Effect of adverse slope on performance of USBR II stilling basin

1 Introduction

Hydraulic jump, which occurs when a flow transfers from supercritical to subcritical, is the most significant variable of spatial flow. This phenomenon is characterized by an abrupt increase in the free surface of the water and a significant loss of kinetic energy [1,2]. The classical hydraulic jump (CHJ) occurs in wide, rectangular horizontal channels and has attracted considerable attention [3–5]. A free hydraulic jump occurs when the tailwater depth equals the sequent depth of the jump. If the tailwater depth exceeds the sequent depth, the hydraulic jump is moved upstream to be submerged jump (SHJ) [6].

Depending on the degree of submergence, SHJ creates a horizontal vortex with strong countercurrent-free surface velocities and air trapped within the vortex core [7–9]. Habibzadeh et al. [10] found that depending on the submergence degree, an SHJ can dissipate less energy than a free jump.

Stilling basins are integral structures constructed downstream of numerous hydraulic structures such as spillways, gates, and weirs to dissipate the high kinetic energy of the incoming flow [11]. The excess energy is assumed to be completely dissipated within a hydraulic jump stilling basin due to the formation of a hydraulic jump, which is referred to as a forced hydraulic jump (FHJ) rather than a CHJ [12]. A more efficient stilling basin can be attained by reducing the sequent depth ratio, shortening the roller length, and increasing the energy losses compared to CHJ [13]. Based on the Froude number of supercritical flow ($F_{FR}$), United States Bureau of Reclamation (USBR) (1968) designed various types of stilling basins. Peterka (1957) recommended the use of chute blocks, baffle blocks, and end sills for reducing the length and sequent depth, stabilizing the jump, and improving the efficiency of USBR stilling basins [14,15]. Several studies have shown that altering the geometry of the stilling basin through implementing an adverse or positive slope can impact the characteristics of the hydraulic jump. McCorquodale and Mohamed studied the adverse hydraulic jump and concluded that it was difficult to establish at $F_{FR} < 9$ and needed continuous tailwater adjustment to...
keep a stationary position for \( F_r < 4 \) [16]. Beirami and Chamani proved that an adverse basin slope decreases the sequent depth ratio, whereas a positive basin slope increases it [17]. Beirami and Chamani demonstrated that the energy loss for the CHJ was greater than that for any other jump forming on positive or negative slopes [13]. Abbas et al. [18] stated that continuous tailgate adjustment is required to control the hydraulic jump on an adverse slope. Also, using a smooth stilling basin with an adverse slope \( (S = -0.06) \) instead of a smooth horizontal bed reduced the energy dissipation ratio of the hydraulic jump, but baffle blocks increased it. Mazumder [11] concluded that for a hydraulic jump in a stilling basin with an adverse slope and a positive end step, the sequent depth ratio and roller length decrease as the adverse slope and height of the positive step increase compared to a traditional jump on flat bed. Based on numerous studies, design engineers believe that the design criteria of the USBR stilling basin resulted in favorable performance [19].

As previously mentioned, numerous studies have examined hydraulic jumps on adverse slopes; however, reviews of earlier studies indicate that neither experimental studies nor mathematical models have yet examined the USBR II stilling basin with an adverse slope. This study aimed to conduct an experimental investigation on the hydraulic performance of the USBR II stilling basin with three different adverse slopes. Six different discharges are applied for each slope. Figure 1 depicts the flowchart of the research methodology.

\[ \text{Sequent depth ratio} = \frac{y_2}{y_1} \]
\[ \text{Relative energy loss (}\%\text{)} = \left( \frac{E_1 - E_2}{E_1} \right) \times 100 \]

**Figure 1:** Flowchart of the research methodology.
2 Method and materials

Experiments were conducted in a rectangular horizontal channel of 20 m length, 0.9 m width, and 0.6 m height. The channel was constructed with a steel bottom and an armored plate-glass sidewall. Under the laboratory floor, a large concrete sump tank was constructed along the side of the flume. This tank stores water, which is then pumped to the flume by a centrifugal pump with an axial flow unit that has a capacity of 72 lps. The inlet and outlet tanks were rigidly attached to the upstream and downstream ends of the flume. The discharge measurements were 3 m from the inlet tank using a v-notch with a notch angle of 90°C. Before starting the experimental work, the standard weir was calibrated using a volumetric flow meter. A tailgate was installed at the downstream end of the flume to control and adjust the tailwater depth. The flume sketch is illustrated in Figure 2.

In accordance with USBR specifications, a rigid foam Ogee spillway with a height of 35 cm and a design head of 7 cm was constructed to generate supercritical flow [20]. The spillway was installed in the middle third of the laboratory flume, 7 m upstream of the tailgate. Moreover, a physical model of the USBR II stilling basin, as depicted in Figure 3, was designed based on the maximum discharge, i.e., 33 lps, in accordance with the Bureau of Reclamation principles and recommendations [15]. These recommendations suggest that the height, width, and distance between adjacent chute blocks should be equal to the depth of flow entering the basin (y1). The height of the dentated sill is 0.2

Figure 2: Sketch of the laboratory flume.

Figure 3: Typical USBR II stilling basin: (a) chute blocks and (b) end sill.
y₂, and its maximum width and spacing is 0.15 y₂. Dentates are recommended on the side walls of the sill, and the continuous portion of the end sill has a slope of 2:1. The length of stilling basin is related to \( \text{Fr}_1 \), and in this study, it is 55 cm in accordance with the design discharge and USBR recommendations [15]. Polylactic acid material was used to create the chute blocks and end sill of a physical model using 3D printing technology. To construct the modified USBR stilling basin, three wooden beds with negative slopes of −0.085, −0.055, and −0.035 were constructed and painted with water-proof paint, as shown in Figure 4. The standard dentated end sill was installed at the end of the adverse slope. The chute blocks were installed at the toe of the spillway for each modified basin. The total horizontal length of the modified basin, including the adverse and positive slope from the end sill, was the same as a typical basin. The procedure used in this study for each test involves running the physical model for various discharge conditions by allowing water to enter the flume gradually. When the required discharge was achieved, the tailgate was slowly closed until the tailwater reached the required depth, at which point the hydraulic jump began near the spillway toe. This situation was maintained for the duration of the data collection. Flow depths were measured using a precision point gauge placed on a trolley. Several depth measurements across the section were taken during each test, and the mean depth was calculated.

3 Dimensional analysis

Dimensional analysis is widely used in engineering applications [21,22]. Generally, several parameters influence the energy dissipation due to hydraulic jump formation in the USBR II with an adverse slope, including the geometric properties of the stilling basin, the physical properties of water, and the hydraulic conditions of the incoming flow. As shown in Figure 5, the energy dissipation efficiency can be written as a function of the following parameters:

\[
e = f(S, h, L, y_1, y_2, v_1, g, \rho, \mu),
\]

where \( e \) is the efficiency, \( S \) is equal to \( \tan \theta \) (dimensionless), \( h \) is the height of end sill \( (L) \), \( L \) is the length of adverse slope \( (L) \), \( y_1 \) is the supercritical depth \( (L) \), \( y_2 \) is the depth of flow downstream positive slope \( (L) \), \( v_1 \) is the flow velocity of supercritical flow \( (L/T) \), \( g \) is the gravitational acceleration \( (L/T^2) \), \( \rho \) is the mass density of water \( (M/L^3) \), and \( \mu \) is the viscosity of water \( (ML^{-1}T^{-1}) \). Using Buckingham’s theory, the dimensionless relationship can be expressed as follows:

\[
e = f \left( S, \frac{h}{y_1}, \frac{L}{y_1}, \frac{y_2}{y_1}, Re, Fr_1 \right).
\]

where \( Fr_1 \) is the Froude number of supercritical flow (dimensionless) and \( Re \) is the Reynolds number (dimensionless). For large Reynolds numbers, the viscosity effect can be ignored [23–26]. As a result, equation (2) can be rewritten as follows:

\[
e = f \left( S, \frac{h}{y_1}, \frac{L}{y_1}, \frac{y_2}{y_1}, Fr_1 \right).
\]

4 Results and discussion

4.1 Typical USBR II stilling basins

In order to study the characteristics of CHJ, different discharge values ranging from 8 up to 33 lps were applied to the Ogee spillway before the installation of the typical stilling basin. The results of the calculation of \( Fr_1 \) and sequent depth ratios for CHJ and FHJ formed in the typical USBR II stilling basin are presented in Table 1. These results demonstrate that the discharge and \( Fr_1 \) values satisfy the requirements for designing a USBR type II stilling basin. In addition,
as the approaching discharge increased, the Froude number decreased for free jumps downstream of the spillway, indicating that the increased rate of the supercritical depth \( y_1 \) is greater than the corresponding increase rate of the velocity \( v_1 \). As a result, the parameter \( y_1 \) is crucial in determining the \( F_{r1} \) values. Previous similar results in the existing literature [10,14,27–32] have confirmed the reduction of \( F_{r1} \) with an increase in \( Q \) for free jumps downstream of the spillway.

Based on continuity and momentum equations, the well-known Belanger equation can be used to calculate the subsequent depth ratio for CHJ as follows [33]:

\[
\frac{y_2}{y_1} = \frac{1}{2} \left( \sqrt{1 + 8F_{r1}^2} - 1 \right),
\]  

(4)

where \( y_1 \) is the supercritical depth (L) and \( y_2 \) is the subcritical depth (L).

In this study, the sequent depth ratios for experimental data of CHJ are slightly lower than those obtained from the Belanger equation due to the bed friction force, which is not considered in this equation [34,35]. In addition, the average sequent depth ratio of the hydraulic jump in USBRII is approximately 10% less than in CHJ. In contrast to a CHJ, a FHJ that is entirely contained within the stilling basin with the toe very close to the end of the spillway requires a lower tailwater [28,36].

The following equation, which is derived from Bernoulli’s equation, can be used to determine the specific energy of fluid flow at any given cross-section:

\[
E = y + \frac{v^2}{2g},
\]  

(5)

where \( y \) represents the flow depth at the section and \( v \) is the velocity at this section. Since the energy loss during the hydraulic jump equals the difference in specific energy before and after the jump [37], the relative energy loss (\( \Delta E \)) can be calculated as follows:

\[
\Delta E \% = \left( \frac{E_1 - E_2}{E_1} \right) \times 100.
\]  

(6)

According to the experimental results shown in Figure 6, the relative energy losses for the hydraulic jump in the typical USBR II stilling basin are slightly greater than those of CHJ under the same flow conditions. These findings are consistent with those reported in the literature. Padulano et al. [28] and Macián-Pérez et al. [38] found that for a typical USBRII stilling basin with \( F_{r1} = 9 \), the relative energy losses were approximately 70–75 and 70.5%, respectively, which is equivalent to 74% in this study. This value of the Froude number gives adequate energy dissipation conditions for hydraulic jumps in the modeled stilling basin according to the USBR [15].

### 4.2 Modified USBR II stilling basins

As previously mentioned, the USBR II stilling basin was modified by changing its horizontal flatbed to an adverse slope bed. The laboratory results indicated that another hydraulic jump formed downstream the positive slope, which extended from the end sill after water left the
stilling basin. As the adverse slope of the stilling basin increases, the end sill height increases, forming a low overflow weir at the downstream end of the stilling basin. According to Chow and Leutheusser and Birk [33,39], depending on the downstream tailwater, four different types of hydraulic jumps can occur at the end of a low-head structure: swept-out jump, optimal jump, SHJ, and washed-out jump. An SHJ occurs when the tailwater depth is slightly greater than the sequent depth. This jump continues until a “washed-out jump” is reached as the tailwater increases. In all tests of this study, the tailgate was adjusted to capture the hydraulic jump toe at the beginning of the stilling basin. Consequently, the modified USBR II stilling basin contains two hydraulic jumps: a free-type jump within the adverse slope surface and a free-to-washout jump downstream end sill.

According to Figure 7, the relative energy losses for \( S = -0.085 \) are lower than those for CHJ and the typical USBR II stilling basin when \( F_{r1} = 6.37, 7.28, \) and \( 8.16 \). Additionally, at the same Froude numbers, the sequent depth ratios are greater than those of CHJ and the typical USBR II stilling basin, as shown in Figure 8.

According to the obtained results, higher tailwater depths are required at \( Q = 33 \) and 28 lps to overcome the additional height of the dentated end sill and to confine the first hydraulic jump within the adverse slope section of stilling basin, with the toe of the jump at the beginning of the basin. On the other hand, at these discharges, the second hydraulic jump becomes washed out, as shown in Figure 9(a). As the discharge decreases to 23 and 18 lps, the required tailwater depths decrease, and the second jump is converted to a submerged type. In this case, the headwater of a positive slope does not have enough energy to push tailwater away from its face. So, water near the bottom of the downstream slope moves downstream, whereas counterintuitive current and reverse rollers on the surface of the water push back upstream toward the adverse slope basin and against the face of the positive slope, reducing the efficiency of the stilling basin, as shown in Figure 9(b). The amount of these upstream-directed surface currents are directly related to the degree of submergence \( (S_m) \), which is computed as follows:

\[
S_m = \frac{y_1 - y_2}{y_1},
\]

where \( y_1 \) is the downstream tailwater depth (L). On the other hand, the modified stilling basin becomes more efficient when the discharge decreases to 8 and 13 lps. For these low discharges, shallow tailwater depth is required, and the first hydraulic jump occurs at a short distance and before reaching the end of the adverse stilling basin, as shown in Figure 9(c). The second hydraulic jump is visible and of the free type, which aids in the dissipation of excess energy. This free hydraulic jump has a length of about 1.8 m for \( Q = 8 \) lps and 2.2 m for \( Q = 13 \) lps. Several authors have observed that as the submergence of a hydraulic jump increases, jet mixing decreases, resulting in a decrease in the wasted energy compared to free hydraulic jumps. Consequently, the downstream region affected by the submerged hydraulic jump is greater than that by the free jump [40,41].

As shown in Figures 10 and 11, for all discharges, the stilling basin with \( S = -0.055 \) is less effective than the CHJ and typical USBR II. According to Figures 12 and 13, it can be observed that the stilling basin having a value of \( S = -0.035 \) has lower efficiency as compared to the standard basin and CHJ for discharges that range from 33 to 18 lps.
However, it demonstrates nearly efficiency to CHJ at a discharge of 13 lps. At the lowest discharge, the basin efficiency increased with a relative energy loss of 1.1% higher and a depth ratio of 5.5% lower than that in the standard basin. In general, the second hydraulic jumps observed at $S = -0.055$ and $S = -0.035$ showed the characteristics of SHJs with a counterintuitive current and reverse roller. These phenomena resulted in a decrease in the efficiency of the stilling basin.

Tables 2 and 3 show the percentage differences in relative energy losses and sequent depth ratios in modified stilling basins compared to CHJ and USBR II stilling basins for six applied discharges. The negative signs represent decreasing in energy dissipation and sequent depth ratio.

Additionally, the inefficiency of the modified stilling basin compared to the typical one may be due to variations in basin length. In accordance with USBR design criteria for stilling basin II, the dimensions and geometries of the stilling basin and its accessories are intended to dissipate the kinematic energy of the water passing with the design discharge. Changing these dimensions will prevent the stilling basin from dissipating this energy. In this study, the chute block and end sill of the modified basin were designed in accordance with standard criteria. In addition, the total

![Figure 9: Hydraulic jump in USBRII stilling basin with $S = -0.085$: (a) 28 lps, (b) 18 lps, and (c) 8 lps.](image)

![Figure 10: Relative energy losses for CHJ and USBR II stilling basins ($S = 0$ and $S = -0.055$).](image)

![Figure 11: Sequent depth ratios for CHJ and USBRII stilling basins ($S = 0$ and $S = -0.055$).](image)
horizontal length \((L_b)\), consisting of the negative and positive slope extending from the end sill, was identical to that of a standard basin.

The stilling basin with slope \(S = -0.085\) has the lowest efficiency at high discharges due to the fact that the length of the adverse slope \((L)\) is approximately 16% shorter than that of the flatbed of the standard basin, resulting in less internal friction with the side walls and bed of the flume. Furthermore, additional tailwater depth is required to maintain the first jump within the adverse slope bed and to overcome the increased height of the dented sill caused by the steeper adverse slope. In this deep tailwater, a washout jump occurs, resulting in less energy loss than in a typical USBR II basin. As the discharge decreases, the efficiency of the basin increases due to the relatively shorter length of the first jump. Thus, the adverse slope bed length is adequate for dispersing the extra energy of the water. In addition, the second jump transforms from a submerged to a free jump, eliminating the counterintuitive current and resulting in greater energy loss.

Because of the reduced silt height, the stilling basin with \(S = -0.55\) becomes more efficient than one with \(S = -0.085\) at high discharges. However, as discharge decreases, the downstream hydraulic jump remains submerged at varying ratios, decreasing the energy dissipation rate compared to the basin \((S = -0.085)\). As the adverse slope reduces to \(-0.035\), the height of the end sill decreases, and the length of the adverse basin becomes 7% shorter than in the typical basin. As a result, the tailwater depth required to catch the first jump within the stilling basin decreases while internal friction losses increase, enhancing the efficiency of the basin. Thus, the efficiency of energy dissipation in a modified stilling basin can be enhanced by extending the length of the inclined floor \((L)\) to match the horizontal length \((L_b)\) used in the standard USBR basin. This is due to the fact...
that the inclined length will be longer, increasing internal friction.

Based on the previous findings, the modified USBR II stilling basin with an adverse slope is less effective than a typical basin. Although the stilling basin with the steepest adverse slope exhibits superior performance compared to the standard one at low discharges, an extra hydraulic jump arises downstream, which requires the construction of an additional stilling basin to confine this jump and protect the bed downstream against scour, thereby increasing the cost. According to Bateni and Yazdandoost [42], adverse slopes have a negligible impact on the relative energy loss in adverse-slope stilling basins.

Moreover, experimental data from Beirami and Chamani in 2006 and 2010 [13,17] indicated that the energy loss for CHJ was greater than that for jump formation on positive and negative slopes because a negative basin slope decreases the sequent depth ratio, while a positive basin slope increases it. The effect of the positive slope prevails over that of the adverse basin slope. Those findings are closely consistent with the results of the current study concerning the USBR II stilling basin.

5 Conclusions

Stilling basins use the characteristics of hydraulic jumps to dissipate large amounts of kinetic energy downstream of hydraulic structures. This study compared the performance of a USBR II basin with three adverse slopes ($S = -0.085$, $-0.055$, and $-0.035$) to that of a typical USBR II basin for discharges ranging from 33 to 8 lps. The results can be summarized as follows:

1. At high discharges, the stilling basin with $S = -0.085$ was the lowest efficiency. Still, it outperformed the standard basin at low discharges because it needed a shallower tailwater depth. A free-type jump occurred at the downstream end sill, which assisted in dissipating energy.

2. A stilling basin was less effective than a typical basin and CHJ ($S = -0.055$ and $-0.035$) because a submerged hydraulic jump was formed downstream with a counterintuitive current and a reverse roller, reducing its efficiency. The strength of upstream-directed surface currents is proportional to submergence ($S_m$) and end sill height.

3. The modified USBR II stilling basin design resulted in a shorter effective length that confined the hydraulic jump downstream spillway. This reduces the internal friction loss and, thus, energy dissipation. It is suggested that for future investigations, the inclined floor length of the modified stilling basin should be equivalent to the horizontal basin length used in the standard USBR basin. This would increase internal friction and thus improve energy dissipation.

4. Finally, a typical USBR II with a horizontal bed is more efficient and economical than an adverse slope.

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References


