in ORC systems using waste heat, with a temperature not exceeding 160°C and pressure at the turbine inlet of 11 bar. Turbines of this type (that is to say, radial-flow turbines) are analysed in this article.

The Brayton–Joule cycle represents the operation of a gas turbine engine (see the diagram in Figure 2). This thermodynamic cycle consists of three basic elements: compressor, combustion chamber/heat exchanger and gas turbine. To generate electricity, an electric generator is also needed. On the left side of Figure 2, a temperature versus

entropy graph can be seen. Line 1-2 represents the isentropic compression of the gas in a compressor and process 2-3 represents an isobaric heat supplied by a heat exchanger or by burning fuel in a combustion chamber. Then the isentropic expansion in a turbine takes place.

Figure 2: Temperature versus entropy graph and scheme of the Brayton–Joule open cycle (C-compressor, CC/HE-combustion chamber/heat exchanger, GT-gas turbine)

The efficiency of the Brayton–Joule cycle depends directly on the ratio of the difference between the work of a turbine $l_t = (i_3 - i_4)$ and the work of a compressor $l_c = (i_2 - i_1)$ to the heat supplied to the cycle $q = i_3 - i_2$ (where i is enthalpy). It is also indirectly connected with a temperature difference at the inlet and outlet of the turbine. When $\dot{m}$ (mass flow rate) is constant the efficiency can be expressed by the following equation 1:

$$\eta = \frac{l_t - l_c}{q} = \frac{(i_3 - i_4) - (i_2 - i_1)}{i_3 - i_2} = 1 - \frac{i_4 - i_1}{i_3 - i_2}, \quad (1)$$

where η – efficiency.

The enthalpy is dependent on the temperature in the following way:

$$i = \dot{m} c_p T, \quad (2)$$

where c_p – specific heat, T – temperature.

When equation 2 is used in equation 1, equation 1 can be simplified as shown below.

$$\eta = 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad (3)$$

There is a dependency between the temperatures described in equation 3, and it is expressed by the following formula (equation 4):

$$\left(\frac{T_2}{T_1} \right)^{\frac{\kappa}{\kappa-1}} = \left(\frac{T_3}{T_4} \right)^{\frac{\kappa}{\kappa-1}}, \quad (4)$$

where κ – heat capacity ratio.

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad (5)$$

When equation 5 is used in equation 3, a higher temperature at the turbine inlet gives a higher efficiency of the cycle, as described in equation 6.

$$\eta = 1 - \frac{T_4}{T_3} \quad (6)$$

The above equations are basic formulas used to describe the efficiency of the simple Brayton–Joule cycle and are presented to show how temperature affects the operation of the thermodynamic cycle of the turbine. Even a small increase in the efficiency can lower the amount of fuel needed. So it is possible to obtain the same electric power output, while significantly reducing the amount of pollution emitted to the atmosphere and the operating costs of such a device. What is more, such an increase in the temperature is also beneficial in the case of CHP because the higher the inlet temperature is, the easier it is to use heat from the outlet of a turbine to heat the working medium in the heat exchanger [15]. Unfortunately, the increase in temperature mentioned above has a negative impact on the strength of the turbine wheel due to the fact that the yield strength of the material decreases as the temperature increases. As far as microturbines that could be used in households are concerned, their ability to operate at high temperatures while maintaining good mechanical properties is very important. In addition, many existing microturbines could be retrofitted in order to improve their efficiency, while making them more resistant to harsh operating conditions and more environmentally-friendly.

Several stress optimisation methods for radial and radial-axial gas turbines have already been developed, for example Fu *et al.* proved that the mass of the disc of a radial-flow turbine can have a significant influence on the stress distribution. The authors also changed the meridional profiles of the disc, which reduced its mass [16]. Barsi *et al.* described a multidisciplinary method for designing the compressor and disc of a radial-axial microturbine. The model made it possible to carry out aerodynamic and mechanical calculations [17]. Mueller *et al.* also created coupled CFD and FEA algorithms [18]. What Kaczmarczyk *et al.* and Passar have demonstrated for radial-flow turbines, is that the change in their flow system has a great influence on the efficiency of the systems tested [19, 20]. Żywica *et al.* made calculations of a similar-sized radial turbine but only kinetostatic stresses were determined and the disc was not optimised [21]. The material from which most modern turbines are made (Inconel 738) was also used to manufacture the guide vanes of the axial gas turbine, which was analysed in detail in the article by Charlang *et al.* [22]. Unfortunately, in the literature, there is

a lack of analyses of the discs made of nickel-based alloys, which are parts of radial or radial-axial gas turbines.

The development of CFD methods and manufacturing technologies has made it possible to create fairly complex machine flow systems. To ensure their proper functioning, these systems must be carefully designed to have excellent mechanical properties. This paper focuses on developing a method for designing the discs of radial gas microturbines, which can withstand very high temperatures and high rotational speeds without any changes in their aerodynamics. To develop this method, general information about strength calculations was provided to understand the importance of certain parameters. Also, a review of suitable materials was done in order to have a wider perspective on the latest trends and get to know their impact on the stress reduction in turbine discs. A basic numerical model, developed during flow optimisation, was used to make the calculations necessary to determine the stresses within the turbine disc. Then, a number of changes to the geometry of the disc were suggested to reduce its stresses. All the changes had no effect on the turbine fluid-flow system and their relevance was confirmed by FEM calculations.

2 Strength calculations of turbine discs

In general, radial turbine discs can be treated as rotational hollow discs. This is, of course, a fairly large assumption, which is useful in order to understand the growth of tensions in certain places and which is also irreplaceable when looking for methods to reduce these tensions.

The fundamental formula for the rotational loading acting on a disc and related to the stress is shown below (equation 7).

$$F = -\rho r \omega^2 \quad (7)$$

It can be seen that the rotational loading (F) acting on a disc depends on the density of the material (ρ) and the size of the disc (r), but also depends largely on the angular velocity (ω). To explain the stress distribution in simple terms, axisymmetric problems should be taken into consideration. In an axisymmetric plane, there are two stress components: radial tensile stress σ_{rr} and tangential tensile stress $\sigma_{\theta\theta}$. Their distribution on a disc can be seen in Figure 3.

These radial (σ_{rr}) and tangential ($\sigma_{\theta\theta}$) tensile stresses can be defined using the following differential equation (8):

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} (\sigma_{rr} - \sigma_{\theta\theta}) = -\rho r \omega^2 \quad (8)$$

Figure 3: Stress distribution (based on [23])

And the radial (ϵ_{rr}) and tangential strains ($\epsilon_{\theta\theta}$) can be expressed by the following equation 9:

$$\epsilon_{rr} = \frac{\partial u_r}{\partial r}, \quad \epsilon_{\theta\theta} = \frac{u_r}{r} \quad (9)$$

Using the dependencies between stress and strain, which are based on Hooke's law, these equations take the following form:

$$\epsilon_{rr} = \frac{1}{E} [\sigma_{rr} - \nu \sigma_{\theta\theta}], \quad \epsilon_{\theta\theta} = \frac{1}{E} [\sigma_{\theta\theta} - \nu \sigma_{rr}], \quad (10)$$

where E – Young's modulus, ν – Poisson ratio

After substituting and simplifying equation 10, the following deformation formula (equation 11) can be obtained:

$$u_r = C_1 r + C_2 \frac{1}{r} - \frac{1}{8} \frac{1 - \nu^2}{E} \rho r^3 \omega^2 \quad (11)$$

Boundary conditions for hollow discs were defined using equation 12:

$$\sigma_{rr}(a) = 0, \quad \sigma_{rr}(b) = 0 \quad (12)$$

Where a is the internal diameter and b is the outer diameter, and introducing two constants A , C (equation 13):

$$A = \frac{-EC_2}{1 + \nu} = -\frac{1}{8} (3 + \nu) \rho \omega^2 a^2 b^2, \quad (13)$$

$$C = \frac{EC_1}{2(1 - \nu)} = \frac{1}{16} (3 + \nu) \rho \omega^2 (a^2 + b^2)$$

The system of equations 14-16 describing radial tensile stresses, tangential tensile stresses and the deformation in the radial direction is obtained [23].

$$\sigma_{rr}(r) = \frac{3 + \nu}{8} \rho \omega^2 \left[a^2 + b^2 - r^2 - \frac{a^2 b^2}{r^2} \right] \quad (14)$$

$$\sigma_{\theta\theta}(r) = \frac{3 + \nu}{8} \rho \omega^2 \left[a^2 + b^2 - \frac{1 + 3\nu}{3 + \nu} r^2 + \frac{a^2 b^2}{r^2} \right] \quad (15)$$

$$u(r) = \frac{3 + \nu}{8} \rho \omega^2 \frac{1 - \nu}{E} \left[a^2 + b^2 - \frac{1 + \nu}{3 + \nu} r^2 \right] \quad (16)$$

Figure 4: Stress versus the ratio of r to b , where $a=0.1$ and $b=1$

$$+ \frac{1 + \nu}{1 - \nu} \frac{a^2 b^2}{r^2} \Big]$$

These more accurate equations show that the stresses in the hollow discs mainly depend on: rotational speed, hole diameter, outer diameter and also the type of material that is defined by Poisson ratio and density. However, the deformations depend on the parameters mentioned above and on Young's modulus E . The highest values of the stresses described should occur near the hole, which is explained by the fact that the smaller the radius r , the higher the stresses, as shown in Figure 4.

The above graph was created based on equations 14 and 15, where the constant values of the parameters were set apart from the radius r , which was changed in the range from a to b . This is why the horizontal axis is designated as the ratio of the radius of the place where the calculations were made to the outer diameter. The highest stresses occurring around the hole ($r=a$) are particularly visible in the graph showing the tangential stress and von-Mises stress curves (i.e. reduced stresses).

This analytical method has been mentioned only to explain the meaning of certain parameters, which can be used to modify the stress conditions of a turbine disc. Nevertheless, a calculation method should be easy from a technical point of view. That is why programs using FEM could be implemented. In this paper, the ANSYS Workbench 19.2 program was used. To confirm that stresses of a disc mainly depend on mass and rotational speed, the finite element analysis was made. This analysis concerns the disc shown in Figure 5, as the basis of the optimisation. The geometry of this disc is original and was created by the authors of this paper in cooperation with the employees of the Department of Turbine Aerodynamics.

The mentioned disc rotates at a speed of 96,000 rpm, and so is a part of a high-speed device. Basic calculations

Figure 5: 3D model with dimensions (in millimetres) of turbine disc

were made to assess the reliability of the FEM model. A discrete model, in which a tetragonal mesh was used, was developed using the ANSYS software. First, the mesh for a single blade was created, which consisted of approximately 250,000 nodes and 150,000 elements. Then, to reflect the actual operation, a “cyclic region” was created, which made it possible to develop a model of the entire disc based on the discretisation of the blade which has just been mentioned. This technique made it possible to reduce the calculation time (from ten minutes for the discretisation of the whole disc to one minute for the discretisation of a single blade, which was duplicated) and ensured a high-quality mesh. During the calculations, the rotational speed was changed and also the mass for the nominal speed was reduced. There is a really strong relationship between mass and stress, as well as between rotational speed and stress (see Figure 6).

Figure 6: Rotational speed and mass of turbine disc vs. stress

The relationship between the stress (σ) and mass is not as strong as between the rotational speed and stress. As we can see the red line has a fairly large angle of inclination in relation to the x-axis (sigma), which results from equation 7, and is, however, still significant. Unfortunately, when considering the reduction of those stresses,

there is no easy way to reduce the rotational speed without exerting any impact on the operation of the entire system. The stress reduction method developed in this paper should be implemented into the existing flow system. That is why the main focus should be given on the mass reduction of the rotating element without exerting any impact on the fluid-flow.

3 Materials review and their selection depending on operating conditions

When considering the stress optimisation, it is very important to carefully select the construction materials. Many turbine components are designed to operate in high temperatures. The turbine disc discussed in this paper should be able to withstand the ambient air temperature of about 600°C. That is the reason why the material must have high yield stress, and must also be heat-resistant.

At high temperatures, which are common in CHP systems, polymers or other plastic materials as well as certain metals and their alloys cannot be used. Both axial-flow and radial-flow turbines can be used as elements of these types of systems. To find an optimal construction material, it is necessary to examine carefully how axial-flow turbines operate at high temperatures, where special attention must be focused on their blades. It can be seen in Figure 7 that the temperatures at the inlet of turbines have increased considerably over the years. According to equations mentioned before, their efficiency has also increased

Figure 7: Increase in the inlet temperature of gas turbines over the period 1940–2020 [24]

The inlet temperature of the turbines has increased over the period 1940–2020 due to the development of alloys and also because of the fact that the blades of the

Table 1: Mechanical properties of Inconel 738 at 600°C

Yield strength	Tensile strength	Young's modulus	Shear modulus	Poisson ratio
910 MPa	1,055 MPa	167,034 MPa	64,827 MPa	0.3

Table 2: Mechanical properties of ceramics at 20°C [29]

Name	Yield strength	Tensile strength	Young's modulus	Poisson ratio	Density	Max. working temperature
Alpha-SiC	310 MPa	324 MPa	476 GPa	0.19	3,210 kg/m ³	1,400°C
Alumina (Al ₂ O ₃)	300 MPa	400 MPa	370 GPa	0.22	3,960 kg/m ³	1,460°C
Si ₃ N ₄	375 MPa	830 MPa	310 GPa	0.27	3,290 kg/m ³	1,000°C
Zirconia ZrO ₂	310 MPa	620 MPa	210 GPa	0.28	5,720 kg/m ³	1,200°C

Figure 8: Dependency between yield strength, ultimate strength and temperature for Inconel 738

axial-flow turbines have been cooled since 1960. Cooling the blades increases the inlet temperature of turbines and thus their efficiency. Moreover, the material of which the blades are made does not necessarily have to withstand very high temperatures. With regard to radial-axial turbines, it is not easy to cool their blades. Therefore, brand new methods for reducing stress are crucial for the development of gas microturbines.

Nickel- and cobalt-based alloys are widely used in power engineering and aeronautics. A good example is Inconel 738 which has great mechanical properties but also can be used at very high temperatures. Other well-known materials are steel alloys and Nimonic 90, Inconel 718 or Inconel 625 [25–27]. Inconel 738 was chosen as the construction material for the turbine disc discussed herein because it has the best mechanical properties. As far as the geometry of the turbine disc is concerned, the type of material can also affect the optimisation process. Nickel-based alloys have a fairly high density and high yield strength. This is why the turbine disc that is to be made of such an alloy should be optimised in a completely different manner from that which must be made of steel.

From the point of view of the strength optimisation, mechanical properties are more important. A graph presented in Figure 8 shows how these properties change depending on the temperature. The yield strength is really high at temperatures near to 600°C and its value is about 910 MPa. When the temperature is higher, yield stress and tensile strength start to decrease rapidly. In the turbine which is examined in this paper, the temperature of the working medium is lower than 600°C. As can be seen in Table 1, the yield stress and tensile strength still have fairly high values and therefore INCONEL 738 superalloy is considered to be an excellent material for the blades.

Regarding high temperatures and high rotational speeds, ceramics which started to be used in the XX century (according to Figure 7) should be taken into consideration. These are inorganic materials with an ionic and covalent bonding, which are made in high-temperature processes [28]. Ceramics have great mechanical properties and, what is important for the strength of a disc, they have a very low density. When considering using these materials for the radial disc of a gas microturbine, only machinable ceramics should be considered. This is important because machining (for example milling or turning) is less expensive and more widely used than other manufacturing techniques.

The most popular machinable ceramics are:

- Silicon carbide – SiC
- Alumina – Al₂O₃
- Silicon nitride – Si₃N₄
- Zirconia – ZrO₂

Table 2 shows the properties of the above-mentioned ceramics.

As can be seen in Table 2, the yield strength of these materials is quite similar. The best materials for the turbine are those which not only have a high yield strength

Figure 9: Yield strength vs. temperature for ceramics (based on [27])

and Young's modulus but also have a low density (which results from equations 14-16). As can be seen in Figure 9, the yield strength is not a constant value, but it decreases with temperature. Si_3N_4 has the highest yield strength at 20°C, which decreases with increasing temperature. Alumina and silicon carbide also have quite good properties at temperatures of up to 800°C.

However, silicon nitride has the highest yield strength at 600°C out of all four ceramics. Therefore, this material was considered for use in the gas microturbine and was chosen for further calculations. What is interesting, different producers of silicon nitride (Kyocera, Honeywell and Saint-Gobain) give different information on the material properties. Figure 10 shows the properties of different types of silicon nitride produced by Kyocera.

Figure 10: Flexural strength vs. temperature for different types of silicon nitride produced by Kyocera (based on [30])

What is important, the producer does not give information about yield strength because of the brittle struc-

ture of silicon nitride, but information on flexural strength is available. The flexural strength values given by the producer are much higher than those presented in Figure 9. However, in the case of hollow discs, the highest stresses are expected to occur in the shaft hole. Therefore, yield strength is much more important than the flexural one because flexural strength is defined based on bending while yield strength is defined based on extension.

4 Calculations of the basic version of the turbine disc

First of all, calculations for the basic turbine disc were made. "Basic" means that the geometry was created and optimised from the point of view of the fluid flow and the stresses were not taken into account yet. Dimensions of the turbine disc are shown in Figure 11. The external diameter of the disc is 109 mm whereas the diameter of the hole (into which the shaft will be inserted) is 15 mm. The thickness of the wall at the rear part of the turbine is 5 mm.

Figure 11: Dimensions (in millimetres) of the basic version of the turbine disc

A tetrahedral mesh was used to create the numerical model. This type of finite elements was chosen because of the rather complex geometry (small transition radii, blades). About 52,000 finite elements and 90,000 nodes were used in the model. Making the mesh denser did not improve the accuracy of the results but increased the computation time. This is why the discretisation that is depicted in Figure 12 was used in the calculations. The orthogonal quality of the mesh was identified as good. To simplify the model, a "cyclic region symmetry" was used, which means that a fragment of the disc was discretised and then duplicated 13 times since that was the number of

blades. Figure 12 shows the FEM model of a fragment of the disc, which was used in the calculations.

Figure 12: Mesh of the disc fragment

The calculations were made for the following boundary conditions:

- A cylindrical support in the hole, with fixed constraints in the radial and tangential direction, and free constraints in the axial direction
- B displacement of the back part of the disc; the constraint of the displacements in the axial direction is equal to 0 mm
- C rotational speed – 96,000 rpm
- D cyclic region symmetry

The boundary conditions described are presented in Figure 13.

Figure 13: Assumed boundary conditions

These boundary conditions were applied to reflect the real conditions as accurately as possible. In addition, test-

ing calculations were made as follows: the shaft was modelled and contact was defined between the shaft journal and the disc hole, taking into account the operating conditions with and without friction (where the coefficient of friction was 0.2). The results obtained did not vary significantly compared to those obtained on the basis of the previously described model, where the cylindrical support condition was applied inside the hole. The calculations aimed to show the stresses that were solely dependent on the high rotational speed, excluding the working medium which expands in the turbine.

As already mentioned, the material called Inconel 738 was used in the numerical model. As the operating temperature of the turbine is approximately 600°C, the following values were set for this alloy: Poisson ratio – 0.3, Young's modulus – 1.67×10^5 MPa, tensile yield strength – 910 MPa and tensile ultimate strength – 1,075 MPa.

The calculation results that are presented in Figure 14 were treated as reference values for the purposes of the optimisation. This figure shows the equivalent von Mises stress for the nominal rotational speed (96,000 rpm). The maximum stress is 1,288.2 MPa at the footing of the blade, from the rear side of the disc. This value is much higher than the ultimate tensile strength of the used material. The main cause of such a high value is the rotating mass of the turbine. Figure 15 shows the distribution of deformations of the turbine disc. The highest displacement (0.2 mm) was observed at the edge of the hole. It is an acceptable value. The nominal rotational speed of the disc is 96,000 rpm and this is a really high value, but Klonowicz *et al.* concluded that in microturbines the higher the rotational speed, the higher the internal efficiency [31]. Nevertheless, after taking a look at Figure 6, one can notice how strongly the maximum stress is dependent on the rotational speed, and this also results from equation 7. The rotational speed has the biggest influence on the stress (its value increases exponentially with the speed).

5 Influence of mass and shape on stress distribution

To start a discussion on the optimisation, we have to return to equation 7. The forces are dependent on mass, radius and rotational speed. Only one of these parameters can be decreased, namely the mass. After doing so, the forces will decrease and so will the stresses. Moreover, it will be possible to manufacture the turbine disc using machining. A graph that shows the stress versus the mass of the turbine disc is presented in Figure 6. As can be seen, the stress is

Figure 14: Equivalent von Mises stress of base turbine disc (rotational speed 96,000 rpm; temperature 600°C)

strongly dependent on the mass. A decrease in the mass by approximately one percentage point causes the stress to also by about one per cent. Inconel 738, which was chosen as the construction material of the turbine disc, has a density of 8,110 kg/m³. To reduce the mass, some material was removed from the back of the base disc, on the basis of the results of calculations described in the previous chapter. The highest and the lowest wall thickness was 5 mm and 1.5 mm respectively. The stress distribution was very similar to the one shown in Figure 14, but the obtained values were lower by about 24%.

Still focusing on the mass reduction which, as already mentioned, is a very important issue of the optimisation discussed herein, one can notice that there are more places where the material could be removed, for example, idle places. Several such places are located at the back of the disc. In Figure 14, they are semicircular in shape (blue colour) and the stress is really low there. Furthermore, if some more material were removed from spots in which deformations are the highest, it would decrease the mass, and consequently the stress.

As it can be seen in Figure 16, the stresses are significantly reduced from 1,288 MPa to 838 MPa, which was achieved by removing material from the rear part of the turbine disc. It is obviously related to the fact that the removal of material from the area between the blades decreases the mass of the disc. Nevertheless, doing so can be a way to reduce deformations, as shown in Figure 17. Now, the highest deformation can be found at the top of the blade. The

highest total displacement at the rear part of the disc is now 0.08 mm, and before it was 0.2 mm; it really makes a difference. Is the deformation in this place so important? Yes, it is. And not only from a mechanical point of view. The axial and radial clearance is strictly connected with the efficiency of the turbomachine. The lower the clearance, the higher the efficiency. The reduction of the deformation on the rear side of the turbine disc makes it possible to decrease the clearance, so the efficiency is higher because the pressure loss is lower. The mass influence is enormous, as was shown above. It also has an impact on the efficiency of the turbine. Scalloping (the cutting out of material) can have a negative effect on the efficiency and operation of the turbine. The impact of the above-mentioned tip clearance and scalloping is described in detail in the article by Ping He *et al.* [32]. Unfortunately, the first step in the process of optimising the shape and reducing the stresses of the disc of the high-speed radial-axial turbine should be to reduce the mass as much as possible. This is why scalloping is necessary in the case discussed. The authors of the article plan to carry out a study to evaluate the impact of this design technique on the operation and efficiency of the turbine. In the case under discussion, other treatments should also be carried out.

As it was stated, mass has the greatest impact on the stress values in the turbine disc. Besides, even the shape that is created by removing material has an impact too. Lei Fu *et al.* [16] made calculations of a turbine disc that was quite similar in shape and size. They also changed the

Figure 15: Deformation distribution of base turbine disc (rotational speed – 96,000 rpm; Temperature – 600°C)

Figure 16: Von Mises stress of turbine disc -optimisation 1 (rotational speed – 96,000 rpm; temperature – 600°C)

meridional profile and decided not to change the fluid-flow system of the turbine. By changing the shape of the area between the blades and the shape of the rear part of the turbine disc, they reduced the stresses by about three times. Nevertheless, as it was mentioned, deformations have to be taken into account because they are very important for internal efficiency. The base model described in the article in question was considerably simplified because the hole needed to insert a shaft was removed from the disc. As

shown in Figure 16, quite high stresses are present around the hole of the disc. High stresses are also present on the back area of the turbine disc (especially near the hole). Therefore, during the development of the numerical model (and thus during the development of the optimisation algorithm), the hole was left intact. Very often, in radial-flow and axial-flow microturbines, the diameter of the shaft is quite small [33, 34]. In fact, the turbine disc is mounted at the end of the assembly process. This is why the hole,

Figure 17: Deformation distribution of turbine disc - optimisation 1 (rotational speed – 96,000 rpm; temperature – 600°C)

which is made for assembling the shaft, cannot be bigger. Very often, the torque of the turbine is transmitted to the shaft by a pin. This method can be used when the rear part of the turbine disc abuts the shaft. Unfortunately, this poses another problem. If the compressor is assembled first, its mounting hole must be larger than the turbine disc hole, and also larger than the pivot between the turbine and the compressor. The dimensions of the compressor disc are similar to those of most gas microturbines [35]. This is the reason why the mounting hole in the compressor cannot be larger than the one in the turbine disc. In this case, the shaft at the rear part of the turbine disc has a diameter of 18 mm, while the diameter of the mounting hole is about 15 mm. It is possible to reduce the diameter of the mounting hole and the stresses would decrease, but if it is not possible from a technological point of view, the shape of the rear part of the disc should be taken into account. To compare any results, the turbine disc that has the smallest possible mass and whose rear part has a flat shape must be considered as the reference disc.

The first approach was to use a randomly generated shape. A certain small part of the rear side of the disc was cut out. As shown in Figure 18 this simple operation made it possible to reduce the stresses by about 11%. It is not much, but it is proof that the shape of the rear part of the disc is really important. The highest stresses occur in the mounting hole and in the footings of the blades.

Next, the hyperbolic shape was made and strength calculations were done again. This shape was created by adding some material to the flat surface on the rear side of

the base turbine disc. Several views of the modified rotor disc are presented in Figure 19.

The highest stresses occur in the footing of the blade and the highest value is approximately 705 MPa. Although the mass of this disc is lower than the mass of the disc with a flat rear part, the stress distribution seems better. This is why the equivalent von Mises stress is lower by about 9%. What is really important is the fact that the stresses in the mounting hole are lower and do not exceed a value of 680 MPa. In the previous case, they almost reached the maximum permissible value of 838 MPa. Figure 20 shows the turbine disc 3D model after its optimisation.

As it was mentioned, the hyperbolic shape was created by adding material to the base model. The function of the line is asymptotic in relation to the vertical projection of the back of the turbine disc and is made by using a fixed radius. Its dimensions were optimised. The most optimal hyperbolic shape was created using a variable radius at the rear part of the disc. Changing the shape of this part of the disc is a practical method of reducing the stress, and it is not as effective as mass reduction, but can be very useful. The calculation results are proof that the variable-radius shape is the best. This shape, which resembles that of a compressor disc, can be really useful when optimising to reduce stress, as shown in this paper.

Figure 18: Front, side and rear view of turbine disc - optimisation 2 (rotational speed – 96,000 rpm; temperature – 600°C)

Figure 19: Front, side and rear view of turbine disc - optimisation 3 (rotational speed – 96,000 rpm; temperature – 600°C)

Figure 20: Turbine disc 3D model (after optimisation)

6 Comparison of developed methods

There are many methods that can be used for reducing stress. Five of these methods are described in this paper. The first and most important method is through mass reduction. It should be considered as a rule and not as a method. It is the basis of any other method and should be used as the main step in reducing stress. Figure 21 shows the effectiveness of the developed methods.

Figure 21: Comparison of the stress optimisation methods used

Five methods were considered. Below is a short description of each one of them.

- Method 1 – Base turbine disc
- Method 2 – Mass reduction
- Method 3 – Mass reduction by removing some material from the areas with high deformations (optimisation 1), scalloping. The variable parameters are R1 and R2, as shown in Figure 22.

Figure 22: Variable parameters used during optimisation using method 3

- Method 4 – Method 3 + cutting the rear part of the disc at a certain angle (optimisation 2). Two additional variable parameters are presented in Figure 23: A1 – the cutting angle of the blade; H1 – the

height of the element which comes into contact with the journal of the shaft.

Figure 23: Additional variable parameters used during optimisation using method 4

- Method 5 – Method 3 + hyperbolic shape of the rear part of the disc, developed using a variable radius (optimisation 3). Three additional parameters are presented in Figure 24: R3 – the radius of the arc on the back side of the disc; L3 – the distance between the end of the disc and its basis; L2 – the height of the start of the arc from the axis.

Figure 24: Additional variable parameters used during optimisation using method 5

Many programs allow the parameters mentioned to be changed automatically. However, this is not a crucial issue. In order to fully understand the phenomenon of the concentration of stresses depending on the alteration of the geometry, the authors manually changed the values of the

Figure 25: Stress reduction optimisation algorithm for radial-axial gas microturbine

particular parameters and also used special tools which allowed these parameters to be modified automatically. Very similar results were obtained in both cases.

As it can be seen, the reduced stresses are at an acceptable level in method 3, but the most effective method consists in reducing the total mass by removing some of the material from the areas with high deformations (Method 2). Methods 3 and 4 are other alternative versions of the mass reduction method; that is why they are not considered as the most effective.

Based on these methods, an algorithm for optimising highly loaded radial-axial turbines was developed and can be seen in Figure 25. This algorithm, presented in the form of a block diagram, systematises the information contained in the text and illustrates the procedure which can be used to optimise the discs of radial-flow microturbines.

The algorithm was developed based on the disc of a radial-flow gas turbine. This disc was made of Inconel 738. As it was proved at the beginning, the mass of the disc has a great influence on its strength. To check the correctness of this thesis, calculations were made for the disc identi-

Table 3: Mechanical properties of silicon nitride at 600°C

Yield strength	Tensile strength	Young's modulus	Poisson ratio	Density
400 MPa	550 MPa	249,935 MPa	0.24	3.26 g/cm ³

B: Static Structural

Equivalent Stress
 Type: Equivalent (von-Mises) Stress
 Unit: MPa
 Time: 1
 19.12.2019 15:24

537,1 Max
 477,43
 417,75
 358,08
 298,41
 238,73
 179,06
 119,38
 59,711
 0,037029 Min

Figure 26: Front, side and rear view of turbine disc – base ceramic with reduced stress distribution (rotational speed – 96,000 rpm; temperature – 600°C)**B: Static Structural**

Equivalent Stress
 Type: Equivalent (von-Mises) Stress
 Unit: MPa
 Time: 1
 19.12.2019 16:13

358,18 Max
 300
 262,51
 225,02
 187,54
 150,05
 112,56
 75,073
 37,585
 0,097263 Min

Figure 27: Front, side and rear view of turbine disc – method 3 ceramic (rotational speed – 96,000 rpm; temperature – 600°C)

cal to the base disc, based on the developed algorithm, and the material was changed to silicon nitride (Si_3N_4). As it can be seen in Figure 10, SN240 by Kyocera has the best mechanical properties. Because there was no information about the Yield strength, calculations had to be made based on typical silicon nitride, whose mechanical properties are shown in Table 3.

As it can be seen in Figure 26, von Mises stresses are more than half lower compared to those obtained for Inconel 738. This is due to the mass of the disc, which is significantly lower (by about 40%). The yield strength is also

much lower compared to the nickel-based alloys. The optimisation should therefore not end at this point. Methods 2 and 3 can be used to reduce the stresses below a value of 400 MPa (Figure 27).

The same mass reduction method was used as for Inconel 738. In this case, the decrease in stress was the same as for the nickel-based alloy (34%), which shows that the mass reduction method is useful and the model used is reliable. After reducing the mass, stresses decreased to 358 MPa, which is a downward trend. Therefore, after a further reduction in mass, it was possible to decrease the stresses

to 280 MPa; this value was acceptable and there was no need to use other stress reduction methods.

To sum up, the developed stress optimisation method can be implemented with different materials and is supported by equations that show what must be done step by step during a stress optimisation process. Optimisation methods are often accompanied by experimental research. For safety reasons, this was not possible in the present case. The drift velocity of the blade tip (which is an important operating parameter not only in gas microturbines but also in wind turbines [36]) is very high. Therefore, destructive tests (where the rotational speed of the rotor is increased until the rotor disc is destroyed) are extremely dangerous. Our method was developed to avoid any phenomena that might threaten the proper operation of the machine; it was also important to guarantee the safety of its operator(s). The turbine described in this paper is a real device which is currently in the design phase. The next stages of work on this device will consist in manufacturing the rotor disc (based on the algorithm discussed in this paper) and in carrying out experimental tests with operating parameters within their nominal ranges.

7 Conclusions

Designers try to constantly improve fluid-flow machines. The efficiency of such machines depends largely on their construction materials and durability. There is no easy way to reduce stress when the nominal rotational speed is very high, but it is possible. As was shown, there are a number of ways to make it happen. However, advanced calculation tools and proper optimisation procedures are required. But it is necessary to bear in mind that every case is different. Blade thickness, channel shape or other aspects of the optimisation, which can eventuate after making CFD calculations, are usually unique and inimitable. But there are some hints, which are always handy. As it was mentioned, the first and most helpful advice is to reduce mass. This method was developed after analysing equation 7. The easiest way to reduce the mass of the disc is to remove some material from its rear part, but also to choose an appropriate construction material. Inconel 738, which was used in the case in question, has a really high density equal to $8,110 \text{ kg/m}^3$ and is resistant to high temperatures. Paradoxically, the material with the highest yield strength is not always the best choice. In the case of high-speed turbomachines, it might sometimes be preferable to choose a material with inferior mechanical properties and with a lower density. This conclusion was drawn after a strength analy-

sis was carried out using an advanced FEM model (which accurately reproduced the geometry of the rotor disc). As shown in Figure 6, the rotational speed and mass have the biggest impact on the stress obtained. High-speed gas microturbines have some crucial spots where the stresses are the highest. These spots are the mounting hole and the footings of the blades. As was shown, it has been possible to decrease the high stresses at the mounting hole by changing the shape of the rear part of the turbine disc. In general, non-flat shapes are better than flat ones. The best shapes are the hyperbolic shapes that are demonstrated in Figure 18. The use of these shapes allows for better stress distributions to be obtained. The stresses at the footings of blades always depend on the mass of the turbine disc. Simple methods are sometimes not efficient enough and the question arises as to whether the designed turbine disc should have the highest efficiency possible from the point of view of the working medium's flow since it could lead to damage after a short period of operation. An ideal situation would be to design a turbine with high durability and the highest efficiency possible – from both thermal and mechanical point of view. A device that is out of order has no efficiency at all. Therefore, using the FEM when optimising the stresses in high-speed turbines is so important.

The design optimisation of the heavily loaded rotor disc of a gas turbine, which has just been presented in this article, was carried out using an advanced FEM model. It was therefore possible to perform an analysis on the disc whose blades had a very accurately reproduced shape and evaluate the impact of the minor modifications of this shape on the stress distribution within the whole element. It would certainly not be possible to achieve such a large reduction in stresses either with the use of analytical methods or with more simplified FEM models. The geometry optimisation procedure that has been proposed for the rotor disc of a gas turbine is pretty universal and could also be used for heavily loaded rotating parts of other fluid-flow machines such as vapour microturbines and turbo-compressors. The strength optimisation methods used for the rotors of these machines are not widespread and remain a well-kept secret (in other words, a know-how which gives a competitive advantage over other producers). The approach to this problem suggested in this paper has not yet been published in the literature, and could bring tangible benefits. This would make it possible to increase the rotational speed and other operating parameters of turbomachines, and this may be of interest to scientists and engineers from all over the world.

In general, both radial and radial-axial gas microturbines could be used to create small power networks (microgrids) in dispersed power generation systems. As men-

tioned, most microturbines cannot be cooled as easily as axial turbines. That is why the methods presented in this paper are so important for the development of gas microturbines, contributing to the increase in their popularity.

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