Applied Geomorphology 2019

Research Article

Simon Rabarijoely*

Rigidity of “Warsaw clay” from the Poznań Formation determined by in situ tests

https://doi.org/10.1515/geo-2020-0030
received July 22, 2019; accepted November 07, 2019

Abstract: The character of morphogenetic processes occurring within river valleys depends on the mechanical and hydrogeological properties of deposits that build up such landforms. In the case of the Polish Lowlands, a series of Pliocene clay lake sediments (so-called Poznań mottled clays) plays a special role. Their current locations and properties are associated with specific sedimentation conditions and glaciotectonic deformations, which the soils have been subjected to many times in Pleistocene. Their presence in the lithological profile influences dynamics of slope processes in valleys’ escarpment zones and channel erosion stabilization. In this article, the “Warsaw clay” from the Poznań Series Formation is presented in comparison with other cohesive Quaternary sediments, which are mostly building neighboring outcrops. This article analyzes the Seismic Dilatometer (SDMT) results and a method of interpretation to determine $I_0$, plasticity index, and liquid limit (LL) parameters. The undrained shear strength $S_d$ was determined based on the Cone Penetration Test (CPT), SDMT, and laboratory test results. Spatial variability of the strength and deformation parameters in the study area were determined using dilatometer test (SDMT) results. Finally, the nomogram chart is proposed to obtain the rigidity index $(I_0)$ of the preconsolidated Pliocene clays, depending on both $p_0$ and $p_1$ pressures from SDMT and effective vertical stress $\sigma'$ and pore water pressure $u_0$.

Keywords: cohesive soils, rigidity index, undrained shear strength, shear modulus, CPT/DMT

1 Introduction

Subsoil properties are the result of the geological history of the area, comprising the deposition and diagenesis processes and the nature and intensity of denudation processes. They are affected by, apart from the specific lithological profile and pattern of exposure, the landscape morphology. In the Polish Lowlands, the most clear relationship between the morphological features and the lithological/sedimentological diversity is represented by the river valley environment. Valleys’ morphodynamics, especially in escarpment zones and in the riverbed, is conditioned, i.e., directly by the features of the soils that build them and by their position. In the case of the Polish Lowlands, a series of Pliocene clay lake sediments (so-called Poznań mottled clays) plays a special role. Their features are connected with specificity of the deposition processes [1]. The location of the outcrops of clays of the Poznań Series very often has a glaciotectonic genesis. These deposits, which often constitute the direct basis for the Quaternary System, have been subject to glaciotectonic deformations many times in the Pleistocene. The Warsaw region is an example of such a zone. Pliocene clays are exposed in a valley’s western margin along the section called “Warsaw corset” [1,2], which is the narrowing of the floodplain surface. The Neogene (Pliocene) deposits form folds [3], which are called “Warsaw folds” [4]. To the east of the riverbed, Pliocene clays form small culminations in the subsoil of the Vistula alluvial series. Greater glaciotectonic structures are found in the moraine plateau east of the Vistula valley in the vicinity of Wólka Młądzka [5,6]. The presence of Poznań clay folds in the western margin of the Vistula valley is one of the reasons for the activity of the slope processes both in historical times and in the Pleistocene [7]. Today, due to the lowering of groundwater levels in the plateau (which is connected with urbanization), the slope processes have been stabilized. The most morphogenetically effective impact of clays of the Poznań Series is observed in the Vistula riverbed in Warsaw. Throughout the entire urban section of the channel, glaciotectonically disturbed Pliocene and glacial formations form protrusion of very complex morphology [2,8]. This form affects the current lines spatial distribution of flood flows and destabilize the balance of the existing morphological form [9].

* Corresponding author: Simon Rabarijoely, Warsaw University of Life Sciences, Department of Geotechnical Engineering, Warsaw, Poland, e-mail: simon_rabarijoely@sggw.pl
In the last decade, intensive development of the municipal infrastructure including high-rise buildings and road network is observed in Warsaw. To recognize geotechnical conditions in the Warsaw area, generalized and detailed investigations for each design structure, comprehensive field, and laboratory were carried out. According to the Geotechnical Investigation Reports, the Pleistocene clay layer was indicated as the softest and most difficult to locate design foundations. Stiffness of clay soils was characterized using the rigidity index ($I_R$). It is defined as the ratio of the shear modulus ($G_0$) to the undrained soil strength ($S_u$) and is a critical parameter for estimating the consolidation coefficient ($c_h$) using cone data of Krage et al. Comprehensive investigation of “Warsaw clay” from the Poznań Formation by CPT and SDMT allows determination of the $I_R$ distribution depending on the preconsolidation ratio overconsolidation ratio ($OCR$). The aim of this study was to determine the values of stiffness, one of the parameters of Pliocene clay of the Poznań series, which is considered as important in the morphogenetic processes in this study.

In designing facilities (as well as in Warsaw), the stiffness of the subsoil is considered more important than its shear strength. The critical condition in building design is settlements or differences in subsoil settlement and not shear strength. According to Eurocode 7, the limited displacement and deformation of buildings are $s_{\text{max}} = 50$ mm and $\Delta s_{\text{max}} = 10$ mm, respectively.

The full characteristics of the subsoil can be obtained from the right combination and selection of tests for different conditions. A variety of methods are now increasing and are still being developed. Correct selection of the parameters requires the use of methods calibrated to the local conditions and interpretation tested results in practice. The correct design of the foundation requires knowledge of the properties of both the material forming the foundation and the type of the subsoil. Generally, the aboveground part and the underground part with the foundation of a building are designed by the same designer. Thus, when designing the above foundation level part, the designer should be aware of the restrictions on digging, earthwork, and excavation of foundation and its drainage, as well as the protection of building foundations against groundwater.

This article presents the characteristics of Pliocene clays in the Warsaw region and analysis of the results of the CPTU and SDMT investigations to determine the distribution of the rigidity index $I_R$, defined as the ratio of the shear modulus ($G_0$) to the undrained soil strength ($S_u$) in the ground of the designed objects. The possible range of $I_R$ (50–500 for clays according to Krage et al. 2014) creates a significant uncertainty in estimating proper values of $I_R$ for Warsaw clay from the Poznań Formation. In the case of soils in Poland, the essential influence of the rigidity index value was displayed by factors such as cementation and anisotropy of macrostructure of certain sediments [10].

2 Properties of Warsaw clay from the Poznań Formation

The research sites discussed in this article are located in the south of the Warsaw moraine plateau, where the Neogene
deposits occur in the basement of the quaternary, represented by lacustrine Pliocene “motley clays.” Above the Pliocene deposits, preglacial (Eopleistocene) alluvial deposits occur, comprising quartz gravels, sands, and silts with lydites. Above the preglacial deposits, or directly on the lacustrine Pliocene clays, lies a series assigned to the south Polish Glaciation – boulder clays, glaciofluvial sediments, and ice-dammed deposits, which form noncontinuous layers. Moreover, clay discovered in Warsaw ground was classified as expansive (Figure 1).

2.1 Stegny site

Ground and water conditions are known to a depth of 246 m b.g.l. (below ground level). This is the result of the research conducted in boreholes for the intake of Oligocene waters. The ground, comprising Neogene clays that belong to the Pliocene, occurs below the alluvial sands, i.e., 4.3 m b.g.l. The “Stegny” experimental site was founded by an academic center, following three research projects of the Scientific Research Committee implemented by the Institute of Hydrogeology and Applied Geology at the Warsaw University [13]; the Department of Geotechnics and Foundation, Building Research Institute, Warsaw [14]; and the Department of Geotechnics at Warsaw University of Life Sciences – SGGW [15]. It is located in the southern part of Warsaw city in the Mokotów district in the Stegny housing estate. The grounds that are subject to research here are Mio-Pliocene clays, belonging to the Poznań Formation. The test area is shown in Figure 1. It occurs on the Praga terrace in the Vistula river valley. The terrace in the area of the Stegny site is located at an elevation of 86 m. The location of the terrace is 5–10 m above the average water level in the Vistula River.

### Table 1: Properties of the tested Warsaw clays

<table>
<thead>
<tr>
<th>Soil parameters</th>
<th>Soil type</th>
<th>Stegny site</th>
<th>Warsaw subway [16]</th>
<th>Hotel building [17]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grain size (%)</td>
<td>Gravel</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>EN ISO 14688-1</td>
<td>Sand</td>
<td>3/8</td>
<td>4/14</td>
<td>0/1</td>
</tr>
<tr>
<td></td>
<td>Silt</td>
<td>40/64</td>
<td>17/50</td>
<td>18/38</td>
</tr>
<tr>
<td></td>
<td>Clay</td>
<td>32/56</td>
<td>46/73</td>
<td>55/61</td>
</tr>
<tr>
<td>Specific density, $\rho_s$ [t/m³]</td>
<td>2.72</td>
<td>2.68</td>
<td>2.67</td>
<td></td>
</tr>
<tr>
<td>Preconsolidation ratio, OCR [-]</td>
<td>2/3 2/5</td>
<td>2/5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plasticity limit, $w_p$ [%]</td>
<td>24.97/31.16</td>
<td>29.2</td>
<td>28.0/39.8</td>
<td></td>
</tr>
<tr>
<td>Liquidity limit, $w_l$ [%]</td>
<td>67.60/88.11</td>
<td>72.9</td>
<td>63.1/118.4</td>
<td></td>
</tr>
<tr>
<td>Plasticity index, $I_p$ [%]</td>
<td>52.49/67.1</td>
<td>43.7</td>
<td>35.1/77.0</td>
<td></td>
</tr>
<tr>
<td>Activity</td>
<td>0.87/1.87</td>
<td>0.59</td>
<td>0.59</td>
<td></td>
</tr>
</tbody>
</table>
tests. CAUC tests on high-quality samples are recommended for more precise estimations of $I_R$.

4 Data and methods

The piezocone penetration CPTU and SDMT dilatometer tests are commonly applied in the recognition of geotechnical conditions of building foundations, and in the case of buildings qualified for the third geotechnical category, SCPTU and SDMT are also used [19–22]. Due to the earlier introduction into practice [5,6] and much more extensive rules of interpretation, probing CPT and then CPTU tests were initially considered more reliable in the assessment of geotechnical conditions in the building structure designed. Currently, despite significant differences in test methodology, CPTU and SDMT soundings are considered equivalent. In Poland, research studies using CPTU and SDMT tests were carried out in the following centers: Department of Geotechnics, the Poznań University of Life Sciences [19]; Department of Geotechnics, Geology and Marine Civil Engineering, Gdańsk University of Technology [23]; Institute of Hydrogeology and Applied Geology at the University of Warsaw [9]; Department of Geotechnics and Foundation, Building Research Institute, Warsaw [14]; Department of Geotechnic at Warsaw University of Life Sciences – SGGW [13,24]; and the Institute of Geology at Adam Mickiewicz University in Poznań [10].

At the Stegny site, 10 DMT tests were carried out at a depth of 10–18 m. The dilatometer investigation consists of measurements at selected depths of gas pressure acting on the membrane of the flat dilatometer (Figure 4). At the Stegny site from the surface of the ground to about 4.0 m, there is a layer of medium sand, below which is Pliocene clay. Groundwater stabilized at a depth of about 3.5 m from the ground surface, two pressures were measured ($A$ and $B$), which force the membrane center to move by 0.05 mm to the ground (reading $A$) and the center of the membrane to the ground by approximately 1.05 mm (reading $B$) (Figure 4). The values of $A$ and $B$ readings were improved due to the inertia of the membrane, and the corrected pressure values are marked as $p_0$ and $p_1$, respectively. The $p_0$ and $p_1$ pressures and the value of the vertical effective stress component $\sigma'_{v0}$ were used to determine the dilatometer indexes [22]: material index, $I_D$; horizontal stress index, $K_D$.

Figure 2: Grain-size distribution of clay from the test sites.

Figure 3: Consolidated undrained triaxial tests, effective stress paths (TXCIU) from the “Stegny” experimental site in Warsaw.
dilatometer modulus, \( E_D \); and water pressure index, \( U_D \), using the following formulas:

- 0.05 mm corrected pressure reading in DMT \( p_0 \):
  \[
p_0 = 1.05(A - Z_m + \Delta A) - 0.05(B - Z_m - \Delta B),
  \]

- 1.10 mm corrected pressure reading in DMT \( p_1 \):
  \[
p_1 = B - Z_m - \Delta B,
  \]

- corrected third reading in DMT \( p_2 \):
  \[
p_2 = C - Z_m - \Delta A,
  \]

- Material index, \( I_D \):
  \[
  I_D = \frac{p_1 - p_0}{p_0 - u_0},
  \]

- Horizontal stress index, \( K_D \):
  \[
  K_D = \frac{p_0 - u_0}{\sigma'_0},
  \]

- Dilatometer modulus, \( E_D \):
  \[
  E_D = 34.7(p_1 - p_0),
  \]

- Water pressure index, \( U_D \):
  \[
  U_D = \frac{p_2 - u_0}{p_0 - u_0}
  \]

where for \( p_0 \): \( A \) is the pressure reading corrected for \( Z_m \) and \( \Delta A \) is the membrane stiffness at 0.05 mm expansion and 0.05 mm expansion itself and used to estimate the total soil stress acting normal to the membrane immediately before its expansion into the soil (0.00 mm expansion); for \( p_1 \): \( B \) is the pressure reading corrected for \( Z_m \) and \( \Delta B \) is the membrane stiffness at 1.10 mm expansion to give the total soil stress acting normal to the membrane at 1.10 mm membrane expansion; for \( p_2 \): \( C \) is the pressure reading corrected for \( Z_m \) and \( \Delta A \) is the membrane stiffness at 0.05 mm expansion and used to estimate pore water pressure; \( \sigma'_0 \) is the vertical effective stress at the center of the membrane before insertion of the DMT blade; \( u_0 \) is the pore water pressure acting at the center of the membrane before insertion of the DMT blade (often assumed as hydrostatic below the water table surface); and \( Z_m \) is the gauge pressure deviation from zero when vented to atmospheric pressure (offset used to correct pressure readings to the true gauge pressure).

The test results of \( p_0, p_1, p_2, I_D, K_D, E_D, \) and \( V_s \) profiles from DMT investigations were used to determine the practical usefulness of the created chart (Figure 4). These studies were carried out at the Geotechnic Department of the Warsaw University of Life Sciences, concerning the Stegny site. Based on the results from the laboratory tests, a grain size distribution curve for cohesive soils was created for the analyzed site.

4.1 Evaluation of the rigidity index

To evaluate the \( I_R \) parameters for Warsaw clay from the Poznań Formation, comprehensive in situ and laboratory tests were carried out. In frame of in situ tests, an SDMT investigation was performed. Based on the SDMT results, the shear modulus (\( G_0 \)) and the undrained shear strength (\( S_u \)) were obtained. Using the SDMT test results, the undrained shear strength was determined by applying the formulas presented in Table 3. The distribution of dilatometer indexes is shown in Figure 4.
This article presents mainly the properties of the shear modulus $G_0$. Like soil strength, the shear modulus is a function of many variables, which are shown in formula (8) [23]. It is commonly known that $I_R$ is a function of many variables. Until now, $I_R$ was calculated exclusively using the parameters obtained from laboratory tests, which is presented in formula (8). The factors that determine the value of $I_R$ are as follows [25]:

$$I_R = f (L, I, P_l, OCR, \sigma_0', \sigma_{60}', e, S, C, A, F, T, \theta, K). \quad (8)$$

The factors that determine the value of the shear modulus $G_0$ include $\sigma_0'$ – effective vertical stress, $e$ – initial void ratio, $S$ – degree of saturation, $C$ – grain characteristics, $A$ – amplitude of vibrations, $F$ – frequency of vibrations, $T$ – effects depending on time, $\theta$ – soil structure, and $K$ – temperature [25].

The rigidity index ($I = G_0/S_0$) was originally defined by Vesic [26] as the ratio of the shear modulus to the undrained shear strength. It provides a description of soil compressibility and was used by Vesic [26] to distinguish between different modes of failure for shallow foundations. It has been shown to influence a large number of geotechnical problems, from in situ testing, cavity expansion, tunneling, shallow foundations, pile foundations, etc. It is also a very useful parameter if only shear strength data are available and/or the disturbed samples preclude the accurate determination of stiffness moduli.

Although a range of different tests have been proposed to determine the rigidity index, the most commonly reported benchmark results use standard triaxial compression test data [17,27], with the secant shear modulus at 50% ($G_{50}$) of the peak undrained shear strength ($S_0$). The values of the rigidity index have been found to be in the range from 25 to 600 [27,28]. More common usage of geophysical tests in the laboratory and in situ tests has expanded the measurement range of the rigidity index, and very low strain rigidity index values ($G_0/S_0$) for different soils are now available in the literature and may span the range from 200 to 3,000 [19]. The empirical formulas that are commonly used in the literature are presented in Table 2.

This article mainly presents the properties of $I_R$. Like soil strength and shear modulus, the rigidity index is a function of many variables, which is presented in formula (9). Using in situ tests such as CPTu or SDMT, it was found that the effect on the tips of a cone or dilatometer blades is influenced by the $I_R$ value. In this case, the $I_R$ ground rigidity index depends on additional factors from CPTu and SDMT (Table 2). The factors that determine the value of $I_R$ in this case are as follows:

$$I_R = f (\sigma_0', \sigma_{60}', e, S, C, A, F, T, \theta, K, CPTu, SDMT). \quad (9)$$

The formula modified by Hardin and Black [25] reveals that the shear modulus $G_0$ is influenced by a number of subsoil properties, related to its structure ($\theta$), lithology ($P_l$), stress state ($\sigma_0'$), consistency (liquidity index [LI]), plastic volumetric strain ratio $\Delta = (C_c - C_s)/C_c$, compression index ($C_c$), recompression index ($C_s$), critical state parameter ($M$), or slope of critical state line, and is equal to $6 \sin \phi'/(3 - \sin \phi')$ and origin ($\sigma_0'$) [eq. (12)].

Again, this relationship depends on parameters measured from the CPTu test, $B_t = (u_2 - u_0)/(q_1 - \sigma_0')$; the obtained $I_R$ values can be restricted to a narrow range: $0.50 < B_t < 0.70$; $N_{at}$ is the cone factor, which can be estimated by $N_{at} = q_{tip}/S_0$; $\Delta$ is the normalized in situ deviator stress, which can be estimated by $\Delta = (\sigma_0'(1 - K_0)/(2S_0))$; $K_0$ is the earth pressure coefficient, which can be estimated by $K_0 = (1 - \sin \phi')OCR\sin \phi' [29$ and $\alpha_c$ is the cone face roughness, which varies from 0 to 1 for perfectly smooth and rough interfaces.

When using in situ tests (CPTu and SDMT), it is possible to extend these factors to determine $I_R$ by $p_0$, $p_1$, and $a_d$ (strain rate factor) parameters measured from the DMT test. The factors that determine the value of $I_R$ based on the CPTu and SDMT tests in this case are as follows (Table 2):

$$I_R = (fp_0, p_1, a_d, B_t, N_{at}, K_0, M, K_0, A, LI, P_l, OCR, \sigma_0', \sigma_{60}', e, S, C, A, F, T, \theta, K). \quad (10)$$

Selected formulas for calculating the shear modulus ($G_0$) and the undrained shear strength ($S_0$) values based on the SDMT test indexes are shown in Table 3. In these equations, pressures $p_0$ and $p_1$ from SDMT, as well as $u_0$ and $\sigma_0'$, largely correspond to the quantity and quality of the parameters $G_0$ and $S_0$.

5 Results and discussion

5.1 Results and discussion – determination of the rigidity index of Pliocene clays from the Poznań Formation in the Warsaw region based on SDMT

The next task was to seek a direct correlation between the results of seismic tests of the rigidity index and reading pressures from DMT for mineral soils (sand, silt, and clay) in the range of stresses from normally consolidated to heavy preconsolidated. The main
Table 2: Selected equations of the rigidity index formula, $I_R$, according to different authors

<table>
<thead>
<tr>
<th>Authors</th>
<th>$I_R = f(\sigma_\nu', e, S, C, A, F, T, \theta, K)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Keaveny and Mitchell (1986) [30]</td>
<td>$I_{R0} = \frac{\exp\left(\frac{\sigma_\nu'}{\sigma_\nu}3.4\right)}{1 + \ln\left[1 + \ln\left(OCR - 1\right)\exp(A)\right]}A(1 - A)OCR^0$ (11)</td>
</tr>
<tr>
<td>Kulhawy and Mayne (1990) [31]</td>
<td>$I_{R} = \left(\frac{2}{3}\right)\left(1 + \frac{e_0}{c_l}\right)\ln(\frac{1}{1 - \ln(OCR)\exp(A)})$ (12)</td>
</tr>
<tr>
<td>Młynarek et al. (2018) [32]</td>
<td>Organic and young clay: $I_{R} = 4.73\ PI - 14.04\ LI - 4.78$ - OCR + 151.74</td>
</tr>
<tr>
<td>Cao (1997) [30], Cao et al. (2001) [33]</td>
<td>(DMT): $\rho_0 = \sigma_{v0} + \frac{2}{3}\sigma_\nu'M_{v0} \left(\frac{OCR}{2}\right)^{A} \left(\ln I_{R} + 1\right)$ (14)</td>
</tr>
<tr>
<td>Cao (1997) [30]; Cao et al. (2001) [33]</td>
<td>(DMT): $\rho_1 = \sigma_{v0} + \frac{2}{3}\sigma_\nu'M_{v0} \left(\frac{OCR}{2}\right)^{A} \left(\ln I_{R} + 1\right)$ (15)</td>
</tr>
<tr>
<td>Mayne (2001) [34]</td>
<td>(CPTu): $I_{R} = e^{\left[\frac{15}{11} I_{R}-2.925\right]} \left(\frac{OCR}{2}\right)^{2.925}$ (16)</td>
</tr>
<tr>
<td>Lu et al. (2004) [35]</td>
<td>$M = \frac{q}{p'} = 6\ \sin\phi_{vs} + 3 - \sin\phi_{vs}$ (17)</td>
</tr>
<tr>
<td>Krage et al. (2014) [18]</td>
<td>(CPTu): $I_{R50} = \left[\frac{1.81G_0}{\sigma_{v0}^{0.75}(OCR)^{0.25}}\right]$ (19)</td>
</tr>
<tr>
<td>Keaveny and Mitchell (1980) [30], Krage et al. (2014) [18]</td>
<td>(CPTu): $I_{R} = 0.26G_0 \left(\frac{OCR}{2}\right)^{0.33} \left(0.33Q_{r}^{0.75}\right)$ (20)</td>
</tr>
<tr>
<td>Mayne (2016) [36]</td>
<td>(CPTu): $I_{R} = \exp\left(\frac{2.93B_q}{1 - B_q}\right)$ (21)</td>
</tr>
</tbody>
</table>

where $B_q = \frac{\omega_{v0} - \omega_0}{\omega_{v0}}$, $0.50 < B_q < 0.70$

Table 3: Empirical coefficients in single- and multifactor relationships to evaluate the rigidity index, $G_0$, and undrained shear strength, $\tau_{ur}$, from the dilatometer test

<table>
<thead>
<tr>
<th>$G_0 = f(\sigma_\nu', e, S, C, A, F, T, \theta, K)$</th>
<th>$S_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marchetti et al. (2008) [37]</td>
<td>Marchetti (1980) [22]</td>
</tr>
<tr>
<td>For Pliocene clay: $G_0 = M_{DMT-15}, 686K_0^{0.921}$ (22)</td>
<td>$S_u = 0.22\sigma_{v0}(0.5K_0)^{1.22}$ (24)</td>
</tr>
<tr>
<td>Galas (2013) [38]</td>
<td>$\sigma_\nu' - vertical\ effective\ stress$</td>
</tr>
<tr>
<td>For high plasticity (40% &lt; $I_p$ &lt; 60%)</td>
<td>$K_0 = \frac{\rho_0 - \omega_0}{\sigma_\nu'}$ horizontal\ stress\ index</td>
</tr>
<tr>
<td>$\frac{G_0}{E_0} = 3.25K_0^{0.8}$ (23)</td>
<td>$\omega_0$ - hydrostatic\ water\ pressure</td>
</tr>
<tr>
<td>$\rho_0, p_1$ - pressure\ readings\ in\ SMDT\ tests</td>
<td>$\rho_0, \rho_1$ - pressure readings in SMDT tests</td>
</tr>
<tr>
<td>Rabarijoely (2000) [39]</td>
<td>$S_u = \sigma_0\sigma_{v0}^{\alpha_0}(\rho_0 - \omega_0)^{\alpha_1}(\rho_1 - \omega_0)^{\alpha_2}$ (25)</td>
</tr>
<tr>
<td>$\sigma_0, \sigma_0, \alpha_0, \alpha_2$ - empirical\ coefficients (for\ Warsaw\ clay: $\alpha_0 = 0.18, \alpha_1 = 0.14, \alpha_2 = 0.20,$ and $\alpha_3 = 0.15$)</td>
<td>$\sigma_0, \sigma_0, \alpha_0, \alpha_2$ - empirical\ coefficients (for Warsaw\ clay: $\alpha_0 = 0.18, \alpha_1 = 0.14, \alpha_2 = 0.20,$ and $\alpha_3 = 0.15$)</td>
</tr>
</tbody>
</table>
impulse for searching for a new form of dependence to determine the rigidity index based on DMT readings was to extend the range of soil types, including organic soils. Therefore, a comprehensive series of multiple regression analyses were performed using both arithmetic and logarithmic scaling. A full set of regression attempts were not included here because they were too large for the discussion. Analysis of the dilatometer pressure readings has shown that parameters $p_0$ (kPa), $p_1$ (kPa), $u_0$ (kPa), and $\sigma_{00}$ are sufficient to obtain a reasonable estimate $I_R$ and thus do not need to be based on DMT indexes such as $I_D$, $K_D$, or $E_D$, without the loss of statistical significance.

The values of groundwater pressures $u_0$ (kPa) and $\sigma_{00}$ (kPa) can be calculated by correlation of data from Table 4 by combining dilatometer pressures such as $p_0$ (kPa) and $p_1$ (kPa). Multiple regression analysis combined with the results gave full success of the calculated results correlating with data obtained from seismic tests based on the following formula:

$$I_R = f(u_0, \sigma_{00}, p_0, p_1),$$

(26)

where $p_0$, $p_1$, $u_0$, and $\sigma_{00}$ are expressed in kPa.

The rigidity indexes of the tested soils from the Warsaw region (Poland) depend on $p_0$ and $p_1$ pressure values in clay based on simplified theoretical solutions of cavity expansion in modified Cam clay. Using the dilatometer test results with $p_0$, $p_1$, $u_0$, and $u_0$, the rigidity index ($I_R$) values can be determined using the following formula:

$$I_R = \frac{\exp\left(\frac{4.6 \cdot p_1}{p_0}\right)}{\log\left(2.714 + \left(0.318 \cdot \frac{p_0 - u_0}{\sigma_{00}}\right)^{5.0}\right)}.$$  

(27)

To check the distribution of the normality of parameters in the composition of the proposed formula, a correlation matrix was performed between each parameter and between dependent and independent variables. The results of these analyses are presented in Table 5 and Figure 5.

Continuing the transformation process for formula:

$$\log(I_R) = \log\left(\frac{\exp\left(\frac{4.6 \cdot p_1}{p_0}\right)}{\log\left(2.714 + \left(0.318 \cdot \frac{p_0 - u_0}{\sigma_{00}}\right)^{5.0}\right)}\right),$$

(28)

the following modification was introduced: $p_0 \to \log(p_0)$, $p_1 \to \log(p_1)$, $u_0 \to \log(u_0)$, and $\sigma_{00} \to \log(\sigma_{00})$. A separate analysis was carried out for Pliocene clays, $u_0 = 0$ kPa. Then, the normality was checked using the Shapiro–Wilk method for Pliocene clay from the Poznań Formation. After the transformation, a normal distribution was obtained. The results of the analysis are presented in Table 6.

$$\varepsilon = \log(I_R) - \log\left(\frac{\exp\left(\frac{4.6 \cdot p_1}{p_0}\right)}{\log\left(2.714 + \left(0.318 \cdot \frac{p_0 - u_0}{\sigma_{00}}\right)^{5.0}\right)}\right),$$

(29)

Additionally, the least square method was used to build the new correlations like eqs. (26) and (27), and it does not require the use of normal distribution (Figures 6 and 7), but the normality of the rest of the model is worth checking for the stability of the equation’s parameters [eq. (28)]. After calculating the value of the rest based on eq. (28), the value of $\varepsilon$ is in the interval $(-0.17/0.14)$ (Figure 8).

The proposed chart nomogram was developed based on the SDMT test results and from the triaxial compression test for the clays in the Warsaw region (Figure 9). In the laboratory, the values of undrained shear strength $S_u$ for non-disturbed samples in the triaxial compression apparatus (TXCIU) were determined. By contrast, $G_0$ values were determined by in situ tests (SDMT). With the two parameters ($G_0$ and $S_u$), the $I_R = G_0/S_u$ value was

---

**Table 4:** List of dilatometer pressures and laboratory test results

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Depth (z)</th>
<th>$r_u$ (MPa)</th>
<th>G (MPa)</th>
<th>$I_R$ meas. (%)</th>
<th>$I_R$ cal. (%)</th>
<th>$\gamma$ (kN/m$^3$)</th>
<th>$u_0$ (kPa)</th>
<th>$p_0$ (kPa)</th>
<th>$p_1$ (kPa)</th>
<th>$\sigma_{00}$ (kPa)</th>
<th>PI (%)</th>
<th>LI (―)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pliocene clay</td>
<td>6.00</td>
<td>0.107</td>
<td>50</td>
<td>467</td>
<td>467</td>
<td>19.5</td>
<td>28</td>
<td>508.98</td>
<td>887.50</td>
<td>84.50</td>
<td>52.49</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>9.00</td>
<td>0.083</td>
<td>70</td>
<td>843</td>
<td>843</td>
<td>19.6</td>
<td>48</td>
<td>858.75</td>
<td>1158.00</td>
<td>116.16</td>
<td>56.95</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>12.00</td>
<td>0.183</td>
<td>100</td>
<td>546</td>
<td>546</td>
<td>19.6</td>
<td>78</td>
<td>951.25</td>
<td>1408.00</td>
<td>142.12</td>
<td>42.09</td>
<td>-0.135</td>
</tr>
<tr>
<td></td>
<td>15.00</td>
<td>0.128</td>
<td>120</td>
<td>938</td>
<td>938</td>
<td>21.0</td>
<td>118</td>
<td>1222.98</td>
<td>2147.50</td>
<td>179.70</td>
<td>62.44</td>
<td>0.058</td>
</tr>
</tbody>
</table>

---
Table 5: Correlation coefficients of the soil parameters

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean values</th>
<th>Standard deviation</th>
<th>$I_R = G_0/\tau_0$ (−)</th>
<th>$u_0$ (kPa)</th>
<th>$p_0$ (kPa)</th>
<th>$p_1$ (kPa)</th>
<th>$(\sigma_v')$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_R = G_0/\tau_0$ (−)</td>
<td>1,153</td>
<td>699</td>
<td>1</td>
<td>0.15</td>
<td>−0.27</td>
<td>−0.43</td>
<td>0.31</td>
</tr>
<tr>
<td>$u_0$ (kPa)</td>
<td>30</td>
<td>41</td>
<td>0.15</td>
<td>1</td>
<td>0.31</td>
<td>0.11</td>
<td>0.80</td>
</tr>
<tr>
<td>$p_0$ (kPa)</td>
<td>809</td>
<td>454</td>
<td>−0.27</td>
<td>0.31</td>
<td>1</td>
<td>0.93</td>
<td>0.53</td>
</tr>
<tr>
<td>$p_1$ (kPa)</td>
<td>1,389</td>
<td>794</td>
<td>−0.43</td>
<td>0.11</td>
<td>0.93</td>
<td>1</td>
<td>0.32</td>
</tr>
<tr>
<td>$(\sigma_v')$ (kPa)</td>
<td>113</td>
<td>62</td>
<td>0.31</td>
<td>0.80</td>
<td>0.53</td>
<td>0.32</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 5: Comparisons between the calibration database and the first 10,000 simulated data points.

Table 6: Scatterplots between horizontal axis = column log of variables and vertical axis = row log of variables

<table>
<thead>
<tr>
<th>Variables</th>
<th>log($I_R$) (−)</th>
<th>log($u_0$) (kPa)</th>
<th>log($p_0$) (kPa)</th>
<th>log($p_1$) (kPa)</th>
<th>log $(\sigma_v')$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log($I_R$) (−)</td>
<td>1</td>
<td>0.46</td>
<td>0.46</td>
<td>0.16</td>
<td>0.86</td>
</tr>
<tr>
<td>log($u_0$) (kPa)</td>
<td>0.46</td>
<td>1</td>
<td>0.96</td>
<td>0.92</td>
<td>0.80</td>
</tr>
<tr>
<td>log($p_0$) (kPa)</td>
<td>0.46</td>
<td>0.96</td>
<td>1</td>
<td>0.90</td>
<td>0.77</td>
</tr>
<tr>
<td>log($p_1$) (kPa)</td>
<td>0.16</td>
<td>0.92</td>
<td>0.90</td>
<td>1</td>
<td>0.55</td>
</tr>
<tr>
<td>log($\sigma_v'$) (kPa)</td>
<td>0.86</td>
<td>0.80</td>
<td>0.77</td>
<td>0.55</td>
<td>1</td>
</tr>
</tbody>
</table>
calculated, which is also considered as the reference value for in situ tests. For each $I_R$ value, the $p_0$, $p_1$, $u_0$, and $\sigma_v^0$ values are exactly combined with the depth data, from which the samples for testing in the triaxial apparatus were taken. Because the $I_R$ parameter is highly dependent on OCR parameters and PI, where the stress history is the $K_D$ function from the SDMT, and the plasticity index is also related to the dilatometer pressures $p_0$ and $p_1$. In this connection, the $I_R$ value is indirectly related to dilatometer pressures, hence the idea of $I_R$ dependence on $p_0$ and $p_1$ pressures (Figure 9). The method of using the chart nomogram is very simple (Figure 9). First, we need to calculate the $p_1/p_0$ ratio to locate the linear curve represented by this value. Then, on the horizontal axis, we need to find the value of $K_D$ (point A), and from this point, we should lead the vertical line up to the intersection with the curve with

**Figure 6:** Comparison of the rigidity index: measured values and calculated values using eq. (27).

**Figure 7:** Comparison of rigidity index: measured values and calculated values after transformation process for eq. (28).

**Figure 8:** Comparison of $\varepsilon$ between before and after transformation of the proposed formula.

**Figure 9:** Proposed nomogram chart for estimation of the rigidity index ($I_R$) based on $K_D$, $p_0$ and $p_1$ from DMT tests.
the $p_t/p_0$ value (point $B$). The last step to be performed in this chart nomogram is to lead the horizontal line to the left until it intersects with the vertical axis (point $C$); the last point is the $I_R$ value that we are looking for (Figure 9).

6 Conclusions

Based on the literature reviews, it can be concluded that the opinion presented by numerous authors [17,28,32,34] has shown that the variability of the rigidity index is related to the variability of the parameters determining properties and diverse soil quality and origin.

Specific geomorphological units, to some extent, predict the sequence of geotechnical layers and the global distribution of soil parameters [40]. It is assumed that similar geotechnical conditions can be distinguished (type and order of geotechnical layers, general deformation distribution, and strength parameters) in similar geomorphological forms. Under the influence of water, soil stiffness as a geotechnical parameter deteriorates. For anti-erosion and landslide protection, including deep and surface drainage, stabilization of engineering structures (slopes and retaining wall) must be taken into account. Extensive knowledge of soil rigidity has provided the engineer with additional information on a large multiyear project since 2006, SOPO – Landslide Protection System, in which the Polish Geological Institute, acting in Poland as the state geological service, has begun implementation.

The conclusions drawn in this article were compiled based on a comparison of only two segments with a distinct geomorphological form – the edge of the moraine plateau. It should be emphasized that the geomorphological shape of the zone selected for verification of the assumptions is very complex and difficult to analyze.

Like soil strength $S_u$ and shear modulus $G$, the $I_R$ stiffness index is a function of many factors. $I_R$ of pre-consolidated Pliocene clays can be determined based on SDMT tests.

Using the hybrid spherical expansion of the inside of the hole – a critical condition – it was shown that the operational value of the rigidity index ($I_R$) is obtained by means of pressure from the membrane displacement, water pore pressure, and effective vertical stress.

A new chart nomogram for the estimation of $I_R$ of Pliocene clays in the Warsaw region based on the SDMT test was proposed. Further research should be undertaken to verify the suitability of SDMT for investigating the stiffness index of Pliocene clays in Poland and beyond.

References


