Abstract: Euler deconvolution is widely used for interpreting magnetic anomalies as it estimates the edges and depths of magnetic sources. Since this method was proposed, there has been an intensive effort to mitigate its primary deficiencies, namely, the generation of many spurious solutions and the high noise sensitivity. To select the most significant solutions, we adopt the strategy of constraining the moving window to the source edges, whose locations are estimated using the enhanced horizontal gradient amplitude method. On the other hand, we reduce noise propagation by performing a stable calculation of the vertical derivatives. For this purpose, we use the $\beta$-VDR method, a finite-difference method that yields a robust approximation of the vertical derivatives of magnetic data. The accuracy of the proposed technique is demonstrated on synthetic magnetic anomalies, providing the depths more precisely and being insensitive to noise. Application of this technique is also demonstrated on aeromagnetic anomalies from the Olympic Peninsula (USA), where the obtained result is in good agreement with known information of the study region.

1 Introduction

Euler deconvolution is one of the most often used quantitative interpretation techniques for gravity and magnetic datasets [1,2]. Both the source edge location and depth can be obtained from the Euler deconvolution without information on magnetization intensity or density contrast [3]. It was proposed by Thompson [4] for 2D sources and generalized for 3D sources by Reid et al. [5], who described the technique as a deconvolution based on Euler’s homogeneity equation and coined the term “Euler deconvolution.” Recently, the application of the Euler deconvolution to gravity and magnetic datasets has shown great success in mapping geological structures [6–10]. The Euler deconvolution has gained popularity mostly as a result of how easy it is to use and implement, giving it an excellent choice for a quick initial interpretation.

The deconvolution takes place in a user-defined scanning window, where the source location and regional field are determined by solving a linear least-squares problem [4]. As the window is usually much smaller than the study area, many least-square solutions are generated, most of them being spurious [1,11].

A great deal of research has been devoted to filtering out spurious Euler solutions, among other aspects, for instance, choosing the structural index relating to the source geometry [12] and the deconvolution of other fields than the gravity/magnetic anomaly [13,14]. The approaches for selecting solutions may be classified into two branches, namely, the use of error estimates [4,15,16] and an a-priori selection of horizontal locations based on enhancement filters [1,11,14,17,18].

We focus on the latter class, which takes advantage of an extensive literature on edge-detection methods [19–28] and has a lower computational cost as the scanning window works on a reduced set of points from the study area grid.
In this scope, we use the peaks of the enhanced horizontal gradient amplitude (EHGA) filter that has been applied to map structural boundaries in many recent studies [29–32], to remove spurious solutions from the Euler deconvolution. In addition, we use the vertical derivative obtained from the β-VDR method [33] instead of those from the usual frequency domain method to provide more stable results for the Euler deconvolution, analogously to Pašteka and Kušnirák [34]. We remark that data with low signal-to-noise ratio can be denoised before applying the Euler deconvolution, as long as the filtering is carefully performed to avoid producing incorrect depth estimates [16].

The accuracy and effectiveness of the presented technique are demonstrated on both synthetic magnetic datasets and a real dataset of the Olympic Peninsula (USA). We compare the proposed approach with other constraining approaches based on the peaks of the analytic signal (AS) and horizontal gradient amplitude (HGA).

2 Method

The 3D Euler deconvolution was introduced to determine the edges and depths of magnetization structures [5]. The 3D form of Euler’s homogeneity equation is given by

\[(x - x_0) \frac{\partial F}{\partial x} + (y - y_0) \frac{\partial F}{\partial y} + (z - z_0) \frac{\partial F}{\partial z} = N(B - F), \quad (1)\]

where \(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \) and \(\frac{\partial F}{\partial z}\) are the derivatives of the field \(F\) measured at \((x, y, z)\); \((x_0, y_0, z_0)\) are the source locations; \(B\) is the regional magnetic field; and \(N\) is the structural index that characterizes the source geometry.

In general, the vertical derivative of magnetic anomaly is calculated in the frequency domain [35]. However, this approach is sensitive to noise [33]. Here it is suggested to use the vertical derivative obtained from the β-VDR method [33] to provide more stable values than the usual frequency domain technique. According to this method, the vertical derivative of magnetic data is given by

\[ \frac{\partial F}{\partial z} = \frac{c_0 F(h_0) + c_1 F(h_1) + c_2 F(h_2) + c_3 F(h_3)}{\Delta h}, \quad (2)\]

where \(c_1, ..., c_5\) are given by

\[
\begin{align*}
c_1 &= (2\beta^3 + 15\beta^2 + 35\beta + 25)/12, \\
c_2 &= (-8\beta^3 - 54\beta^2 - 104\beta - 48)/12, \\
c_3 &= (12\beta^3 + 72\beta^2 + 114\beta + 36)/12, \\
c_4 &= (-8\beta^3 - 42\beta^2 - 56\beta - 16)/12, \\
c_5 &= (2\beta^3 + 9\beta^2 + 11\beta + 3)/12,
\end{align*}
\]

and \(F(h_i)\) is upward-continued data at height \(h_i = h_0 - \beta \Delta h - (i - 1) \Delta h\) with \(h_0\) being the height of the observation plane, \(\Delta h\) is smaller than the grid spacing (i.e., 1/100 or 1/10 of grid spacing for noise-free and noisy data, respectively, as reported by Tran and Nguyen [36] for this kind of calculation) and \(\beta\) is a user-defined stabilizing parameter. In this work, we use \(\beta = 40\) for computing the vertical derivative [33].

Since the Euler deconvolution method tends to generate many spurious solutions, Ruppel et al. [18] used the located Euler deconvolution to remove spurious solutions. Their method uses the peak locations in the AS map for selecting the window locations. The AS is created by combining derivatives of magnetic data, and is given by [37]

\[AS = \sqrt{\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2 + \left(\frac{\partial F}{\partial z}\right)^2}. \quad (4)\]

However, its peaks are moved inward from the borders for deeper bodies, so that the window locations are not located over the true boundaries in this case. A related approach is to use the peak locations of the HGA [14,38].

\[HGA = \sqrt{\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2}. \quad (5)\]

Likewise, the HGA is not able to accurately delineate the borders of deep sources. Moreover, peak detection methods are usually not able to detect all maxima of the HGA [39,40].

In this study, we used the peak locations of the EHGA to select the window locations. The method is given by [25]

\[EHGA = R \left\{ \frac{\partial \text{EHGA}}{\partial z} \right\} - 1 \left[ \frac{\partial G_A}{\partial x} \right] + 1, \quad (6)\]

where \(k\) is a constant defined by the researcher. Here we used \(k = 2\) to compute the EHGA [25]. Similar to other methods based on the HGA [41,42], the EHGA works best for reduced to pole (RTP) anomalies. For this reason, magnetic data should be reduced to pole before applying the proposed approach. Moreover, depth estimates by the Euler deconvolution are usually more accurate for data which have been reduced to pole [4].

A remarkable difference between the EHGA and HGA or AS is that the EHGA is able to balance the anomaly of shallow and deep sources, which allows us to use a threshold value to select for selecting the centers of the sliding window. We used the locations of the grid values where the EHGA is greater than or equal to 0.5 for the proposed method. On the other
hand, since the HGA and AS are not balanced filters, it is difficult to use a threshold value for detecting the peak locations. In these cases, we use the Blakely and Simpson technique [43] to locate their peaks, then use the locations of these peaks to select the windows for the Euler deconvolution, discarding the peaks whose value is lower than 1% of the maximum value of the HGA (AS).

3 Results and discussion

The efficacy of the technique, which we refer to as ED-EHGA from here on, is demonstrated with a synthetic example and a real field dataset from the Olympic Peninsula (USA). The Euler method was performed through a window containing 10 × 10 grid points, which is a commonly used size for this type of calculation [1], and a structural index of zero because our target structures are contacts. We also compared the proposed method with the located Euler deconvolution [18], the Euler deconvolution with window locations at the peaks of the HGA [14], which we refer to as ED-HGA, and Tilt-depth technique [44]. In all the methods, we remove the isolated solutions, which are more than five grid points away from other solutions [45].

3.1 Theoretical example

We consider a model with two prisms located at depths of 3 km (source A) and 5 km (source B). The parameters of the prisms are given in Table 1. Figure 1a and b present 3D and plan views of the bodies. Figure 2a presents the magnetic anomaly of the sources. Figure 2b and c shows the HGA and the AS of the magnetic anomaly. We can see that these filters do not clearly delineate the deeper prism, although the edges in the HGA are slightly more clear than AS. Figure 2d shows the EHGA of magnetic data. It can be observed that the EHGA yields more accurate edges than the HGA and AS. For this reason, the use of the EHGA can provide better window locations for the Euler deconvolution. Figure 3a–d depict the results of applying the located Euler deconvolution, ED-HGA, Tilt-depth, and the proposed method, respectively. One can see that the located Euler deconvolution and ED-HGA generate smaller values for the depths of both sources, while the Tilt-depth technique produces false solutions between the sources and larger depth values compared to the real values (Figure 3c). In this case, the location and depth of the bodies mapped by the proposed technique are consistent with the real values. According to these results, we can say that the ED-EHGA (Figure 3d) provides much better performance in locating magnetic sources compared with other methods.

To verify the stability of the suggested approach, magnetic data in Figure 2a were contaminated with Gaussian noise with a standard deviation of 1 nT. This amount corresponds to about 0.5% of the maximum absolute amplitude of

Table 1: Parameters of the synthetic model

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</table>

Figure 1: 3D view (a) and plan view (b) of the synthetic model.
Figure 2: Magnetic anomaly calculated from the theoretical model from Figure 1a (a) and transformed anomaly maps obtained using the enhancement filters HGA (b), AS (c), and EHGA (d).

Figure 3: Depth estimates obtained for the data in Figure 2a using the ED-HGA (a), located Euler deconvolution (b), tilt-depth (c), and ED-EHGA (d).
Figure 4: Magnetic anomaly from Figure 2a contaminated with Gaussian noise with a standard deviation of 1 nT (a) and transformed anomaly maps obtained using the enhancement filters HGA (b), AS (c), and EHGA (d).

Figure 5: Depth estimates obtained for the data in Figure 4a using the ED-HGA (a), located Euler deconvolution (b), tilt-depth (c), and ED-EHGA. (d)
the data, similar to Melo and Barbosa [16]. Figure 4a displays the noise-corrupted magnetic data. Figure 4b–d shows the results of using the HGA, AS, and EHGA to noise-corrupted magnetic data, respectively. Even though EHGA is more affected by noise than the HGA and AS, it is still able to delineate the border of both prisms. Figure 5a–d shows the depths obtained from the application of the located Euler deconvolution, Tilt-depth, and proposed technique, respectively. For the chosen noise level, there are too many less significant maxima in the HGA and AS maps that are related to noise, leading to many spurious windows for the located Euler deconvolution. For this reason, the ED-HGA and the located Euler deconvolution bring many false solutions (Figure 5a and b). Since the Tilt-depth technique is based on the Tilt angle filter that produces balanced signals for both magnetic data and noise, it is also sensitive to noise (Figure 5c). In this case, the estimates shown in Figure 5d indicate that the presented technique is less noise sensitive than others. The obtained depths from the presented technique are close to the real depths. Clearly, the use of the vertical derivative from the β-VDR and the window locations from the EHGA peaks for the Euler deconvolution allows us to reduce the noise and to determine magnetic sources more accurately. The spurious solutions from the other three methods could be eliminated using auxiliary selection criteria, while the ED-EHGA method does not need these additional resources.

3.2 Real example

We applied the suggested technique to aeromagnetic dataset of the Olympic Peninsula (USA) (Figure 6a). The study region

Figure 6: (a) Location map and (b) Geological map (UTM coordinates, zone 10N) of the study area.
is situated in the forearc of the Cascadia subduction zone involving the northeastward subduction of the Juan de Fuca plate beneath North [46]. Figure 6b presents the geology of the Olympic Peninsula. The region is an east-plunging anticlinorium and is dominated by two major geologic terranes: a non-magnetic tertiary sedimentary core and a peripheral terrane of early Eocene basalts and marine sedimentary rocks wrapping around the eastern portion of the mountains [47]. The peripheral belt is primarily composed of Crescent Formation basalts of early to middle Eocene age and associated volcanic rocks and sediments [48]. The Crescent Formation includes a lower member of massive submarine basalts and an upper member consisting of subaerial basalts with sparse interbeds of sediments [49].

Aeromagnetic data (Figure 7) of the study region were measured in 1997 by the United States Geological Survey with flight lines of 0.4 km spacing [50]. The nominal flight altitude was 0.3 km over flat to moderate terrain, but much higher over mountainous territory [50]. The data were reduced to pole using $I$ and $D$ of 69.399° and 18.975°, respectively. The RTP aeromagnetic data are displayed in Figure 8a. Figure 8c–d shows the HGA, AS, and EHGA of RTP aeromagnetic data, respectively. The high amplitude responses from the large anomalies caused by the basalts of the Eocene Crescent Formation dominate the HGA and AS, while the EHGA can determine all edges of the magnetized structures with different anomalies.

Figure 9a shows the result of the ED-HGA. The ED-HGA uses the windows determined from the locations of the HGA peaks. Since the Blakely and Simpson technique is usually not able to map all peaks of the HGA [40], the structures determined by the ED-HGA are discontinuous. Figure 9b shows the result of the located Euler deconvolution. This method uses the windows determined from the AS peak locations. The AS map (Figure 8c) of RTP aeromagnetic data does not always provide peaks over the magnetization contrasts. In fact, the AS often has a single bell-shaped anomaly over the center of the magnetized structures, as is the case with the field over the Eocene Crescent Formation. For this reason, the structures determined by the located Euler deconvolution are discontinuous. Figure 9c presents the result of the Tilt-depth. We can see that the technique is useful in mapping a wide range of magnetized structures in the study region. However, several nearby structures found using this technique are linked, leading to spurious small circular-shaped bodies. Additionally, as demonstrated by the second synthetic model, the Tilt-depth technique yields some incorrect results, making geologic interpretation more complex. The reason for producing false solutions is that the Tilt-depth is based on the tilt derivative filter that generates spurious structures in the output map [51,52]. Figure 9d shows the result of the proposed technique. The technique uses the windows determined from the peak locations in the EHGA map (Figure 8d). Since the EHGA can provide the edge locations more accurately than the traditional methods, it is helpful to select the appropriate area to be scanned. Figure 9d shows that the majority of the magnetic sources found using the suggested method had depths of less than 3 km, which is similar to the depths estimated by the ED-HGA (Figure 9a), located Euler deconvolution (Figure 9b), and Tilt-depth method (Figure 9c). However, the solutions of the proposed technique show more continuous linear features and is less susceptible to noise than the located Euler deconvolution. The proposed method can also avoid bringing the spurious sources that appear in the output map of the Tilt-depth method. Using the proposed technique, the boundaries of the Eocene Crescent Formation were also detected. The solutions of the proposed technique also show that most of the contacts along the Eocene Crescent Formation are very shallow. This is consistent with the geological data

![Figure 7: Aeromagnetic anomaly map of the study area.](image-url)
demonstrating the surface exposure of this Formation. In addition, the results of the proposed technique also show a linear structure at the northwest corner, while the Tilt-depth method does not provide any solutions for this region, and those from the located Euler deconvolution are less continuous. Moreover, the estimates of the proposed method demonstrate the existence of structures with high susceptibilities in the eastern part, which generate the anomaly with a fairly circular shape. These structures are not observed on the surface, and cannot be determined by the Tilt-depth method, but they can be determined by applying some enhancement filters to magnetic data, as reported by some other studies [27,33,48]. We also note that the estimates obtained from the proposed technique ED-EHGA have revealed the presence of additional magnetized sources located at depths from 1 to 3.5 km, which are obscured by the sediments in the western edge, and just yield only subtle magnetic signals. Again, the use of the Euler deconvolution with the vertical derivative from the β-VDR and the window locations from the EHGA peaks allows for more accurate identification of source locations.

4 Conclusion

We verified that the Euler deconvolution can provide sharper results if the vertical derivatives are computed from the stable β-VDR method and the window locations are determined from the peaks of the EHGA filter,
especially for locating geological features from magnetic anomalies. The findings of the application of the presented method on model studies indicate that it can successfully estimate the locations and depths of magnetization structures. We compared our method with the Euler deconvolution constrained by HGA peaks, the located Euler deconvolution, and the Tilt-depth technique, showing that the proposed method can identify source locations more clearly and with higher accuracy. Moreover, the method is less sensitive to noise and avoids introducing any artifacts into the output map. We applied the technique to RTP magnetic dataset from the Olympic Peninsula, USA. The findings show that the Euler deconvolution based on the β-VDR and EHGA methods is not only able to provide source locations more clearly and with higher accuracy, but also reveals the presence of some additional magnetized sources which are obscured by sediments.

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Figure 9: Depth estimates obtained for the data in Figure 8a using the ED-HGA (a), located Euler deconvolution (b), tilt-depth (c), and ED-EHGA (d).
process. D.V.N., Q.T.V., and A.M.E. carried out the data selection and discussion. The authors applied the SDC approach for the sequence of authors.

Conflict of interest: The authors declare no conflict of interest.

Data availability statement: The datasets are available from the corresponding author on reasonable request.

References


[28] Prasad KND, Pham LT, Singh AP. A novel filter “ImpTAHG” for edge detection and a case study from Cambay Rift Basin, India. Pure Appl Geophys. 2022;179(6–7):2351–64.


Oliveira SP, Pham LT. A stable finite difference method based on upward continuation to evaluate vertical derivatives of potential field data. Pure Appl Geophys. 2022;179(12):4555–66.


