Evaluation of measurement uncertainty in a static tensile test

Abstract: This article presents a scheme of the procedure for assessing the measurement uncertainty of material constants determined in the static tensile test. The key analysis was preceded by a discussion of the concept of uncertainty of measurement, equated with absolute error, the concept of relative error and methods of assessment of these errors, depending on the type of mutual dependence of the analyzed quantity and the measured quantities. The next step in this article shows examples of stress–strain curves, which were used to calculate errors, as well as specific material constants. By importing the spreadsheets of the Mathcad package, which was used in the analysis, the procedure for assessing the measurement uncertainty was specified.

Keywords: static tensile test, measurement uncertainty, absolute error, relative error, material constant, mechanical properties

1 Introduction

The basic test in assessing the strength of construction materials is a static tensile test [1,2], which provides basic information about the material constants required to describe the properties of the material or structural elements during strength analyzes, carried out either by analytical or numerical method. These parameters include Young’s modulus E, Poisson’s ratio ν, yield strength σ₀, tensile strength σₚ, breaking stress σᵤ, strain hardening constant a and strain hardening exponent n in Ramberg–Osgood (RO) or other power law used to describe the stress–strain curve in its actual approach, in the range from the yield strength from refs. [1,2]. The previous studies [3–5] give various aspects of the use of material constants determined during a static tensile test. Having the yield point and tensile strength specified in the right way and with accurate accuracy, we can determine the power exponent in the R–O law and for materials with a clear yield point (i.e., the yield point elongation or discontinues the yield point) the length of the plasticity stop (i.e., lower yield or Luder’s strain) [3–5]. Procedures [3–5] allow estimating almost the entire stress–strain curve, based only on the yield strength, using the empirical formulas given in these procedures.

However, none of these documents recommended for use during static tensile tests [1,2] or used for assessing the strength of various structural elements [3–5] that mention about the measurement uncertainty. As we know, each measurement is made with certain accuracy. In the first definition, measurement inaccuracy is resulting from a specific measuring scale of a measuring tool, and it can be considered as measurement uncertainty. This uncertainty is reduced by using more accurate methods of measuring specific quantities, but it can never be eliminated completely. Therefore, in the first approximation, referring to direct measurements, the measurement uncertainty ΔW of the measured value denoted by W can be understood as the accuracy of the measuring tool. In many previous studies refs. [6–8], the measurement uncertainty is referred to as the absolute error. While in the case of measuring a single quantity, there is no problem with the interpretation of the absolute error, in the case of measuring the size W, which is the algebraic sum of other approximate quantities Wᵢ:

\[ W = W₁ + W₂ + ... + Wₙ, \]  

(1)

Absolute error is the sum of the absolute errors of its individual components:

\[ ΔW = ΔW₁ + ΔW₂ + ... + ΔWₙ. \]  

(2)

However, this method may exaggerate the importance of individual measurement errors. Therefore, when calculating the absolute error, partial error compensation of different signs should be taken into account:

\[ ΔW = \sqrt{(ΔW₁)^2 + (ΔW₂)^2 + ... + (ΔWₙ)^2}. \]  

(3)
The basic science of measurement introduces the concept of relative uncertainty $\delta W$, which is the quotient of the measurement uncertainty $\Delta W$ and the value of the measured value $W$:

$$\delta W = \frac{\Delta W}{W}. \quad (4)$$

From the previous studies refs. [6–8], one can introduce the concept of the relative error of the approximate value of $W$, which is the ratio of the absolute error $\Delta W$ to the absolute value of $W$:

$$\delta W = \frac{\Delta W}{|W|}. \quad (5)$$

It should be noted that the relative error of the difference in positive numbers is greater than the relative errors of these numbers, especially when these numbers differ very little from each other [6–8].

There is often a situation in the laboratory where the measurement result is a function of independent variables that are multiplied or divided. Then, the relative errors of these numbers should be added up. If we are dealing with an expression that allows calculating the quantity $f$,

$$f = \frac{W_1 \cdot W_2 \ldots W_m}{Z_1 \cdot Z_2 \ldots Z_n}, \quad (6)$$

and the relative error of the expression should be estimated as follows:

$$\delta f = \delta W_1 + \delta W_2 + \ldots + \delta W_m + \delta Z_1 + \delta Z_2 + \ldots + \delta Z_n. \quad (7)$$

In the case of such a large number of variables, we must also compensate for errors with different signs, using the relationship:

$$\delta f = \sqrt{\left(\delta W_1\right)^2 + \left(\delta W_2\right)^2 + \ldots + \left(\delta W_m\right)^2 + \left(\delta Z_1\right)^2 + \left(\delta Z_2\right)^2 + \ldots + \left(\delta Z_n\right)^2}. \quad (8)$$

The absolute error of the functions of the variables $f$ ($W_i$) is calculated a little differently, $i = 1, 2, \ldots, n$, which is differentiable in some area, assuming that this error is caused by small argument errors. Then, the total differential method should be used, according to the scheme (9):

$$\Delta W = \left|\frac{\partial W}{\partial W_1}\right| \Delta W_1 + \left|\frac{\partial W}{\partial W_2}\right| \Delta W_2 + \ldots + \left|\frac{\partial W}{\partial W_n}\right| \Delta W_n. \quad (9)$$

However, the aforementioned method exaggerates the importance of individual measurement errors. Ref. [7] proposes a method of considering compensation of individual errors. Based on the relationships given earlier, the value of the absolute error should be calculated as follows [6–8]:

$$\Delta W = \sqrt{\left(\frac{\partial W}{\partial W_1} \cdot \Delta W_1\right)^2 + \left(\frac{\partial W}{\partial W_2} \cdot \Delta W_2\right)^2 + \ldots + \left(\frac{\partial W}{\partial W_n} \cdot \Delta W_n\right)^2}. \quad (10)$$

The introduction of error calculus to engineering issues requires an appropriate record of the measured quantities, which should be given taking into account the absolute error value $\Delta W$ and a commentary on the relative error made (given in percentage).

2 Experimental data used in the paper, determination of material constants in a static tensile test

Returning to the first paragraph of this article in Section 1, this article will familiarize the reader with the subject of estimating measurement errors for quantities determined in a static tensile test. The study will use research material collected and presented in refs. [9–12], which will concern the five different stress–strain curves. Figure 1 presents curves recorded during the experimental tests, illustrating the change of force acting on the specimen as a function of extensometer displacement, and Figure 2 presents determined engineering tensile curves in the
stress system as a function of deformation. The same measurement base \( l_0 = 50 \text{ mm} \) was used for all tests carried out. Table 1 presents the selected quantities necessary to determine selected material constants measured during experimental tests [9–12], and Table 2 presents the selected mechanical quantities determined in accordance with applicable standards.

Determination of Young’s modulus \( E \), Poisson’s ratio \( \nu \), yield stress \( \sigma_0 \) and corresponding strains \( \varepsilon_0 \), tensile strength \( \sigma_m \) and corresponding strains \( \varepsilon_m \), breaking stress \( \sigma_u \) and corresponding strains \( \varepsilon_u \), or elongation at break \( A_e \) raises no doubts, and it is described in detail in the standards [1,2]. Due to the simplicity of these analyzes, this will not be discussed in this article. However, due to the fact that the stress–strain curve sometimes requires that it be described by appropriate constitutive relationships, which is required in the case of analysis of strength of materials based on appropriate hypotheses, here the method of estimation of the strain hardening exponent \( n \) in the Ramberg–Osgood law will be indicated. In a general form, Ramberg–Osgood law is expressed as follows:

\[
\varepsilon / \varepsilon_0 = \alpha / \sigma_0 + \alpha \cdot (\sigma / \sigma_0)^n,
\]

(11)

where \( \sigma_0 \) is the yield point (\( R_e \) or \( R_{0.2} \)); \( \varepsilon_0 \) is the strain corresponding to the yield point \( (\varepsilon_0 = \sigma_0 / E) \); \( E \) is Young’s module; \( \alpha \) is a constant, referred to as the strain hardening constant; \( n \) is the power exponent, referred to as the strain hardening factor (i.e., strengthening factor) or strain hardening exponent (strengthening exponent).

The full analysis of stress distribution and strains near the front of crack in a nonlinear material, carried out by Hutchinson [13], was based on the constitutive relationship, which is a three-dimensional generalization of equation (11). However, in a number of articles on fracture mechanics and analysis of stress and strain fields near the crack tip [14–18], the elastic–plastic material is described by another version of equation (11):

\[
\begin{align*}
\frac{\varepsilon}{\varepsilon_0} &= \frac{\sigma}{\sigma_0} & \text{for } \sigma \leq \sigma_0 \\
\frac{\varepsilon}{\varepsilon_0} &= \alpha \cdot \left(\frac{\sigma}{\sigma_0}\right)^n & \text{for } \sigma > \sigma_0
\end{align*}
\]

(12)

In the aforementioned power laws (11) and (12), often in engineering calculations, the strain hardening constant \( \alpha \) is taken as equal to unity. The degree of material hardening is then determined only on the basis of the strain hardening exponent \( n \). Figure 3 shows the

![Figure 2: Engineering tensile diagrams \( \sigma = f(\varepsilon) \) for materials used in the analysis (based on refs. [9–12]).](image)

**Table 1:** The quantities necessary to determine selected material constants measured during experimental tests [9–12] (selected results)

<table>
<thead>
<tr>
<th>Measured value</th>
<th>Steel symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma ) ( \text{[MPa]} )</td>
<td>( \varepsilon ) ( \text{[mm/mm]} )</td>
</tr>
<tr>
<td>( E ) ( \text{[GPa]} )</td>
<td>( \nu )</td>
</tr>
<tr>
<td>207.12</td>
<td>0.3</td>
</tr>
<tr>
<td>219.00</td>
<td>0.3</td>
</tr>
<tr>
<td>208.00</td>
<td>0.3</td>
</tr>
</tbody>
</table>

**Table 2:** The quantities necessary to determine selected material constants measured during experimental tests [9–12] (selected results)

<table>
<thead>
<tr>
<th>Measured value</th>
<th>Steel symbol</th>
<th>145Cr6</th>
<th>S355J2</th>
<th>41Cr4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E ) ( \text{[GPa]} )</td>
<td>( \nu )</td>
<td>( \sigma_0 ) ( \text{[MPa]} )</td>
<td>( \varepsilon_0 )</td>
<td>( \sigma_m ) ( \text{[MPa]} )</td>
</tr>
<tr>
<td>207.12</td>
<td>0.3</td>
<td>934.08</td>
<td>0.00451</td>
<td>1044.21</td>
</tr>
<tr>
<td>219.00</td>
<td>0.3</td>
<td>400.18</td>
<td>0.01572</td>
<td>548.31</td>
</tr>
<tr>
<td>208.00</td>
<td>0.3</td>
<td>473.06</td>
<td>0.00224</td>
<td>703.33</td>
</tr>
</tbody>
</table>
differences in model curves described by equations (11) and (12) [19].

If the R–O law described by the formula (11) or power law (12) is used to describe the stress–strain curve, then using any computer program, we can use the least squares method to adjust the parameters \( a \) and \( n \) in obtaining the best possible convergence of the presented equation and experimental results according to the proper criterion. This adjustment of the measured points can be done for “reasonably selected” points from the beginning of the recorded experimental curve until the maximum on the experimental curve is reached. Simple methods of analytical determination of parameters \( a \) and \( n \) are also possible, which was presented in ref. [19].

It turns out that the “reasonable choice” of points used for approximation is becoming crucial. It may depend on several factors, among which following are the most important [19]:

(a) The purpose and scope of the analysis (small or large deformations, the advantage of elastic or plastic deformations)
(b) The nature of the stress–strain curve (explicit or arbitrary yield point).

Analyzes carried out years ago ref. [19] showed that the method of selecting points used for approximation significantly changes the obtained values of \( a \) and \( n \). Therefore, it is important to determine the criterion for choosing this “proper” pair \( a \) and \( n \). By testing several different options for selecting points for approximation, suggestions were made as to the selection of the best option based on comparing the results of numerical calculations using the real uniaxial stress–strain curve and the model curve for different pairs, considering the \( J = f(P/P_0) \) curves, stresses in front of the crack at a distance \( r \) equal to \( r = 2.0J/\sigma_0 \), averaged stresses in front of the crack after a distance from od \( 1.0J/\sigma_0 \) to \( 6.0J/\sigma_0 \) and the comparison of the real \( \sigma = f(\varepsilon) \) curve with the model curve [19] (Figure 4).

As a result, the basic conclusion of ref. [19] is the recommendation according to which material constants for the use of power law (12) should be determined when the strain hardening constant \( \alpha = 1 \), looking for the strain hardening exponent \( n \) based on the point corresponding to the tensile strength \( \sigma_m \) [19]. Thus, assuming \( \alpha = 1 \) and then transforming the relationship (12), we obtain the formula that allows us to estimate the strain hardening exponent \( n \), which was used for this article:

\[
   n = \left( \ln \frac{\varepsilon_m}{\varepsilon_0} \right) / \left( \ln \left( \frac{\sigma_m}{\sigma_0} \right) \right). \tag{13}
\]

Section 3 presents a method of assessing relative and absolute errors for material constants and characteristic points determined in a static tensile test (Figure 5).

3 Analysis of error calculus of quantities determined in a static tensile test

Among the quantities that were analyzed and for which it was decided to estimate the absolute error and the relative error include:

- Yield stress: \( \sigma_0 = F_0/S_0 \).
- Ultimate strength: \( \sigma_m = F_m/S_0 \).
- Breaking stress: \( \sigma_u = F_u/S_0 \).
- Strain corresponding to: yield stress \( \varepsilon_0 \), ultimate strength \( \varepsilon_m \) and breaking stress \( \varepsilon_u \) (the general deformation formula
will be used $\varepsilon = (l - l_0)/l_0$, where the actual value of the specimen length will be denoted by $l$.

- Strain hardening exponent $n$ in the R–O law, which will be calculated using equation (13).
- Elongation at break denoted by $A_{\text{sa}}$ using the general formula for elongation $A = l - l_0$.

The calculation of errors was carried out both in the assessment of the absolute error level $\Delta W$ and in the assessment of the relative error of the measured values $\delta W$. Hence, in this study, in the first approximation, it was decided to estimate the absolute error $\Delta W$ according to formula (9) and the relative error $\delta W$ according to formula (5) based on the complete differential method. In the second approximation, the absolute error $\Delta W$ was estimated according to the relationship (10), taking into account the compensation of individual errors.

When analyzing the material presented in paragraph 1 in Section 1 of this article, it may seem that the error calculation is not a complicated process, but it requires an appropriate level of mathematical knowledge that guarantees correct conduct in the field of differential calculus, which is a rather time-consuming process. However, by using the right tools to support the work of the engineer – the Mathematica package or the Mathcad package, the analysis time is significantly reduced. In this article, the error calculation was carried out using the Mathcad package. The calculation scheme will be presented later for the selected parameters – only for quantities that are stress. All results of the calculation are presented in the following section in a graphic form.

### 3.1 The scheme for evaluating the measurement uncertainty for a quantity being stress

![Figure 5: Comparison of the selected strain values determined in the static tensile test and elongation $A_0$ for all steels used in the analysis.](image)

Estimation of the measurement uncertainty (absolute and relative error) of the stress value – a rectangular specimen (a part of the Mathcad spreadsheet):

**Basic equations:**

\[
S_0 = a_0 \cdot b_0 \quad \sigma = F_{S0}
\]

\[
\delta \sigma_F = \frac{d \sigma}{d F} \quad \delta \sigma_{a_0} = \frac{d \sigma}{d a_0} \quad \delta \sigma_{b_0} = \frac{d \sigma}{d b_0}
\]

\[
\delta \sigma_F = \frac{1}{a_0 \cdot b_0} \quad \delta \sigma_{a_0} = -\frac{F}{a_0^2 \cdot b_0} \quad \delta \sigma_{b_0} = -\frac{F}{a_0 \cdot b_0^2}
\]

Absolute error $\Delta \sigma$ using equation (9):

\[
\Delta \sigma = |\delta \sigma_F| \cdot |\Delta F| + |\delta \sigma_{a_0}| \cdot |\Delta a_0| + |\delta \sigma_{b_0}| \cdot |\Delta b_0|
\]

\[
\Delta \sigma = \frac{\Delta F}{|a_0 \cdot b_0|} + \frac{\Delta a_0 \cdot |F|}{(|a_0|)^2 \cdot |b_0|} + \frac{\Delta b_0 \cdot |F|}{|a_0| \cdot (|b_0|)^2}
\]

Relative error $\delta \sigma$ using equation (5):

\[
\delta \sigma = \frac{\Delta \sigma}{|\sigma|} = \frac{|a_0| \cdot |b_0|}{|F|} \left( \frac{\Delta F}{|a_0 \cdot b_0|} + \frac{\Delta a_0 \cdot |F|}{(|a_0|)^2 \cdot |b_0|} + \frac{\Delta b_0 \cdot |F|}{|a_0| \cdot (|b_0|)^2} \right)
\]

Absolute error $\Delta \sigma$ including compensation of individual errors of component quantities according to the formula (10):

\[
\Delta \sigma = \sqrt{(\delta \sigma_F \cdot \Delta F)^2 + (\delta \sigma_{a_0} \cdot \Delta a_0)^2 + (\delta \sigma_{b_0} \cdot \Delta b_0)^2}
\]

\[
\Delta \sigma = \sqrt{\frac{\Delta F^2}{a_0^2 \cdot b_0^2} + \frac{F^2 \cdot \Delta a_0^2}{a_0^4 \cdot b_0^2} + \frac{F^2 \cdot \Delta b_0^2}{a_0^2 \cdot b_0^4}}
\]

Relative error $\delta \sigma$ using equation (5) on taking into account the compensation of individual component errors (based on equation (10)):

\[
\delta \sigma = \frac{\Delta \sigma}{|a|} = \frac{|a_0| \cdot |b_0|}{|F|} \left( \frac{\Delta F}{a_0 \cdot b_0} + \frac{F^2 \cdot \Delta a_0^2}{a_0^3 \cdot b_0^2} + \frac{F^2 \cdot \Delta b_0^2}{a_0^2 \cdot b_0^3} \right)
\]

### 3.2 Results of the measurement errors analysis

Uncertainty of the performed measurements (see data given in Tables 1 and 2) – estimated according to the
The measurement error generally does not exceed 0.25%.

4 Discussion

The error values calculated using the compensation method are generally slightly lower than the error values calculated using the total differential method. Despite the low accuracy of the selected measuring tools, the absolute and relative error values obtained can be considered acceptable. Certainly, the introduction of error calculations into engineering practice in the field of laboratory measurements may seem natural; however, as noted at the beginning of this article, this is not shown in scientific papers. Although such analyzes are found in typical papers in the field of metrology of geometrical quantities and angle, they are difficult to find in scientific papers dealing with the subject of determining physical quantities considered as material constants or material characteristics. Perhaps this is due to the fact that the entire analysis scheme is based on painstaking calculations in

![Figure 6: Comparison of the absolute errors Δσ for selected stress values determined in the static tensile test using equation (9).](image-url)

<table>
<thead>
<tr>
<th>Measured value</th>
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<tbody>
<tr>
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</tr>
<tr>
<td>σ₀ (MPa)</td>
<td>934.08</td>
<td>400.18</td>
<td>473.06</td>
</tr>
<tr>
<td>Δσ₀ (MPa) (Equation (9))</td>
<td>1.867</td>
<td>2.400</td>
<td>2.838</td>
</tr>
<tr>
<td>δσ₀ (Equations (9) and (5))</td>
<td>0.20%</td>
<td>0.60%</td>
<td>0.60%</td>
</tr>
<tr>
<td>Δσ₀ (MPa) (Equation (10))</td>
<td>1.867</td>
<td>2.040</td>
<td>2.412</td>
</tr>
<tr>
<td>δσ₀ (Equations (10) and (5))</td>
<td>0.20%</td>
<td>0.51%</td>
<td>0.51%</td>
</tr>
<tr>
<td>A₀ (mm)</td>
<td>5.31373</td>
<td>10.37424</td>
<td>7.65000</td>
</tr>
<tr>
<td>ΔA₀ (mm) (Equation (9))</td>
<td>1.00 × 10⁻²</td>
<td>1.00 × 10⁻²</td>
<td>1.00 × 10⁻²</td>
</tr>
<tr>
<td>δA₀ (Equations (9) and (5))</td>
<td>0.188%</td>
<td>0.096%</td>
<td>0.131%</td>
</tr>
<tr>
<td>ΔA₀ (mm) (Equation (10))</td>
<td>1.00 × 10⁻²</td>
<td>1.00 × 10⁻²</td>
<td>1.00 × 10⁻²</td>
</tr>
<tr>
<td>δA₀ (Equations (10) and (5))</td>
<td>0.188%</td>
<td>0.096%</td>
<td>0.131%</td>
</tr>
</tbody>
</table>

Table 3: Determined in accordance with the method presented in paragraph 3 and absolute and relative error values of selected quantities determined during a static tensile test (selected results of the calculations)
the field of differential calculus, and this seems to be time consuming and labor intensive.

However, it is suggested that the research practice be consistent with the canons of science to give results of error calculus, which certainly says a lot about the accuracy of measuring physical quantities. The analysis can be simplified by using the Mathcad package or equivalent to automate calculations, as shown in this article. The authors do not exclude the development of an application that allows estimating the level of uncertainty in a static tensile test after entering the relevant data. The author of this article intends to refer the presented method of estimating measurement errors to other tests in the field of structural strength – for example, fracture mechanics or material fatigue.
That is why, in the Appendix, the author shows the method of determining the measurement uncertainty of the hardening exponent $n$ in the power law (equation (12)). In the future, author plans to include the impact of measurement error of the strain hardening exponent in the power law in the FEM calculations in the field of elastic-plastic fracture mechanics.

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**Conflict of interest:** The author declares no conflict of interest.

**References**

Appendix A: Calculation of the uncertainty of the strain hardening exponent

It seems much more complicated to estimate the uncertainty of measurement of a power exponent in the RO law (equation (11)) or in the power law described by the formula (12), which, taking into account the assumption that the strengthening constant $\alpha = 1$, is quite often used for approximation stress–strain curve by many researchers, including the author of this publication. Generally, based on two points—the yield strength and tensile strength (as well as the corresponding deformations), the strengthening exponent $n$ assuming that the constant $\alpha = 1$ is determined using formula (13). The determined value of the strain hardening exponent $n$ determines the tensile curve used in the FEM numerical calculations, which in turn affects, as has already been mentioned, the value of the $J$-integral calculated numerically, or the level of stress near the crack tip, if you consider issues in the field of fracture mechanics, as shown in ref. [19].

In view of this fact, it was decided to include in the Appendix the calculation scheme carried out in the Mathcad package, which allows estimating the uncertainty of the measurement of the strain hardening exponent $n$ in the power law described by the formula (12).

By using the “solve” function available in Mathcad, we automatically obtain the formula (13), using the second part of the formula (13), applicable for points above the yield point on the real stress–strain curve (below are excerpts of the calculation sheet from the MathCad package):

$$n = \frac{\varepsilon_m}{\varepsilon_0} = \left(\frac{\sigma_m}{\sigma_0}\right)^n \quad \text{solve,} \quad n = \frac{\ln\left(\frac{\varepsilon_m}{\varepsilon_0}\right)}{\ln\left(\frac{\sigma_m}{\sigma_0}\right)}.$$

As we can see, the exponent $n$ is a function of four variables ($\sigma_0$, $\sigma_m$, $\varepsilon_0$, and $\varepsilon_m$). In view of this fact, material constants determined for the steels used in refs. [9–12] will be used in further analysis, as well as the corresponding errors generally indicated as $\Delta W$, which was shown earlier. In the next step, according to the theory in paragraph 1, the partial derivatives will be estimated:

$$\delta n_{\varepsilon_0} = \frac{d}{d\varepsilon_0} - n \quad \delta n_{\varepsilon_m} = \frac{d}{d\varepsilon_m} - n \quad \delta n_{\sigma_0} = \frac{d}{d\sigma_0} - n \quad \delta n_{\sigma_m} = \frac{d}{d\sigma_m} - n,$$

which can be written as follows:

$$\delta n_{\varepsilon_0} = -\frac{1}{\varepsilon_0 \cdot \ln\left(\frac{\sigma_m}{\sigma_0}\right)} \quad \delta n_{\varepsilon_m} = \frac{1}{\varepsilon_m \cdot \ln\left(\frac{\sigma_m}{\sigma_0}\right)}$$

$$\delta n_{\sigma_0} = \frac{\ln\left(\frac{\varepsilon_m}{\varepsilon_0}\right)}{\sigma_0 \cdot \ln\left(\frac{\sigma_m}{\sigma_0}\right)^2} \quad \delta n_{\sigma_m} = -\frac{\ln\left(\frac{\varepsilon_m}{\varepsilon_0}\right)}{\sigma_m \cdot \ln\left(\frac{\sigma_m}{\sigma_0}\right)^2}.$$

In the next step, we can save the final formulas for relative and absolute errors made in determining the exponent $n$, both without and with compensation for component errors:

- Relative error (equation (9)):

$$\delta n = \frac{\Delta n}{|n|} = \left|\ln\left(\frac{\sigma_m}{\sigma_0}\right)\right| \cdot \left(\frac{\Delta \varepsilon_0}{\varepsilon_0} \cdot |\varepsilon_0| + \frac{\Delta \varepsilon_m}{\varepsilon_m} \cdot |\varepsilon_m| + \frac{\Delta \sigma_0}{\sigma_0} \cdot \left(\ln\left(\frac{\sigma_m}{\sigma_0}\right)\right)^2 \cdot |\sigma_0| + \frac{\Delta \sigma_m}{\sigma_m} \cdot \left(\ln\left(\frac{\sigma_m}{\sigma_0}\right)\right)^2 \cdot |\sigma_m|\right|.$$  

- Absolute error (equations (9) and (5)):

$$\delta n = \frac{\Delta n}{|n|} = \left|\ln\left(\frac{\sigma_m}{\sigma_0}\right)\right| \cdot \left(\frac{\Delta \varepsilon_0}{\varepsilon_0} \cdot |\varepsilon_0| + \frac{\Delta \varepsilon_m}{\varepsilon_m} \cdot |\varepsilon_m| + \frac{\Delta \sigma_0}{\sigma_0} \cdot \left(\ln\left(\frac{\sigma_m}{\sigma_0}\right)\right)^2 \cdot |\sigma_0| + \frac{\Delta \sigma_m}{\sigma_m} \cdot \left(\ln\left(\frac{\sigma_m}{\sigma_0}\right)\right)^2 \cdot |\sigma_m|\right|.$$  

- Relative error, taking into account individual errors of measured quantities (equation (10)):

$$\Delta n = \sqrt{\left(\delta n_{\varepsilon_0} \cdot \Delta \varepsilon_0\right)^2 + \left(\delta n_{\varepsilon_m} \cdot \Delta \varepsilon_m\right)^2 + \left(\delta n_{\sigma_0} \cdot \Delta \sigma_0\right)^2 + \left(\delta n_{\sigma_m} \cdot \Delta \sigma_m\right)^2}$$

$$\Delta n = \sqrt{\frac{\Delta \varepsilon_0^2}{\varepsilon_0^2} \cdot \ln\left(\frac{\sigma_m}{\sigma_0}\right)^2 + \frac{\Delta \varepsilon_m^2}{\varepsilon_m^2} \cdot \ln\left(\frac{\sigma_m}{\sigma_0}\right)^2 + \frac{\Delta \sigma_0^2}{\sigma_0^2} \cdot \ln\left(\frac{\sigma_m}{\sigma_0}\right)^2 + \frac{\Delta \sigma_m^2}{\sigma_m^2} \cdot \ln\left(\frac{\sigma_m}{\sigma_0}\right)^2}.$$
• Absolute error, taking into account individual errors of measured quantities (equations (10) and (5)):

\[
\delta n = \left| \frac{\Delta n}{|n|} \right| = \left| \frac{\Delta n}{|n|} \right| = \left| \frac{\ln \left( \frac{\sigma_m}{\sigma_0} \right)}{\ln \left( \frac{\sigma_0}{\sigma_0} \right)} \right|
\]

By using the algorithm shown earlier, for the steels considered in refs. [9–12], all absolute and relative error values were determined, and then, they were summarized in Table A1 and illustrated in Figure A1.

As can be seen, absolute and relative errors, estimated taking into account the compensation of individual errors of measured quantities (formulas (10) and (5)), are almost half smaller than errors determined in accordance with formulas (9) and (5). It can be stated that the higher the value of the strain hardening exponent (i.e., the lower the level of material hardening), the greater the measurement uncertainty value. The more brittle the material—the small value of the strain hardening exponent \( n \), the smaller the uncertainty of measurement of this quantity. It can be arbitrarily stated that for the materials considered in this article, tested in the same laboratory conditions, with the same instruments (testing machine, extensometer, calliper), smaller errors in the determination of the strain hardening exponent \( n \) depending on the tensile curve obtained in the laboratory are observed for strongly hardening materials. An increase in the value of the strain hardening exponent results in a decrease in the hardening of the material and generates an increase in measurement uncertainty when determining this quantity.

The analysis of the assessment of measurement uncertainty in determining the strain hardening exponent \( n \), carried out by the author for the purposes of this article, for the materials considered in this article, shows that for all steels tested in this article, the stress–strain curve may differ by several percent (Figures A2 and A3). In the summary presented in Figures A2 and A3, the difference between the model real curve determined for the estimated value of the strain hardening exponent and the curves drawn based on the strain hardening exponent reduced or increased by the value of absolute error, corresponding to the tensile strength, is more than 2%.

Table A1: The absolute and relative error values of strain hardening exponent \( n \) for steels used in this article

<table>
<thead>
<tr>
<th>Steel</th>
<th>( n )</th>
<th>( \Delta n ) (Equations (9) and (5))</th>
<th>( \delta n ) (%)</th>
<th>( \Delta n ) (Equations (10) and (5))</th>
<th>( \delta n ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>165Cr6</td>
<td>27.43</td>
<td>1.404</td>
<td>5.119</td>
<td>0.803</td>
<td>2.926</td>
</tr>
<tr>
<td>S355J2</td>
<td>6.90</td>
<td>0.309</td>
<td>4.48</td>
<td>0.163</td>
<td>2.367</td>
</tr>
<tr>
<td>41Cr4</td>
<td>9.69</td>
<td>0.525</td>
<td>5.415</td>
<td>0.286</td>
<td>2.950</td>
</tr>
<tr>
<td>14MoV63</td>
<td>9.16</td>
<td>0.232</td>
<td>2.536</td>
<td>0.154</td>
<td>1.676</td>
</tr>
<tr>
<td>X30Cr13</td>
<td>4.11</td>
<td>0.085</td>
<td>2.061</td>
<td>0.054</td>
<td>1.306</td>
</tr>
</tbody>
</table>

Figure A1: Comparison of the absolute and relative errors (denoted as \( \Delta n \) and \( \delta n \), respectively) for strain hardening exponent \( n \), for all steels used in this article.

Appendix B: A list of formulas

helpful in assessing the measurement uncertainty

Appendix B presents a list of all formulas derived during the preparation of this article, which may prove helpful
Table B1: Determination of the absolute and relative error values of strain hardening exponent \( n \) for steels used in this article

## Estimation of the measurement uncertainty of the stress quantity \( \sigma \) for a specimen with a rectangular cross-section

**Equations (9) and (5)**

\[
\Delta \sigma = \frac{\Delta \sigma_1}{|O_1|} + \frac{\Delta \sigma_2}{|O_2|} + \frac{\Delta \sigma_3}{|O_3|}
\]

\[
\delta \sigma = \sqrt{\frac{(\Delta \sigma_1)^2}{|O_1|^2} + \frac{(\Delta \sigma_2)^2}{|O_2|^2} + \frac{(\Delta \sigma_3)^2}{|O_3|^2}}
\]

**Equations (10) and (5)**

\[
\Delta \sigma = \sqrt{\left(\frac{\Delta \sigma_1^2}{|O_1|^2} + \frac{\Delta \sigma_2^2}{|O_2|^2} + \frac{\Delta \sigma_3^2}{|O_3|^2}\right)}
\]

\[
\delta \sigma = \frac{\Delta \sigma}{|F|}
\]

## Estimation of the measurement uncertainty of the stress quantity \( \sigma \) for a specimen with a circular cross-section

**Equations (9) and (5)**

\[
\Delta \sigma = \frac{4 \cdot \Delta \sigma_1}{\pi (|O_1|)^2} + \frac{8 \cdot \Delta \sigma_2}{\pi (|O_2|)^2}
\]

\[
\delta \sigma = \frac{\Delta \sigma}{|F|}
\]

**Equations (10) and (5)**

\[
\Delta \sigma = 4 \sqrt{\left(\frac{\Delta \sigma_1^2}{|O_1|^2} + \frac{\Delta \sigma_2^2}{|O_2|^2}\right)}
\]

\[
\delta \sigma = \frac{\Delta \sigma}{|F|}
\]

## Estimation of the measurement uncertainty of the quantity being a contraction \( Z \) for a specimen with a rectangular cross-section

**Equations (9) and (5)**

\[
\Delta Z = \Delta a_0 \left[ \frac{1}{a_0} + \frac{a \cdot b \cdot a_0 \cdot b_0}{a_0 \cdot b_0} \right] + \Delta b_0 \left[ \frac{1}{b_0} + \frac{a \cdot b \cdot a_0 \cdot b_0}{a_0 \cdot b_0} \right] + \Delta a \left[ \frac{a}{|a|} + \frac{b \cdot a_0 \cdot b_0}{|a| \cdot |b_0|} \right] + \Delta b \left[ \frac{b}{|b|} + \frac{a \cdot a_0 \cdot b_0}{|a| \cdot |b|} \right]
\]

\[
\delta Z = \left\{ \frac{\Delta a_0}{|a_0|}, \frac{\Delta b_0}{|b_0|}, \frac{\Delta a}{|a|}, \frac{\Delta b}{|b|} \right\}
\]

**Equations (10) and (5)**

\[
\Delta Z = \sqrt{\left(\frac{\Delta a_0^2}{|a_0|^2} + \frac{\Delta b_0^2}{|b_0|^2} + \frac{\Delta a^2}{|a|} + \frac{\Delta b^2}{|b|}\right)}
\]

\[
\delta Z = \frac{\Delta Z}{|F|}
\]

## Estimation of the measurement uncertainty of the quantity being a contraction \( Z \) for a specimen with a circular cross-section

**Equations (9) and (5)**

\[
\Delta Z = \Delta d_0 \left[ \frac{1}{d_0} + \frac{2 \cdot n \cdot d_0^2 - 2 \cdot n \cdot d_0^2}{|d_0|^2} \right] + \frac{2 \cdot \Delta d \cdot |d|}{|d_0|^2}
\]

\[
\delta Z = \sqrt{\left(\frac{\Delta d_0^2}{|d_0|^2} + \frac{2 \cdot (\Delta d)^2 \cdot |d|}{|d_0|^2}\right)}
\]

**Equations (10) and (5)**

\[
\Delta Z = \sqrt{\left(\frac{\Delta d_0^2}{|d_0|^2} + \frac{2 \cdot (\Delta d)^2 \cdot |d|}{|d_0|^2}\right)}
\]

\[
\delta Z = \frac{\Delta Z}{|F|}
\]

## Estimation of the measurement uncertainty of the quantity which is the strain \( \varepsilon \)

**Equations (9) and (5)**

\[
\Delta \varepsilon = \Delta l_0 \left[ \frac{1}{l_0} + \frac{1 - l_0 b}{l_0} \right] + \frac{1}{l_0}
\]

\[
\delta \varepsilon = \frac{\Delta \varepsilon}{|F|}
\]

**Equations (10) and (5)**

\[
\Delta \varepsilon = \sqrt{\left(\frac{\Delta l_0^2}{|l_0|^2} + \frac{2 \cdot \Delta l \cdot |l|}{|l_0|^2}\right)}
\]

\[
\delta \varepsilon = \frac{\Delta \varepsilon}{|F|}
\]
Table B1: Continued

Estimation of the measurement uncertainty of the quantity, which is the elongation $A$

Equations (9) and (5)

$\Delta A = \Delta l + \Delta l_0$

$\delta A = \frac{\Delta l + \Delta l_0}{l - l_0}\quad\forall l > l_0$

Equations (10) and (5)

$\Delta A = \sqrt{\Delta l^2 + \Delta l_0^2}$

$\delta A = \frac{\sqrt{\Delta l^2 + \Delta l_0^2}}{l - l_0}\quad\forall l > l_0$

Estimation of the measurement uncertainty of the strain hardening exponent $n$ in the power law – formula (12) for the power constant $a = 1$

Equations (9) and (5)

$\Delta n = \frac{\Delta \sigma_0}{\ln \left( \frac{\sigma_0}{\sigma_m} \right)} + \frac{\Delta \sigma_0}{\ln \left( \frac{\sigma_0}{\sigma_m} \right)} + \frac{\Delta \sigma_0}{\ln \left( \frac{\sigma_0}{\sigma_m} \right)} + \frac{\Delta \sigma_0}{\ln \left( \frac{\sigma_0}{\sigma_m} \right)}$

$\delta n = \frac{\Delta \sigma_0}{\ln \left( \frac{\sigma_0}{\sigma_m} \right)}$

Equations (10) and (5)

$\Delta n = \frac{\Delta \sigma_0^2}{\ln \left( \frac{\sigma_0}{\sigma_m} \right)} + \frac{\Delta \sigma_0^2}{\ln \left( \frac{\sigma_0}{\sigma_m} \right)} + \frac{\Delta \sigma_0^2}{\ln \left( \frac{\sigma_0}{\sigma_m} \right)} + \frac{\Delta \sigma_0^2}{\ln \left( \frac{\sigma_0}{\sigma_m} \right)}$

$\delta n = \frac{\Delta \sigma_0}{\ln \left( \frac{\sigma_0}{\sigma_m} \right)}$

$a$ – stress; $\sigma_0$ – yield stress; $\sigma_m$ – ultimate strength; $\epsilon$ – strain; $\epsilon_0$ – strain corresponding to the yield stress; $\epsilon_m$ – strain corresponding to the ultimate strength; $a$ – strain hardening constant in power law equation (12) – in this calculation $a = 1$; $n$ – strain hardening exponent in power law equation (12); $F$ – force; $a_0$, $b_0$ – initial dimensions of the specimen with rectangular cross-section ($a_0$ and $b_0$); $a$, $b$ – final dimensions of the specimen with rectangular cross-section ($a$ and $b$); $d_0$ – initial diameter of the specimen with circular cross-section ($\pi d_0^2/4$); $d$ – final diameter of the specimen with circular cross-section ($\pi d^2/4$); $A$ – elongation of the specimen; $Z$ – contraction of the specimen.
in assessing the measurement uncertainty of the quantities determined during the uniaxial tensile test. These formulas were derived with the use of the MathCad package, using the symbolic calculation module.

Figure A2: Comparison of the real stress–strain curve determined for the estimated value of the strain hardening exponent and the curves drawn based on the strain hardening exponent reduced or increased by the value of absolute (example graph).

Figure A3: Comparison of the real stress–strain curve determined for the estimated value of the strain hardening exponent and the curves drawn based on the strain hardening exponent reduced or increased by the value of absolute (zoom of the graph).