Research Article

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Equipment selection based on two different fuzzy multi criteria decision making methods: Fuzzy TOPSIS and fuzzy VIKOR

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Abstract: Deciding on equipment from a set of feasible alternatives for excavation purpose in open pit mines is a critical task for the engineers and contractors since it considerably affects the project costs and productivity. Alternatives of rock excavation equipment, which are deemed suitable for the diggability of rock units in open pits, can have many technical specifications or factors that should be considered simultaneously. It can be successfully selected in a scientific way by employing multi-criteria decision making methods by means of taking the advantage of engineering knowledge, intuition and past experiences. In this study, we select the most convenient hydraulic excavator for a magnesite mine by fuzzy multi criteria decision making methods: fuzzy TOPSIS and fuzzy VIKOR. The fuzzy approach allows the experienced engineers to compare the multiple criteria (technical factors) and the possible alternatives of rock excavation machines by simply using linguistic expressions in their natural language.

Keywords: Equipment selection; hydraulic excavator; fuzzy multi-criteria decision making; fuzzy TOPSIS; fuzzy VIKOR

1 Introduction

The excavability or diggability of rocks in open pit mines has a major importance on selecting of excavation equipment. The right equipment selection by taking into account site specific conditions is the most important phenomena affecting the productivity in surface mining oper-

ations. It mainly depends upon mine site related considerations including several factors such as annual production target, formation properties to be excavated, face height and angle, bench width, maneuvering space, floor conditions and blasting restrictions. It is also affected by intact strength of the rock, bulk density, bulking factor and water content which influences the adhesion or stickiness of the excavated material.

Hydraulic excavators are considered as productive and practical machines used for rock excavation purposes in open pit mines. The field engineers frequently encounter with a major challenge of deciding on a hydraulic excavator to meet the technical, operating and economic requirements of their mines. Due to the complexity of taking into account the equipment related technical considerations and mine conditions, it is extremely important to formulate the decision process for evaluating the excavator alternatives and pick the best option before surface mining operation. In such cases, multi criteria decision making (MCDM) methods should be utilized to find sufficiently good alternative from a set of feasible alternatives attaining the goal of each criteria [1, 2]. Many different MCDM methods such as AHP [3] (Analytic Hierarchy Process), TOPSIS [4] (The Technique for Order of Preference by Similarity to Ideal Solution), FMADM (fuzzy multiple attribute decision making), VIKOR [5] (VlseKriterijumska Optimizacija I Kompromisno Resenje), ELECTRE [6] (ELimination Et Choix Traduisant la REalité) and PROMETHEE [7] (Preference Ranking Organization Method for Enrichment Evaluations) have been developed to cope with different multi-criteria decision problems.

As an important research branch of decision-making theory, MCDM has gained great success in various fields such as engineering, business, medical and related health sciences, environmental science, and the natural sciences [8]. In the field of mining engineering, many scholars have introduced successful MCDM applications such as equipment selection for open pit mines [9, 10], drilling technology investment analysis, ground support

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Table 1: MCDM methods applied for equipment selection in mining.

Author(s)	Employed method(s)	Application purpose
Kesimal and Bascetin (2002) [9]	FMADM	Equipment selection in an open pit mine.
Samanta <i>et al.</i> (2002) [10]	AHP	Opencast mining equipment selection.
Kazakidis (2004) [11]	AHP	Drilling technology investment analysis, ground support design, tunneling systems design, shaft location selection and mine planning risk assessment.
Bascetin (2004) [12]	AHP	Loading-hauling system selection.
Bascetin <i>et al.</i> (2006) [22]	AHP, Yager's method	Open-pit mining method selection, underground mining method selection.
Aghajani and Osanloo (2007) [13]	AHP, TOPSIS	Loading-haulage equipment selection.
Bazzazi <i>et al.</i> (2008) [14]	Fuzzy TOPSIS	Loading-haulage equipment selection.
Yavuz (2008) [23]	TOPSIS	Hydraulic excavator selection.
Bazzazi <i>et al.</i> (2009) [15]	AHP, TOPSIS	Loading-haulage equipment selection.
Bascetin (2009) [16]	AHP	Loading-hauling system selection.
Bazzazi <i>et al.</i> (2011) [17]	Fuzzy AHP	Loading-haulage equipment selection.
Bazzazi <i>et al.</i> (2011) [18]	VIKOR	Loading-hauling system selection.
Owusu-Mensah and Musungwini (2011) [24]	AHP	Ore transport system selection.
Lashgari <i>et al.</i> (2012) [19]	Fuzzy TOPSIS	Loading and hauling equipment selection.
Kun <i>et al.</i> (2013) [25]	AHP, TOPSIS	Wheel loader selection.
Yazdani-Chamzini (2014) [20]	AHP, TOPSIS, Fuzzy AHP, Fuzzy TOPSIS	Handling equipment selection in mining.
Yavuz (2015) [26]	AHP, Yager's method	Loader selection.
Yavuz (2016) [27]	Fuzzy TOPSIS	Truck selection for an open pit.
Rahimdel (2018) [21]	Fuzzy DEMATEL, Fuzzy TOPSIS	Haulage system selection in an open pit.

design, tunneling systems design, shaft location selection, mine planning risk assessment [11], loading-hauling system/equipment selection [12–21], mining method selection [22], hydraulic excavator selection [23], ore transport system selection [24], wheel loader selection [25, 26], and truck selection for an open pit [27]. Table 1 lists the MCDM methods preferred by the scholars and their application purposes in mining.

In many decision making process, it is very hard task for a decision maker to express his/her opinions by giving crisp values. Experts usually rely on their domain knowledge and experience to evaluate the alternatives and criteria. A decision maker can easily evaluate a criterion by linguistic terms such as “very low; low; medium; high; very high”. Therefore, the decision theories and methods based on linguistic terms have wide application potentials and huge benefits [8].

Noticing that the fuzzy set theory is much closer to human's subjective cognition and representation is so powerful in a decision making problem, we are motivated to use fuzzy logic based MCDM methods in equipment selection

for a mine. For this purpose, two fuzzy MCDM methods, namely; fuzzy TOPSIS and fuzzy VIKOR, are used to select a suitable hydraulic excavator from the equipment alternatives in the market for an open pit mine. In this way, fuzzy logic approach allows the decision maker to evaluate the decision criteria in linguistic expressions based on the experts' engineering knowledge, experience, intuition, and judgement.

The rest of this paper is organized as follows: In the next section, we explain the computational process of fuzzy-TOPSIS and fuzzy-VIKOR methods. Afterward, we describe the study area together with decision problem in the studied mine. Then we show how these MCDM methods are applied in the selection of hydraulic excavator for the studied site. We conclude with some main findings in the last section.

2 Materials and Methods

Rock mass classification systems or their extensions for estimating ease of excavation, excavability or diggability are often used in mining since rock excavation is highly dependent on the nature of rock mass [28]. These systems allow field engineers to get a singular index obtained by summing the ratings assigned for several parameters about the site conditions to quantify rock mass conditions. Such a rating system can only provide an indication of the ease of digging and propose the related equipment for the considered site. However, this type of classification systems should be updated so as to include newer and more powerful equipment for mechanical excavation since they evolve depending on the today's technological developments. In other words; when the equipment selection is made based on these classification systems, mechanical excavation cannot be proposed, but the rocks can be economically excavated by more powerful equipment which is not currently available in the classification systems [29].

The assessment about the ease of excavation of rocks is usually being made according to the experience of the operators since there is no generally acceptable quantitative measure of diggability at present [30]. However, a fairly reliable indication can be obtained from similar excavations in the same materials in the area or the behaviour of the ground excavated in trial pits. In conclusion, specific mine conditions should be evaluated for the success of proper selection of excavation equipment in open pits. Inconvenient equipment choice will definitely lead to increase in unit production costs and should be avoided. Therefore, selection of the most suitable equipment for open pit mining projects should be carefully analyzed [31].

Hydraulic excavators are considered as productive and practical machines used for rock excavation purposes in open pit mines. The digging and lifting power together with the production capabilities of hydraulic excavators have been improved due to the technological developments in machinery in last decades. Many manufacturers have also introduced computer controlled engine management systems and hydraulic support technology. For this reason, these machines are preferred in the direct excavation of rocks in open pit mines without employing drilling and blasting operations.

Deciding on the most suitable hydraulic excavator is not an easy task since there are many alternative machines which can be used in mine's operational conditions. In addition, many machine related technical factors should be considered in this decision making problem. As a result,

mining companies and contractors face major challenge to meet the technical, operating and economic requirements of their mines during decision making process.

Most engineers working in civil and mining industries, in which soil and rock excavation operation is involved, perform in environments where there is a need for continuous information seeking, and where the consequences for errors are costly. Given these constraints it becomes paramount that engineers make optimal decisions [32]. For this reason, it is difficult to find the best alternative or make the right decision under this uncertain conditions and fuzzy environment.

Most of the real-world decision making problems involve imprecision or vagueness, uncertainties and fuzziness in the criteria and in the evaluations made by the experts or decision makers. Therefore, classical MCDM method may encounter serious constraints in decision making process [33]. For example, the expert or decision maker may express his/her engineering knowledge by linguistic terms (fuzzy numbers) rather than using a crisp number when assigning criterion weight during the pairwise comparisons between the criteria and alternatives. The method of pairwise comparison has also been generalized in many different ways by allowing estimates to be expressed as fuzzy sets [34]. Fuzzy set theory plays an important role to represent the human thinking when decision maker's or expert's evaluations may be vague and subjective in decision making [33]. It can cope with these uncertainties in a scientific and mathematical way by allowing the decision makers to use linguistic evaluations about the decision criteria.

One of the challenging decision making problems encountered in open pit mining operations is to choose the best equipment from a set of possible alternatives. This process is of great importance for the mine management since it involves the interaction of several subjective factors or criteria. If there is a criterion expressed in linguistic terms in the selection criteria, fuzzy logic based decision making methods should be employed. Therefore, hydraulic excavator selection problem for an open pit is solved by fuzzy-TOPSIS and fuzzy-VIKOR methods. The application procedure of these methods is briefly introduced.

2.1 Fuzzy TOPSIS method

The TOPSIS method was first developed by Hwang and Yoon [4], based on the concept that the chosen alternative should have the shortest distance from the positive ideal solution and the furthest from the negative ideal solution for solving a MCDM problem. Thus, the best alternative

should not only have the shortest distance from the positive ideal solution but also have the largest distance from the negative ideal solution [35].

The TOPSIS method evaluates a decision matrix containing J alternatives associated with n attributes or criteria. It requires the following successive steps [4]:

Step 1: Find the following normalized measures to construct the normalized decision matrix (R):

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^n x_{ij}^2}} \quad (1)$$

where, x_{ij} denotes the performance measure of the j^{th} alternative in terms of the i^{th} criterion for $i=1, \dots, n$ and $j=1, \dots, J$. The weighted normalized matrix is computed as:

$$v_{ij} = r_{ij} \cdot W_i \quad (2)$$

where W_i is the normalized weight for criterion i .

Step 2: Construct the weighted normalized decision matrix as:

$$V = R \cdot W = \begin{bmatrix} w_1 \cdot r_{11} & w_2 \cdot r_{12} & \dots & w_n \cdot r_{1n} \\ w_1 \cdot r_{21} & w_2 \cdot r_{22} & \dots & w_n \cdot r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_1 \cdot r_{J1} & w_2 \cdot r_{J2} & \dots & w_n \cdot r_{Jn} \end{bmatrix} \quad (3)$$

$$= \begin{bmatrix} v_{11} & v_{12} & \dots & v_{1n} \\ v_{21} & v_{22} & \dots & v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{J1} & v_{J2} & \dots & v_{Jn} \end{bmatrix}$$

Step 3: Determine the ideal (A^*) and negative ideal (A^-) solutions:

$$A^* = \left\{ \left(\max_j v_{ij} \mid i \in I \right), \left(\min_j v_{ij} \mid i \in I' \right), j = 1, 2, \dots, J \right\} \quad (4)$$

$$= \{v_1^*, v_2^*, \dots, v_n^*\}$$

$$A^- = \left\{ \left(\min_j v_{ij} \mid i \in I \right), \left(\max_j v_{ij} \mid i \in I' \right), j = 1, 2, \dots, J \right\} \quad (5)$$

$$= \{v_1^-, v_2^-, \dots, v_n^-\}$$

Step 4: Calculate the separation measure (S_j^*) and the distance from the negative-ideal solution (S_j^-) as:

$$S_j^* = \sqrt{\sum_{i=1}^n (v_{ij} - v_i^*)^2}, \text{ for } j = 1, 2, 3, \dots, J \quad (6)$$

$$S_j^- = \sqrt{\sum_{i=1}^n (v_{ij} - v_i^-)^2}, \text{ for } j = 1, 2, 3, \dots, J \quad (7)$$

Step 5: Calculate the relative closeness to the ideal solution:

$$C_j^* = \frac{S_j^-}{S_j^* + S_j^-} \quad (8)$$

$C_j^* = 1$ if an alternative A_j is closer to the ideal solution ($A_j = A^*$) and $C_j^* = 0$ if $A_j = A^-$.

Step 6: Rank the preference order according to the order of C_j^* . The best alternative is the one with the shortest distance to S_j^* and with the longest distance to S_j^- .

Chen and Hwang [36] have extended the TOPSIS method, based on the fuzzy set theory to be used for solving fuzzy MCDM problems. Chen [37] suggested using the linguistic variables and fuzzy numbers given in Table 2 for evaluating the weights of the criteria and the ratings of alternatives.

The steps of the fuzzy TOPSIS method proposed by Chen and Hwang [36] are as follows:

Step 1: Construct the fuzzy decision matrix regarding to the alternatives ($A_1, A_2, \dots, A_J$) and criteria ($C_1, C_2, \dots, C_n$), and identify the weight vector.

$$\tilde{D} = \begin{matrix} & C_1 & C_2 & \dots & C_n \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_J \end{matrix} & \begin{bmatrix} \tilde{f}_{11} & \tilde{f}_{12} & \dots & \tilde{f}_{1n} \\ \tilde{f}_{21} & \tilde{f}_{22} & \dots & \tilde{f}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{f}_{J1} & \tilde{f}_{J2} & \dots & \tilde{f}_{Jn} \end{bmatrix} \end{matrix} \quad (9)$$

$$i = 1, 2, \dots, n; j = 1, 2, \dots, J$$

$$\tilde{W} = [\tilde{w}_1 \quad \tilde{w}_2 \quad \dots \quad \tilde{w}_n] \quad (10)$$

where, $\tilde{f}_{ij} = (l_{ij}, m_{ij}, r_{ij})$ represents fuzzy rating of alternative A_j with regard to criterion C_i and $\tilde{w}_i$ denotes the fuzzy weights for the criteria.

Step 2: Normalize the fuzzy decision matrix.

$$\tilde{R} = [\tilde{r}_{ij}] \quad (11)$$

where;

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_i^*}, \frac{b_{ij}}{c_i^*}, \frac{c_{ij}}{c_i^*} \right), \quad i \in B; \quad (12)$$

$$\tilde{r}_{ij} = \left(\frac{a_i^-}{c_{ij}}, \frac{b_i^-}{b_{ij}}, \frac{a_i^-}{a_{ij}} \right), \quad i \in C; \quad (13)$$

$$c_i^* = \max_j c_{ij} \text{ if } i \in B; \text{ (benefit criteria)} \quad (14)$$

Table 2: Linguistic variables and fuzzy numbers used in the comparison matrix [34].

Weights for criteria		Ratings for alternatives	
Linguistic variables	Fuzzy numbers	Linguistic variables	Fuzzy numbers
Very Low (VL)	(0, 0, 0.1)	Very Poor (VP)	(0, 0, 1)
Low (L)	(0, 0.1, 0.3)	Poor (P)	(0, 1, 3)
Moderately Low (ML)	(0.1, 0.3, 0.5)	Medium Poor (MP)	(1, 3, 5)
Moderate (M)	(0.3, 0.5, 0.7)	Fair (F)	(3, 5, 7)
Moderately High (MH)	(0.5, 0.7, 0.9)	Medium Good (MG)	(5, 7, 9)
High (H)	(0.7, 0.9, 1.0)	Good (G)	(7, 9, 10)
Very High (VH)	(0.9, 1.0, 1.0)	Very Good (VG)	(9, 10, 10)

$$a_i^- = \min_j a_{ij} \text{ if } i \in C; \text{ (cost criteria)} \quad (15)$$

Step 3: Calculate the weighted normalized fuzzy decision matrix.

$$\tilde{V} = [\tilde{v}_{ij}] \quad (16)$$

$$\tilde{v}_{ij} = \tilde{r}_{ij} \cdot w_i \quad (17)$$

Step 4: Calculate the Fuzzy Positive Ideal Solution (FPIS), A^* and Fuzzy Negative Ideal Solution (FNIS), A^- .

$$A^* = (\tilde{v}_1^*, \tilde{v}_2^*, \dots, \tilde{v}_n^*) \quad (18)$$

$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^-) \quad (19)$$

$$\tilde{v}_i^* = (1, 1, 1) \quad (20)$$

$$\tilde{v}_i^- = (0, 0, 0) \quad (21)$$

Step 5: Compute the distances from each alternative to the FPIS, d_j^* and FNIS, d_j^- .

$$d_j^* = \sum_{ii} d(\tilde{v}_{ij}, \tilde{v}_i^*) \quad (22)$$

$$d_j^- = \sum_{ii} d(\tilde{v}_{ij}, \tilde{v}_i^-) \quad (23)$$

where, $d(\cdot, \cdot)$ is the distance measurement between two fuzzy numbers computed by vertex method.

Step 6: Calculate the closeness coefficient CC_j for each alternative.

$$CC_j = \frac{d_j^-}{d_j^* + d_j^-} \quad (24)$$

Step 7: Rank the CC_j values in descending order, and select the best alternative having maximum CC_j value.

2.2 Fuzzy VIKOR method

The VIKOR method was firstly proposed for multi criteria optimization of complex systems by Opricovic [5]. The method determines the compromise ranking-list, the compromise solution, and the weight stability intervals for preference stability of the compromise solution obtained with the initial (given) weights. It introduces the multi criteria ranking index based on the particular measure of closeness to the ideal solution [38].

The various J alternatives are denoted as $A_1, A_2, \dots, A_J$. For alternative A_j , the rating of the i th aspect is denoted by f_{ij} , i.e. f_{ij} is the value of i th criterion function for the alternative A_j ; is the number of criteria. Development of the VIKOR method starts with the following form of L_p -metric:

$$L_{p,j} = \left\{ \sum_{i=1}^n \left[w_i \left(f_i^* - f_{ij} \right) / \left(f_i^* - f_i^- \right) \right]^p \right\}^{1/p} \quad (25)$$

where; $1 \leq p \leq \infty$; $j = 1, 2, \dots, J$.

Within the VIKOR method L_{1j} (as S_j in Eq. 26) and $L_{\infty j}$ (as R_j in Eq. 27) are used to formulate ranking measure. The solution obtained by $\min_j S_j$ is with a maximum group utility (majority rule), and the solution obtained by $\min_j R_j$ is with a minimum individual regret of the opponent.

The compromise solution F^c is a feasible solution that is the closest to the ideal F^* , and compromise means an agreement established by mutual concessions, as is illustrated in Figure 1 by $\Delta f_1 = f_1^* - f_1^c$ and $\Delta f_2 = f_2^* - f_2^c$.

The compromise ranking algorithm VIKOR has the following steps [39]:

Step 1: Determine the best f_i^* and the worst f_i^- values of all criterion functions, $i = 1, 2, \dots, n$. If the i th function represents a benefit then: $f_i^* = \max_j f_{ij}$, $f_i^- = \min_j f_{ij}$.

Figure 1: Ideal and compromise solutions.

Step 2: Compute the values of S_j and R_j ; $j = 1, 2, \dots, J$:

$$S_j = \sum_{i=1}^n \left[w_i \left(f_i^* - f_{ij} \right) / \left(f_i^* - f_i^- \right) \right] \quad (26)$$

$$R_j = \max_i \left[w_i \left(f_i^* - f_{ij} \right) / \left(f_i^* - f_i^- \right) \right] \quad (27)$$

where, w_i is the criterion weight expressing its relative importance.

Step 3: Compute the Q_j values, $j = 1, 2, \dots, J$:

$$Q_j = \frac{\vartheta (S_j - S^*)}{(S^- - S^*)} + \frac{(1 - \vartheta) (R_j - R^*)}{(R^- - R^*)} \quad (28)$$

where; $S^* = \min_j S_j$, $S^- = \max_j S_j$, $R^* = \min_j R_j$, $R^- = \max_j R_j$ and ϑ is introduced as weight of the strategy of “the majority of criteria” (or “the maximum group utility”).

Step 4: Rank the alternatives sorting by the values of S , R and Q , in descending order.

Step 5: Propose as a compromise solution the alternative (A') which is ranked the best by the measure Q (minimum) if the following two conditions are satisfied:

Condition 1 (“Acceptable advantage”)

$$Q(A'') - Q(A') \geq DQ \quad (29)$$

where, A'' is the alternative with second position in the ranking list by Q ; $DQ = 1/(J - 1)$; J is the number of alternatives.

Condition 2 (“Acceptable stability in decision making”)

Alternative A' must also be the best ranked by S or/and R . This compromise solution is stable within a decision making process, which could be: “voting by majority rule”

(when $\vartheta > 0.5$ is needed), or “by consensus” $\vartheta \approx 0.5$, or “with veto” ($\vartheta < 0.5$). Here, ϑ is the weight of the decision making strategy “the majority of criteria” (or “the maximum group utility”).

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of: Alternatives A' and A'' if only condition 2 is not satisfied, Alternatives A' , A'' , $\dots$, $A^{(M)}$ if condition 1 is not satisfied; and $A^{(M)}$ is determined by the relation $Q(A^{(M)}) - Q(A') < DQ$ for maximum M (the positions of these alternatives are “in closeness”).

The best alternative, ranked by Q , is the one with the minimum value of Q . The main ranking result is the compromise ranking list of alternatives, and the compromise solution with the “advantage rate”.

The VIKOR method has been extended into many different forms, such as the fuzzy VIKOR (F-*VIKOR*) [40, 41], the interval valued VIKOR (IV-*VIKOR*) [42], the intuitionistic fuzzy VIKOR (IF-*VIKOR*) [43], the interval-valued intuitionistic fuzzy VIKOR (IVIF-*VIKOR*) [44], the hesitant fuzzy VIKOR (HF-*VIKOR*) [45], and the hesitant fuzzy linguistic VIKOR (HFL-*VIKOR*) [46].

The steps of fuzzy VIKOR method are as follows:

Step 1: Construct the fuzzy decision matrix and identify weight vector. Determine the alternatives ($A_1, A_2, \dots, A_J$) and criteria ($C_1, C_2, \dots, C_n$).

$$\tilde{D} = \begin{matrix} & \begin{matrix} C_1 & C_2 & \dots & C_n \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_J \end{matrix} & \begin{bmatrix} \tilde{f}_{11} & \tilde{f}_{12} & \dots & \tilde{f}_{1n} \\ \tilde{f}_{21} & \tilde{f}_{22} & \dots & \tilde{f}_{2n} \\ \vdots & \vdots & & \vdots \\ \tilde{f}_{J1} & \tilde{f}_{J2} & \dots & \tilde{f}_{Jn} \end{bmatrix} \end{matrix} \quad (30)$$

$i = 1, 2, \dots, n; j = 1, 2, \dots, nJ$

$$\tilde{W} = \begin{bmatrix} \tilde{w}_1 & \tilde{w}_2 & \dots & \tilde{w}_n \end{bmatrix} \quad (31)$$

where; $\tilde{f}_{ij} = (l_{ij}, m_{ij}, r_{ij})$ denotes fuzzy rating corresponding to alternative A_j with regard to criterion C_i , $\tilde{w}_i$ denotes the fuzzy weights for the criteria.

Step 2: Determine the fuzzy best $\tilde{f}_i^* = (l_i^*, m_i^*, r_i^*)$ and the fuzzy worst $\tilde{f}_i^- = (l_i^-, m_i^-, r_i^-)$ values of all criteria regarding to their benefit or cost functions.

$$\tilde{f}_i^* = \max_j \tilde{f}_{ij} \quad (32)$$

$$\tilde{f}_i^- = \min_j \tilde{f}_{ij} \quad (33)$$

Step 3: Compute the values of $\tilde{S}_j$ and $\tilde{R}_j$,

$$\tilde{S}_j = \sum_{i=1}^n \left[\tilde{w}_i \left(\tilde{f}_i^* - \tilde{f}_{ij} \right) / \left(\tilde{f}_i^* - \tilde{f}_i^- \right) \right] \quad (34)$$

$$\tilde{R}_j = \max_i \left[\tilde{w}_i \left(\tilde{f}_i^* - \tilde{f}_{ij} \right) / \left(\tilde{f}_i^* - \tilde{f}_i^- \right) \right] \quad (35)$$

Step 4: Compute the values of $\tilde{Q}_j$,

$$\tilde{Q}_j = \frac{\vartheta \left(\tilde{S}_j - \tilde{S}^* \right)}{\left(\tilde{S}^- - \tilde{S}^* \right)} + \frac{(1 - \vartheta) \left(\tilde{R}_j - \tilde{R}^* \right)}{\left(\tilde{R}^- - \tilde{R}^* \right)} \quad (36)$$

where; $\tilde{S}^* = \min_j \tilde{S}_j$, $\tilde{S}^- = \max_j \tilde{S}_j$, $\tilde{R}^* = \min_j \tilde{R}_j$, $\tilde{R}^- = \max_j \tilde{R}_j$ and $\vartheta = 0.5$.

Step 5: Rank the alternatives for $\tilde{S}$, $\tilde{R}$ and $\tilde{Q}$ values in ascending order, and construct three lists based on $\tilde{S}$, $\tilde{R}$ and $\tilde{Q}$ values. The best alternative in each ranked list is the one with the lowest value.

Step 6: Propose a compromise solution, the alternative (A^*) ranked the best by the measure $\tilde{Q}$ (minimum) if the two conditions mentioned above are satisfied.

2.3 Comparison of VIKOR and TOPSIS methods

Although the two MCDM methods TOPSIS and VIKOR have been proposed for the problem of equipment selection in this study, there are some differences between the two methods when compared to each other [1, 39]. These differences are listed below as a guide to a decision maker for the selection of appropriate equipment:

1. Linear normalization is used by the VIKOR method, whereas vector normalization is used by the TOPSIS method. Unlike the VIKOR method, the normalized value in the TOPSIS method could depend on the evaluation unit.
2. The VIKOR method considers an aggregating function of all criteria, the relative importance of the criteria, and a balance between total and individual satisfaction. This function represents the distance from the ideal solution. On the other hand, the TOPSIS method is based on the ranking index including the distances from the ideal point and from the negative-ideal point. These distances in TOPSIS are simply summed without considering their relative importance.

3. While the closest one to the ideal solution is the alternative with the highest ranked by the VIKOR method, the highest ranked alternative by the TOPSIS method is not always the closest to the ideal solution, although it is the best in terms of the ranking index. Also the VIKOR method proposes a compromise solution with an advantage rate.

3 Case Study

The fuzzy MCDM methods are applied to select a suitable hydraulic excavator, which is one of the important equipment, for a magnesite mine (Figure 2) owned by Yamac mining company. The open pit, in which raw magnesite of 20000 tonnes per year has been extracted, is located in Mihaliccik district of Eskisehir in Turkey as seen in Figure 3.

A hydraulic excavator is requested for the open pit by the mine management to excavate the rocks and magnesite ore without using drilling and blasting operation. The past experiences and engineering knowledge show that direct excavation in the mine is possible with the usage of a hydraulic excavator having 300-350 HP of installed power and 40-50 tonnes of weight. However, there are many excavator alternatives in the market having the same technical specifications in different brands' product range. For this reason, it requires the use of one of the MCDM methods to select the most suitable hydraulic excavator for this open pit mine by evaluating the criteria and the alternatives. The technical specifications of alternative hydraulic excavators, which are considered to be in accordance with the mine conditions, are given in Table 3. The alternative excavators in Table 3 are assigned with A_1 , A_2 , A_3 , A_4 and

Figure 2: A view of Yamac magnesite mine.

Figure 3: Location map of Yamac magnesite mine in Mihaliccik, Eskisehir.

Table 3: Technical specifications of hydraulic excavators.

Criteria	Excavator Alternatives				
	A ₁	A ₂	A ₃	A ₄	A ₅
C ₁ Operating weight (t)	44.9	49.2	45.1	40.2	42.6
C ₂ Net flywheel power (HP)	345	326	321	307	345
C ₃ Bucket capacity (m ³)	2.8	3	2	1.9	2.8
C ₄ Digging force (kN)	210	248	224.6	215	275
C ₅ Lifting capacity (t)	24.5	27	25	11.5	24.8
C ₆ Max. digging depth (mm)	8920	8550	8250	8285	8445
C ₇ Max. reach ground level (mm)	12960	12300	12290	11665	12565
C ₈ Max. loading height (mm)	7930	7750	7840	7420	7715
C ₉ Max. travel speed (km/h)	4.4	4.8	4.8	5.3	5.5
C ₁₀ Fuel tank capacity (l)	705	700	685	620	650
C ₁₁ Hyd. Sys. max. press (kPa)	38000	38400	31400	32400	37300
C ₁₂ Swing speed (RPM)	8.6	5.6	8.5	10.2	9.1
C ₁₃ Fuel consumption (l/h)	Medium	High	Low	Low	High
C ₁₄ Attachments	High	Medium	Low	Medium	High
C ₁₅ Sound performance	High	Low	Medium	Low	High
C ₁₆ Break performance	Low	Medium	High	Low	High
C ₁₇ Machine price	High	Low	Medium	Medium	High

A₅ in order not to let a conflict of interest about the excavator brands in the market.

An expert group consisting of five engineers, whose expertise area is rock excavation in open pit mines, is formed to decide on the best convenient hydraulic excavator for the studied site.

As a first step, the decision makers are requested to evaluate the relative importance (weight) of the criteria by linguistic variables based on the terms defined in Table 2. Their evaluations are given in Table 4.

And then, the expert group members also evaluate the alternative hydraulic excavators depending on the criteria

Table 4: Linguistic evaluations for the criteria by the expert group.

Criteria	Decision Makers (Expert Group)				
	1	2	3	4	5
C_1	VH	H	H	VH	VH
C_2	VH	VH	H	H	VH
C_3	H	M	MH	H	M
C_4	VH	MH	H	VH	H
C_5	H	M	MH	ML	M
C_6	L	M	MH	M	ML
C_7	H	VH	MH	H	M
C_8	M	MH	M	VH	H
C_9	M	ML	ML	VL	L
C_{10}	H	VH	H	VH	VH
C_{11}	H	M	H	M	M
C_{12}	VH	H	H	VH	MH
C_{13}	H	H	M	M	M
C_{14}	M	ML	M	ML	VL
C_{15}	M	MH	L	VL	L
C_{16}	M	H	VH	M	MH
C_{17}	MH	M	MH	M	M

as shown in Table 5 using the ratings given as linguistic terms in Table 2.

4 Results and discussion

Fuzzy TOPSIS and fuzzy VIKOR methods to decide on the best convenient hydraulic excavator for the studied site are introduced in this section. From the linguistic evaluations of expert group mentioned above, the step by step calculation procedure is illustrated.

4.1 Implementation of fuzzy Topsis method

Step 1: The fuzzy decision matrix regarding to the alternatives is constructed as shown in Eq. 9. For this purpose, the evaluations stated in linguistic forms by the decision makers for every criterion in Table 4 and for alternatives in Table 5 are turned into fuzzy numbers by considering the corresponding weights and ratings in Table 2.

The mean fuzzy numbers for the criteria are calculated as follows since 5 decision makers assigned VH (0.9, 1.0, 1.0), H (0.7, 0.9, 1.0), H, VH and VH linguistic values for C_1 criterion:

$$(0.9 + 0.7 + 0.7 + 0.9 + 0.9)/5 = 0.82,$$

$$(1 + 0.9 + 0.9 + 1 + 1)/5 = 0.96,$$

$$(1 + 1 + 1 + 1 + 1)/5 = 1.$$

The calculated fuzzy weights for all criteria are given in Table 6 as the weight vector identified in Eq. 10.

For alternative of A_1 considering C_1 criterion, 5 decision makers assigned G (7, 9, 10), MG (5, 7, 9), G, VG (9, 10, 10) and MG linguistic variables. For this reason, the mean fuzzy numbers are calculated as follows:

$$(7 + 5 + 7 + 9 + 5)/5 = 6.60,$$

$$(9 + 7 + 9 + 10 + 7)/5 = 8.40,$$

$$(10 + 9 + 10 + 10 + 9)/5 = 9.60.$$

The fuzzy ratings for all alternatives with regard to all criteria are calculated, and the fuzzy decision matrix in Eq. 9 is composed of mean fuzzy numbers (Table 7).

Step 2: The fuzzy decision matrix in Table 7 is normalized by using Eq. 11, 12 and 14. The first fuzzy number of 6.60 for C_1 criterion based on A_1 alternative is divided to the maximum value of fuzzy rating (9.80) in Table 7 for C_1 criterion ($6.60/9.80 = 0.673$).

Step 3: The weighted and normalized fuzzy decision matrix in Eq. 16 is then obtained by using Eq. 17. The normalized values in step 2 are then multiplied with the fuzzy weights in Table 6. The normalized value of 0.673 for C_1 criterion is then multiplied with the corresponding fuzzy weight of 0.82 for C_1 criterion in Table 6 ($0.673 \times 0.82 = 0.552$). When the similar calculations are made for the other elements of the matrix, the weighted and normalized fuzzy decision matrix is constructed as shown in Table 8.

Step 4: As the fuzzy ratings for each alternative in the weighted normalized fuzzy matrix in Table 8 have a value between 0 and 1, the FPIS A^+ is (1, 1, 1) and FNIS A^- is (0, 0, 0) according to Eq. 18, 19, 20, and 21.

Step 5: The distances from each alternative to the FPIS, d_j^* and FNIS, d_j^- are computed by using Eq. 22 and Eq. 23.

For A_1 alternative with regard to C_1 criterion, d_1^* and d_1^- are calculated as follows:

By using vertex method, first subtract the fuzzy numbers in weighted normalized fuzzy decision matrix for C_1 criterion (0.552; 0.823; 0.980) from the FPIS of 1, then find the square root of the mean value of sum of the square of these values to reach $d(\tilde{v}_{11}, \tilde{v}_1^*)$.

Table 5: Linguistic evaluations for the alternatives depending on the criteria.

Criteria	Alternatives	Decision Makers (Expert Group)				
		1	2	3	4	5
C_1	A_1	G	MG	G	VG	MG
	A_2	G	MG	MG	G	G
	A_3	G	MG	G	VG	G
	A_4	F	MP	MG	G	MP
	A_5	MG	F	MG	G	F
C_2	A_1	G	VG	G	VG	G
	A_2	MG	G	G	G	MG
	A_3	MG	G	G	G	MG
	A_4	F	MG	MG	G	MG
	A_5	G	VG	G	VG	G
C_3	A_1	G	G	VG	G	G
	A_2	VG	G	VG	VG	VG
	A_3	F	MG	G	MG	G
	A_4	F	MG	G	MG	G
	A_5	G	G	VG	G	VG
C_4	A_1	F	MG	G	F	MP
	A_2	G	VG	VG	G	MG
	A_3	MG	G	G	MG	F
	A_4	F	G	G	MG	F
	A_5	VG	VG	VG	G	G
C_5	A_1	G	VG	MG	G	G
	A_2	VG	VG	G	G	VG
	A_3	G	VG	MG	G	G
	A_4	P	MP	F	VP	VP
	A_5	G	VG	MG	G	G
C_6	A_1	VG	G	VG	G	VG
	A_2	G	MG	G	MG	G
	A_3	MG	G	G	F	MG
	A_4	MG	G	G	F	MG
	A_5	G	MG	G	MG	G
C_7	A_1	VG	G	VG	VG	VG
	A_2	G	G	VG	G	G
	A_3	G	G	VG	G	G
	A_4	MG	F	G	MG	MG
	A_5	G	G	VG	G	VG
C_8	A_1	VG	G	VG	G	VG
	A_2	G	G	G	G	G
	A_3	VG	G	VG	G	G
	A_4	G	G	G	F	MG
	A_5	VG	G	VG	G	G
C_9	A_1	G	G	MG	MG	F
	A_2	G	G	G	MG	MG
	A_3	G	G	G	MG	MG
	A_4	VG	VG	VG	G	G
	A_5	VG	VG	VG	G	VG
C_{10}	A_1	VG	VG	G	G	VG

Continued on the next page

Table 5: ... continued.

Criteria	Alternatives	Decision Makers (Expert Group)				
		1	2	3	4	5
	A_2	VG	VG	G	G	VG
	A_3	VG	G	G	MG	G
	A_4	MG	G	MG	MG	G
	A_5	G	MG	G	F	MG
C_{11}	A_1	VG	G	G	VG	G
	A_2	VG	G	VG	VG	G
	A_3	MG	F	MG	MG	F
	A_4	MG	F	MG	MG	MG
	A_5	VG	G	G	VG	G
C_{12}	A_1	G	MG	MG	MG	G
	A_2	F	MP	MP	P	F
	A_3	G	MG	MG	MG	G
	A_4	VG	VG	G	VG	VG
	A_5	VG	G	MG	G	G
C_{13}	A_1	G	F	G	F	G
	A_2	VP	P	VP	P	VP
	A_3	VG	G	VG	G	VG
	A_4	VG	G	VG	G	VG
	A_5	VP	P	VP	P	VP
C_{14}	A_1	VG	G	VG	G	G
	A_2	P	MG	P	F	MG
	A_3	VP	F	VP	P	F
	A_4	P	MG	P	F	MG
	A_5	VG	G	VG	G	G
C_{15}	A_1	VP	P	VP	P	MP
	A_2	P	G	P	G	VG
	A_3	F	F	F	F	MG
	A_4	P	G	P	G	VG
	A_5	VP	P	VP	P	MP
C_{16}	A_1	MG	MP	MP	G	MG
	A_2	F	F	F	MG	F
	A_3	VG	G	G	VG	VG
	A_4	MG	MP	MP	G	MG
	A_5	VG	G	G	VG	VG
C_{17}	A_1	VP	P	F	VP	P
	A_2	G	G	MG	MG	G
	A_3	F	F	MG	F	MG
	A_4	F	F	MG	F	MG
	A_5	VP	P	F	VP	P

Concluded

Table 6: Fuzzy weights for the criteria.

Criteria	Fuzzy weights		
C_1	0.82	0.96	1.00
C_2	0.82	0.96	1.00
C_3	0.50	0.70	0.86
C_4	0.74	0.90	0.98
C_5	0.38	0.58	0.76
C_6	0.24	0.42	0.62
C_7	0.62	0.80	0.92
C_8	0.54	0.72	0.86
C_9	0.10	0.24	0.42
C_{10}	0.82	0.96	1.00
C_{11}	0.46	0.66	0.82
C_{12}	0.74	0.90	0.98
C_{13}	0.46	0.66	0.82
C_{14}	0.16	0.32	0.50
C_{15}	0.16	0.28	0.46
C_{16}	0.54	0.72	0.86
C_{17}	0.38	0.58	0.78

$$\left(\sqrt{\left((0.552 - 1)^2 + (0.823 - 1)^2 + (0.980 - 1)^2 \right) / 3} \right) = 0.278$$

When $d(\tilde{v}_{i1}, \tilde{v}_i^*)$ value for every criterion is found, the summed value of them gives the distance from A_1 alternative to the FPIS, $d_1^* = 8.776$.

In order to find d_1^- for A_1 alternative based on C_1 criterion, the difference between fuzzy numbers for C_1 criterion (0.552;0.823;0.980) and FNIS of 0 is found, and $d(\tilde{v}_{11}, \tilde{v}_1^-)$ is calculated as;

$$\left(\sqrt{\left((0 - 0.552)^2 + (0 - 0.823)^2 + (0 - 0.980)^2 \right) / 3} \right) = 0.805$$

After computing $d(\tilde{v}_{i1}, \tilde{v}_i^-)$ value for every criterion, the computed values are summed and d_j^- value (9.343) is obtained as the distance from A_1 alternative to the FNIS.

Table 9 shows the d_j^* and d_j^- values for all of the alternatives.

Step 6: The final decision is made by considering closeness coefficient of CC_j for each alternative. For this purpose, CC_j coefficients are calculated for every alternative by using Eq. 24.

The CC_j coefficient for A_1 alternative is found as by using the values of 8.776 and 9.343 for d_j^* and d_j^- , respectively:

$(9.343)/(9.343+8.776) = 0.516$. The closeness coefficient for each alternative is calculated as shown in Table 10. These values are sorted by the CC_j values in descending order to find the rank of the alternatives.

The results obtained from fuzzy TOPSIS method based on expert group decisions in linguistic forms reveal that the alternative of A_3 hydraulic excavator is the best solution for the studied site.

4.2 Implementation of fuzzy VIKOR method

The fuzzy VIKOR method uses the same fuzzy decision matrix (Table 7) constructed by linguistic evaluation of the experts' opinions and knowledge as mentioned in Section 4.1. Therefore, it is possible to determine the fuzzy best $\tilde{f}_i^*$ and the fuzzy worst $\tilde{f}_i^-$ values without constructing another decision matrix as defined in Eq. 30.

Step 1: The best ($\tilde{f}_i^*$) and the worst ($\tilde{f}_i^-$) values for all criteria functions are calculated as shown in Table 11 by applying maximum and minimum operator in Eq. 32 and 33. For C_1 criterion in Table 11, the values of 7.0, 8.8 and 9.8 (the first, second and third fuzzy numbers) for $\tilde{f}_i^*$ are the maximum fuzzy numbers assigned for 5 alternatives according to C_1 criterion in Table 7. Similarly, the values of 3.4, 5.4 and 7.2 for the worst ($\tilde{f}_i^-$) are the minimum ones.

Step 2: The values of $\tilde{S}_j$ are the sum of S_j values, which mean the distance for j th alternative to the best fuzzy solution according to all of the criteria, by using Eq. 34. For example, the $\tilde{S}_j$ value for A_1 alternative is found as follows:

$$\begin{aligned} S_{A1} = & (0.82 \times (7 - 6.6)/(7 - 3.4)) + (0.82 \times (7.8 - 7.8)/(7.8 - 5)) + (0.50 \times (8.6 - 7.4)/(8.6 - 5.4)) \\ & + (0.74 \times (8.2 - 3.8)/(8.2 - 3.8)) + (0.38 \times (8.2 - 7)/(8.2 - 0.8)) + (0.24 \times (8.2 - 8.2)/(8.2 - 5.4)) \\ & + (0.62 \times (8.6 - 8.6)/(8.6 - 5)) + (0.54 \times (8.2 - 8.2)/(8.2 - 5.8)) + (0.10 \times (8.6 - 5.4)/(8.6 - 5.4)) \\ & + (0.82 \times (8.2 - 8.2)/(8.2 - 5.4)) + (0.46 \times (8.2 - 7.8)/(8.2 - 4.2)) + (0.74 \times (8.6 - 5.8)/(8.6 - 1.6)) \\ & + (0.46 \times (8.2 - 5.4)/(8.2 - 0)) + (0.16 \times (7.8 - 7.8)/(7.8 - 1.2)) + (0.16 \times (4.2 - 0.2)/(4.2 - 0.2)) \\ & + (0.16 \times (4.6 - 0.2)/(4.6 - 0.2)) + (0.54 \times (8.2 - 3.8)/(8.2 - 3.4)) + (0.38 \times (6.2 - 0.6)/(6.2 - 0.6)) \\ = & 2.714. \end{aligned}$$

The values of $\tilde{S}_j$ for every alternative are similarly calculated, and the results are given in Table 12 together with the mean $\tilde{S}_j$ values.

Table 7: Fuzzy decision matrix.

	A ₁			A ₂			A ₃			A ₄			A ₅		
C ₁	6.60	8.40	9.60	6.20	8.20	9.60	7.00	8.80	9.80	3.40	5.40	7.20	4.60	6.60	8.40
C ₂	7.80	9.40	10.00	6.20	8.20	9.60	6.20	8.20	9.60	5.00	7.00	8.80	7.80	9.40	10.00
C ₃	7.40	9.20	10.00	8.60	9.80	10.00	5.40	7.40	9.00	5.40	7.40	9.00	7.80	9.40	10.00
C ₄	3.80	5.80	7.60	7.40	9.00	9.80	5.40	7.40	9.00	5.00	7.00	8.60	8.20	9.60	10.00
C ₅	7.00	8.80	9.80	8.20	9.60	10.00	7.00	8.80	9.80	0.80	1.80	3.40	7.00	8.80	9.80
C ₆	8.20	9.60	10.00	6.20	8.20	9.60	5.40	7.40	9.00	5.40	7.40	9.00	6.20	8.20	9.60
C ₇	8.60	9.80	10.00	7.40	9.20	10.00	7.40	9.20	10.00	5.00	7.00	8.80	7.80	9.40	10.00
C ₈	8.20	9.60	10.00	7.00	9.00	10.00	7.80	9.40	10.00	5.80	7.80	9.20	7.80	9.40	10.00
C ₉	5.40	7.40	9.00	6.20	8.20	9.60	6.20	8.20	9.60	8.20	9.60	10.00	8.60	9.80	10.00
C ₁₀	8.20	9.60	10.00	8.20	9.60	10.00	7.00	8.80	9.80	5.80	7.80	9.40	5.40	7.40	9.00
C ₁₁	7.80	9.40	10.00	8.20	9.60	10.00	4.20	6.20	8.20	4.60	6.60	8.60	7.80	9.40	10.00
C ₁₂	5.80	7.80	9.40	1.60	3.40	5.40	5.80	7.80	9.40	8.60	9.80	10.00	7.00	8.80	9.80
C ₁₃	5.40	7.40	8.80	0.00	0.40	1.80	8.20	9.60	10.00	8.20	9.60	10.00	0.00	0.40	1.80
C ₁₄	7.80	9.40	10.00	2.60	4.20	6.20	1.20	2.20	3.80	2.60	4.20	6.20	7.80	9.40	10.00
C ₁₅	0.20	1.00	2.60	4.60	6.00	7.20	3.40	5.40	7.40	4.60	6.00	7.20	0.20	1.00	2.60
C ₁₆	3.80	5.80	7.60	3.40	5.40	7.40	8.20	9.60	10.00	3.80	5.80	7.60	8.20	9.60	10.00
C ₁₇	0.60	1.40	3.00	6.20	8.20	9.60	3.80	5.80	7.80	3.80	5.80	7.80	0.60	1.40	3.00

Table 8: Weighted normalized fuzzy decision matrix.

	A ₁			A ₂			A ₃			A ₄			A ₅		
C ₁	0.552	0.823	0.980	0.519	0.803	0.980	0.586	0.862	1.000	0.284	0.529	0.735	0.385	0.647	0.857
C ₂	0.640	0.902	1.000	0.508	0.787	0.960	0.508	0.787	0.960	0.410	0.672	0.880	0.640	0.902	1.000
C ₃	0.370	0.644	0.860	0.430	0.686	0.860	0.270	0.518	0.774	0.270	0.518	0.774	0.390	0.658	0.860
C ₄	0.281	0.522	0.745	0.548	0.810	0.960	0.400	0.666	0.882	0.370	0.630	0.843	0.607	0.864	0.980
C ₅	0.266	0.510	0.745	0.312	0.557	0.760	0.266	0.510	0.745	0.030	0.104	0.258	0.266	0.510	0.745
C ₆	0.197	0.403	0.620	0.149	0.344	0.595	0.130	0.311	0.558	0.130	0.311	0.558	0.149	0.344	0.595
C ₇	0.533	0.784	0.920	0.459	0.736	0.920	0.459	0.736	0.920	0.310	0.560	0.810	0.484	0.752	0.920
C ₈	0.443	0.691	0.860	0.378	0.648	0.860	0.421	0.677	0.860	0.313	0.562	0.791	0.421	0.677	0.860
C ₉	0.054	0.178	0.378	0.062	0.197	0.403	0.062	0.197	0.403	0.082	0.230	0.420	0.086	0.235	0.420
C ₁₀	0.672	0.922	1.000	0.672	0.922	1.000	0.574	0.845	0.980	0.476	0.749	0.940	0.443	0.710	0.900
C ₁₁	0.359	0.620	0.820	0.377	0.634	0.820	0.193	0.409	0.672	0.212	0.436	0.705	0.359	0.620	0.820
C ₁₂	0.429	0.702	0.921	0.118	0.306	0.529	0.429	0.702	0.921	0.636	0.882	0.980	0.518	0.792	0.960
C ₁₃	0.248	0.488	0.722	0.000	0.026	0.148	0.377	0.634	0.820	0.377	0.634	0.820	0.000	0.026	0.148
C ₁₄	0.125	0.301	0.500	0.042	0.134	0.310	0.019	0.070	0.190	0.042	0.134	0.310	0.125	0.301	0.500
C ₁₅	0.004	0.038	0.162	0.099	0.227	0.448	0.074	0.204	0.460	0.099	0.227	0.448	0.004	0.038	0.162
C ₁₆	0.205	0.418	0.654	0.184	0.389	0.636	0.443	0.691	0.860	0.205	0.418	0.654	0.443	0.691	0.860
C ₁₇	0.024	0.085	0.244	0.245	0.495	0.780	0.150	0.350	0.634	0.150	0.350	0.634	0.024	0.085	0.244

Table 9: Distances from positive and negative fuzzy ideal solution.

	Positive					Negative				
	A ₁	A ₂	A ₃	A ₄	A ₅	A ₁	A ₂	A ₃	A ₄	A ₅
C ₁	0.278	0.300	0.252	0.518	0.418	0.805	0.790	0.834	0.548	0.658
C ₂	0.216	0.310	0.310	0.396	0.216	0.861	0.775	0.775	0.682	0.861
C ₃	0.426	0.384	0.522	0.522	0.412	0.656	0.682	0.560	0.560	0.664
C ₄	0.520	0.284	0.402	0.431	0.240	0.550	0.791	0.679	0.644	0.832
C ₅	0.530	0.493	0.530	0.874	0.530	0.543	0.573	0.543	0.162	0.543
C ₆	0.618	0.663	0.690	0.690	0.663	0.442	0.406	0.376	0.376	0.406
C ₇	0.301	0.351	0.351	0.485	0.334	0.763	0.730	0.730	0.596	0.741
C ₈	0.377	0.420	0.391	0.486	0.391	0.686	0.659	0.677	0.589	0.677
C ₉	0.808	0.792	0.792	0.768	0.765	0.243	0.261	0.261	0.281	0.282
C ₁₀	0.194	0.194	0.262	0.337	0.367	0.876	0.876	0.817	0.746	0.710
C ₁₁	0.443	0.430	0.608	0.585	0.443	0.629	0.637	0.468	0.494	0.629
C ₁₂	0.375	0.702	0.375	0.221	0.304	0.713	0.359	0.713	0.845	0.778
C ₁₃	0.549	0.944	0.430	0.430	0.944	0.523	0.087	0.637	0.637	0.087
C ₁₄	0.708	0.845	0.910	0.845	0.708	0.345	0.197	0.118	0.197	0.345
C ₁₅	0.935	0.756	0.771	0.756	0.935	0.096	0.295	0.294	0.295	0.096
C ₁₆	0.603	0.625	0.377	0.603	0.377	0.463	0.443	0.686	0.463	0.686
C ₁₇	0.887	0.539	0.653	0.653	0.887	0.150	0.552	0.427	0.427	0.150
Sum	8.766	9.034	8.624	9.600	8.934	9.343	9.114	9.595	8.541	9.145

Table 10: The rank of the alternatives.

Alternatives	Positive distance, d_j^+	Negative distance, d_j^-	CC_j	Rank
A ₁	8.766	9.343	0.516	2
A ₂	9.034	9.114	0.502	4
A ₃	8.624	9.595	0.527	1
A ₄	9.600	8.541	0.471	5
A ₅	8.934	9.145	0.506	3

The values of $\tilde{R}_j$ are the sum of R_j values, which mean the maximum distance for j th alternative to the worst fuzzy solution according to the all criteria, by using Eq. 35. For $\tilde{R}_j$ value with regard to A₁ alternative is found as follows:

$$\begin{aligned}
 R_{A1} = & \max((0.82 \times (7 - 6.6)/(7 - 3.4)) + (0.82 \times (7.8 - 7.8)/(7.8 - 5)) + (0.50 \times (8.6 - 7.4)/(8.6 - 5.4)) \\
 & + (0.74 \times (8.2 - 3.8)/(8.2 - 3.8)) + (0.38 \times (8.2 - 7)/(8.2 - 0.8)) + (0.24 \times (8.2 - 8.2)/(8.2 - 5.4)) \\
 & + (0.62 \times (8.6 - 8.6)/(8.6 - 5)) + (0.54 \times (8.2 - 8.2)/(8.2 - 5.8)) + (0.10 \times (8.6 - 5.4)/(8.6 - 5.4)) \\
 & + (0.82 \times (8.2 - 8.2)/(8.2 - 5.4)) + (0.46 \times (8.2 - 7.8)/(8.2 - 4.2)) + (0.74 \times (8.6 - 5.8)/(8.6 - 1.6)) \\
 & + (0.46 \times (8.2 - 5.4)/(8.2 - 0)) + (0.16 \times (7.8 - 7.8)/(7.8 - 1.2)) + (0.16 \times (4.2 - 0.2)/(4.2 - 0.2)) \\
 & + (0.16 \times (4.6 - 0.2)/(4.6 - 0.2)) + (0.54 \times (8.2 - 3.8)/(8.2 - 3.4)) + (0.38 \times (6.2 - 0.6)/(6.2 - 0.6))) \\
 = & \max(0.091, 0, 0.188, 0.740, 0.062, 0, 0, 0, 0.100,
 \end{aligned}$$

Table 11: The fuzzy best $\tilde{f}_i^*$ and worst $\tilde{f}_i^-$ values.

Criteria	$\tilde{f}_i^*$			$\tilde{f}_i^-$		
C ₁	7.0	8.8	9.8	3.4	5.4	7.2
C ₂	7.8	9.4	10.0	5.0	7.0	8.8
C ₃	8.6	9.8	10.0	5.4	7.4	9.0
C ₄	8.2	9.6	10.0	3.8	5.8	7.6
C ₅	8.2	9.6	10.0	0.8	1.8	3.4
C ₆	8.2	9.6	10.0	5.4	7.4	9.0
C ₇	8.6	9.8	10.0	5.0	7.0	8.8
C ₈	8.2	9.6	10.0	5.8	7.8	9.2
C ₉	8.6	9.8	10.0	5.4	7.4	9.0
C ₁₀	8.2	9.6	10.0	5.4	7.4	9.0
C ₁₁	8.2	9.6	10.0	4.2	6.2	8.2
C ₁₂	8.6	9.8	10.0	1.6	3.4	5.4
C ₁₃	8.2	9.6	10.0	0.0	0.4	1.8
C ₁₄	7.8	9.4	10.0	1.2	2.2	3.8
C ₁₅	4.6	6.0	7.4	0.2	1.0	2.6
C ₁₆	8.2	9.6	10.0	3.4	5.4	7.4
C ₁₇	6.2	8.2	9.6	0.6	1.4	3.0

$$\begin{aligned}
 & - 7.8)/(7.8 - 1.2)) + (0.16 \times (4.2 - 0.2)/(4.2 - 0.2)) \\
 & + (0.16 \times (4.6 - 0.2)/(4.6 - 0.2)) + (0.54 \times (8.2 - 3.8)/(8.2 - 3.4)) + (0.38 \times (6.2 - 0.6)/(6.2 - 0.6))) \\
 = & \max(0.091, 0, 0.188, 0.740, 0.062, 0, 0, 0, 0.100,
 \end{aligned}$$

Table 12: $\tilde{S}_j$ and $\tilde{R}_j$ values.

Alternatives	$\tilde{S}_j$			Mean	$\tilde{R}_j$			Mean
A_1	2.714	3.477	3.782	3.324	0.740	0.900	0.980	0.873
A_2	3.374	4.141	3.894	3.803	0.740	0.900	0.980	0.873
A_3	3.587	4.441	4.273	4.100	0.500	0.700	0.860	0.687
A_4	6.371	8.231	9.162	7.921	0.820	0.960	1.000	0.927
A_5	3.168	3.918	3.912	3.666	0.820	0.960	1.000	0.927

Table 13: $\tilde{S}_j^*$, $\tilde{S}_j^-$, $\tilde{R}_j^*$ and $\tilde{R}_j^-$ values.

$\tilde{S}_j^*$ (Min.)	2.714	3.477	3.782
$\tilde{S}_j^-$ (Max.)	6.371	8.231	9.162
$\tilde{R}_j^*$ (Min.)	0.500	0.700	0.860
$\tilde{R}_j^-$ (Max.)	0.820	0.960	1.000

Table 14: $\tilde{Q}_j$ values.

Alternatives	$\tilde{Q}_j$			Mean
A_1	0.375	0.385	0.429	0.396
A_2	0.465	0.454	0.439	0.453
A_3	0.119	0.101	0.046	0.089
A_4	1.000	1.000	1.000	1.000
A_5	0.562	0.546	0.512	0.540

$$0, 0.046, 0.296, 0.157, 0, 0.160, 0.495, 0.380) \\ = 0.74.$$

For other alternatives, the same procedure is followed for obtaining the $\tilde{R}_j$ values (Table 12).

Step 3: To calculate $\tilde{Q}_j$ values, the minimum and maximum $\tilde{S}_j$ and $\tilde{R}_j$ values for five alternatives should be determined (Table 13).

The first, second and third fuzzy numbers of $\tilde{Q}_j$ values are calculated for A_1 alternative by using Eq. 36 as follows:

$$\begin{aligned} \tilde{Q}_{A1} &= 0.5 \times (2.714 - 2.714) / (6.371 - 2.714) \\ &\quad + (1 - 0.5) \times (0.74 - 0.500) / (0.820 - 0.500) \\ &= 0.375. \end{aligned}$$

The $\tilde{Q}_j$ values calculated in a similar way for other alternatives are given in Table 14 together with the mean values.

Step 4: The alternatives are sorted in ascending order according to $\tilde{Q}_j$, $\tilde{S}_j$ and $\tilde{R}_j$ values as shown in Table 15.

It is determined that alternative of A_3 is the best solution according to the $\tilde{Q}_j$ index. However, it should be checked whether it satisfies the best and consensus solu-

tion (condition 1: acceptable advantage in Eq. 29). It is concluded that A_3 alternative is the best solution since A_3 alternative is also at the first rank according to $\tilde{R}_j$ index.

With the numerical examples presented above, it has been shown that both Fuzzy MCDM methods can be used effectively for equipment selection problem. Although both methods may currently propose the same alternative (A_3), as the most appropriate option, for the considered equipment selection problem in this study, it is important to consider that a decision maker can achieve different results for different equipment selection problems. For this reason, it is suggested that the decision maker decide which proposed MCMD method is more appropriate, by taking into account the differences given in Section 2.3.

5 Conclusions

Open pit equipment selection problem is an important issue having strategic effects on open pit design and production planning. On the other hand, decision makers often consider only a single or a few non-judgmental decision criteria, known as deterministic. Thus, not only cost and time loss but also loss of productivity occurs frequently as a result of wrong choices that are consisting of only objective criteria but do not include any personal judgment and experience. The problem of equipment selection is a multi-criteria decision making problem due to its complex nature. Also, it is necessary for the decision makers to take into consideration all of the decision criteria as a whole, based on their experiences and personal judgments. Aiming this concept, two different fuzzy MADM methods, fuzzy TOPSIS and fuzzy VIKOR in both of which all quantitative and qualitative decision criteria can be evaluated as subjective as necessary, are presented for the problem of equipment selection in open pit mining in this study.

The following points are some main findings and contributions of the study:

1. In general; if a decision process includes subjectivity, at least 2 different multi criteria decision making

Table 15: $\tilde{Q}_j$, $\tilde{S}_j$ and $\tilde{R}_j$ index values.

Alternatives	$\tilde{Q}_j$		$\tilde{S}_j$		$\tilde{R}_j$	
	Index	Rank	Index	Rank	Index	Rank
A_1	0.396	2	3.324	1	0.873	2
A_2	0.453	3	3.803	3	0.873	2
A_3	0.089	1	4.100	4	0.687	1
A_4	1.000	5	7.921	5	0.927	4
A_5	0.540	4	3.666	2	0.927	4

methods are preferred to compare the consistency of the results. For this reason, two fuzzy multi criteria decision making methods based on group decision making are presented for the selection of open pit mining equipment. The fuzzy methods can enable the decision makers to use their linguistic expressions (fuzzy numbers) rather than using crisp numbers when comparing the criteria and alternatives in question.

- Both fuzzy TOPSIS and fuzzy VIKOR methods use the same comparison matrix construction based on the experts' evaluations in linguistic terms. In this way, decision makers can quickly proceed with the decision making process and simply compare the results of two different methods in equipment selection.
- A decision making process is provided in which the decision criteria and available alternatives, affecting the correct equipment selection can be evaluated not only with the objective measures but also with decision makers' subjective measures as personal experiences and judgments.
- A case study in which the selection of an important equipment, a hydraulic excavator, together with the seventeen criteria and five alternatives by the proposed methods is provided, and the most convenient hydraulic excavator is selected for the studied open pit mine. In this way, it is demonstrated that the presented fuzzy methods are considerably effective in practice for equipment selection purpose.
- When the presented decision making process and the provided case study are evaluated together, it can be easily seen that the proposed fuzzy MADM methods are also applicable to the selection of any equipment used in earth sciences.
- The experts use simple linguistic terms, such as "high, very high", to evaluate the criteria and the alternatives in the proposed MCDM models. When an expert is uncertain or hesitant about his opinion, he may say "it is between high and very high" during

pairwise comparisons. In this case, we advise to use hesitant fuzzy linguistic models proposed by Liao et al. [46].

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