Research Article

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Estimation of permeability and saturation based on imaginary component of complex resistivity spectra: A laboratory study

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Abstract: Low-frequency resistivity logging plays an important role in the field of petroleum exploration, but the complex resistivity spectrum of rock also contains a large amount of information about reservoir parameters. The complex resistivity spectra of 15 natural sandstone cores from western China, with different water saturations, were measured with an impedance analyzer. The pore space of each core was saturated with NaCl solution, and measurements were collected at a frequency range of 40–15 MHz. The results showed a linear relationship between the real resistivity at 1 kHz and the maximum values of imaginary resistivity for each core with different water saturations. The slopes of the linear best-fit lines had good linear relationships with the porosity and the permeability of cores. Based on this, a permeability estimation model was proposed and tested. In addition, the maxima of imaginary resistivity had power exponential relationships with the porosity and the water saturation of the cores. A saturation evaluation model based on the maxima of imaginary resistivity was established by imitating Archie's formula. The new models were found to be feasible for determining the permeability and saturation of sandstone based on complex resistivity spectrum measurements. These models advance the application of complex resistivity spectrum in petrophysics.

Keywords: natural sandstone cores, complex resistivity, permeability, water saturation, maxima of imaginary resistivity

1 Introduction

Archie's formula uses the electrical resistivity of rocks to provide a quantitative evaluation of oil and gas saturation [1]. As a result, resistivity logging occupies an irreplaceable position in the petroleum industry. Conventional resistivity logging tools use relatively low frequencies for their measurements. It was not until Snyder et al. [2] discovered that the resistivity dispersion spectrum of a formation contains abundant useful information that scientists began to pay attention to the complex resistivity spectra of formations. Klein and Sill [3] manufactured artificial shaly sandstones and explored the resistivity dispersion characteristics of the samples in the frequency range of (1/128)–1,024 Hz. They found that the resistivity dispersion of the samples in this frequency band was consistent with the Cole–Davidson model, and the time constant τ had a strong correlation with the pore size distribution of the sample. The complex resistivity spectra of natural shaly sandstone samples in the frequency range of 1–1,000 Hz were measured, and a dispersion model that accounted for the shale content, water salinity, and oil saturation was established in ref. [4]. Khairy and Harith [5] studied the effects of pore geometry, confining pressure, and water saturation on the complex resistivity of argillaceous sandstones with frequencies ranging from 1 Hz to 0.2 MHz. Kavian et al. [6] measured the complex resistivity of natural cores at four frequencies (105 kHz, 332 kHz, 665 kHz, and 1.05 MHz) and found that the relationships between their real components of each frequency, porosity, and water saturation follow Archie's law. Zisser et al. [7,8] studied the complex resistivity characteristics of sandstones and tried to use this information to predict their permeability. The resistivity and dielectric dispersion of natural cores in the

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frequency range of 0.1 Hz to 1 kHz were studied by Liu et al., who defined the degree of frequency dispersion of the permittivity or resistivity values to evaluate the water saturation of the cores [9]. Norbistrath et al. [10] estimated the permeability of a dolomite formation based on its complex resistivity spectra in the frequency range of 0.1 Hz to 100 kHz. Most similar studies used induced polarization exploration, so the frequency bands in their research are relatively low and are not suitable for well logging. Few studies have focused on the complex resistivity dispersion of the high-frequency band. Tong and Tao [11] measured the complex resistivity spectra of natural shaly cores from 100 to 20 MHz, and they found that the slope of the real component curve of the complex resistivity spectrum is strongly correlated with porosity and permeability. Li et al. [12] produced artificial pure sandstones and shaly sandstones and measured their complex resistivity spectra in the frequency range of 40–15 MHz. The experimental data were fitted with a second-order Cole–Cole equation, and a saturation evaluation model was established based on the time constant. Jiang et al. [13] designed a set of logging tools to obtain the complex resistivity spectrum of a formation in the frequency range of 1–500 kHz, and they used the frequency corresponding to the bottom of the imaginary component to evaluate the water-filled porosity.

The information extracted from the complex resistivity spectra in the previous research is immensely important to geophysical exploration. However, the existing literature has not considered all characteristics related to reservoir parameters contained in the complex resistivity spectra. In addition, previous research used complex resistivity spectra to calculate reservoir parameters mostly based on various

empirical resistivity dispersion models, such as the Cole–Cole model. While such models can accurately describe the resistivity dispersion of formation in most cases, they are not universal and sometimes yield results that diverge from the measured data. Therefore, the accuracy of their evaluation method used depends largely on the applicability of the dispersion model that is selected. In this study, the spectra of 15 pure sandstone cores from western China in the frequency range of 100–15 MHz were analyzed, and some useful information were discovered from the spectra.

2 Spectrum measurement

2.1 Core features

We collected 15 cores from western China (Table 1). All core contained less than 10% shale, so they can be considered as pure sandstone cores. The depths of these cores ranged from 4731.25 to 4780.48 m, with porosity of 4.8–26.5% and permeability of 2.9–669.12 mD.

2.2 Experimental methodology

We used the two-electrode method to measure the complex resistivities of the core samples with a 4194A impedance analyzer (HP, USA) operating with measurement frequencies from 100 to 15 MHz (Figure 1). The cores were saturated with NaCl solution with a salinity of 1,00,000 ppm, and then measurements were made at room temperature and at a constant confining pressure

Table 1: Parameters of core samples

Sample number	Depth (m)	Shale contents (%)	Porosity (%)	Permeability (mD)	Group
YD-12	4738.5	1.90	5.10	2.9	A
YD-38	4750.68	3.60	7.70	15.49	
YD-18	4741.39	8.60	22.80	23.16	
YD-43	4752.93	8.30	18.50	112.25	
YD-50	4759.18	5.20	23.80	221.36	
YD-49	4758.6	6.10	23.90	468.66	
YD-2	4731.25	2.90	22.60	550.23	
YD-37	4750.05	5.60	20.30	669.12	B
YD-47	4757.73	3.00	4.80	10.81	
YD-67	4769.38	2.00	6.70	17.64	
YD-65	4768.24	3.10	10.30	24.12	
YD-84	4780.48	5.00	21.80	179.38	
YD-35	4749.56	4.70	22.90	433.56	
YD-23	4743.43	2.40	24.50	505.7	
YD-61	4766.33	3.40	24.00	585.12

Figure 1: Experimental schematic.

of 10 MPa. The solution in the core was displaced with kerosene by a pump during the measurement to change the water saturation of the core. The metering tube was used to collect the liquid displaced from the core, which allowed us to calculate the water saturation of the core at different moments. The complex resistivity spectra of each core were collected at different water saturation levels by the impedance analyzer.

3 Results

3.1 Spectrum characteristics

Figure 2 shows the complex resistivity spectra of core YD-43, with different levels of water saturation. When the measurement frequency was lower than 10 kHz, both the imaginary part and the real part (absolute value) of the complex resistivity decreased slowly as the frequency used to make the measurements increased, but were relatively stable overall. When the measurement frequency reached about 10 kHz, the imaginary resistivity began to increase gradually with the frequency until it reached an extreme value and then began to decrease again, a pattern reflected in the local minimum, labeled R_{xb} , in the figure. This variable represents the maximum imaginary resistivity for a given core. At the same time, the real resistivity decreased sharply until it approached zero. The maximum value of the imaginary resistivity aligned with the most rapid decrease in real resistivity.

Because the impedance of the rock matrix is much greater than that of aqueous pore solution, higher water saturation will result in a smaller real resistivity measurement in the low-frequency band. Similarly, the impedance of kerosene is much larger than that of aqueous solution, and displacement of water will cause an intensification of interfacial polarization in the pores because of the more complex surface morphology of

Figure 2: Complex resistivity spectra of core YD-43. Each level of water saturation is depicted by a matching colored line (real resistivity) and colored dashed line (imaginary resistivity).

aqueous pore solution. The migration path of ions becomes complicated, and it takes longer to complete the polarization process, so smaller water saturation will cause higher R_{xb} and smaller corresponding frequencies. A power exponential relationship was identified between the maxima of the imaginary resistivity of each core and the saturation values (Figure 3a). Figure 3b shows the maxima of the imaginary resistivity of different cores with water saturation of 100% and the power exponential relationship between these values and porosity.

The phenomenon can be explained qualitatively by means of the polarization mechanism. When an alternating current is applied to water-bearing rocks, various polarization phenomena occur in the rocks, which will cause the amplitude of the current to decay and the phase to shift [14]. When this happens, complex models are needed to characterize the impedance of rocks [15]:

$$\rho^*(\omega) = \rho'(\omega) + j\rho''(\omega) = \frac{\sigma}{\sigma^2 + (\omega\varepsilon)^2} - j\frac{\omega\varepsilon}{\sigma^2 + (\omega\varepsilon)^2}, \quad (1)$$

where $\rho^*(\omega)$, $\rho'(\omega)$, and $\rho''(\omega)$ represent the complex resistivity and its real and imaginary components, respectively; ω is the angular frequency; σ is the conductivity of the core; ε is the dielectric constant of the core; and $j = \sqrt{-1}$.

The intensities of various polarization phenomena change in response to the frequency of the alternating current, so the complex resistivity of the core changes with the frequency used to make the measurement. When the frequency is lower than 1 MHz, the interfacial

Figure 3: Plots of the power exponential relationships between the maxima of imaginary resistivity and (a) water saturation and (b) porosity.

polarization phenomenon (Maxwell–Wagner effect) in the core is most obvious [10]. This is mainly caused by the accumulation of ions on the inner surface of pores under the action of the electric field.

The interface polarization leads to the generation of the imaginary component of the rock's impedance. The position of the maximum value of the imaginary resistivity corresponds to the frequency at which the interface polarization is the strongest. When the porosity of the core is small, or after the kerosene enters the pore space, the interface morphology of the solution with other components becomes more complicated, and it takes longer for the ions to accumulate at the interface. Thus, as water saturation and porosity decrease, interface polarization is more intense, and rocks have smaller R_{xb} at a smaller frequency.

A capacitance model can also be used to understand the effects of porosity and water saturation on the imaginary part of the complex resistivity spectrum. A pore of rock can be imagined as a capacitor, where the fluid in the pore is the dielectric and the pore inner surface constitutes the capacitive plates. The phases of the voltage and the current across the capacitor are always 90° different, so the reactance of the capacitor contributes to the imaginary component of the complex resistivity. The larger the porosity of the core is, the larger the distance between the plates of the capacitor is and the smaller the capacitive reactance of the capacitor will be, resulting in a smaller maximum in imaginary resistivity. The dielectric constant of water is larger than that of hydrocarbons and matrix, and smaller water saturation will also result in great imaginary resistivity maximum values just like the porosity is getting smaller.

3.2 New characterization parameters

Our measurement results revealed good power exponential relationships between the maxima of imaginary resistivity, porosity, and the water saturation of the core. These are similar to the relationships between porosity, water saturation, and the low-frequency resistivity of the formation obtained by traditional resistivity logging tools. Therefore, the real resistivity corresponding to 1 kHz was extracted from the complex resistivity spectrum to investigate the relationship with the maximum of imaginary resistivity. Surprisingly, there was a nearly perfect linear relationship between the maxima of imaginary resistivity and the real resistivity at 1 kHz, for a core with different water saturation levels, as shown in Figure 4. In other words, when the porosity and the permeability of the core are held constant, the ratio between the real resistivity at 1 kHz and the maxima of the imaginary resistivity is constant and independent of the water saturation of the core. This slope of fitted line is expressed as β_{RX} .

It should be noted that the frequencies corresponding to R_{xb} and R_{1k} are not equivalent: R_{1k} is, by definition, always at 1 kHz, whereas R_{xb} corresponds to the frequency with the strongest interface polarization. The real resistivity is relatively stable in the frequency domain centered on 1 kHz, whereas R_{xb} is not fixed and depends on the porosity of the rock and the nature of the fluid in the pore. Therefore, β_{RX} is different from the traditional phase angle of the complex resistivity.

Figure 5 shows the relationship between β_{RX} and porosity in these natural cores, revealing a good linear relationship between porosity and β_{RX} . When the porosity is high, the distribution of data points in the figure is

Figure 4: A plot of the real resistivity at 1 kHz (R_{ik}) and the maxima of imaginary resistivity (R_{xb}) of cores YD-43 and YD-12, showing their nearly perfect linear relationship.

disorderly. The inset figure shows the relationship between permeability and β_{RX} for the seven cores with porosities greater than 22%, showing another good linear relationship. Therefore, the disorderly distribution of data points when the porosity is greater than 22% can be attributed to the differences in their permeability values.

There is also a good linear relationship between β_{RX} and the permeability of the cores, as shown in Figure 6. The distribution of data points in the figure deviates from the fitting line when the permeability is less than 100 mD. The linear relationship between porosity and β_{RX} of the six cores with permeability less than 100 mD is also strong, as shown in the inset (Figure 6). Therefore,

Figure 5: An illustration of the linear relationship between β_{RX} and core porosity. The inset shows the good linear relationship between β_{RX} and the permeability for those cores with porosities greater than 22%.

Figure 6: A plot demonstrating the linear relationship between β_{RX} and the permeability of the cores. There is also a good linear relationship between β_{RX} and porosity when the permeability is less than 100 mD (inset).

this deviation when the permeability is less than 100 mD is driven by their different porosities.

4 Discussion

The experimental results of this study confirm strong linear relationships between porosity, permeability, and β_{RX} . These relationships provide a new idea for estimating permeability based on complex resistivity spectra. Porosity and permeability of cores can be treated as independent variables, with β_{RX} as a dependent variable for binary linear fitting. Many logging curves can provide the porosity of a formation, such as acoustic logging, neutron-density and NMR logging, so permeability can be estimated based on the fitting result when β_{RX} is determined from the complex resistivity spectra.

Since this study did not use actual logging data, half of the cores were selected to build the model and the remaining cores were used to verify the model. The 15 cores were ranked by permeability from smallest to largest, and the eight cores with an odd ranking (group A in Table 1) were selected to fit the relationship between the three parameters. The resulting equation is given as follows:

$$\beta_{RX} = 1.20664^{-4} \times K + 0.4151 \times \phi + 0.24205, \quad (2)$$

where K and ϕ are the permeability and the porosity, respectively.

A new slope (β_{RXa}) for the remaining seven cores (group B in Table 1) was obtained by plugging their porosity and permeability values into equation (2). As shown in Figure 7,

Figure 7: A plot of β_{RX} , which was extracted directly from the complex resistivity spectra, and β_{RXa} , which was calculated from group B cores with equation (2). The blue line is the angular bisector of the first quadrant.

the relative error between β_{RXa} and β_{RX} , which was extracted directly from the complex resistivity spectra, is small. The data points in Figure 7 are roughly distributed on the angular bisector of the first quadrant, which proves that the relationships between porosity, permeability, and β_{RX} can be described by a binary first-order equation.

Figure 8 shows the relationship between the measured permeability (K) and the calculated permeability (K_a) of the seven cores in group B. While these two permeability metrics were derived from very different sources, the data points still distribute along the angular bisector of the first quadrant. This demonstrates the feasibility of estimating permeability with equation (2).

Figure 8: A plot of measured permeability (K) and calculated permeability (K_a) of the seven cores of group B. The blue line is the angular bisector of the first quadrant.

Figure 9: A plot of the correlation between porosity and permeability of the 15 cores used in this study.

There should be a strong positive correlation between porosity and permeability for sedimentary rocks from the same sedimentary environment. However, the cores used in this study were collected from a range of depths and exhibited a poor correlation between porosity and permeability (Figure 9). This may impact the accuracy of the permeability estimation method based on β_{RX} .

Estimating permeability has always been a difficult problem in well-logging interpretation, and the various existing permeability estimation methods struggle to achieve the desired accuracy. Although the accuracy of the permeability estimates made with the method developed here is not perfect in this example, the results are sufficient to show a strong correlation between porosity, permeability, and β_{RX} in these sandstone cores. This approach may encourage other researchers and industry scientists to pay attention to β_{RX} , to obtain more accurate permeability estimates, and to further refine these methods.

4.1 Saturation estimation

The maxima of imaginary resistivity have a power exponential relationship similar to that of porosity and permeability in cores like the real resistivity of low frequency. Therefore, we can try to calculate the water saturation based on the maxima of the imaginary resistivity spectra. Imitating the formation factor and resistivity increase factor, the following definitions were made:

$$F' = \frac{R_{xb0}}{R_{xbw}} = \frac{a'}{\phi^{m'}}, \quad (3)$$

$$I' = \frac{R_{xb}}{R_{xb0}} = \frac{b'}{S_w^{n'}}, \quad (4)$$

where R_{xb} is the maximum of the imaginary part of core's complex resistivity spectrum, F' is the formation factor related to R_{xb} , I' is the increase factor related to R_{xb} , R_{xb0} is the maximum of the imaginary part of the complex resistivity when the core is saturated with solution, R_{xbw} is the maximum of the imaginary part of the solution's complex resistivity spectrum, m' is the cementation exponent related to R_{xb} , n' is the saturation exponent related to R_{xb} , and a' and b' are lithology-related coefficients related to R_{xb} .

To obtain the imaginary resistivity of the solution, a plastic pipe with an inner diameter of 3 mm and a length of 2.4 m was filled with NaCl solution, and the complex resistivity spectrum of the solution was measured by an impedance analyzer with the electrodes inserted into the nozzles of the plastic pipe. The R_{xbw} value was calculated from the complex resistivity spectrum.

By fitting the relationship between I' and S_w with a power function, the values of n' and b' were obtained. Figure 10 shows the relationship between I' and S_w of core YD-43. According to the definition, $R_{xb} = R_{xb0}$ and $I' = 1$ when water saturation reaches 100%. Therefore, before fitting the relationship between I' and S_w for each core, ten additional points with coordinates (1,1) were added to the measured datasets to bring the curve closer to the theoretical relationship.

Figure 10: A plot of the power relationship between I' and S_w in core YD-43, with ten artificial data points added at (1,1).

Figure 11: A plot of the power relationship between F' and ϕ for the eight cores of group A.

The eight cores of group A were selected to build the model, and the seven cores of group B were used to verify the model. Figure 11 shows the relationship and the fitting result between F' and ϕ for the cores of group A. In an approach similar to that used for I' and S_w , ten additional points with coordinates (1,1) were added to the datasets because $R_{xbw} = R_{xb0}$ and $F' = 1$ when ϕ is equal to 1.

We then used a model similar to Archie's formula to calculate the water saturation of the seven cores of group B by using the imaginary part of complex resistivity:

$$S_{wa} = \sqrt[n']{\frac{a'b'R_{xbw}}{\phi^{m'}R_{xb}}}. \quad (5)$$

Any calculated saturation values that were greater than 1 were set to exactly 1. Figure 12 shows the relationship between the water saturation calculated from the complex resistivity spectrum (S_{wa}) and the water saturation recorded during the experiment (S_w). The correlation between the two datasets is very high, and the data points are distributed along the angular bisector of the first quadrant. This model can be used to effectively calculate the water saturation sandstone cores.

Although the structure of the new model is similar to Archie's formula, it is different in several essential ways. For example, Archie's formula uses the real resistivity of low frequencies to evaluate the water saturation, whereas the model in this article is based on imaginary resistivity. Real resistivity is largely affected by the conductivity of the medium being analyzed, while imaginary resistivity is mainly affected by the dielectric constant. In addition, the real resistivity used in Archie's formula comes from a constant, predetermined determined

Figure 12: A plot of water saturation as calculated from the complex resistivity spectrum (S_{wa}) against water saturation recorded during the experiment (S_w) for the seven cores of group B. The red line depicts the angular bisector of the first quadrant.

frequency, whereas the frequency corresponding to the maximum value of imaginary resistivity varies, and it is related to the water saturation of the medium. The frequency corresponding to the maximum value of imaginary resistivity varies with the water saturation and other parameters of the medium, so the model proposed in this study is based on a dimension of the frequency domain that is different from that used by Archie's formula.

5 Conclusion

The complex resistivity spectra of 15 natural sandstone cores in western China were measured with a frequency range of 100–15 MHz. Our analysis revealed a nearly perfect linear relationship between the real resistivity at 1 kHz and the maxima of imaginary resistivity. Moreover, the slopes of the linear fitting lines had strong linear correlations with both the porosities and permeabilities of the cores. We developed a permeability estimation model based on these slopes, and it proved effective at estimating the permeability of sandstone samples, even the samples with poor correlations between porosity and permeability.

As with low-frequency resistivity, the maximum values of imaginary resistivity had power relationships with porosity and water saturation. The saturation evaluation model demonstrated high accuracy and reliability in its saturation predictions by using the maxima of imaginary resistivity. These models and the

conceptual understanding that underlies them should help popularize the use of complex resistivity spectra in the field of geophysics.

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