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The Impact of Paying for Milk Solids on the Performance of the Dairy Supply Chain and Consumers

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Abstract: An important particularity of the dairy chain is that many times the main interest of the dairy industry relies on milk components, the so-called milk solids. Paying for milk solids content is a way of trying to create incentives for farms to invest in improving the solids content. However, little is known about the effects of this type of payment on the dairy supply chain. The paper proposes a microeconomic model to analyze the effects of paying for milk solids content on the performance of farms, dairy processors and consumer welfare. Based on the model, we find that this mechanism improves the yield of milk in producing dairy products and benefits farms, processor and consumers simultaneously. Extensions demonstrate the robustness of results and provide a generalized model and conditions for which these results are valid.

Keywords: milk, dairy products, milk solids content, milk quality payment

1 Introduction

The dairy chain is a peculiar one. Raw milk is produced by farms and delivered to cooperatives and dairy processors, which in turn produce and deliver processed dairy products to wholesalers, retailers and consumers. The way the productive chain links are organized varies from countries and regions, but milk intrinsic characteristics evoke some discussions that are relevant everywhere.[1]

An important issue relies on the fact that many times the main interest of the dairy industry is on milk components, the so-called milk solids. For instance, butter is basically milk fat, while milk powder and cheeses are composed by fats, proteins, among other components. As informed by Draaiyer et al. (2009), the greater the amount of fat and solids-non-fat in milk the greater the yield of milk in producing dairy products. It is not by coincidence that efforts are made to create incentives for farmers to increase the milk solids content (MSC from now on). As pointed out by Sutton (1989) and Palmquist et al. (1993), this is possible by investing in animal breeding, feeding, among other aspects that affect the amount of milk solids.

Paying for MSC is a way of trying to create this incentive by valuing the yield of milk in producing dairy products. Instead of a unique price for a liter of milk, there is a bonus according to the amount of some selected solids, as the weight of fat and proteins. Dairy Report (2015) highlights that this type of payment is usual in United States, especially for cheese, butter and dry whey production. Sneddon et al. (2013) presents a review of milk payment systems adopted in several countries and regions, including New Zealand, Australia, United States and European countries, and highlights that fat and total protein are the most usual compositional criteria in price formulas.

Despite the importance of the biological and veterinary aspects, little is known about the impacts of paying for MSC on the economic performance and incentives of the dairy supply chain and on the welfare of consumers. This analysis is fundamental for countries where milk and dairy production play an important role on the subsistence of families, profitability of smallholder farmers, employment,

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1 The paper was thought for cow's milk, but it is also valid for other types, as goat, sheep and buffalo milk.

agriculture production, among other aspects.[2] Furthermore, milk is an essential low-cost protein for children, adults and the elderly in developing countries, especially considering the ongoing fight against poverty and hunger in places like India, Brazil, China and the African continent. Considering these aspects, finding ways of improving the welfare of consumers and milk/dairy producers becomes a key task for researchers and policy makers.

This paper aims to analyze how paying for MSC affects the economic performance of the dairy supply chain and consumers. We develop a microeconomic model where a dairy processor and farm(s) maximize profits based on the model parameters and variables. We compare two scenarios, in the presence and in the absence of a payment for MSC, to check if this type of payment increases the yield of milk in producing dairy products, if it enhances farms and dairy processors profits and if it increases the welfare of consumers. Based on the proposed model, we find that this mechanism benefits everyone simultaneously. We provide two extensions to demonstrate the robustness of results, while a third extension provides a generalized model and general conditions for which these results are valid.

This paper is related to the literature on milk quality payment systems. Basically, milk quality includes two aspects: the MSC and anti-hygienic components, mainly bacteria count. Draaiyer et al. (2009) presents many types of milk quality payment systems, some of them considering only one aspect and some considering both. As previously mentioned, a review of milk payment systems adopted in developed countries can be obtained in Sneddon et al. (2013).

More specifically on the MSC, there is a traditional literature focused on the United States that aimed to understand the industry of that time, which includes Brog (1971), Ladd and Dunn (1979), Bangstra et al. (1988) and Keller and Allaire (1989). Nevertheless, more recently only a few papers have focused on this issue. One of them is Botaro, Gameiro, and Santos (2013), where authors analyzed econometrically milk quality payment systems in dairy cooperatives of southern Brazil and find no evidence that the program contributed to increase fat and protein content. Meneghini et al. (2016) proposed a linear programming model to price the raw milk and determine the optimal mix of dairy products that maximizes dairies margin in Brazil, concluding that optimal schemes remunerate producers based on the quantity and quality of raw milk, including the MSC. Edwards et al. (2019) analyzes the effects of fat and protein milk prices on the profitability of two important breeds in New Zealand, Jersey and Holstein-Friesian, finding conditions for which both are equally profitable and for which one is more profitable than the other.

The remainder of the paper is organized as follows. Section 2 presents the main model, while Section 3 provides three model extensions. Section 4 presents the conclusions and the paper ends after references.

2 The Model

Suppose one dairy processor buying raw milk from $n \geq 1$ milk farms. The inverse demand for the processor's product is $p = a - bq$, where $p > 0$ is the price, $q > 0$ is the quantity and a, b are positive coefficients. The product can be any processed dairy product that uses milk components as main inputs, as cheese, milk power, butter and so on. Assume $q = \theta x$, where x is the quantity of milk used and $\theta > 0$ is the coefficient of the yield of milk, directly related to the MSC. We state $\theta \in (0, 1/b]$.[3]

Denote the milk price by I. The processor can pay a bonus according to the MSC, defined by $I\,(1 + \beta\theta)$, where $\beta \geq 0$ is the bonus coefficient. Note that if $\beta = 0$ the milk price is unique.

Farms have identical quadratic costs functions, given by $C_i = \theta x_i^2/2$, $i = 1, \ldots, n$. They can set the yield of milk, but higher values increase the total cost.[4] The marginal cost per firm is $CMg_i = \theta x_i$, $i = 1, \ldots, n$, which represents the milk

2 Examples include: United States, where dairy industry supports more than three million direct and indirect jobs that generate $159 billion in wages and $620 billion in overall economic impact (IDFA 2020); Australia, where dairy industry accounted for around 7% of the gross value of agricultural production and around 6% of agricultural export income in 2018–2019, besides its importance in international dairy market as an exporter (Weragoda and Frilay 2020); European Union countries in general, since dairy sector is the second biggest agricultural sector in European Union and represents more than 12 % of total agricultural output (Augère-Granier 2018); many other important players in milk and dairy production, as New Zealand, Brazil, China and India.

3 It is reasonable to assume an upper bound to the yield of milk. In practice, the main findings of the paper are the same in the absence of this assumption, however we could not calculate exactly values for variables and parameters. This case is considered later at the model extensions.

4 We assume that the cost structure is unaltered for distinct values of θ, that is, total costs change only according to θ and x_i, $i = 1, \ldots, n$. This is the case of improving animal feed or increasing the number of cows without economy of scale, for example. In practice, it is possible for farms to modify the cost structure in order to improve profits. This issue is addressed in Extension 3.3.

supply function per firm. As farms are identical, we have $x_i = x/n$, $i = 1, \ldots, n$, and since the milk supply function of the industry is the sum of the individuals supplies, we have $I = \sum_{i=1}^{n} \theta x_i = \theta x$.

Consider first the dairy processor decision. The processor's profit is given by $\Pi = pq - (1+\beta\theta)Ix$, which can be expressed as follows:[5]

$$\Pi = (a - b\theta x)\theta x - (1+\beta\theta)Ix \quad (1)$$

The first order condition regarding x yields the following milk demand function:

$$x = \frac{a\theta - (1+\beta\theta)I}{2b\theta^2} \quad (2)$$

Solving the demand-supply system of equations we find the milk quantity and milk price that maximize the dairy processor profit:

$$x = \frac{a}{(2b\theta + 1 + \beta\theta)} \quad (3)$$

$$I = \frac{a\theta}{(2b\theta + 1 + \beta\theta)} \quad (4)$$

Now consider the farms decision. The amount of milk delivered can be expressed as:

$$x_i = \frac{a}{(2b\theta + 1 + \beta\theta)n}, \quad i = 1, \ldots, n \quad (5)$$

While the farm's profit function is $\pi_i = x_i I(1+\beta\theta) - (\theta x_i^2/2)$, $i = 1, \ldots, n$. Note that the farm's profit includes the bonus in the total revenue. Replacing the values of I and x_i, $i = 1, \ldots, n$ obtained in (4) and (5), the profit for each farm is expressed as follows:

$$\pi_i = \frac{a^2\theta[2n(1+\beta\theta) - 1]}{2n^2(2b\theta + 1 + \beta\theta)^2}, \quad i = 1, \ldots, n \quad (6)$$

We analyze each scenario separately hereafter.

2.1 No Payment for MSC

The first proposition is stated below.

Proposition 1: *With no payment for MSC ($\beta = 0$) farms set $\theta = 1/2b$.*

Proof: For $\beta = 0$, the farm's profit is resumed to $\pi_i = a^2\theta(2n-1)/2n^2(2b\theta+1)^2$. The first derivative with respect to θ yields the following:

$$\frac{\partial \pi_i}{\partial \theta} = \frac{a^2(2n-1)(1-2b\theta)}{2n^2(2b\theta+1)^3}, \quad i = 1, \ldots, n \quad (7)$$

The right-hand side in (7) is zero when $\theta = 1/2b$, thus it is a critical point. The second derivative is as follows:

$$\frac{\partial^2 \pi_i}{\partial \theta^2} = -\frac{4a^2b(2n-1)(1-b\theta)}{n^2(2b\theta+1)^4}, \quad i = 1, \ldots, n \quad (8)$$

Note that $\partial^2\pi_i/\partial\theta^2 < 0 \; \forall \; \theta < 1/b$ and $\partial^2\pi_i/\partial\theta^2 = 0$ for $\theta = 1/b$, therefore $\theta = 1/2b$ is a maximum of the function. It is enough to conclude that farms set $\theta = 1/2b$. □

Figure 1 below expresses the behavior of farms profits in case of no payment for MSC.

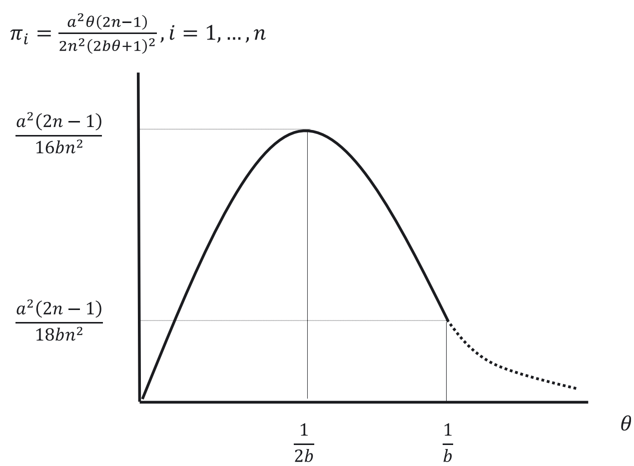

Figure 1: The farm's profit behavior with no payment for MSC ($\beta = 0$). *Note: If $\theta \geq 1/b$ was allowed the equality would be an inflexion point, i.e., a threshold between a negative and a positive second derivative. In any case, $\theta = 1/2b$ is the maximum of the function.

To sum up, with no payment for MSC ($\beta = 0$) farms set $\theta = 1/2b$. The quantity delivered by each farm is $x_i = a/2n$, $i = 1, \ldots n$, while the profit is $\pi_i = a^2(2n-1)/16bn^2$, $i = 1, \ldots, n$. The milk price is $I = a/4b$ and the amount of milk demanded by the processor is $x = a/2$. The processor produces $q = a/4b$ units of processed dairy product, sold in the market at the price $p = 3a/4$. Lastly, the processor's profit is $\Pi = a^2/16b$. The following figures show the equilibriums in this scenario for farms and processor, respectively (Figures 2 and 3).

5 Expressing the profit as a function of the input is common in monopsony and oligopsony models. Examples include Durham and Sexton (1992) and Muth and Wohlgenant (1999). In our case, we consider both streams for the processor, an inverse demand function for the processed dairy product given by the market and a demand function for the raw milk.

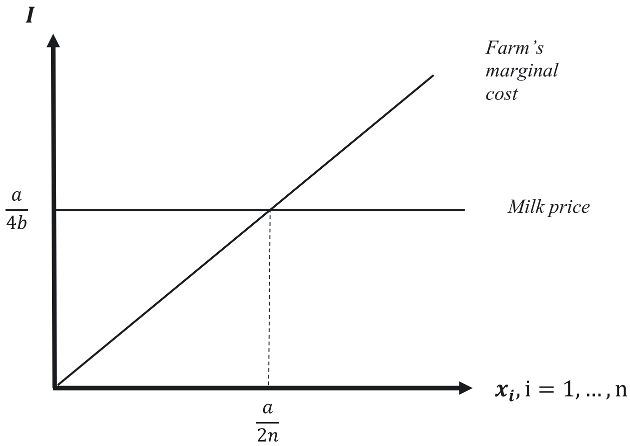

Figure 2: Farm(s) equilibrium(s) with no payment for MSC.
*Note: Milk price is determined on the whole market and farms produce a quantity of milk that equals the milk price to the marginal cost. Replacing the values obtained in this scenario on milk supply function and putting in function of x_i, $i = 1, \ldots, n$, each farm marginal cost is represented by $I = nx_i/2b$, $i = 1, \ldots, n$.

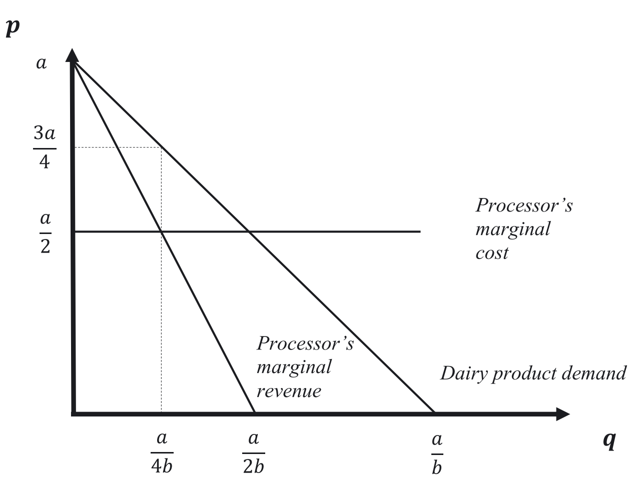

Figure 3: The processor equilibrium with no payment for MSC.
*Note: The dairy product demand is represented by $p = a - bq$. Replacing the values obtained in this scenario in processor's profit and putting in function of q, marginal revenue is $a - 2bq$, while total and marginal costs are $aq/2$ and $a/2$, respectively.

2.2 Paying for MSC

The second proposition of the paper is the following.

Proposition 2: *With a payment for MSC ($\beta > 0$) farms set $\theta = (2n-1)/(4bn - 2\beta n - 2b - \beta)$ and the dairy processor states $\beta = b(2n-1)/(2n+1)$, then $\theta = 1/b$.*

Proof: For $\beta > 0$, the first derivative of the farm's profit as in (6) regarding θ is as follows:

$$\frac{\partial \pi_i}{\partial \theta} = \frac{a^2(\beta\theta + 2b\theta + 2\beta n\theta - 4bn\theta + 2n - 1)}{2n^2(2b\theta + 1 + \beta\theta)^3}, \quad i = 1, \ldots, n \tag{9}$$

The first derivative above is zero when $\theta = (2n-1)/(4bn - 2\beta n - 2b - \beta)$. As $\theta \in (0, 1/b]$, the bonus is bounded by $\beta \in (0, b(2n-1)/(2n+1)]$. To prove that $\theta = (2n-1)/(4bn - 2\beta n - 2b - \beta)$ is a maximum we calculate the second derivative:

$$\frac{\partial^2 \pi_i}{\partial \theta^2} = -\frac{a^2(\beta^2\theta + 4b^2\theta + 4\beta b\theta + 2\beta^2 n\theta + 8bn + 2\beta n - 8b^2 n\theta - 4b - 2\beta)}{n^2(2b\theta + 1 + \beta\theta)^4}, \quad i = 1, \ldots, n \tag{10}$$

A simple algebraic manipulation shows that $\partial^2\pi_i/\partial\theta^2 < 0 \; \forall \, \theta \in (0, 1/b]$, thus $\theta = (2n-1)/(4bn - 2\beta n - 2b - \beta)$ is a maximum.

Now we check the optimal value of β for the processor. For $\theta = (2n-1)/(4bn - 2\beta n - 2b - \beta)$ we have the following values from (3) and (4), respectively:

$$x = \frac{a(4bn - 2\beta n - 2b - \beta)}{(8bn - 4b - 2\beta)} \tag{11}$$

$$I = \frac{a(2n-1)}{(8bn - 4b - 2\beta)} \tag{12}$$

Replacing these values in (1) results in the following processor's profit:

$$\Pi = \frac{a^2 b(2n-1)^2}{(8bn - 4b - 2\beta)^2} \tag{13}$$

The first derivative in (13) regarding β is $\partial\Pi/\partial\beta = 4a^2 b(2n-1)/(8bn - 4b - 2\beta)^3$. Note that $\partial\Pi/\partial\beta > 0 \; \forall \, \beta \in (0, b(2n-1)/(2n+1)]$, therefore the processor will set $\beta = b(2n-1)/(2n+1)$. It follows that $\theta = 1/b$. □

For $\beta = b(2n-1)/(2n+1)$ and $\theta = 1/b$ we have the following values: the demand of milk is $x = a(2n+1)/(8n+2)$; the milk price is $I = a(2n+1)/b(8n+2)$; the processed dairy product price and quantity are $p = a(6n+1)/(8n+2)$ and $q = a(2n+1)/b(8n+2)$; the processor`s profit is $\Pi = a^2(2n+1)^2/b(8n+2)^2$; the amount of milk delivered by each farm is $x_i = a(2n+1)/\ n(8n+2), i = 1, \ldots, n$; the farm's profit is $\pi_i = a^2(2n+1)\ \ (8n^2 - 2n - 1)/$

$2bn^2(8n+2)^2$, $i = 1, \ldots, n$. Figures below present the equilibriums for farms and processor in this scenario (Figure 4).

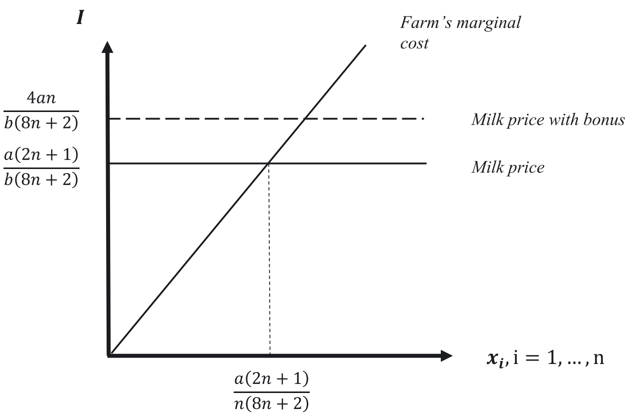

Figure 4: Farm(s) equilibrium(s) in the presence of MSC payment.
*Note: Replacing the values obtained in this scenario on milk supply function and putting in function of x_i, $i = 1, \ldots, n$, each farm marginal cost is represented by $I = \frac{nx_i}{b}$, $i = 1, \ldots, n$.

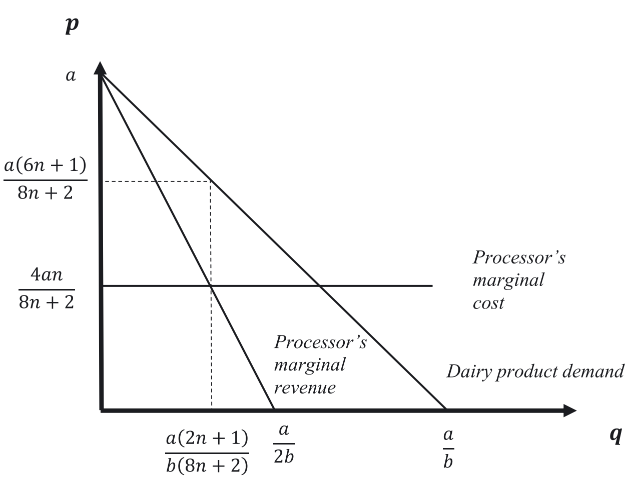

Figure 5: The processor equilibrium in the presence of MSC payment.
*Note: The dairy product demand is represented by $p = a - bq$. Replacing the values obtained in this scenario in processor's profit and putting in function of q, marginal revenue is $a - 2bq$, while total and marginal costs are $\frac{4anq}{8n+2}$ and $\frac{4an}{8n+2}$, respectively.

2.3 Comparing the Scenarios

Table 1 below presents the values for each scenario.

Table 1: Calculated values of parameters and variables in both scenarios.

Values	No paying for MSC	Paying for MSC
β	–	$\frac{b(2n-1)}{(2n+1)}$
θ	$\frac{1}{2b}$	$\frac{1}{b}$
X	$\frac{a}{2}$	$\frac{a(2n+1)}{(8n+2)}$
I	$\frac{a}{4b}$	$\frac{a(2n+1)}{b(8n+2)}$
P	$\frac{3a}{4}$	$\frac{a(6n+1)}{(8n+2)}$
q	$\frac{a}{4b}$	$\frac{a(2n+1)}{b(8n+2)}$
Π	$\frac{a^2}{16b}$	$\frac{a^2(2n+1)^2}{b(8n+2)^2}$
x_i, $i = 1, \ldots, n$	$\frac{a}{2n}$	$\frac{a(2n+1)}{n(8n+2)}$
π_i, $i = 1, \ldots, n$	$\frac{a^2(2n-1)}{16bn^2}$	$\frac{a^2(2n+1)(8n^2-2n-1)}{2bn^2(8n+2)^2}$

Source: own calculations.

Briefly, paying a bonus of $\beta = b(2n-1)/(2n+1)$ encourages farms to invest on the MSC. Consequently, the milk demand (and each farm supply) decreases,[6] as well as the processed dairy product price, at the same that the milk price and the processed dairy product quantity increase. Lastly, a simple algebraic manipulation shows that both processor and farms profits are higher in case of paying for MSC.[7]

Now consider the consumer scenario. Based on Figure 3, the consumer surplus for the scenario with no payment for MSC is $a^2/32b$. This is easily obtained by calculating the triangle area above the price and below the processed dairy product demand function. The same procedure for the scenario with the payment for MSC in Figure 5 results in a consumer surplus of $a^2(2n+1)^2/2b(8n+2)^2$, higher than before. We conclude that consumers are also in a better situation with the payment for MSC.

6 Note the trade-off for farms: investing in MSC increases the yield of milk, which is the "milk quality" regarding solids, but consequently the processor will need a less amount of raw milk to maximize its profit. This trade-off is beneficial for farms in the main model, while in Extension 3.3 we show for which conditions farms keep investing in MSC.

7 It is possible to express every variable/parameter in function of β to check the effects of paying the bonus. In our model, for $\beta \in \left(0, \frac{b(2n-1)}{(2n+1)}\right)$, we have x, x_i, $i = 1, \ldots, n$ and p strictly decreasing, while θ, I, q, Π, π_i, $i = 1, \ldots, n$ are strictly increasing in β. In a general model, with Π and π_i, $i = 1, \ldots, n$ in function of β, increasing β benefits the processor as long as the added revenue from the processed dairy product is higher than the added cost from paying the bonus, while it benefits farms while the added revenue from receiving the bonus is higher than the added cost from investing in milk solids. See Extension 3.3 for further details.

3 Model Extensions

3.1 No Upper Bound for the Yield of Milk

Observe the Proposition 1 and Figure 1. We have $\theta = 1/2b$ as a maximum for $\theta \in (0, 1/b]$, but it still valid for $\theta > 0$, thus the absence of the upper bound would not change the analysis for the scenario with no payment for MSC.

Now consider the scenario with the payment for MSC. As $\theta = (2n-1)/(4bn-2\beta n-2b-\beta)$ and θ is strictly positive (and not undetermined) we have $2b(2n-1)/(2n+1) > \beta$. From (13), the dairy processor will set β as close as possible to $2b(2n-1)/(2n+1)$, since the profit is increased, however this value cannot be reached. Observe that the limit of $\theta = (2n-1)/(4bn-2\beta n-2b-\beta)$ as β approaches $2b$ $(2n-1)/(2n+1)$ from the left is infinite, which means that we would not have defined values of β and θ. We could only conclude that the processor would pay a bonus as close as possible to $2b$ $(2n-1)/(2n+1)$ and farms would invest up to the infinite on the yield of milk. Without these values we could not calculate quantities, prices and profits.

3.2 Considering the Milk Testing Cost

Until now we have been neglecting the milk testing cost. A test is required to define the MSC, then the processor can set the bonus according to the yield of milk. Even considering the development and popularization of technologies, which tends to decrease their costs, the milk quality test incurs costs that must be borne by someone.[8]

Suppose the dairy processor buys raw milk periodically. Following the steps informed by Draaiyer et al. (2009), in the beginning of each period a sample of raw milk is tested, then the processor sets the bonus for the entire amount of milk. Draaiyer et al. (2009) argue that testing the milk many times is impractical due the cost, time involved and inconvenience, thereby the bonus payment is usually defined based on an initial sample.

Assume first that the processor pays the milk testing. Defining this cost by F, the processor's profit as in (1) is now the following:

$$\Pi = (a - b\theta x)\theta x - (1 + \beta\theta)Ix - F \tag{14}$$

Observe that the analysis starts from the first derivative regarding x, thus the fixed cost does not change the variables values. In other words, values in Table 1 are the same in this case. The exception is the processor`s profit: now a payment for MSC results in $\Pi = (a^2(2n+1)^2/b(8n+2)^2) - F$. Comparing this scenario the one with no payment for MSC the processor is better if $(a^2(2n+1)^2/b(8n+2)^2) - F > a^2/16b$, or $a^2(8n+3)/4b(8n+2)^2 > F$ after some manipulation. Remember that $p = a - bq$, thus a/b is the potential demand for the processed dairy product[9] and b is the demand slope. Practically all dairy products are highly commercialized and have a high potential demand, while the demand slope tends not to be high,[10] therefore we presume this condition is easily satisfied. For example, if $b = 1$, $n = 2$ and $F = 10$ it is enough that $a > 27$, approximately, meaning that 27 units of potential demand are enough to compensate the milk testing cost. This is a derisive quantity in terms of dairy products commercialization.

Now assume the milk quality test is borne by the farmer. The new farm's profit is the following:

$$\pi_i = x_i I(1 + \beta\theta) - \left(\frac{\theta x_i^2}{2}\right) - F, \ i = 1, \ \ldots, \ n \tag{15}$$

The framework is similar. The parameters and variables are obtained from the first derivative in relation to θ, thus the fixed cost does not change any value in Table 1, except farms profits. Now it is $\pi_i = [a^2(2n+1)(8n^2-2n-1)/2bn^2(8n+2)^2] - F, i = 1, \ldots, n$, while in the scenario with no payment for MSC it is $a^2(2n-1)/16bn^2$. Farms are in a better situation if $[a^2(2n+1)(8n^2-2n-1)/2bn^2(8n+2)^2] - F > a^2(2n-1)/16bn^2$, or $a^2(8n^2-2n-1)/4bn^2(8n+2)^2 > F$. We also presume it is easily satisfied. For example, if $n = 2$, $b = 1$ and $F = 10$ it is enough that $a > 44$, a significantly small quantity for dairy producers.

3.3 General Conditions in a Generalized Model

The main model of the paper required a few assumptions that could limit the validity of the results. We assumed a linear demand for the processed product, a linear relation between the q and x, quadratic costs for identical farms, a

8 An online search informs that milk-testing price for a sample in United States is usually less than USD 10.00. See DHIA Laboratories (2019) and Dairy One (2019), for example.

9 The value of q when p approaches zero.

10 For example, in general Heien and Wessells (1988) and Bouamra-Mechemache et al. (2008) find inelastic demand price elasticities for dairy products.

common value of θ and a linear relation between β and θ. In this section, we extend our analysis for a generalized model that provides conditions for which our results are valid, that is, paying for MSC benefits farms, processors and consumers as long as the following conditions are observed.

First, we relax the assumption of farms homogeneity by setting θ_i, $i = 1, \ldots, n$ as functions of β, denoted by $\theta_i(\beta)$, $i = 1, \ldots, n$, while $\partial\theta_i/\partial\beta > 0, i = 1, \ldots, n$. Assume the following:

i. $x(\theta_1(\beta), \theta_2(\beta), \ldots, \theta_n(\beta))$, with $\partial x/\partial\theta_i < 0, i = 1, \ldots, n$;
ii. $I(x(\theta_1(\beta), \theta_2(\beta), \ldots, \theta_n(\beta))$, with $\partial I/\partial x > 0$;
iii. $q(\beta)$, with $\partial q/\partial\beta > 0$;[11]
iv. $p(q(\beta))$, with $\partial p/\partial q < 0$;
v. $x_i(\theta_i(\beta))$, $i = 1, \ldots, n$, with $\partial x_i/\partial\theta_i < 0, i = 1, \ldots, n$;
vi. $C_i(x_i(\theta_i(\beta)))$, $i = 1, \ldots, n$, with $\partial C_i/\partial x_i > 0, i = 1, \ldots, n$.[12]

The processor's profit is the following:

$$\Pi = p(q(\beta))q(\beta) - I(x\theta_1(\beta),\ \theta_2(\beta), \ldots, \theta_n(\beta))x(\theta_1(\beta),\ \theta_2(\beta), \ldots, \theta_n(\beta)) \quad (16)$$

While the first derivative regarding β provides the following, after some algebraic manipulation:

$$\frac{\partial\Pi}{\partial\beta} = \frac{\partial q}{\partial\beta}\left(p + q\frac{\partial p}{\partial q}\right) + \left(\sum_{i=1}^{n}\frac{\partial x}{\partial\theta_i}\right)\left(\sum_{i=1}^{n}\frac{\partial\theta_i}{\partial\beta}\right)\left(x\frac{\partial I}{\partial x} + I\right) \quad (17)$$

Remember that $x = \sum_{i=1}^{n} x_i$, and this is the reason for the sums in (17). The first term of the right-hand side above is related to the additional revenue of selling more q, while the second term represents the decrease in processor's costs for using less x. For example, suppose that an increase in β results in one additional unit of q, sold in the market for five monetary units, while the processor demands one unit less of x that would cost four monetary units. Expression (17) would result in nine monetary units of additional profit. Observe the processor will pay the highest possible value of β, as the additional profit is always positive given the general model assumptions.[13]

For farms, each one's profit can be expressed as the following:

$$\pi_i = I(x(\theta_1(\beta), \theta_2(\beta), \ldots, \theta_n(\beta))x_i(\theta_i(\beta)) - C_i(x_i(\theta_i(\beta))), i = 1, \ldots, n \quad (18)$$

The first derivative regarding θ_i can be expressed as follows:

$$\frac{\partial\pi_i}{\partial\theta_i} = -\left(\frac{\partial I}{\partial x}\left|\frac{\partial x}{\partial\theta_i}\right|x_i + I\left|\frac{\partial x_i}{\partial\theta_i}\right|\right) + \frac{\partial C_i}{\partial x_i}\left|\frac{\partial x_i}{\partial\theta_i}\right|,\ i = 1, \ldots, n \quad (19)$$

The term inside parentheses of the right-hand side above is related to the decrease in farm's revenue from increasing θ_i, $i = 1, \ldots, n$ and consequently decreasing x_i, $i = 1, \ldots, n$ and I. The second term is the change in total costs from increasing θ_i, $i = 1, \ldots, n$ and reducing x_i, $i = 1, \ldots, n$. We can conclude that a specific farm will invest in MSC while total costs decrease in a greater amount than the revenue.[14]

Lastly, the consumer surplus (CS) is the following:

$$CS = \frac{q(\beta)}{2}\left[p_{\max} - p(q(\beta))\right] \quad (20)$$

In which $p_{\max}$ is the price of processed product when quantity is zero. The first derivative regarding β can be written as the following:

$$\frac{\partial CS}{\partial\beta} = \frac{1}{2}\frac{\partial q}{\partial\beta}\left[p_{max} - \left(q\frac{\partial p}{\partial q} + p\right)\right] \quad (21)$$

The subtraction inside brackets represents the difference of the maximum price and the marginal revenue for the processor. Since $p_{\max}$ is the maximum, it follows that the marginal revenue is always lower, resulting in a positive

11 Rigorously speaking, q is a function of x and $(\theta_1(\beta), \theta_2(\beta), \ldots, \theta_n(\beta))$ simultaneously. In the main model, we assumed a linear relation on the form $q = \theta x$. Here, to avoid extra assumptions, it is enough to assume $q(\beta)$ and $\partial q/\partial\beta > 0$.

12 Rigorously speaking, C_i, $i = 1, \ldots, n$ is a decreasing function of x_i, $i = 1, \ldots, n$, but increases in relation to θ_i, $i = 1, \ldots, n$, as in the main model. The point is that $\partial x_i/\partial\theta_i < 0, i = 1, \ldots, n$, thus increasing θ_i, $i = 1, \ldots, n$ enhances total costs by one side and decreases by the other. The model requires a negative net effect because the impact of increasing the MSC on farm's revenue is negative (as shown later in Eq. (19)), thus for a positive value of θ_i, $i = 1, \ldots, n$ we need initially that total costs decrease in a greater amount than the revenue, otherwise investing in MSC would never be profitable and $\theta_i = 0, i = 1, \ldots, n$. For this purpose, it is enough to assume $\partial C_i/\partial x_i > 0, i = 1, \ldots, n$.

13 The unique possibility of a negative $\partial\Pi/\partial\beta$ is a negative additional revenue from selling more q greater than the costs decrease from using less x. As price is a non-negative variable, a negative additional revenue is illogical.

14 It is possible for the farm to change the cost function structure itself, and not only according to θ_i, $i = 1, \ldots, n$ and x_i, $i = 1, \ldots, n$. For example, increasing the bonus for MSC may incentive farms to change the animal breed, cattle confinement, types of pasture, number of cows (with economy of scale) or any other factor that tends to modify the cost structure besides the milk production and MSC. Assuming that the farm will change the cost structure only if it increases profits, there will be a threshold between an old and a new cost structure for some $\theta_i(\beta)$, $i = 1, \ldots, n$. In any case, it does not modify the findings in this section: while total costs decrease in a greater magnitude than the revenue, farms will enhance $\theta_i(\beta)$, $i = 1, \ldots, n$.

derivative. Based on the general model assumptions, it is possible to conclude that increasing the payment for MSC is always beneficial to consumers.

4 Conclusions

Several dairy products use milk components as main inputs. The amount of milk solids determines the yield of milk, while dairy companies try to incentive milk producers to invest in MSC by paying a bonus. However, it is not clear how this mechanism affects the behavior of dairy supply chain and consumers. We developed a model to analyze how paying for MSC impacts the performance of dairy supply chain actors and the consumer welfare.

We found that paying for MSC increases the yield of milk, increases farms and dairy processor profits and enhances consumer welfare. In other words, everyone is benefitted, and even considering the milk testing cost there is no reason to suppose a different result. We also extended our analysis to a generalized model that finds conditions for which our results are valid.

The main suggestion of this paper is that companies that produce dairy products from milk components should use this type of payment instead of paying for milk indistinctly. Some topics for future studies include analyzing other aspects of the milk quality payment, as bacteria count. It is also possible to study a milk quality payment system that covers both milk solids and anti-hygienic components. Empirical papers are also important, mainly to check if these mechanisms are in fact beneficial for dairy agents and consumers.

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