Research Article

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Characteristics of selected measures of stress triaxiality near the crack tip for 145Cr6 steel - 3D issues for stationary cracks

<https://doi.org/10.1515/eng-2020-0042>

Received Nov 25, 2019; accepted Mar 06, 2020

Abstract: In the paper the numerical analysis of the stress fields for 145Cr6 steel, near crack tip is presented, based on three-dimensional finite element method (FEM) analysis. The FEM analysis is focused on SEN(B) specimens with relative crack length $a/W \approx 0.30$. In addition to the presentation of the normal components of the stress tensor, the paper presents selected measures of stress triaxiality parameters, measured for the value of the J -integral, corresponding to the experimentally determined fracture toughness, denoted as J_{IC} , which is considered to be a material constant or material characteristic [1, 2]. Presented topic is a continuation of papers [3, 4], which were based on experimental analysis, presented in [5].

Keywords: 145Cr6 steel, stress triaxiality, fracture, fracture toughness, stress fields, stationary cracks

1 Introduction (based on [3–6])

The 145Cr6 steel is the basic material used for the production of cold working tools. In the previous nomenclature, 145Cr6 steel was designated as NC6. This material is considerably resistant to abrasion, is characterized by high hardness, shows medium hardenability. The 145Cr5 steel characterized by as well as low tendency to deformation and significant regularity of dimensional changes during hardening. In addition, 145Cr6 steel is steel with good machinability and blade durability, but only under operating conditions that do not cause excessive heating of the working tool. This material also tends to form a carbide mesh at the boundaries of austenite grains, which remains after soft annealing and hardening, reducing tool ductility. The 145Cr6 steel is used to produce tools and workpieces up

to 15 mm thick. It is used to make cutting tools, pressing and measuring tools that should not change dimensions after hardening, such as gauges, taps, dies, reamers, file cutters, paper and tobacco knives, circular saws for cold metal cutting, deep drawing cold stamping dies, cutting dies. In some cases, this steel is sometimes used for the production of bolts, dowels, small beams that carry quite significant loads.

Professional literature and industry portals provide some characteristics of the mechanical properties of this steel, however, information about the parameters characterizing the fracture of 145Cr6 steel is quite poor, which results from the fact that this material does not appear in Eurocodes [6] as the basic construction material used in the construction of machinery and equipment or in the construction of buildings (civil engineering, road engineering, etc.). A rather interesting fact about 145Cr6 steel is shown in [5], when after quenching and next after tempering, this material characterized by the decrease in hardness as the tempering temperature increases and will become a material more and more plastic [5], with quite a high yield point and a high ultimate strength.

2 About experimental research and critical values of the J - integral – J_{IC} (based on [3–6])

In papers [3, 4], the topic of assessment of parameters characterizing the fracture of 145Cr6 steel was discussed, using experimental tests and numerical calculations. In the first paper [3], the experimental research program was discussed, while in the second paper the details of numerical calculations were presented, which were carried out for the case of three-dimensional models and the case of plane strain state dominance, after which the results of preliminary numerical calculations were presented. Among the experimental results presented in [3], one can find information on the determined J - R curves for

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three groups of SEN(B) specimens made of 145Cr6 steel, characterized by different thickness. For each thickness, experimental program was included the use of specimens with different relative crack length a/W . In addition, the critical value of the J -integral, determined by J_C , was estimated for each specimen, which, according to the requirements of ASTM [7], can be considered to be fracture toughness for plane strain state domination. Among the results of numerical calculations, which were given in [4] for critical moments (characterized by critical value of J -integral denoted as J_C), values of measures of geometrical constraints denoted as T_m and Q_m^* [8–10] were determined:

- T_m – the average value of the T_z parameter, which is definition of the level of stress triaxiality according to the T_z formula $T_z = \sigma_{xx}/(\sigma_{yy} + \sigma_{zz})$ [11–13];
- Q_m^* – the thickness-averaged and normalized by the yield stress difference between the actual stress value – estimated by numerical calculations, and the solution proposed by Guo Wanlin [11–13].

Both parameter can be used to more accurately describe the stress field near crack tip in elastic-plastic materials in the case of analysis of three-dimensional problems [8], or to propose new fracture criteria [9, 10], allowing the estimation of real fracture toughness.

The concept of critical moment in fracture mechanics requires clarification here. The critical moment is the point on the J - R curve, which according to ASTM assumptions [7] corresponds to the increase in the crack length – crack growth – $da = 0.2\text{mm}$. It is, according to ASTM recommendations [7], the conventional moment of initiation of the crack growth, for which the critical value of fracture toughness is determined – in this case, the critical fracture toughness is critical value of the J -integral, denoted as J_C .

The paper [4] also presents in graphic form the changes in these measures of geometric constraints along with the increasing external load, expressed by J -integral, as well as the impact of the relative crack length and specimen thickness on the changes in the $T_m = f(J)$ and $Q_m^* = f(J)$ curves [4].

The results presented in [3, 4] may prove useful in solving engineering problems in the field of assessing the actual fracture toughness, determined according to one of many fracture criteria, as well as in estimating the stress distribution near the crack front in structural elements made of 145Cr6 steel. However, for all real specimens analyzed in [3–5], for which bidirectional numerical analysis was performed, the stress state prior to the crack point was not assessed (both for calculations in the case of assumption of plane deformation and calculations for 3D cases), as well as changes in the values of stress triaxiality param-

eters for three-dimensional cases were not discussed. For this reason, in recent years the area of analysis of collected experimental and numerical data has been expanded and it was decided to supplement the conducted research with new elements. This paper presents selected results of numerical analysis conducted for three-dimensional cases, focusing on stress distribution near the crack tip and the distribution of selected measures of stress triaxiality parameters.

Presented in [3] laboratory tests data, show, that each specimen used in the experimental research program ensures domination of the plane strain state. That means, that determined for all specimens critical value of the J -integral, denoted as J_C , may be considered as fracture toughness for plane strain state domination, determined in accordance with standard as J_{IC} . For the purposes of this study, in order to analyze the level of stress triaxiality, it was decided to use the experimental results for three SEN(B) specimens characterized by different thicknesses $B = \{5, 10, 15\}$ mm, characterized by relative crack lengths $a/W \approx 0.30$ – for subsequent thicknesses the relative crack length was $a/W = \{0.32, 0.35, 0.29\}$ respectively. The width of the specimens W was equal to $W = 25\text{mm}$ and the support spacing $S = 100\text{mm}$. The total length of the specimen was $L = 120\text{mm}$. Figure 1a [3, 5] shows the geometry of the SEN(B) specimen, while Figure 1b shows the stress-strain curves registered in the laboratory, on the basis of which the material constants required to perform the numerical analysis were determined [3, 5]. For the purpose of numerical calculations, the stress-strain curves were described by the power law (1):

$$\frac{\epsilon}{\epsilon_0} = \begin{cases} \frac{\sigma}{\sigma_0} & \text{for } \sigma \leq \sigma_0 \\ \alpha \cdot \left(\frac{\sigma}{\sigma_0}\right)^n & \text{for } \sigma > \sigma_0 \end{cases} \quad (1)$$

it was assumed that the constant $\alpha = 1$.

For the purposes of this paper, the 145Cr6 steel used in the tests can be characterized by the following values of material constants: Young's modulus $E = 200\text{GPa}$, yield point $\sigma_0 = 927\text{MPa}$, strain corresponding to the yield point $\epsilon_0 = 0.00464$, tensile strength $\sigma_m = 1040\text{MPa}$, corresponding strain tensile strength $\epsilon_m = 0.09775$ Poisson's ratio $\nu = 0.3$ and the strain hardening exponent in the Ramberg-Osgood law $n = 28$. The assumed values of material constants guarantee obtaining a model of stress-strain curve which is the lower boundary of the experimentally determined charts $\sigma = f(\epsilon)$, shown in Figure 1b [3–5], which is consistent with the desire to obtain conservative solutions, referred to in [1, 2, 11, 12].

Figure 2 presents examples of the results of experimental tests, which are presented in [3, 5] – all the curves pre-

Figure 1: a) The geometry of the SEN(B) specimen used to assess the fracture toughness of 145Cr6 steel; b) Engineering tensile diagrams of 145Cr6 steel, together with the determined values of material constants assumed in further FEM analysis and the stress-strain curve which was used in FEM analysis; (based on [3–5])

Figure 2: The comparison of the signals (quantities) of: a) potential drop ϕ in the function of the load line displacement v_{LL} ; b) force P in the function of the load line displacement v_{LL} ; c) J -integral (determined based on laboratory tests) as a function of the increase of the crack length da ; for three SEN(B) specimens made of 145Cr6 steel; (based on [3–5])

sented relate to specimens with a relative crack length a/W close to $a/W \approx 0.3$. Figure 2a shows the change in the potential drop signal ϕ as a function of load line displacement v_{LL} – the potential drop signal in accordance with [7, 13] is used to determine the increase in crack length denoted as da , which is required to plot J - R curves, which are necessary to determine the critical value J -integral - J_C . Figure 2b presents a comparison of forces P acting on the specimen, as a function of load line displacement v_{LL} – the thicker the specimen, the greater the force value. In

contrast, Figure 2c is a summary of the determined J - R curves for the three mentioned SEN(B) specimens. There are slight differences between J - R curves for specimens with thickness $B = 10\text{ mm}$ and $B = 15\text{ mm}$, for crack length increments $da > 5\text{ mm}$. These curves significantly differ in terms of the increase in crack length $da = \langle 0; 3 \rangle\text{ mm}$ [3, 5].

Figure 3 is a summary of the signals registered in the laboratory and the J - R curves determined on their basis, separately for each of the SEN(B) specimen analyzed in the numerical program, of different thickness B . These quanti-

Figure 3: List of signals (quantities) registered during experimental tests ($\phi = \phi(v_{LL})$, $P = P(v_{LL})$) and determined J - R curves ($J = J(da)$) for three SEN(B) specimens made of 145Cr6 steel: a) $B = 5\text{mm}$, $a/W = 0.32$; b) $B = 10\text{mm}$, $a/W = 0.35$; c) $B = 15\text{mm}$, $a/W = 0.29$; (based on [3–5])

Table 1: List of results of numerical calculations for SEN(B) specimens with relative crack length $a/W \approx 0.30$ for the moment of crack initiation [3–5]

B [mm]	a_0 [mm]	$b = W - a_0$ [mm]	a_0/W	J_C [kN/m]	$25 \times J_C / \sigma_0$ [mm]	Q (plane strain)	P/P_0 (plane strain)	T_m (3D)	Q_m^* (3D)
5	8.080	16.920	0.32	138.985	3.736	−0.398	0.851	0.130	−0.122
10	8.634	16.366	0.35	122.805	3.301	−0.340	0.826	0.183	−0.129
15	7.162	17.838	0.29	168.924	4.541	−0.475	0.889	0.182	−0.135

ties can be found on the comparative graphs shown in Figure 2, however in Figure 3, the change of some quantities can be analyzed much more clearly. Table 1 presents a summary of the results of experimental tests and selected results of numerical calculations in the scope of assessment of geometrical constraints measures – values averaged over thickness (T_m – measure of constraints in the thickness direction (the out-of plane constraints), Q_m^* – measure of in-plane constraints, Q – the Q -stress defined by O’Dowd and Shih [14, 15] – also a measure of in-plane constraints determined at the dominance plane strain state), for SEN(B) specimens (used further in the numerical program), for the moment of crack initiation, which corresponds to the moment for which critical value of the J -integral, denoted by J_C [3–5]. The impact of thickness on the change in the critical value of the J -integral and the measures of geometrical constraints determined in [4] are presented in Figure 4 [3–5].

As can be seen (Figure 4), the increase in specimen thickness is usually accompanied by a decrease in the value of Q -stress determined for the dominance of the plane strain state – the lower the value of the Q -stress, the lower the level of geometric constraints. As the thickness

Figure 4: The influence of thickness on the change of the critical value of the J -integral denoted as J_C and the measures of geometrical constraints determined in [4] (based on [3–5])

of the specimen increases, a slight increase in the T_m parameter is observed, which is almost constant for the thickness $B=10\text{mm}$ and $B = 15\text{mm}$. The increase in thickness is also accompanied by a decrease in the value of the Q_m^* parameter, which is a measure of in-plane constraints de-

terminated for the case of three-dimensional structural elements [8] – this change may seem insignificant, but it is worth noting here.

The parameters of geometrical constraints, mentioned above, can be considered as some measures of stress triaxiality, however, currently in the literature various researchers are increasingly focusing on stress triaxiality parameters determined directly from the estimated FEM of stress distributions to be independent in this undertaking from the adopted material model, which generally determines quantities that can be further analyzed. Below in the next section the characteristics of the quantities considered to be a stress triaxiality parameters will be clarified, which will be analyzed in the following sections of this paper.

2.1 About selected parameters of stress triaxiality (based on [16, 17])

In the specialist literature, the parameters T_z and Q^* are known to be measures of the so-called geometric constraints, *i.e.* constraints of a material during the occurrence of plastic deformations under external loads [2]. These parameters are not the only parameters used as measures of constraints in the fracture criteria. In 1968, McClintok [18] proposed to use the ratio of the average normal stresses σ_m to the yield strength σ_0 , designated by σ_m/σ_0 , in the fracture criterion. A year later, Rice and Tracey [19] employed the ratio of the average stresses σ_m to the effective stresses σ_{eff} , calculated according to the Huber-Mises-Hencky (HMH) hypothesis, σ_m/σ_{eff} . Some researchers have considered the influence of geometric constraints on the distribution of stresses for three-dimensional cases, analyzing the actual stresses responsible for the crack opening [20], or the differences between the actual description obtained through FEM analysis and that obtained on the basis of the HRR solution for a case of plane strain [21, 22]. The author of the paper has previously dealt with various measures of geometric constraints and measures of the level of stress triaxiality, for details of which can be found in [34–43].

In the literature on the elastic-plastic fracture mechanics we can find expressions defining the measures of the geometric constraints for three-dimensional cases, which may be considered as the stress triaxiality parameters:

- the ratio of the average normal stresses σ_m to the yield strength $\sigma_0 - \sigma_m/\sigma_0$:

$$\frac{\sigma_m}{\sigma_0} = \frac{1}{\sigma_0} \cdot \frac{(\sigma_{11} + \sigma_{22} + \sigma_{33})}{3} \quad (2)$$

where σ_{11} , σ_{22} , and σ_{33} designate the normal constituents of the stress tensor; this parameter sometimes is called as “mean stress” or “hydrostatic stress”;

- the ratio of the effective stresses σ_{eff} calculated according to the HMH hypothesis to the yield strength – σ_{eff}/σ_0 [18];
- the ratio of the average normal stresses σ_m to the effective stresses σ_{eff} according to the HMH hypothesis – σ_m/σ_{eff} [19];
- the stress triaxiality coefficient T_z [23–25], calculated as:

$$T_z = \frac{\sigma_{33}}{(\sigma_{11} + \sigma_{22})} \quad (3)$$

The considerations regarding the characteristics of the state of stress near the crack tip, requires the isotropic material model, what allows to formulate the invariants of the stress tensor $[\sigma]$, defined, respectively, by following equations [26, 27]:

$$p = -\sigma_m = -\frac{1}{3} \text{tr}([\sigma]) = -\frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3) \quad (4)$$

$$q = \bar{\sigma} = \sigma_{eff} = \sqrt{\frac{3}{2} [S] : [S]} \quad (5)$$

$$= \sqrt{\frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}$$

$$r = \left(\frac{9}{2} [S] \cdot [S] : [S] \right)^{\frac{1}{3}} = \left[\frac{27}{2} \det([S]) \right]^{\frac{1}{3}} \quad (6)$$

$$= \left[\frac{27}{2} (\sigma_1 - \sigma_m)(\sigma_2 - \sigma_m)(\sigma_3 - \sigma_m) \right]^{\frac{1}{3}}$$

wherein the $[S]$ denotes the deviatoric stress tensor according to Equation (7):

$$[S] = [\sigma] + p[I] \quad (7)$$

where the $[I]$ is the identity tensor; σ_1 , σ_2 , σ_3 are the principal stresses, fulfilling the following condition: $\sigma_1 \geq \sigma_2 \geq \sigma_3$ [26, 27]. The σ_m and σ_{eff} denote the mean stress and effective stress respectively, and by r we understand the third invariant of the stress deviator [26, 27].

Based on the given considerations [27], we can define the stress triaxiality parameter, which is generally given in the form of dimensionless hydrostatic stress denoted by η :

$$\eta = \frac{-p}{q} = \frac{\sigma_m}{\bar{\sigma}} \quad (8)$$

It is a quotient of mean stresses and effective stresses, quite often used in literature for discussion about ductile fracture [18, 19]. In some papers, this quantity is indicated by a

quotation in the form of a quotient σ_m/σ_{eff} . This parameter is also often used in the discussion of triaxiality of stresses, just like Lode angel denoted as θ , which can be related to the normalized third stress invariant denoted as ξ by following Equation [26, 27]:

$$\xi = \left(\frac{r}{q}\right)^3 = \cos(3\theta) \quad (9)$$

wherein the Lode angel range is $0 \leq \theta \leq \pi/3$ and the range of the normalized third stress invariant is $-1 \leq \xi \leq 1$. In some papers [27], the normalized Lode angel denoted as $\bar{\theta}$ is used in the following form [26, 27]:

$$\bar{\theta} = 1 - \frac{60}{\pi} = 1 - \frac{2}{\pi} \arccos(\xi) \quad (10)$$

The values of the normalized Lode angel $\bar{\theta}$ should be included in the range $-1 \leq \bar{\theta} \leq 1$ [26, 27].

Bai and Wierzbicki [28] also used another definition of the Lode parameter, denoted as L , which may be presented in the following form [26]:

$$L = LODE = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} \quad (11)$$

where the σ_1 and σ_3 are the maximum and the minimum principal stresses respectively. In this paper, the Lode parameter L will be referred to as an expression $LODE$. The Lode parameter may be related to the normalized third stress invariant ξ through the following relationship [26, 27]:

$$\xi = L \frac{9 - L^2}{\sqrt{(L^2 + 3)^3}} \quad (12)$$

The parameters mentioned above can be successfully used to describe the level of triaxiality of stress near the crack tip [29], or calibration the constitutive equations, what was presented by Bao and Wierzbicki or Bai and Wierzbicki [28, 30, 31], where the yield stress denoted as σ_{yld} were defined according to the next formula [26, 27]:

$$\sigma_{yld} = \bar{\sigma}(\bar{\epsilon}_p) [1 - c_\eta (\eta - \eta_0)] \cdot \left[c_\theta^s + (c_\theta^{ax} - c_\theta^s) \left(\gamma - \frac{\gamma^{m+1}}{m+1} \right) \right] \quad (13)$$

where the function $\bar{\sigma}(\bar{\epsilon}_p)$ describes the relation between effective real stress and accumulated plastic strain; η_0 denotes the reference value of the triaxiality coefficient η (see Eq. (8) – it can be noted, that for the case of uniaxial tensile test, value of the η_0 is equal to 1/3 for cylindrical specimen); the parameters c_η , c_θ^s , c_θ^{ax} and m must be determined for specific material experimentally; the function γ may be calculated by Bai and Wierzbicki [28, 30, 31] formula (this function satisfies the inequality $0 \leq \gamma \leq 1$, and

$\gamma = 0$ for plane strain or pure shear states, or $\gamma = 1$ for axial symmetry) [26]:

$$\gamma = 6.464 \left[\sec \left(\theta - \frac{\pi}{6} \right) - 1 \right] \quad (14)$$

In the proposed formula (13) the expression appeared the function $\bar{\sigma}(\bar{\epsilon}_p)$, which “describes the relation between effective real stress and accumulated plastic strain” [29]. In this relationship, the function $\bar{\sigma}(\bar{\epsilon}_p)$ is the strain hardening term, which “represents the stress–strain curve in zero hydrostatic pressure, for example in the torsion test. In practice, tensile tests of a smooth round bar or a dog-bone specimen are very commonly used to calibrate the stress–strain curve” [28]. In other words, the $\bar{\sigma}(\bar{\epsilon}_p)$ function, is “material strain hardening function from the reference test” [28].

Information on other parameters of geometrical constraints, which can be considered as measures of stress triaxiality, can be found in [34–43].

3 Some aspects of numerical calculation (based on [4])

In the 3D Finite Element Method (FEM) numerical analysis, the geometry of the SEN(B) specimens used in the experimental program was accurately reproduced. All calculations were carried out assuming small deformations and small displacement. Calculations for the 3D case were also made with the help of the ADINA SYSTEM 8.4 program [32, 33], solving the contact problem. A quarter of the specimen was modelled [4]. The standard twenty-node three-dimensional finite elements (FEs) with 27 numerical integration points, assuming “MIXED” interpolation formulation in FE. The region close to crack tip, with a radius equal to $(0.5 \div 1)$ mm, was divided into $(36 \div 50)$ FEs, the smallest of which at the crack tip was $(20 \div 100)$ times smaller than the last. This meant that this element was $(1/2000 \div 1/10000)$ of the specimen width W , and the largest one modelling the crack tip region was $(1/100 \div 1/1000)$ of the specimen width. The crack tip was modelled as a quarter of an arc with a radius of $r_w = (1 \div 2) \cdot 10^{-6}$ m. It turned out to be 40000 times smaller than the specimen width. This arch is divided into 7 equal parts [4].

The thickness quadrant of the SEN(B) specimen was divided into nine layers, located in the following coordinates: $x_3/B = \{ 0; 0.119; 0.222; 0.309; 0.379; 0.434; 0.472; 0.483; 0.494; 0.5 \}$, whereas the thickness of the layers decreased in the direction from the specimen axis ($x_3/B = 0.0$) to the edge of the specimen ($x_3/B = 0.5$). It should be noted that the layer located in the specimen axis was

Figure 5: Exemplary numerical model of the SEN(B) specimen used in the FEM 3D calculations; b) A fragment of the mesh around the crack tip for the FEM 3D analysis case; (based on [4])

(20÷50) times larger than the layer on the edge of the specimen, which is associated with a large stress gradient in this area. In total, the specimen was modelled using 2488 FEs, which consisted of 12142 nodes [4].

All parameters and details of the numerical model (such as the radius of rounding at the crack tip, size of the FE, type of FE, division of the structural element into FEs, division in the direction of thickness) are the result of a number of tests carried out during calculations (attempts were made to determine such parameters that the results obtained coincide), as well as the guidelines contained in [8]. The calculation used a homogeneous, isotropic material model with the Huber-Misses-Hencky plasticity condition. The constitutive relationship describing the material analyzed is described by the relationship (1). The J -integral during calculations was determined using the virtual increase in crack length. It is the J -integral calculated in accordance with Guo's recommendations [25], i.e. it is the far field J -integral (J^{far}) determined using the integration contour passing through the area dominated by plane stress state. Figure 5a presents an example of the numerical model of the SEN(B) specimen used in the 3D FEM calculations [4].

4 Numerical results

The analysis of the obtained numerical results was carried out for such a load level that corresponds to the moment of reaching the critical point on the J - R graph, which is identified with the conventional moment of crack initiation. As part of the analysis, the distribution of the main compo-

nents of the stress tensor (which determine the value of normal stresses, values of stress triaxiality parameters, effective stress values) in a limited range of the normalized distance from the crack tip $\psi = \langle 0; 10 \rangle$ (where $\psi = r \cdot \sigma_0 / J$, where r is the physical distance of the measuring point from the crack tip) for each model layer. In addition to assessing the normal component distributions of the stress tensor, the analysis of the distribution of the mentioned effective stresses, mean stresses and selected measures of the level of stress triaxiality, listed in section 1.2 (among them the ratio of mean stress and effective stress, parameter T_z and parameter $LODE$) were done. The analysis of all these quantities was illustrated in graphical form using charts, which are shown below.

As can be seen, as the distance from the crack tip increases, the level of normal components of the stress tensor decreases (Figure 6) - as the crack tip approaches, the singularity in the stress distribution over the crack tip is observed. As expected, the highest normal components of the stress tensor reach in the specimen axis ($x_3/B = 0.000$) - Figure 6, where theoretically we are dealing with the dominance of the plane strain state. With the distance from the specimen axis, going towards the edge of the specimen ($x_3/B = 0.000$), the value of stress decreases, hypothetically striving to achieve the values characteristic of a plane stress state. Analyzing Figures 6c, 6f and 6i, it can be seen that the value of the component σ_{zz} on the edge of the specimen is almost 0, which is in line with expectations and confirms the correctness of the numerical calculations.

Figure 7 shows a summary of selected stress triaxiality measures. For the 145Cr6 steel considered in the paper, the level of effective stress (Figures 7a, 7d, 7g) at a nor-

Figure 6: The normal stress distributions near the crack tip for three SEN(B) specimens made of 145Cr6 steel: a-c) $B = 5\text{ mm}$, $a/W = 0.32$; d-f) $B = 10\text{ mm}$, $a/W = 0.35$; g-i) $B = 15\text{ mm}$, $a/W = 0.29$ ($x_3/B = 0.500$ – the edge of the specimen; $x_3/B = 0.000$ – the middle plane of the specimen thickness)

Figure 7: The distributions of the effective stress (σ_{eff}/σ_0), mean stress (σ_m/σ_0) and the ratio of the mean and effective stress (σ_m/σ_{eff}) – (which is also called as triaxiality parameter), near the crack tip for three SEN(B) specimens made of 145Cr6 steel: a-c) $B=5\text{mm}$, $a/W=0.32$; d-f) $B=10\text{mm}$, $a/W=0.35$; g-i) $B=15\text{mm}$, $a/W=0.29$ ($x_3/B=0.500$ – the edge of the specimen; $x_3/B=0.000$ – the middle plane of the specimen thickness)

Figure 8: The distributions of the T_z parameter defined by Guo Wanlin [23–25] and the $LODE$ parameter, near the crack tip for three SEN(B) specimens made of 145Cr6 steel: a-b) $B = 5\text{mm}$, $a/W = 0.32$; c-d) $B = 10\text{mm}$, $a/W = 0.35$; e-f) $B = 15\text{mm}$, $a/W = 0.29$ ($x_3/B = 0.500$ – the edge of the specimen; $x_3/B = 0.000$ – the middle plane of the specimen thickness)

malized distance from the crack tip $\psi = \langle 1; 3 \rangle$ (in which the maximum stress opening the crack surfaces generally occurs when analyzing large deformations and large displacements), is at the level of $\sigma_{eff}/\sigma_0 \approx 1.0$, in the layer thickness range of the specimen $x_3/B \leq 0.472$. As the specimen edge approaches, a local decrease in effective stress is observed, whose value in the considered range is about $\sigma_{eff}/\sigma_0 \approx 0.7$.

As can be seen in Figures 7a, 7d, 7g, in the presented range of normalized distances from the crack tip, the level of effective stress is generally almost $\sigma_{eff}/\sigma_0 \approx 1.0$, for layers characterized by the location $x_3/B \leq 0.472$. As the crack tip rises farther away, in each subsequent layer of the specimen thickness, the level of mean stress (also called hydrostatic stress) decreases, and as it approaches the edge of the specimen ($x_3/B = 0.500$), it tends to the

value of $\sigma_m/\sigma_0 = 0.5$ (see Figures 7b, 7e, 7h). Similar conclusions can be drawn from the analysis of Figures 7c, 7f, 7i, presenting changes in the ratio of average stresses and effective stresses σ_m/σ_{eff} , which results from the fact that the value of effective stresses generally for layers characterized by the location $x_3/B \leq 0.472$ almost is equal to the yield point. The average stress decreases as the specimen edge approaches ($x_3/B = 0.500$) – their largest value is observed in the specimen axis ($x_3/B = 0.000$), which results from the previously described normal components of the stress tensor.

The value of the T_z triaxiality stress parameter, which was defined by Guo Wanlin [23–25] - formula (3) decreases with distance from the crack tip, with the highest values being achieved in the layer located in the specimen axis ($x_3/B = 0.000$), and the lowest on the edge of the speci-

Figure 9: Changes in normal components of the stress tensor (σ_{yy}/σ_0 , σ_{zz}/σ_0 , σ_{xx}/σ_0) and selected measures of stress triaxiality parameters (σ_{eff}/σ_0 , σ_m/σ_0 , σ_m/σ_{eff} , T_z , LODE) along the crack front, for two selected distances from the crack tip ($r = 1.0 \cdot J/\sigma_0$ and ($r = 2.0 \cdot J/\sigma_0$)) for three SEN(B) specimens made of 145Cr6 steel: a-c) $B = 5\text{ mm}$, $a/W = 0.32$; d-f) $B = 10\text{ mm}$, $a/W = 0.35$; g-i) $B = 15\text{ mm}$, $a/W = 0.29$ ($x_3/B = 0.500$ – the edge of the specimen; $x_3/B = 0.000$ – the middle plane of the specimen thickness)

Figure 10: Comparison of: a) the effective stress (σ_{eff}/σ_0) and mean stress (σ_m/σ_0); along the crack front; b) stress triaxiality parameter (σ_m/σ_{eff}) and Guo Wanlin stress triaxiality parameter (T_z); c) stress triaxiality parameter (σ_m/σ_{eff}) and *LODE* parameter (*LODE*); for two selected distances from the crack tip ($r = 1.0 \cdot J/\sigma_0$ and $r = 2.0 \cdot J/\sigma_0$) for three SEN(B) specimens made of 145Cr6 steel ($x_3/B = 0.500$ – the edge of the specimen; $x_3/B = 0.000$ – the middle plane of the specimen thickness)

men, tending with the distance from the crack tip to the value of $T_z = 0$, which is characteristic for the dominance of the plane stress state (Figure 8a-8c). Quite interesting are the changes in the *LODE* parameter near the crack tip for the moment of crack initiation (Figure 8d-8f). The *LODE* parameter in close proximity to the crack tip $\psi < 2.0$, takes negative values, then increases, striving for a set value of $LODE \approx 0.20$. It should be noted that the value of the *LODE* parameter changes with the distance from the specimen axis ($x_3/B = 0.000$) – the closer the specimen edge ($x_3/B = 0.500$), the higher the value of the *LODE* parameter (Figure 8d-8f).

Figure 9 presents changes along the front of the crack, the quantities previously analyzed (normal components of the stress tensor, effective stresses, mean stresses and selected measures of stress triaxiality) for the moment of crack initiation, equated when the *J*-integral reaches the critical value denoted as J_C . These charts were prepared for two normalized distances from the crack tip $\psi = 1.0$ and $\psi = 2.0$, where $\psi = r \cdot \sigma_0/J_C$.

The analysis of Figures 9a, 9d and 9g leads to the following conclusions: the highest values near the crack tip are achieved by the stresses opening the crack surfaces, marked by σ_{zz} , and the smallest stresses by σ_{xx} . The further from the crack tip, the lower the stress value. Along the crack front, slight changes in effective stress are observed (Figures 9b, 9e, 9h), whose value is usually equal to the yield stress of the material and practical in the considered range $\psi = 1.0$ and $\psi = 2.0$ is independent of the

distance from the crack tip. The nature of changes in mean stress normalized by the yield stress or effective stress, presented in Figures 9b, 9e and 9h, coincides with the nature of changes in the normal components of the stress tensor – faster changes in the values of the parameters σ_m/σ_0 and σ_m/σ_{eff} are observed for specimens with smaller thickness $B = 5\text{mm}$. In the case of specimens with a thickness of $B = 15\text{mm}$, in the range of distance $x_3/B = \langle 0.0, 0.35 \rangle$ from the specimen axis, one can speak of an almost constant value of the parameters σ_m/σ_0 and σ_m/σ_{eff} (Figure 9h). Figures 9c, 9f and 9i show changes along the crack front of the T_z and *LODE* parameters. Faster changes in both parameters are observed for a specimen with thickness $B = 5\text{mm}$ (Figure 9c). Both parameters maintain a nearly constant value for the distance $x_3/B < 0.4$ from the specimen axis in the case of a specimen thickness $B = 15\text{mm}$ (Figure 9i). The further from the crack tip, the smaller the value of the T_z parameter and the greater the value of the *LODE* parameter (Figure 9c, 9f, 9i). A detailed analysis of mutual dependencies is left to the reader.

The analysis of the obtained numerical results indicates that in the considered thickness range of SEN(B) specimens, for the analyzed 145Cr6 steel, the effective stresses practically do not depend on the specimen thickness (Figure 10a). In close proximity to the specimen axis ($x_3/B < 0.1$), mean stresses also do not depend on the specimen thickness (Figure 10a). Moving along the front of the crack from the specimen axis ($x_3/B = 0.000$) to its edge ($x_3/B = 0.500$) is accompanied by the fact that the

mean stress value depends on the specimen thickness (Figure 10a-c) - the thicker the specimen, the greater the mean stress value. For the material considered in the paper, it can be stated also, that the T_z parameter along the front of the crack starting from the specimen axis is initially almost independent of thickness, however, after exceeding the position $x_3/B > 0.300$, the value of the T_z parameter begins to decrease and tends to zero, wherein with the thicker the specimen, the greater the value of the T_z parameter (Figure 10b). The thicker the specimen, the lower the $LODE$ parameter value (Figure 10c). For specimens of greater thickness, we should note about a constant value of the $LODE$ parameter along the front of the crack, for layers distant from the specimen axis that meet the condition $x_3/B < 0.450$.

5 Conclusions and summary

The paper presents a summary of experimental tests that allow assessing the fracture toughness of 145Cr6 steel (based on [3–5]), as well as analysis of the selected measures of stress triaxiality parameters. Based on the numerical calculations scheme presented in [4], the most important aspects of modeling three-dimensional problems in the field of fracture mechanics in the finite element method were indicated, details of the numerical model were given, and then the values were selected that will be analyzed in quantitative and qualitative assessment [4]. Based on the recommendations in the assessment of the measures of stress triaxiality parameters given in [16, 17, 26, 27], for three three-dimensional SEN(B) specimens with different thicknesses ($B = \{5, 10, 15\}$ mm) and relative crack length a $a/W \approx 0.30$, diagrams of normal components of the stress tensor, mean stresses, effective stresses and selected measures of stress triaxiality (T_z and $LODE$ parameters, quotient of mean stress and effective stress) were prepared. The moment when the value of the J -integral, considered as the crack driving force, reaches the critical value J_C , determined by the ASTM standard [7], was evaluated. Changes in these quantities were analyzed along with the distance from the crack tip, as well as changes in these sizes along the crack front for two normalized distances from the crack tip – $\psi = 1.0$ and $\psi = 2.0$ – for these distances O'Dowd and Shih [14, 15] recommended determination of Q -stress, while in [8] a catalogue of numerical solutions is given in the scope of changes in the parameters of geometrical constraints – T_z and Q_m^* parameters, which were discussed in section 1.1.

The numerical analysis and evaluation of the obtained results, lead to the following conclusions:

- the increase in distance from the crack tip is accompanied by a decrease in the values of stress tensor components, mean stresses, effective stresses and the T_z parameter – while the value of the $LODE$ parameter increases in the considered range of distances from the crack tip;
- among the normal components of the stress tensor, the highest value is observed for the component opening the crack surfaces, and the lowest for the stresses acting along the thickness of the specimen;
- the increase in specimen thickness is usually accompanied by an increase in the value of normal stresses, mean stresses or the T_z parameter;
- effective stresses are not sensitive to changes in specimen thickness;
- an increase in specimen thickness reduces the value of the $LODE$ parameter in the same normalized position relative to the crack tip and relative to the specimen axis;
- smaller thickness of the specimen, means that the determined along the cracks front quantities (stress components, stress triaxiality parameters) change faster – in the case of large thicknesses ($B = 15$ mm), starting from the specimen axis, initially the stability of the determined values is observed, after which the change (increase / decrease) occurs in the layer characterized by relative position $x_3/B > 0.400$.

In summary, carrying out a comprehensive numerical analysis of the mechanical fields near crack tip, should include not only an analysis of stress tensor component distribution, but also an assessment of stress triaxiality parameters. The stress triaxiality parameters can be considered as the as measures of geometric constraints [34–43], affecting the distribution of mechanical fields near the crack tip, and but real fracture toughness. Such an analysis should be carried out in a quantitative and qualitative way, and all the conclusions drawn can be used in the correct description of mechanical fields, or in predicting actual fracture toughness using proper fracture criteria [1, 2, 9, 10, 12, 25] or assessing the strength of a structure containing certain defects, using FAD (Failure Assessment Diagrams) or CDF (Crack Driving Force) diagrams [1, 2, 11, 12].

Acknowledgement: The research reported herein was supported by a grant from the Faculty of Mechatronics and Machine Design at Kielce University of Technology (project No. 01.0.09.00/2.01.01.00.0000 SUBB.MKTM.20.002).

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