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Experimental and theoretical investigation of CVT rubber belt vibrations

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Abstract: This article contains the results of experimental tests of vibrations of a continuously variable transmission (CVT) belt transmission driven by a two-stroke internal combustion engine. The measurements were made with the use of a high-speed camera, which allowed to obtain results impossible to obtain with other measurement methods. The nonlinear integro-partial differential equation of vibrations of the moving belt is given. Based on a simplified linear equation, the observed effects on the amplitude–frequency characteristics obtained from the measurements are explained. An approximate formula is given that allows for determining the transmission belt velocities for which resonances occur.

Keywords: continuously variable transmission, moving beam vibrations, high-speed camera

1 Introduction

Modern vehicles are more and more often equipped with stepless gearboxes, which unlike traditional transmissions, allow for much better utilization of the internal combustion engine power characteristics. These mechanisms, called continuously variable transmissions (CVTs), enable for obtaining any gear ratio within a specified range. An engine may operate in the optimal range of rotational speeds in terms of fuel consumption or vehicle dynamics [1]. In today's cars, CVTs with push belts (steel or hybrid) are in common use, but in small vehicles like

scooters or snowmobiles, the powertrain is based on a rubber V-belt. Dry rubber V-belts are usually used because of the high friction coefficient created between a belt and pulleys so that clamping force can be much smaller than it is in lubricated variants. The main problem in using dry rubber belt CVTs is the lack of cooling in the contact area, limiting the torque capacity of the CVT and its application mainly to small, two-wheeled vehicles like scooters. Transmissions with rubber belts are also characterized by lower efficiency compared to classic constructions based on toothed gears. In CVT transmissions, power losses usually reach from 20% to even 50% [2]. It is very low in comparison with classic gear transmission where efficiency is higher than 95%. Taking into account the significant importance of the powertrain efficiency on factors such as fuel consumption and pollutants emission from exhaust gases, the raised issue seems to be very important in a global sense. Low-power single-track vehicles like scooters are mainly used in south-east Asia because of their simplicity, cheapness, and reliability. For example, two wheelers are the most economical mode of personal transportation in India. The segment constitutes more than 80% of the market for on-road vehicles [3].

There are two ways to improve efficiency of the rubber belt CVT. The first is using advanced ratio control methods. In a typical scooter, only a simple mechanical centrifugal regulator is used, and the advanced way needs the application of the additional device of electro-mechanical or hydraulic type. This includes disadvantages like cost and large powertrain complication because of the lack of space in transmission housing. The second way to improve CVT efficiency is associated with belt properties and their behavior.

The subject of vibration phenomena in belt transmission has been the object of research by many authors over the last several decades [4–12]; however, the considerations usually did not go beyond the area of the theoretical analyses. The fundamental model to describe the dynamics of the axially moving system is a string model without flexural rigidity (i.e., narrow belts and chains in power transmission) [13–15]. When bending stiffness cannot be neglected, the Euler–Bernoulli, Timoshenko, and Rayleigh beam theories are

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used [16–18]. Comprehensive reviews of different models applied to analyze the dynamics of axially moving systems are presented in previous studies [19,20], which describe both linear and nonlinear mathematical models using the partial differential equations (PDEs) and integro-partial differential equations (IPDEs). The IPDE type equations are obtained, inter alia, from the use of the quasi-static stretch assumption. The Voigt–Kelvin (V–K) and standard (SLS) rheological models are mainly used for the material damping effect [21–24]. Different approaches depend on including the steady dissipation term related to the beam axial motion, which involves the use of the ordinary partial time derivative or the material time derivative of the strain in the constitutive relation, which also affects boundary conditions.

Belt vibrations are one of the reasons for the relatively low efficiency of the belt drive. This article deals with the vibrational phenomena of the rubber CVT drive belts. This study also presents the research methodology and the measurements results of the belt transverse vibrations with the use of a high-speed camera. The presented results distinguish this study from similar studies by the measurements carried out using a tool that allowed for recording the image at a very high frequency of up to 8,300 fps. Hence, it was possible to capture the exact behavior of the belt over the whole engine rotational speed range. Moreover, the tested transmission is driven by a single-cylinder two-stroke engine, which is a typical source of drive for city scooters. Thus, the presented test results very well reflect the actual working conditions of the considered CVT. It is worth emphasizing that the experimental research results known from the literature referred most often to the transmission driven by an electric motor. In addition to presenting the experimental data, this article also gives a theoretical approach to observed phenomena based on a model of the Rayleigh moving beam. The causes

of vibrations of the moving transmission belt and the effects observed after the analysis of the experimental data are explained. The obtained results may be useful in selecting the operating conditions of the CVT transmission, ensuring its maximum efficiency.

2 Test stand

The type of drive is a very important factor for determining the belt behavior (it is most often an electric motor in the literature). The research tests, including the registration of the CVT belt transverse vibrations, were conducted on the test stand, the basis of which was the complete drive system of the TGB 101S scooter, powered by a two-stroke internal combustion engine with a swept volume of 49 cm³. This is a significant advantage of the test stand, which allows for measuring in real conditions, i.e., when the drive source is an internal combustion engine, which is a torque generator with a wide rotational speed band. The scheme of the test stand is shown in Figure 1.

The method of the belt vibrations registration should not affect its behavior. Therefore, only contactless methods, such as image recording or optical distance sensors, can be used. The application of optical rangefinders is a relatively less complicated and cheaper approach, but it does not provide full information about the behavior of the tested object, especially with the use of a single sensor. In the literature, there are few attempts of local registration of belt vibrations with the use of laser sensors [25–27] and electrostatic sensors [25]. The aforementioned studies were carried out on model objects and are therefore devoid of disadvantages related to the actual drive system, such as a lack of mounting space

Figure 1: Test stand scheme.

Figure 2: The view of test stand with a high-speed camera.

or drive unit vibrations negatively affecting the sensor indications. The study by Manin et al. [26] presented the behavior of a multigroove belt recorded with a high-speed camera; however, this material was not the basis for deeper software analysis.

The present article proposes a method of using a high-speed camera to measure the transverse vibrations of a CVT drive belt in a complete drive system of a city scooter. Figure 2 shows the method of conducting the research with the use of a high-speed camera. The location of the eddy current brake and the necessity of setting the lens orthogonal to the plane of the belt movement made it impossible to capture the entire transmission.

The professional high-speed camera Phantom VEO 710S [28], shown in Figure 3, was used to record the footage. The image was recorded at the resolution of $1,280 \times 720$ (HD-SDI) with a frequency of 8,300 frames per second. At the maximum considered engine rotational

speed of 7,500 rpm, such a frequency allows for registration of the image every 5.5° of the rotation angle of the crankshaft. Selected parameters of the camera are presented in Table 1.

Digital image analysis was performed using TEMA Motion 4.2 software (Figure 3). The belt movement analysis was performed based on the displacements of points defined by the software based on the contrast between the belt and the transmission housing, which had been deliberately made white. The algorithm recognizes the position of the points on subsequent frames and determines their displacement sensed in pixels. This distance is then converted according to the predefined scale. The program does not require the previous implementation of markers defining base points. This is especially important in the case of CVT belt vibration measurements because the transmission specificity does not allow for marking the markers on the side (working) V-belt surface.

Figure 3: High-speed camera Phantom VEO 710S and view of TEMA Motion software window.

Table 1: Selected features of high-speed camera Phantom VEO 710S

Item	Parameter
Type	Phantom VEO 710S
Speed at full resolution	7,400 fps at 1,280 × 800
Maximum speed	1 mln fps
Throughput	7 Gpx/s
Minimum exposure	0.3 μs
Maximum RAM configuration	72 GB (6 s of record time at 7,400 fps)

3 Experimental results

The experimental tests were carried out for the fixed operating conditions of the drive system. The measurements were realized at full throttle, selected engine revs and the constant belt transmission ratio close to 1. Figure 4 shows an exemplary photo material illustrating the behavior of the belt during a single rotation of the engine crankshaft at its rotational speed of 4,560 rpm (i.e., the nominal average speed).

Figure 4: Belt vibration during single crankshaft revolution (engine rotational speed: 4,560 rpm): upper fragment – tight belt side, lower fragment – slack belt side.

Figure 5: Transverse displacement measured 70 mm from left belt-pulley contact point at tight part of belt, for selected engine rotational speeds (3,150 and 4,560 rpm), constant transmission ratio (0.9), and full throttle opening.

The observed transverse vibrations of the belt are characterized by a wide range of amplitudes and frequencies, which is confirmed by the results of the software image analysis presented in Figure 5. The presented waveforms illustrate the transverse displacement, measured 70 mm from the left belt-pulley contact point at the tight part of the belt, for different engine rotational speeds. The recorded amplitudes of the transverse vibrations are within a relatively wide range, i.e., 0.5–8 mm. The highest recorded value was obtained for $n_{\text{eng}} = 4,560$ rpm. Because in this case the amplitude definitely stands out from other cases, it can be presumed that vibrations occur near the resonance. The lowest amplitude was recorded at $n_{\text{eng}} = 6,000$ rpm. At this rotational speed, the engine is near the peak of the drive torque characteristic, which directly refers to the maximum longitudinal force in the tight belt side.

Figure 6 presents exemplary results of the amplitude–frequency analysis, which show that in each considered case, it is possible to identify the essential components of the frequency spectrum. The distinguished frequencies correspond to the engine rotational speeds (3,150 rpm – 52.5 Hz, 4,560 rpm – 76.0 Hz) and their multiples.

The instantaneous rotational speed of the internal combustion engine (especially a single-cylinder engine) undergoes certain fluctuations, which are responsible for the variability of the belt longitudinal velocity, being the direct cause of belt transverse vibrations. The variability of the instantaneous drive unit rotational speed results from the cyclical nature of the gas force acting on the piston and (to a lesser extent) from the kinematics of the crank and piston system. The measurement of the instantaneous engine rotational speed changes within a single cycle of operation (crankshaft rotation) is a difficult

Figure 6: Transverse velocity frequency spectrum (velocity measured 70 mm from left belt-pulley contact point at tight part of belt) for selected engine rotational speeds (3,150 and 4,560 rpm), constant transmission ratio (0.9), and full throttle opening.

Figure 7: Approximation of crankshaft instantaneous rotational speed during a single revolution for a nominal average rotational speed equal to 3,150 rpm.

task due to the limited registration frequency of typical rotational speed converters. Thus, the instantaneous engine rotational speed was determined based on the analysis of the motion of a selected point of the CVT drive pulley, which was conducted using TEMA Motion software. Figure 7 shows the results of the analysis of material recorded by the high-speed camera based on the several revolutions of the crankshaft at the nominal rotational speed of 3,150 rpm. The instantaneous rotational speed approximation was done using a ninth-degree polynomial.

It is worth noting that the instantaneous engine rotational speed varies repeatedly with a clearly defined area with a large gradient resulting from a rapid pressure increase in the combustion chamber (range between 200 and 300 degrees in Figure 7). Based on the obtained research results, it should be emphasized that the real longitudinal velocity of the belt does not undergo fluctuations of small amplitude around the mean value, but it is a periodic function requiring taking into account several harmonic components of the Fourier series. It should be included in the theoretical analysis of the transverse vibration of the moving transmission belt.

4 Belt structure

The CVT drive belts have a complex structure that consists of a rubber core with vulcanized cord fibers and an outer layer made of fabric. Figure 8 shows the cross-section of the belt. The theoretical analysis of the belt transverse vibrations requires the determination of basic physical parameters.

The longitudinal stiffness modulus was determined based on the tensile tests by measuring the tensile force and elongation of the analyzed part of the belt. The value of this parameter results mainly from the stiffness of the nylon cord, which is responsible for the tensile forces transfer.

During the CVT work, the belt is bent as a result of winding on the pulley and vibrations of the free part. In the theoretical analysis, it was assumed that the neutral axis of bending is located in the layer of cord threads, which is characterized by high tensile stiffness. Since the belt is a heterogeneous (along the cross-section) composite, whose geometric and physical properties are not symmetrical with respect to the neutral axis, it is not possible

Figure 8: Cross section of CVT rubber belt (Dayco 17.5 × 765).

to obtain analogous bending characteristics directly from the experimentally determined elastic and viscous tensile characteristics. For example, the bending stiffness cannot be determined based on the knowledge of the tensile stiffness and the cross-sectional geometric moment of inertia with respect to the neutral axis. The bending stiffness was determined experimentally by treating a fragment of the belt as a cantilever beam and measuring its bend deflection under applied force.

5 Dynamic model: equation of motion

An accurate analysis of the observed vibration phenomena requires taking into account many factors such as forces in the belt free fragments, longitudinal belt velocity, parameters describing its tensile and bending stiffness, the length of the free fragments, and production errors of the belt and conical pulleys.

The mechanical model applied is an axially moving, with time-dependent velocity, Rayleigh beam (i.e., formally Euler–Bernoulli beam with rotational inertia), and belt-pulley contact boundary conditions. It was assumed that the viscoelastic material of the belt obeyed the Voigt–Kelvin model, with the constitutive stress-strain relation given by $\sigma = E\varepsilon + \alpha(\varepsilon_t + V\varepsilon_x)$, resulted from the material time derivative applied, where E and α are Young's modulus and the viscous damping coefficient, respectively. Because the belt is nonhomogeneous and its cross-sectional area value changes along the length, the following (averaged along the belt length) equivalent coefficients are introduced: $\bar{\rho A}$, $\bar{EI}$, $\bar{\alpha I}$, $\bar{\rho I}$, $\bar{EA}$, and $\bar{A\alpha}$, where ρ represents the density and A and I are the cross-section area and the area moment of inertia, respectively.

As the string model does not allow to study the influence of the belt-pulley boundary conditions on the behavior of the system, the Rayleigh beam model was used to describe vibrations of the moving belt. The vibrating section of the belt is not so short that it is necessary to use, at least in the initial stage, more complex phenomenological models, for example, the Timoshenko beam with consideration of the shear effect.

The transmission belt is a composite structure (Figure 8), in which various phenomena occur, causing energy dissipation during vibrations. The commonly used Voigt–Kelvin rheological model allows for describing the dissipation effects, and it is relatively easy to determine experimentally the physical parameters that define it. In Chapter 4, it is described how to determine the equivalent longitudinal

stiffness $\bar{EA}$ and the equivalent bending stiffness $\bar{EI}$. In a similar way, it is possible to determine the equivalent longitudinal damping $\bar{\alpha A}$ and the equivalent bending damping $\bar{\alpha I}$. The equivalent longitudinal damping $\bar{\alpha A}$ can be determined on the basis of the course of the damped free vibrations of a system with one degree of freedom, in which the elastic-viscous element is made of a section of the transmission belt (vibrations parallel to the belt axis). The equivalent bending damping $\bar{\alpha I}$ can be determined based on the analysis of the damped free vibrations of the cantilever beam, made of the belt material (static deflection first mode approximation of bending vibrations).

The equivalent longitudinal inertia coefficient $\bar{\rho A}$ is the average linear density of the beam, while the equivalent rotational inertia coefficient $\bar{\rho I}$ must be calculated taking into account that the center of mass of the beam cross-section changes its position with regard to the neutral axis with the location change along the belt axis. The observation of vibrations, especially near the resonance, made it possible to draw the conclusion that due to the structure of the belt, it would be advantageous to include in the kinetic energy the component taking into account the rotational inertia of the cross-section [29–31].

The equation of the transverse motion of the moving belt can be derived by using the generalized Hamilton's principle [30,31]. Omitting the details of calculations, the following, governing IPDE of the transverse vibration is obtained:

$$\begin{aligned} & \bar{\rho A}(w_{tt} + \dot{V}w_x + 2Vw_{xt} + V^2w_{xx}) - T_0w_{xx} + \bar{EI}w_{xxxx} \\ & + \bar{\alpha I}(w_{xxxxt} + Vw_{xxxxx}) - \bar{\rho I}(w_{xxtt} + \dot{V}w_{xxx} + 2Vw_{xxt} + V^2w_{xxx}) \\ & - \frac{1}{l}w_{xx} \int_0^l \left(\frac{\bar{EA}}{2}w_x^2 + \bar{\alpha A}w_xw_{xt} + \bar{A\alpha}Vw_xw_{xx} \right) dx = 0, \end{aligned} \quad (1)$$

where $w(x, t)$ denotes the transverse displacement, x and t are the axial and time coordinates, and V and T_0 are the moving (transport) velocity of the belt and the static tension, respectively, assumed to be a constant along the belt.

The Lagrangian strain definition used in the derivation of equation (1) accounts for the geometric nonlinearities, the integral component comes from the assumption of the quasi-static stretching, i.e., the dynamic tension does not vary along the belt. This is reasonable when the following condition is satisfied: $\bar{EA}/T_0 \gg 1$, which means that the longitudinal wave velocity is much more than the velocity of transverse waves in belt (and the belt velocity V). In the considered case, this condition is also satisfied for the tight segment of the belt, for example, for $n_{\text{eng}} = 4,560$ rpm

(when $T_0 = 121\text{N}$): $\bar{E}\bar{A}/T_0 = 583.14$ (for the considered belt, the equivalent longitudinal stiffness is equal: $\bar{E}\bar{A} = 70,560\text{N}$). The boundary conditions assumed are as follows:

$$\begin{aligned} w(0) &= 0, & w(l) &= 0, \\ w_{xx}(0) &= \frac{1}{R_1}, & w_{xx}(l) &= \frac{1}{R_2}, \end{aligned} \quad (2)$$

where R_1 and R_2 are the radiuses of the left and right support pulleys.

The quasi-static stretching assumption adopted in this article seems to be fully justified. It allows in a possible easy way for taking into account the longitudinal vibrations of the axially moving belt. In many cases, the more complex mathematical models should be considered, taking into account the relationship between the axial tension and the axial belt velocity [33,34].

6 Linearization. Equilibrium configuration

In the beginning, the linear approximation of equation (1) is considered. Neglecting the nonlinear terms, the linear equation takes the form:

$$\begin{aligned} \bar{\rho}\bar{A}(w_{tt} + \dot{V}w_x + 2Vw_{xt} + V^2w_{xx}) - T_0w_{xx} + \bar{E}\bar{I}w_{xxxx} \\ + \bar{a}\bar{I}(w_{xxxx} + Vw_{xxxx}) - \bar{\rho}\bar{I}(w_{xxtt} + \dot{V}w_{xxx} + 2Vw_{xxt} \\ + V^2w_{xxx}) = 0. \end{aligned} \quad (3)$$

To study the influence of the nonhomogeneous boundary conditions (2), the equilibrium configuration is first obtained. Neglecting the time-related terms in equation (3) and assuming that $V(t) = V_0 = \text{const.}$, it furnishes with the following equation:

$$(\bar{E}\bar{I} - \bar{\rho}\bar{I}V_0^2)\bar{w}_{xxxx} - (T_0 - \bar{\rho}\bar{A}V_0^2)\bar{w}_{xx} = 0. \quad (4)$$

for which the following boundary conditions are taken:

$$\begin{aligned} \bar{w}(0) &= 0, & \bar{w}(l) &= 0, \\ \bar{w}_{xx}(0) &= \frac{1}{R_1}, & \bar{w}_{xx}(l) &= \frac{1}{R_2}. \end{aligned} \quad (5)$$

Assuming that $a = (\bar{E}\bar{I} - \bar{\rho}\bar{I}V_0^2) > 0$; $b = (T_0 - \bar{\rho}\bar{A}V_0^2) > 0$, the solution of equation (4) is given by

$$\bar{w}(x) = A + Bx + D \exp\left(\sqrt{\frac{b}{a}} x\right) + H \exp\left(-\sqrt{\frac{b}{a}} x\right), \quad (6)$$

where:

$$\begin{aligned} A &= -\frac{a}{b} \frac{1}{R_1}, \\ B &= \frac{a}{bl} \left(\frac{1}{R_1} - \frac{1}{R_2} \right), \\ D &= \frac{a}{b} \frac{\left(\frac{1}{R_2} - \frac{1}{R_1} \exp\left(-\sqrt{\frac{b}{a}} l\right) \right)}{\left(\exp\left(\sqrt{\frac{b}{a}} l\right) - \exp\left(-\sqrt{\frac{b}{a}} l\right) \right)}, \\ H &= \frac{a}{b} \frac{\left(\frac{1}{R_1} \exp\left(\sqrt{\frac{b}{a}} l\right) - \frac{1}{R_2} \right)}{\left(\exp\left(\sqrt{\frac{b}{a}} l\right) - \exp\left(-\sqrt{\frac{b}{a}} l\right) \right)}. \end{aligned} \quad (7)$$

7 Forced and parametric vibration

Since the equilibrium deformation $\bar{w}(x)$ is analytically calculated, after substitution: $w(x, t) = \bar{w}(x) + u(x, t)$, the following equation is derived from equations (3) and (4):

$$\begin{aligned} \bar{\rho}\bar{A}(u_{tt} + \theta\dot{s}u_x + 2(V_0 + \theta s)u_{xt} + (V_0 + \theta s)^2u_{xx}) \\ - T_0u_{xx} + \bar{E}\bar{I}u_{xxxx} + \bar{a}\bar{I}\{u_{xxxx} + (V_0 + \theta s)u_{xxxx}\} \\ - \bar{\rho}\bar{I}\{u_{xxtt} + \theta\dot{s}u_{xxx} + 2(V_0 + \theta s)u_{xxt} + (V_0 + \theta s)^2u_{xxx}\} \\ + \theta\dot{s}(\bar{\rho}\bar{A}\bar{w}_x - \bar{\rho}\bar{I}\bar{w}_{xxx}) + (2V_0\theta s + \theta^2s^2)(\bar{\rho}\bar{A}\bar{w}_{xx} - \bar{\rho}\bar{I}\bar{w}_{xxxx}) \\ + \bar{a}\bar{I}(V_0 + \theta s)\bar{w}_{xxxx} = 0. \end{aligned} \quad (8)$$

In equation (8), it is assumed that the belt velocity is given by the expression: $V(t) = V_0 + \theta s$, where $V_0 = \text{const.}$, $s = s(t)$ is any (reasonable) function of time, and θ is a dimensionless parameter.

In addition, the nonhomogeneous boundary conditions for $w(x, t)$ are replaced by the simply supported boundary conditions:

$$\begin{aligned} u(0) &= 0, & u(l) &= 0, \\ u_{xx}(0) &= 0, & u_{xx}(l) &= 0, \end{aligned} \quad (9)$$

The differential equation (8) is an equation with parameters which are the functions of time; moreover, the components containing derivatives $\bar{w}_x, \dots, \bar{w}_{xxxx}$ form a time-dependent distributed load, resulting from the nonuniform boundary conditions (2).

In most studies on vibrations of moving beams, the variable axial velocity is taken as a constant value with an additional small harmonic disturbance. In the case considered in this article, $s(t)$ is a periodic function with a period equal to the time of one rotation of the belt pulley, which can be obtained from the course presented in Figure 7. The forms of expressions dependent on velocity and appearing in equation (8) show that, in addition to the parametric excitation, the distributed periodic excitation occurs in the system.

This explains the form of the amplitude–frequency characteristics shown in Figure 6, in which there are peaks of amplitudes for the fundamental frequency, equal to the number of revolutions of the belt–pulley per second, and its successive multiples. The reason for the extremely high amplitude value visible on the plot in Figure 6 for $n_{\text{eng}} = 4,560$ rpm (frequency equal to 76.0 Hz) may be the coincidence of the frequency of one of the excitation components with one of the natural frequencies of the moving transmission belt.

Equation (8) with boundary conditions (9) can be solved, for example, by the Galerkin method, assuming approximating functions in the form of $\sin(k\pi x/l)$.

8 Resonance frequencies. Approximate formula

Before performing a full numerical, linear, or nonlinear analysis, it is worth estimating the natural frequencies of the moving belt based on the approximate analytical expressions. Neglecting the rotational inertia, damping and assuming a constant velocity equal to V_0 , the equation of transverse motion takes the following form:

$$\overline{\rho A}(u_{tt} + 2V_0u_{xt} + V_0^2u_{xx}) - T_0u_{xx} + \overline{EI}u_{xxxx} = 0. \quad (10)$$

For the free support boundary conditions (9), the dimensionless angular frequencies of natural vibrations are given by the following formula [32]:

$$\hat{\omega}_k = k\pi \left[1 - \hat{V}_0^2 + \frac{\gamma^2 k^2 \pi^2 (\hat{V}_0^4 + 6\hat{V}_0^2 + 1)}{2} + \dots \right], \quad (11)$$

$$k = 1, 2, 3 \dots,$$

where the following dimensionless notations were introduced:

$$\gamma^2 = \frac{\overline{EI}}{T_0 l^2}, \quad \hat{\omega} = \omega \sqrt{\frac{\overline{\rho A} l^2}{T_0}}, \quad \hat{V}_0^2 = V_0^2 \frac{\overline{\rho A}}{T_0}. \quad (12)$$

The aforementioned approximation is valid for small bending stiffness and low axial belt velocity.

Table 2 presents the first three natural frequencies of the moving belt for the nominal rotational speed of $n_{\text{eng}} = 3,150$ rpm and $n_{\text{eng}} = 4,560$ rpm. These cases differ in the value of the constant tensile force T_0 , which was determined based on the difference of forces in the tight and slack side, resulting from the driving torque measured on the CVT output pulley. The following values of parameters appearing in expressions (12), dependent on the physical features of the belt, were used in calculations:

Table 2: Natural frequencies of moving belt (approximation formula)

$n_{\text{eng}} = 3,150$ rpm ($f_0 = 52.5$ Hz), $T_0 = 98$ N	$n_{\text{eng}} = 4,560$ rpm ($f_0 = 76.0$ Hz), $T_0 = 121$ N
$f_1 = 40.9$ Hz	$f_1 = 34.1$ Hz
$f_2 = 87.7$ Hz	$f_2 = 76.2$ Hz
$f_3 = 146.3$ Hz	$f_3 = 134.1$ Hz

$\overline{EI} = 8.99 \times 10^{-3} \text{ Nm}^2$, $\overline{\rho A} = 0.13 \text{ kg/m}$, $l = 0.255 \text{ m}$. The radius of the drive pulley is equal: $R_1 = 0.0423 \text{ m}$.

Comparing values in Table 2, it can be seen that the basic (fundamental) excitation frequency f_0 for $n_{\text{eng}} = 4,560$ rpm coincides almost exactly with the second natural frequency f_2 of the moving belt, which is the cause of the high vibration amplitude visible in Figure 6. This confirms the correctness of adopting the beam model to describe the dynamics of the considered transmission belt.

After substituting on the left-side of the formula (11) the successive multiplicities of the angular velocity of the shaft, expressed in terms of the belt velocity, an algebraic equation for $\hat{V}_0$ is obtained, enabling to calculate the velocity values of the moving belt for which the resonance occurs (where $R_1 = \hat{R}_1 l$).

$$\frac{\hat{V}_0}{\hat{R}_1} m = k\pi \left[1 - \hat{V}_0^2 + \frac{\gamma^2 k^2 \pi^2 (\hat{V}_0^4 + 6\hat{V}_0^2 + 1)}{2} + \dots \right], \quad (13)$$

$$m = 1, 2, 3 \dots, \quad k = 1, 2, 3 \dots$$

9 Conclusions

This article presents the results of experimental tests of CVT transmission vibrations with the use of the high-speed camera. The tests were carried out on the system in which the movement of the transmission belt was driven by the real internal combustion engine, which allows for accurately discovering the origin of the excitation causing the vibrations. The reasons for vibrations of the moving belt and the course of the amplitude–frequency characteristics are explained.

The results of the conducted experimental and theoretical analysis show that CVT belt resonance may occur as a result of the excitation coming from the internal combustion engine. The operation of the belt in the resonant range is disadvantageous for various reasons, including the reduced drive efficiency. The analysis presented in this article shows that it is possible to select the

belt parameters that ensure operation beyond the resonance range for a specific drive system. The drive belts available on the automotive market are varied significantly in terms of the physical features that determine the natural frequencies. Therefore, it seems possible to select the belt for a particular vehicle in such a way as to exclude the risk of resonance phenomena in the most frequently used engine rotational speed ranges.

The next step will be the numerical analysis of the equations of motion, for both the linear and nonlinear models. The final aim is to create a complete dynamic model describing the operation of the CVT transmission, consisting of the equations of motion of the belts and the rotational motion of the belt pulleys. The purpose of the calculations will be to determine the accurate energy losses resulting from vibrations of CVT belts. This can be used, for example, to find the optimal physical and geometric parameters of the belt minimizing these losses.

In the case of a significant influence of these losses on the overall drive system efficiency, wider model analysis is planned. It will allow for identifying the possible modifications of the transmission design parameters, both geometric and inertial. The selection of pulley diameters (without changing the total gear ratio range) may be significantly important because it determines the belt transport velocity and its static tension. Another geometric parameter that will importantly affect the belt behavior is the distance between pulleys. The development of the CVT dynamic model will also allow for studying the influence of inertial transmission parameters on the belt vibration phenomena. The reduced moments of inertia at the input and output from the transmission as well as the inertia of the belt may be crucial in this case. The application of an additional damper should be also taken into consideration here. A complete analysis may turn out to be very valuable for designers of this type of drive systems.

Conflict of interest: Authors state no conflict of interest.

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