Research Article

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Multimodal Algebra Learning: From Math Manipulatives to Tangible User Interfaces

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Abstract: While manipulatives have played an important role in children's mathematics development for decades, employing tangible objects together with digital systems in the classroom has been rarely explored yet. In a transdisciplinary research project with computer scientists, mathematics educators and a textbook publisher, we investigate the potentials of using tangible user interfaces for algebra learning and develop as well as evaluate a scalable system for different use cases. In this paper, we present design implications for tangible user interfaces for algebra learning that were derived from a comprehensive field study in a grade 9 classroom and an expert study with textbook authors, who also are teachers. Furthermore, we present and discuss the resulting system design.

Keywords: Tangible user interface, smart objects, tabletop interaction, embodied interaction, multimodal feedback, collaborative learning, adaptive system

1 Introduction

In mathematics education, simple passive manipulatives provide valuable "hands-on" approaches to teach students abstract concepts, especially when the students start to learn a novel unit of math, e. g., arithmetic or algebra. Based on the assumption that activity supports learning, these approaches are in-line with models from didactics like Bruner's *concrete-representational-abstract approach* [3] or the constructionist *objects-to-think-with approach* [8] that suggest to use physical objects for abstract concepts, especially for beginners. While a considerable body of research on using tangible user interfaces (TUIs) for learning has been conducted (c.f. [1, 16]), more research efforts are needed to address how TUIs can support the learning of abstract concepts such as algebra, and how the tangibles need to be designed in order to meet the needs of teachers and students.

In our research, we investigate the potentials of smart objects for algebra learning. The objects are based on traditional *algebra tiles*, which are passive manipulatives (see Figure 1) used in many schools in Northern America to support algebra learning. We are extending these tiles to smart tiles by integrating them into an interactive multi-touch surface and by adding multimodal input and output capabilities as well as adaptivity and feedback.

In educational research tactile models are the object of interest for a few decades already, as publications by Bruner [3], Montessori [21], McNeil and Jarvin [18], Carbonneau et al. [4] or Kieran [13] show. For example, Bruner's and Montessori's learning concepts encompass using physical objects to teach small children abstract concepts. Kieran mentions, that "algebraic meaning" can be achieved by modelling the situations with physical objects. McNeil and Jarvin [18] summarize the research regarding manipulatives and draw the conclusions, that it seems to be easier for children to learn these abstract concepts when "the problems draw on children's practical understandings". Common digital learning platforms, such as *Dragonbox*,[1] an App for algebra learning, lack the benefits that come with tangibility. By transforming the algebra tiles into multimodal TUIs, we combine the feedback provided by digital platforms with the benefits of tactile interaction. With this approach, we aim at supporting the transition from the enactive stage via the iconic stage to the symbolic stage – as classified by Bruner [3]. Although these stages have first been established with regard to young

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1 http://dragonbox.com/ (last access: June 22nd, 2018).

Figure 1: Algebra Tiles as commonly used in Northern America.

children, they have in variations proven helpful also where older learners with different abilities are to be addressed in one learning setting, allowing some children to work at their appropriate stage longer than others.

In the following sections of this article, we will provide an overview on related work, introduce the approach of algebra learning with tangible tiles, and then present insights from two comprehensive field studies we conducted with students, teachers and textbook authors in order to derive design implications. Finally, we present and discuss the resulting system design.

2 Related Work

Tangible user interfaces emerged in the 1990s, when Ishii and Ullmer [10, 11] described their vision for "tangible bits", physical objects that are connected to digital data or functions and can be directly manipulated for interaction. Examples for using tangibles for math learning have been provided by Falcaõ et al. [6], Girouard et al. [7], Manches and O'Malley [14], and Marichal et al. [15], amongst many others. Others, like Rick [22], have incorporated touch to be able to directly manipulate math objects presented on a screen. Research about how tangibles can support learning or how learning theories can inform tangible development has for example been presented by the "Tangible Interaction Framework" by Hornecker and Buur [9] or the "Tangible Learning Design Framework" by Antle and Wise [1]. They propose design principles for tangibles and a taxonomy about the relationship between TUIs, interactions and learning. Furthermore, Marichal [15] and Marshall, Price and Rogers [16] have critically discussed how tangibles can support learning. Results from neuroscience research suggest that concept learning is improved when using someone's motor skills [5, 12]. A study by Kiefer et al. for example implies that action information helps conceptual processing.

With this research, we add to this body of work by exploring tangible interaction for algebra learning based on the concept of algebra tiles. Inspired by earlier tangible systems such as the sifteo cubes [19] or Actibles [2], we are especially focusing on the design of multimodal feedback based on smart objects for interaction (e. g., through integrating haptic elements with embedded displays and LEDs).

3 Algebra Learning with Tangible Tiles

Algebra tiles as shown in Figure 1 consist of three types of tiles: single units used as "ones", variable-tiles and variable^2-tiles. For the ease of reading, variables are denoted as x-tiles/x^2-tiles, even though any other symbol could be used as the variable placeholder. Each tile has two sides being differently colored, red represents the negative and the other color the positive sign, the latter color also stands for the type of tiles, yellow for single "one" units, green for x-tiles, blue for x^2-tile. To represent a linear equation, the tiles are placed on a 2×2 grid (see Figure 2), where the two areas on the left represent the left side of an equation and the two areas on the right represent the right side of the equation. The top areas on both sides are the "addition zones" (tiles are connected by addition), while the lower areas are the "subtraction zones" (tiles there are subtracted from the top ones). Figure 2 (top) shows how an equation, in this case $3+(-x)-5=x$, is set up with algebra tiles. After initialization, the goal is to apply a sequence of legal actions in order to transform the equation to finally isolate one remaining x-tile. Therefore, it is allowed to add or subtract by removing the same tiles in value from the same zone on both sides or the same tiles in value from opposite zones on one side of the equation. Division is represented by setting up the same number of equal groups on each side and then continuing to work with only one of them. Additionally, it is allowed to put a negative and a positive tile of the same kind into the same zone (a so called "zero pair" – their values add up to 0) to transform the equation (c.f. step 1 and 2 in Figure 2).

With the traditional tiles, a student can transform the equation but does not get any feedback about the correctness of the actions. Our approach combines tangible algebra tiles with visual and haptic feedback with an interac-

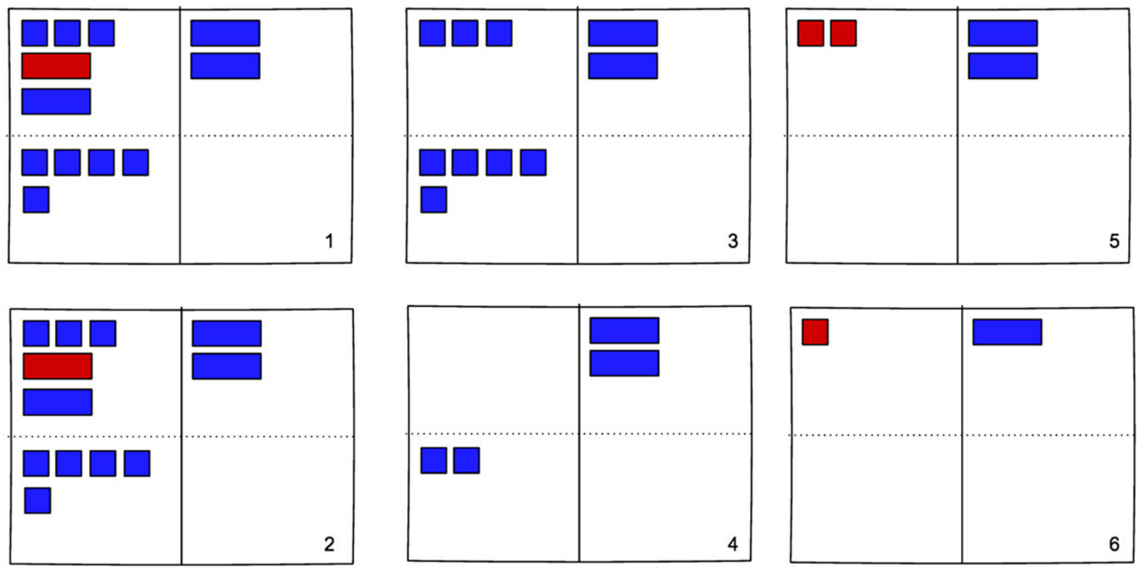

Figure 2: Solving steps of the equation $3 + (-x) - 5 = x$ with algebra tiles.

tive surface to provide an interactive learning experience. To better understand the challenges of and requirements for our system, we conducted two studies. Based on the results of these, we derived design implications for a tangible algebra system and present our design approach.

4 Insights from a Field Study with Paper Tiles in the Classroom

A paper version of the tiles was tested in a realistic setting in school. Over the course of 9 sessions (à 75 to 90 minutes) a group of 12 students of grade 9 (age 14–16) with persisting problems in elementary algebra worked with paper representations of the algebra tiles. The use of self-made paper tiles allowed to easily create a set of tiles and a mat that already addressed some issues that had been identified by the team. Unlike the commercially available algebra tiles, the paper versions were white (ones) and blue (x-tiles), with the negative side printed over in grey. The subtraction zone (as described in the previous paragraph) was an extra sheet to be placed parallelly to the vertical axis inside the addition zone to stress the importance of the left-right-divide and to overcome the obstacle of not being able to divide with tiles in the negative area. To illustrate division, straws were given to the students. The straws function as divider to highlight the equal groups that are necessary for a valid division.

During the field study, the students were instructed to work collaboratively in pairs. All sessions were videotaped with three cameras (each camera filming one pair of students) and observed. The observers took notes during the sessions. The video material was coded regarding (1) usability and user experience problems, (2) value adding elements for the planned system and (3) possible solutions [17].

The video analysis revealed that a digital system could benefit from automatically providing the next task and therefore reducing idle times between the tasks. Furthermore, a common problem was that tasks were either not completed or not performed as intended. The students had to shift their foci between the tasks on paper, the documentation of their steps and the paper tile system, which was a necessary workaround. This clearly led to errors, omitting tasks, and problems in focus. The analysis showed that many of the students would have benefited from feedback regarding tile meaning and legal moves as they took away an x-tile on the left side and a unit tile on the right side or divided unequal groups. Even with prior knowledge regarding negative numbers, they struggled with the negative sides of the tiles and the subtraction zone. The division was confused with the steps of a subtraction. The collaborative task design revealed that in unequal groups one student typically performed all actions and rarely explained his or her actions to the second person.

5 Insights from an Expert Study with Textbook Authors

The second study was an expert study with authors of mathematics textbooks, who are experienced teachers.

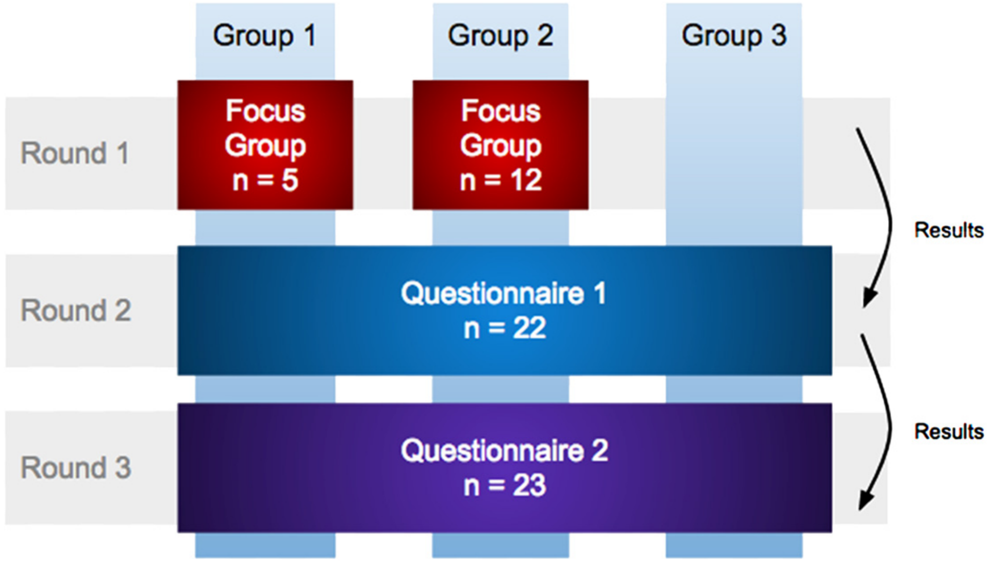

Figure 3: The three rounds of the expert study with the involved groups.

The authors formed fixed teams based on the textbook they contribute to. The study had three phases that built on each other (see Figure 3): first focus groups and then two consecutive questionnaires. Within the 80 minutes of the focus groups (size: 5 and 12) the authors learned the concept of algebra tiles, discussed models as used in textbooks, online tools like *Dragonbox*, and the concept idea of a tangible algebra learning system. The questionnaires built upon the results of the respective prior round – questionnaire one on the focus group, questionnaire two on questionnaire one. Both questionnaires asked for agreement on statements from previous rounds on a 5-point-Likert scale. The first one additionally contained open questions regarding algebra learning models and the advantages or disadvantages of them, 22 and 23 authors from three different textbooks participated.

The experts highlighted the importance of the enactive and iconic aspects of algebra teaching models. Some noted they prefer simpler systems with constraints over complex ones since they need less explanations. While organizational issues like storage space and availability speak against physically available models, they exceed purely digital models in their enactive aspects. A digital one convinced the experts regarding easy availability, flexibility and adaptability, which was emphasized in the context of teaching students with different needs and learning paces. The experts would welcome a system that reduces their workload and allows the students to work on their own and in their individual pace. As 83 % underline the importance of the transition from the enactive/iconic to the symbolic stage, they expect a system supporting this and giving clear feedback. But as costs will likely restrict the availability, teachers do not want to depend on the system. When exposed to the algebra tiles concept, the teachers were slightly confused with the different colors for each of the tiles. Moreover, some disliked that the length of the rectangle shape of the x-tile appeared to be three or four times the unit tiles, as the value of x should not be considered to be three or four due to the length. Problems with equivalent transformations also occurred and the x^2 actions caused some trouble in the beginning as well. The authors liked that negative numbers are represented and began to think about utilizing the inherent riddle aspect for the better students as a challenge. Exposed to the *Dragonbox* game, some teachers commented that for the target group of students this application might be too "childish" regarding design.

6 Design Implications for a Tangible Algebra Tiles System

From the results of the field and the expert study we derive the following design implications (also inspired by Nielsen's 10 heuristics for user interface design [20] and Antle's and Wise's and "tangible learning design framework" [1]):

DI 1. *The system should provide clear guidance through the tasks and thereby aid understanding of the system.*

DI 2. *As both students and teachers had problems with the colors and new ways of interacting with the model, a* *tutorial introducing each functionality step by step is highly recommended.*

DI 3. *Furthermore, hints should be available to provide solutions to the current situation, e. g. when the user is stuck or an action cannot be performed as the user tried.*

DI 4. *This includes going back to a former situation as in keeping track of previous steps.*

DI 5. *The user needs the opportunity to get information on what each tile means and what the possible interactions are.*

Furthermore, the collaboration in the classroom revealed the potential, but also the problem of supporting unequal pairs. As the experts identified supporting children at their own pace and capabilities as an important aspect, we suggest the following design implications:

DI 6. *As collaboration is a great opportunity when working with tangible tiles, the system should support this use case, e.g. by distributed controls as proposed by Antle and Wise [1].*

DI 7. *A tangible system is a tool to support the transition from the enactive/iconic stage to the symbolic stage. Thus, all ways of representations should be available and it should be possible to continuously increase difficulty.*

DI 8. *The continuous increase in difficulty should be also available regarding the tasks, as one goal is to provide benefit for students with different learning capabilities and should therefore be adjustable to each individual student.*

Both studies revealed problems with the color schemes, although different ones were used. This indicates that for a tangible algebra tiles system, a clear and intuitive color scheme is crucial, especially as an intuitive access to the system as mention by the experts.

DI 9. *A clear and intuitive color scheme is important to convey the necessary information without confusing the user.*

7 Design Approach: A Scalable System for Algebra Learning

We combine the concepts from literature, the requirements and the design implications from the studies to create a system for algebra learning. As having a large interactive surface for every student is not feasible, among the project goals was also to create a scalable system ranging from a tablet-based multitouch version mostly focusing on the iconic and symbolic stage to an interactive table version with interactive tangibles and a strong focus on the enactive stage. Here we focus on the tangible system.

In our scalable algebra learning system, we reduced the color set to two colors, one for the positive sign (blue) and one for negative sign (red) (see design implication (DI) no. 9). We incorporated a preview for the tiles – so called “ghost tiles” – to support during equation set up especially during the tangible scenario. Additionally, the ghost tiles are used as hints to guide the user when help is required (one approach to provide help as advised in DI 2 & 3). To support the transfer to the symbolic stage, we also show the symbolic representation of each equation updating in real time while moving tiles on the canvas (DI 7). As maintaining the equivalence is an important aspect, we can give either instant or delayed feedback. In our current design, the feedback is always slightly delayed to allow the user to finish an interaction, for example when removing tiles from both sides. To cope with the different screen sizes of each version, task descriptions are currently placed in a sidebar menu. The current version of our system can be seen in Figure 4.

8 Discussion and Conclusion

Developing a scalable system for touch and tangible use on different screen sizes poses challenges such as the available screen space or that the tangibles occlude text displayed on the screen. There are differences in terms of the possible interaction on pure touch devices compared with a combined touch and tangible approach. One example is the division. On the tablet with pure touch there is a nice solution with a dividing gesture and fading tiles, as it can be seen in Figure 5. As tangibles cannot just disappear, we have to carefully design the interaction affordances to lead to the same result – just the result of the division remains on the canvas while all other tangibles are removed. When focusing on specific concepts like for example zero pairs, tangibles can enrich the interaction equipped with dynamic constraints.

The presented work has the following limitations: The students participating in the field study had prior knowledge in algebra from previous instruction. Therefore we do not know how novice students would have interacted when exposed to the concepts of algebra mediated by the algebra tiles. Additionally, it was a group of students who were selected by their teachers by their need of extra lessons in algebra. Therefore the group likely excluded

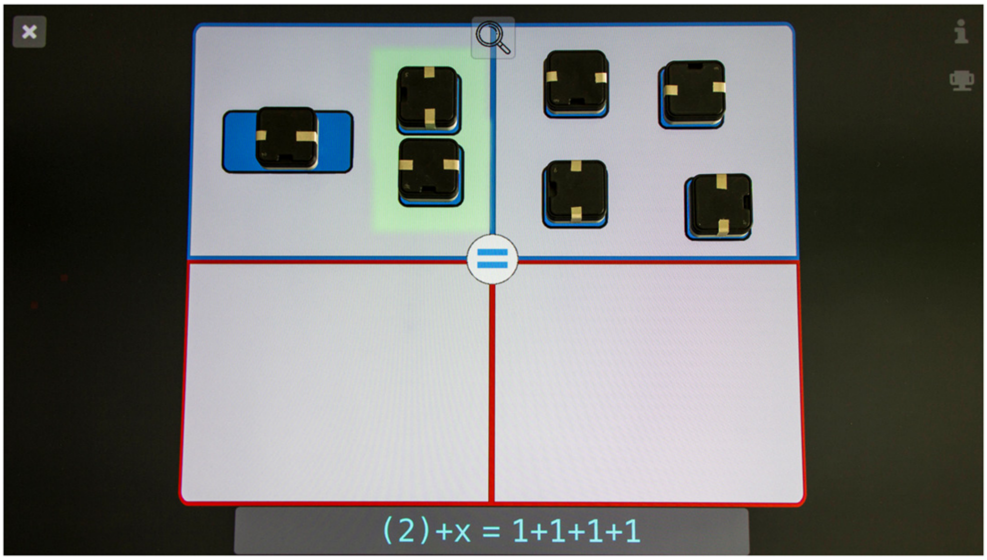

Figure 4: Our current system design with seven tangibles placed on the screen. The green highlight around the two one units indicates that they form a group as also can be seen in the symbolic representation.

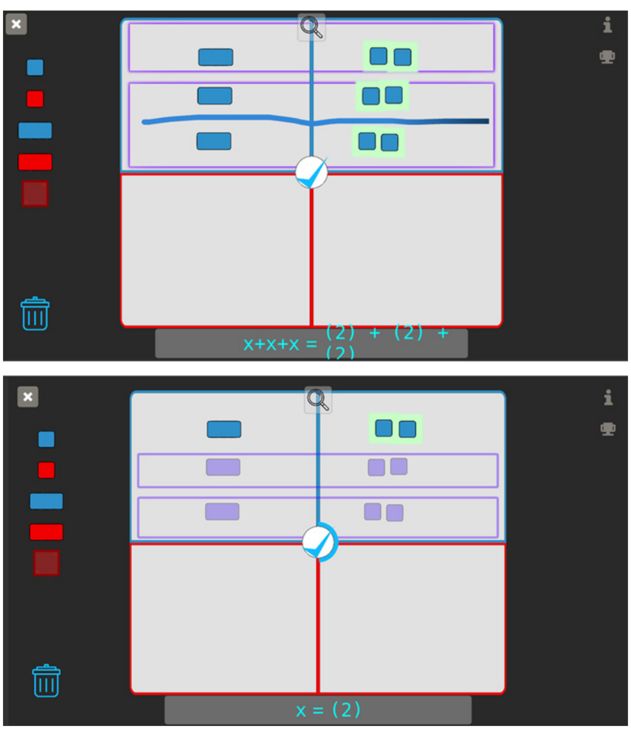

Figure 5: Top: current touch system design with the division gesture – bottom: fading out animation after a successful division resulting in showing only the tiles on the canvas, that are the result of the division.

students who might have been ready to work with symbols alone at an earlier point in instruction. The expert study with textbook authors was a Delphi study [23] designed to lead to different types of consensus. As the textbook authors were representing the existing variety of school types, we only can expect biased results because of the different group sizes. However, due to the teachers' responsibility as textbook authors they might have been in favor for supporting books over technology.

In summary, a multimodal system for algebra learning with tangibles requires a clear strategy, starting with a clear and intuitive color set, an easy to follow introduction into the system and its functionalities and comprehensive didactic concept. Additionally, when developing a scalable system, a consistent and comparable set of interactions is needed in order to ease switching between the systems. As a next step, we aim at testing the interactive prototypes with students to evaluate the benefit of the digital and tangible system. Based on the results we will refine the guidelines and improve our system for future testing.

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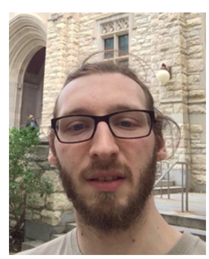

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Angelika Bikner-Ahsbahs is professor of mathematics education at the University of Bremen. One of her main research interests is design based research, specifically addressing the question of how students build new mathematical knolwedge and how conditions, such as tools, tasks and additional resource, can be shaped to foster these learning processes. In this respect, algebra learning plays a key role in school.

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