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Allocation of Adaptation Aid: A Normative Theory

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Abstract: The paper derives an efficient allocation rule of adaptation aid from a donor to the recipient nations, which is different from the traditional development aid literature. The design of the allocation rule takes care of the optimum reaction of the nations on receipt of the aid. It suggests that the allocation rule must take care of (i) distributional weight of the recipients on the donor's welfare, (ii) their vulnerability to climate change, (iii) their efficiency in using the aid and (iv) their mitigation response to the aid. It discusses the way these factors can be incorporated in an implementable rule by use of the available data. The score of a nation derived from the first three factors requires an upward/downward adjustment if the adaptation aid leads to an increase/decrease of the mitigation effort of the country. A nation receiving a higher score is rewarded with higher allocation of aid. The paper suggests as a rule of thumb that the aid in the form of technical assistance requires a downward adjustment and the aid in monetary form requires an upward adjustment in the score.

Keywords: adaptation, mitigation, international climate finance, climate change

JEL Classification: C79, F35, H87, Q54, Q58

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1 Introduction

The research about the allocation of adaptation aid to "vulnerable" developing countries is quite new in the literature as the commitment to such an aid from the developed countries came as late as Copenhagen Summit of the United Nations Framework Convention on Climate Change (UNFCCC) in 2009.[1] In the Summit, beyond USD 30 billion as fast-start finance for 2010–2012, the developed countries pledged to mobilize USD 100 billion every year both for mitigation of climate change and adaptation to climate change in the Global South till 2020 (UNFCCC 2009). The Paris Agreement in 2015 repeated the pledge and called on the developed countries to significantly increase their adaptation finance keeping in mind the *needs and priorities* of developing countries (UNFCCC 2015).[2] According to the data provided by OECD (2021), the adaptation aid flow to the developing countries has almost quadrupled from under USD 5 billion in 2010 to over USD 20 billion in 2019. However, the allocation of adaptation aid had been uneven and seemed to have only partial connection with vulnerability to climate change (Weiler et al. 2018). While some small island countries received disproportionately high per capita transfers, some of the populous countries received the bulk of the adaptation aid. In both these cases we are not sure whether the amount of aid transfer is commensurate to their vulnerability as defined by IPCC (2013), i.e. either to their physical predisposition to be affected or to their lack of adaptive and coping capacities. There is an apprehension that the considerations about the economic need of the recipient and foreign-policy interests of the donor prevail over the recipient's vulnerability to climate change in determination of adaptation aid flows. Here, we identify two types of research questions. One is positive: what are the major determinants of the observed allocation of adaptation aid flows? The other is normative: what should be criteria for allocation of aid? While the positive question has recently been dealt with carefully by Weiler et al. (2018), the normative question needs further research. The present paper deals with the normative question and finds the way the aid should be efficiently and equitably distributed from a donor's perspective.

1 Historically developed countries having a higher share of per capita emissions are responsible for the destruction of global climate, to which the developing countries need to adjust to maintain their livelihood. The aid is thought to be justified as the developing countries are adversely affected both by deteriorated global climate and pressure of adaptation.

2 Article 9 of the Paris Agreement states that the provision of scaled-up financial resources should aim to achieve a balance between adaptation and mitigation. At the Paris COP it was also decided that prior to 2025 a new collective quantified goal from a floor of USD 100 billion per year shall be set.

The plan of the paper is as follows. Section 2 surveys the relevant literature. Section 3 introduces the basic model without including the choice of mitigation effort by the recipient nations and develops the basic criteria (the score) for allocation of adaptation aid. Section 4 extends the model to include the mitigation effort by the recipient nations, the aggregate mitigation game they play with the other nations and shows how it introduces an adjustment in the allocation score derived in Section 3. Section 5 discusses the policy implications by illustrating development of the index with the available data. The final section concludes.

2 The Literature

The nascent literature on allocation of adaptation aid closely follows the already grown literature on allocation of development aid. The normative issues in the development aid literature are led by the theory proposed by papers like Dudley and Montmarquette (1976). The main drawback of this theory is that it takes the recipients' behavior as passive. The literature developed afterwards rightly points out that the flow of foreign aid can change the behavior of the recipient countries which is adjusted to maximize their own objective which may be different from the donor's.[3] Few papers like Lahiri and Raimondos-Moller (2004) also consider the optimization of the aid decision of the donor in anticipation of the behavioral change of the recipient. The present paper belongs to the tradition of Lahiri and Raimondos-Moller (2004). However, unlike them, who work on allocation of development aid, the concern of the present paper is allocation of adaptation aid by a donor. The recipient nation, in our theoretical model, adjusts its choice of adaptation effort and mitigation effort to maximize its own welfare in response to the adaptation aid. The donor anticipates the reaction of the recipients and adjusts its allocation accordingly to maximize its own welfare that depends both on the welfare of the recipient nations and the state of global climate.[4]

3 See papers like Boone (1995), Lahiri and Raimondos-Moller (2004), and Marjit and Mukherjee (2006). The possibility of adjustment lies at the heart of the empirical literature that claims foreign aid fails its purpose. See Mosley et al. (1987), Pack and Pack (1993), Burnside and Dollar (2000), Svensson (2000), and the following papers.

4 In this paper our focus is on the adaptation-aid allocation problem of the donor. The donor has been imagined as an international financial institution funded by a set of countries. The donor is committed to adaptation aid as the OECD countries in UNFCCC. However, it is perfectly possible that the donor is an individual country disbursing aid to more than one recipient. In the case of an individual donor, the country must also decide about the allocation of its budget among its own adaptation and mitigation expenditure and the amount to be given in overseas adaptation assistance. This paper abstracts away from the latter problem in the sense that it assumes that the

The highlight of the theoretical model presented in this paper is that it takes into account the spillover effect of adaptation aid on the mitigation decision of the recipient. While the benefit of the adaptation aid is private to the recipient nation,[5] the benefit of its mitigation effort contributes to improvement of global climate, a global public good that benefits all the nations, both the recipients and the donor of the aid. The mitigation effort, being a case of concentrated cost and distributed benefit, is a subject of free-riding and strategic interaction between the players in the global mitigation game. The model applies the aggregative games approach developed by Cornes and Hartley (2007) for more than two heterogeneous players[6] to solve the problem of the donor. While the simultaneous treatment of mitigation and adaptation decisions in a

domestic allocation problem of the donor country has already been solved and now its sole concern is allocation of its overseas adaptation assistance among recipient nations. While solving the allocation problem, the donor country abstracts away from its own adaptation and mitigation behavior. Buob and Stephan (2013), in a North-South framework, have studied how the commitment of the North in terms of mitigation affects the adaptation aid flow to the South. However, unlike our paper, it abstracts away from the aid-allocation problem arising out of the heterogeneity of the receiving nations. Similar to Buob and Stephan (2013) we also assume the adaptation aid is the only connection between the donor and the recipients, which allows us to abstract away from the terms of trade effect that may arise through transfer, discussed in detail by Stephan and Schenker (2017). Yet, even if the terms of trade effect is taken into account, Stephan and Schenker (2017) highlight the need for a committed adaptation aid flow from the North to the South. The current paper develops a normative theory of such an aid flow.

5 Due to the private good nature of adaptation one may ask for reasons for providing international adaptation aid at all. Why should a donor provide such aid when it does not enjoy beneficial externalities from doing so? As Tol (2005) points out, "international organisation ha[s] little to do with adaptation, and should not try". Pittel and Rübbelke (2013) investigate whether fairness arguments could possibly explain the payment of international adaptation transfers. Eyckmans et al. (2016) analyze the ethical motivation of donors in providing development aid, mitigation finance and adaptation finance. Civelli et al. (2016) stress that altruistic motives of the donor play an important role in a wide range of voluntary transfers.

6 The origins of the aggregative game approach (AGA) date back to the 1960s and 1970s when McManus (1962) and Selten (1970) independently point out the aggregative nature of Cournot oligopoly. They reason that an oligopolist may decide on his strategy by considering the overall amount of the good being offered at the market, rather than every single oligopolist's supply. The number of independent variables can therefore be reduced to one: aggregate supply. McGuire (1974) puts forward a similar argument in his analysis of pure public good provision by an arbitrary number of heterogeneous agents. More recent applications also include contests with many players (Cornes and Hartley 2003, 2005), open access resources and impure public good provision (Cornes 2016; Vicary 2009, 2011), matching schemes for public good provision (Buchholz et al. 2011, 2012) as well as club models (Fraser 2012). For related approaches also see Buchholz and Rübbelke (2017) and Bayramoglu et al. (2018).

theoretical model is not new,[7] most of them assume symmetry except Bayramoglu et al. (2018). The application of the aggregative games approach in the normative theory of aid allocation, as it has been done in this paper, is unique in the literature.

The paper finds that the allocation decision of the donor must be based on the scores obtained by the recipient nations in an index constructed by incorporation of three factors: (i) the weight the welfare of the recipient nation enjoys in the welfare function of the donor; (ii) the vulnerability of the recipient nation to climate change; (iii) the ability of the recipient nation to convert a given amount of adaptation aid into adaptation output; adjusted for the impact of the adaptation aid on the quality of global climate which works through a change in mitigation behavior of the nation. If the 'global climate change effect' is negative, the adaptation aid hurts the global climate and the score is adjusted in the downward direction. The opposite happens if the effect is positive. A nation with higher score receives higher allocation of aid.

The index suggested by the paper can be constructed with available data. The welfare weight can be calculated by using the World poverty index of the nations adjusted by historical proximity/trade relations. A nation with more physical vulnerability to climate change and higher population share of the world can be defined as a more vulnerable nation. Such a nation is expected to gain more in terms of welfare from a rise in its adaptation output. A well governed nation with higher technical ability/education can produce higher adaptation output from a given amount of adaptation aid. The empirical papers in this area like Weiler et al. (2018) have already used the available indices to show that the poorer countries, the countries with greater physical vulnerability and the countries with good governance receive higher allocation of adaptation aid. They also show that the volume of bilateral trade with the donor country matters in receipt of higher allocation of aid. Particularly, Weiler et al. (2018) use indices like Notre Dame Global Adaptation Index (ND-GAIN) and Index of Structural Vulnerability to Climate Change (SVCCI) to capture physical vulnerability of the nations to climate change; World Bank's Worldwide Governance Indicators (WGIs) to capture good governance of the recipient nations; and the UN Comtrade database to capture the volume of the bilateral trade flow between the donor and the recipient nations. Since these papers follow the old development aid literature, our paper suggests some important modifications of their approach. As Weiler et al. (2018) and other similar papers were using the old literature, the 'global climate change effect' as mentioned above, not surprisingly, is completely absent in their approach as a factor affecting adaptation aid. However, dealing with the issue of climate change, it should have been there. Our framework provides a

7 See Kane and Shogren (2000), Lazkano et al. (2016), Pittel and Rübbelke (2015, 2017), Tol (2005), Ingham et al. (2006, 2007, 2013), Zehaie (2009), Buob and Stephan (2011), Heuson et al. (2015), Auerswald et al. (2017), Parry et al. (2001), and Bayramoglu et al. (2018).

correction for this omission and emphasizes the importance of the 'global climate change effect' in determination of the adaptation aid flow. However, one must admit that it is difficult to quantify the 'global climate change effect'. This paper solves the problem by designing a rule of thumb accounting for this effect. If it is the case that the adaptation aid is strongly complementary with the adaptation effort of the recipient nation, its mitigation effort falls that leads to deterioration of global climate. Then, there is a case for downward adjustment of the score. The opposite happens if adaptation aid is a weak complement or substitute of domestic adaptation effort. While the former is the more likely to be the case if the aid takes the form of technical assistance, the latter is more likely to be the case for monetary transfers.

Let us summarize the contribution of the paper to the literature on adaptation aid allocation. First, it attempts to replace the traditional development aid theory which is being used in this field with a normative theory of its own, by introducing three factors: (i) the optimum reaction of the recipient nations in terms of their choice of adaptation effort and mitigation effort in response to adaptation aid; (ii) an aggregative game to determine Nash equilibrium behavior of the nations in the mitigation game in response to adaptation aid; (iii) vulnerability of nations and the 'global climate change effect' as criteria for allocation of adaptation aid. Second, it defines a rule of thumb for allocation of adaptation aid that can be easily implemented by using available data. Third, it suggests some modifications in the empirical approach used in this field which follows the traditional development aid literature. Specifically, it argues for modification of indices used for such studies and introduction of the 'global climate change effect' as a factor for studying the flow of adaptation funding from the North to the Global South.

3 The Basic Model

Consider an allocation of adaptation aid A among a number n of recipient nations around the world such that the ith nation receives an allocation of $A_i \geq 0$. The donor decides to allocate A in the most efficient and equitable way. Therefore, while deciding about the allocation, it takes into account the way the recipient nation i ($\nabla i = 1, 2, \ldots\ldots, n$) uses the aid and derives welfare from it.

The adaptation output x_i in nation i positively depends both on A_i, i.e. the aid received from the donor, and a_i, i.e. the amount of domestic input spent on adaptation. While each input's marginal contribution to adaptation output is positive, it shows diminishing returns. Formally, the adaptation output production function is written as:

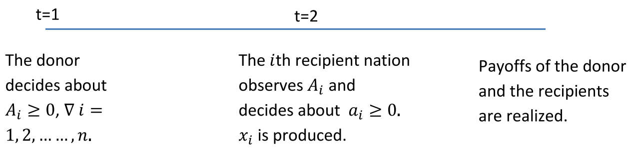

Figure 1: The sequence of decisions in the basic model.

$$x_i = x_i\,(a_i, A_i) \quad \text{with } \frac{\partial x_i}{\partial a_i} > 0, \frac{\partial x_i}{\partial A_i} > 0, \frac{\partial^2 x_i}{\partial a_i^2} < 0 \text{ and } \frac{\partial^2 x_i}{\partial A_i^2} < 0. \tag{1}$$

The sequence of decisions in the basic model is represented in Figure 1 below.

In production of adaptation output the domestic input and aid may either be substitutes $\left(\frac{\partial^2 x_i}{\partial A_i \partial a_i} < 0\right)$ or complements $\left(\frac{\partial^2 x_i}{\partial A_i \partial a_i} > 0\right)$ of each other. However, in the basic model, we show later that the allocation decision of the donor does not depend on the substitutability or complementarity of the domestic input and the aid.

The welfare of the ith recipient nation depends not only on the adaptation output, it also depends on its consumption of a private commodity, the amount of which is represented as z_i, and the state of climate G that depends on the global mitigation effort Q. A higher value of Q translates into an improved climatic condition with diminishing returns. We write it as $\frac{\partial G}{\partial Q} > 0, \frac{\partial^2 G}{\partial Q^2} < 0$. In the basic model we assume nation i does not choose its mitigation effort. We relax this assumption in the extension of the basic model.

Formally, the welfare of recipient nation i is written as $u_i\,(z_i, x_i\,(a_i, A_i), G\,(Q))$ with $\frac{\partial u_i}{\partial z_i}, \frac{\partial u_i}{\partial x_i}, \frac{\partial u_i}{\partial G} > 0$; $\frac{\partial^2 u_i}{\partial z_i^2}, \frac{\partial^2 u_i}{\partial x_i^2}, \frac{\partial^2 u_i}{\partial G^2} < 0$. We also assume $\frac{\partial^2 u_i}{\partial x_i \partial G} = \frac{\partial^2 u_i}{\partial G \partial x_i} < 0$, $\frac{\partial^2 u_i}{\partial z_i \partial G} = \frac{\partial^2 u_i}{\partial G \partial z_i} > 0$ and $\frac{\partial^2 u_i}{\partial x_i \partial z_i} = \frac{\partial^2 u_i}{\partial z_i \partial x_i} > 0$. While the marginal effects of each of the consumption input, adaptation input and climate input are positive, they show diminishing returns. As the climate improves, on the one hand, the marginal welfare gain from adaptation output falls and *vice versa*; on the other hand, the marginal welfare gain from consumption input rises and *vice versa*. In a more adapted nation, the marginal welfare gain from the consumption input rises and *vice versa*.

Nation i maximizes $u_i\,(z_i, x_i\,(a_i, A_i), G\,(Q))$ by choosing $\{z_i, a_i\}$ subject to its budget constraint:

$$z_i + ca_i = I_i \tag{2}$$

where I_i represents the endowment of the commodity of private consumption to nation i and c represents the cost per unit incurred by the nation in converting its consumption input into the adaptation input.

We assume that an interior solution to the nation's welfare maximization problem exists. The solution $\{z_i(c, A_i, I_i, G(Q)) > 0, a_i(c, A_i, I_i, G(Q)) > 0\}$ satisfies:

$$\frac{\frac{\partial u_i}{\partial x_i}\frac{\partial x_i}{\partial a_i}}{\frac{\partial u_i}{\partial z_i}} = c \tag{3}$$

along with the equation of the budget constraint given in (2) above.

Substituting $\{z_i(c, A_i, I_i, G(Q)) > 0, a_i(c, A_i, I_i, G(Q)) > 0\}$ in $u_i(z_i, x_i(a_i, A_i), G(Q))$ the indirect welfare function of nation i is derived as:

$$v_i(c, A_i, I_i, G(Q)) = u_i(z_i(c, A_i, I_i, G(Q)), x_i(a_i(c, A_i, I_i, G(Q)), A_i, G(Q)). \tag{4}$$

Lemma 1: $\frac{\partial v_i}{\partial A_i} = \frac{\partial u_i}{\partial x_i}\frac{\partial x_i}{\partial A_i} > 0.$

Proof: See the appendix. □

The flow of adaptation aid from the donor to the recipient nation i affects the welfare of the recipient nation in two alternative ways. The first one is the direct effect which works through a rise in adaptation output x_i as described in equation (1); the second one is the indirect effect which works through a change in the equilibrium behavior of the nation as its choices of z_i and a_i change. Because of the budget constraint (2), the two indirect effects work in opposite directions and cancel each other so that the direct effect prevails.

We assume that the donor holds a utility function given as[8]

$$W = W(v_1(A_1, G(Q)), \ldots v_i(A_i, G(Q)), \ldots, v_n(A_n, G(Q)), G(Q))$$

where $\frac{\partial W}{\partial v_i} > 0$ for all $i = 1, 2, \ldots\ldots, n$. It maximizes W by choosing an aid allocation $\{A_1, \ldots\ldots\ldots, A_i, \ldots\ldots\ldots, A_n\}$ among the nations such that its budget constraint

$$A_1 + \ldots \ldots + A_i + \ldots \ldots + A_n = A. \tag{5}$$

is satisfied.

8 See footnote 4 above for a discussion of the objective function of the donor. We assume that the adaptation aid is the only connection between the set of recipients and the donor. The donor's sole concern is the allocation of a given amount of adaptation aid among the recipients. We abstract away from the discussion of mitigation effort by the donor. In particular, we assume that while allocating the adaptation aid, the donor is concerned about the aggregate mitigation impact of the recipient nations. But the donor itself does not mitigate.

We assume that an interior solution to the donor's problem exists. The solution $\{A_1 > 0, \ldots\ldots\ldots, A_i > 0, \ldots\ldots\ldots, A_n > 0\}$ satisfies the following set of n equations given by

$$\frac{\partial W}{\partial v_i}\frac{\partial v_i}{\partial A_i} = \lambda \qquad \forall i = 1, 2, \ldots\ \ldots\ \ldots., n; \tag{6}$$

and the equation of the budget constraint (5). In equation (6) on the RHS, λ is the value of Lagrange multiplier used in the process of constrained optimization. At the equilibrium, the value of λ can be shown to be equal to $\frac{\partial W}{\partial A} > 0$, i.e. the benefit the donor derives from relaxation of its budget constraint. The second order condition for maximization is also assumed to be satisfied with $\frac{\partial^2 W}{\partial A_i^2} < 0$ for all $i = 1, 2, \ldots\ \ldots., n$.

Now, using Lemma 1, equation (6) can be rewritten as:

$$\frac{\partial W}{\partial v_i}\left(\frac{\partial u_i}{\partial x_i}\frac{\partial x_i}{\partial A_i}\right) = \lambda\ \forall i = 1, 2, \ldots\ \ldots\ \ldots., n. \tag{7}$$

We are going to argue that equation (7) may be used as a guiding principle by the donor for allocation of adaptation aid among the recipient nations.

Equation (7) balances the marginal utility of the donor from giving 1 extra $ of aid to nation i to the marginal benefit from keeping the extra $ with itself.

Notice that in equation (7) on the LHS $\frac{\partial W}{\partial v_i} > 0$ stands for the weight the welfare of nation i enjoys in the donor's objective function. Following the welfare economics literature, one can imagine that a nation which is poor with a lower rank in the Human Development Index (HDI) would enjoy a higher[9] $\frac{\partial W}{\partial v_i}$. But it is also possible that $\frac{\partial W}{\partial v_i}$ gets influenced by the political/trade relation of the ith nation with the donor. The second term on the LHS of (7) $\frac{\partial u_i}{\partial x_i} > 0$ stands for the donor's evaluation of the importance/value of the adaptation aid to the country i. The higher is the vulnerability of the nation to climate change, the higher is the value of $\frac{\partial u_i}{\partial x_i}$. The third term on the LHS of (7) $\frac{\partial x_i}{\partial A_i}$ stands for the donor's assessment of the physical productivity of the adaptation aid in country i. A low rank of the nation in the governance index or a high rank of the nation in the corruption index would imply a low value of $\frac{\partial x_i}{\partial A_i}$.

Proposition 1: *A nation either with higher weight in the donor's utility function or with higher vulnerability to climate change or with higher physical productivity of adaptation aid or with all the three characteristics together receives higher adaptation aid from the donor.*

9 Like a Bernoulli-Nash type of welfare function (see Hillman 2019).

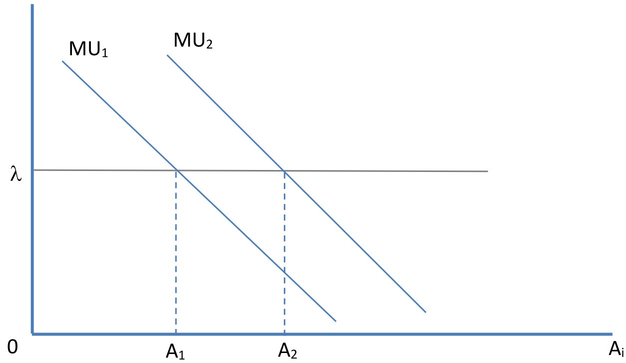

Figure 2: The donor's optimum choice of aid for countries 1 and 2.

Proof: It follows from equation (7) that higher values of $\frac{\partial W}{\partial v_i}$ or $\frac{\partial u_i}{\partial x_i}$ or $\frac{\partial x_i}{\partial A_i}$ imply that at the initial equilibrium $\frac{\partial W}{\partial A_i} > 0$. Since $\frac{\partial^2 W}{\partial A_i^2} < 0$, the statement of the proposition follows. □

We explain the intuition behind the result and its proof with the help of Figure 2.

Figure 2 represents the solution to the allocation problem of the donor when $n = 2$. The marginal utility of the donor from transferring adaptation aid to country $i = 1, 2$ is given by $\mathrm{MU}_1 = \frac{\partial W}{\partial A_1} > 0$ and $\mathrm{MU}_2 = \frac{\partial W}{\partial A_2} > 0$, respectively. The marginal utility curves slope downward owing to concavity of the donor's utility function since it implies $\frac{\partial^2 W}{\partial A_i^2} < 0$. Without loss of generality, the figure assumes $\mathrm{MU}_2 > \mathrm{MU}_1$, which is a result of a higher value of either $\frac{\partial W}{\partial v_2}$ or $\frac{\partial u_2}{\partial x_2}$ or $\frac{\partial x_2}{\partial A_2}$. Therefore, the satisfaction of the first order condition of the donor's optimization problem given in equation (7) implies $A_2 > A_1$.

4 Extension: Mitigation Effort of the Recipient Nations

The basic model is extended in this section by allowing nation i to choose its mitigation effort q_i along with its adaptation effort a_i as before. The chosen amount of q_i adds to the global mitigation effort Q such that if Q_{-i} represents the total mitigation effort put by all nations other than the ith nation, the following equation holds:

$$Q = q_i + Q_{-i}. \tag{8}$$

The nation's welfare maximization problem is now written as:

Maximization of $u_i(z_i, x_i(a_i, A_i), G(Q))$ by choice of $\{z_i, a_i, q_i\}$ subject to its budget constraint

$$z_i + ca_i + dq_i = I_i \tag{9}$$

where I_i represents the endowment of the commodity of private consumption to nation i, c represents the cost per unit of adaptation effort and d represents the cost per unit of the mitigation effort. The problem is solved by first substituting z_i from equation (9) in the objective function of nation i and then maximizing it through the choice of $\{a_i, q_i\}$. We assume an interior solution to the nation's maximization problem exists such that $\{a_i > 0, q_i > 0\}$ satisfies:

$$[a_i] \quad \frac{\frac{\partial u_i}{\partial x_i}\frac{\partial x_i}{\partial a_i}}{\frac{\partial u_i}{\partial z_i}} = c, \tag{10}$$

$$[q_i] \quad \frac{\frac{\partial u_i}{\partial G}\frac{\partial G}{\partial Q}}{\frac{\partial u_i}{\partial z_i}} = d. \tag{11}$$

Equation (10) solves for $a_i(A_i, Q)$. After substituting $a_i(A_i, Q)$ in (11), equation (11) solves for $q_i = R_i(A_i, Q)$ where R_i is defined as the individual replacement function of nation i as in Cornes and Hartley (2007).[10] Since $Q = q_i + Q_{-i}$, notice that the choice of q_i by nation i also depends on Q_{-i}. The ith nation can possibly free-ride on the aggregate mitigation effort put by the other nations and every other nation also possesses a similar incentive. One can imagine that the mitigation effort by each of the nations is decided through a simultaneous move mitigation game among the

10 The replacement function can be thought of as indicating the best response of player i if a certain amount is removed from the aggregate Q. If player i reacts by replacing that precise amount, thereby restoring the original equilibrium, then that best response equals his replacement value. In contrast to the best response function that depends on the *other* players' strategy choices, the replacement function has the aggregate of *all* players' choices, including player i's own, as the independent variable. Therefore, in contrast to the best response function approach, the aggregative game approach allows for the analysis of Nash equilibria in games with more than two heterogeneous agents (Cornes and Hartley 2007). As the individual replacement functions all depend on the same variable, they can easily be added up to an economy's aggregate replacement function. In other words, in a setting with n different agents, there are n different best response functions with n variables, but there is only one aggregate replacement function with the aggregate as the single variable. The Nash equilibrium level of the aggregate can immediately be derived by solving a single equation which requires the value of the aggregate replacement function to equal the amount of the aggregate; this is equivalent to the requirement that the sum of individual best responses has to equal total provision.

nations.[11] The marginal utility from choice of q_i for the ith nation $(\forall i = 1, 2 \ldots\ldots, n)$ depends directly on the adaptation aid A_i it receives, and indirectly on $q_j\ \forall j = 1, 2 \ldots\ldots, n, i \neq j$ and therefore on $(A_1, \ldots\ldots\ldots, A_{i-1}, A_{i+1} \ldots\ldots\ldots, A_n)$. At the Nash equilibrium, the value of Q, therefore, is a function of the vector of adaptation aid $(A_1, \ldots\ldots\ldots, A_i, \ldots\ldots\ldots, A_n)$ defined as A_0. Consequently, the solution to equations (10) and (11) can be written as $\{a_i(A_0), q_i(A_0)\}$. Then, using equation (9), the equilibrium value of z_i can be solved as:

$$z_i(A_0) = I_i - ca_i(A_0) - dq_i(A_0). \tag{12}$$

However, the solution described above crucially depends on the existence of a unique Nash equilibrium in the mitigation game. To check the existence of the unique Nash equilibrium we adopt the method followed by Cornes and Hartley (2007)[12] who use the idea of aggregative games for this purpose. The method has been outlined in footnote 10 above. First, we use the individual replacement functions $q_i = R_i(A_i, Q)\ \forall i = 1, \ldots\ldots\ldots, n$ in equation (8) to define an aggregate replacement function $R(Q, A_0)$ as:

$$Q = \sum_{i=1}^{n} R_i(A_i, Q) = R(Q, A_0). \tag{13}$$

The Nash equilibrium of the mitigation game exists if a solution to equation (13) in terms of Q exists.

Lemma 2: The Nash equilibrium of the mitigation game among the nations exists if the following conditions hold:

$$\frac{\partial u_i}{\partial G}\frac{\partial^2 G}{\partial Q^2} + \left(\frac{\partial G}{\partial Q}\right)^2 \frac{\partial^2 u_i}{\partial G^2} + \frac{\partial x_i}{\partial Q}\left\{\frac{\partial G}{\partial Q}\frac{\partial^2 u_i}{\partial x_i \partial G} - d\frac{\partial^2 u_i}{\partial x_i \partial z_i}\right\} < 0$$

and as $Q \to 0, R_i(Q) \to ve$.

Proof: See the appendix. □

The conditions stated in lemma 2 demand two things: First, as the global mitigation effort (and therefore the global climate) improves, the mitigation

11 Given the assumptions of the model, by application of equations (10) and (11), one can derive the slope of the reaction function of nation i $(\forall i = 1, \ldots\ldots, n)$ from equation (11) as $\frac{\partial q_i}{\partial Q_{-i}} = -\frac{\frac{\partial u_i}{\partial G}\frac{\partial^2 G}{\partial Q^2}}{D} < 0$ where $D = \left[\frac{\partial u_i}{\partial G}\frac{\partial^2 G}{\partial Q^2} - 2d\frac{\partial G}{\partial Q}\frac{\partial^2 u_i}{\partial z_i \partial G} + \left(\frac{\partial G}{\partial Q}\right)^2 \frac{\partial^2 u_i}{\partial G^2} + d^2\frac{\partial^2 u_i}{\partial {z_i}^2}\right] < 0$. Therefore, in the simultaneous move mitigation game the nations' mitigation strategies are strategic substitutes of each other.

12 See Bayramoglu et al. (2018) for another application of this method.

effort of a recipient nation must fall; and second, even if the global mitigation effort falls, a recipient nation does not reduce its mitigation effort to zero. The inequality that appears in lemma 2 is derived from equation (11) where a nation chooses its utility-maximizing mitigation response by balancing its marginal benefit and its marginal cost. As the global mitigation effort rises and the global climate improves, clearly the choice of a higher mitigation effort by the nation lowers its marginal benefit as there is diminishing marginal return from further improvement of global climate and raises its marginal cost as (with the resource constraint) consumption is sacrificed. Although the adaptation output is likely to fall with improved global climate, it is expected to have an insignificant impact on the nation's welfare since adaptation is not important in an improved climatic condition. Therefore, the conditions stated in lemma 2 are likely to hold, ensuring existence of the Nash equilibrium in the mitigation game.

Given the existence of a Nash equilibrium of the mitigation game, how does nation i respond to adaptation aid A_i received from the donor? The answer to this question is found by use of equations (10) and (11).

Lemma 3: (i) $\frac{\partial a_i}{\partial A_i} = \frac{-N\pi + L\theta}{\Delta}$;
(ii) $\frac{\partial q_i}{\partial A_i} = \frac{-K\theta + M\pi}{\Delta}$,
where $K = \frac{\partial x_i}{\partial a_i}\left\{\frac{\partial x_i}{\partial a_i}\frac{\partial^2 u_i}{\partial x_i^2} - 2c\frac{\partial^2 u_i}{\partial z_i \partial x_i}\right\} + \frac{\partial u_i}{\partial x_i}\frac{\partial^2 x_i}{\partial a_i^2} + c^2\frac{\partial^2 u_i}{\partial z_i^2}$, $L = M = \frac{\partial x_i}{\partial a_i}\left\{\frac{\partial G}{\partial Q}\frac{\partial^2 u_i}{\partial x_i \partial G} - d\frac{\partial^2 u_i}{\partial x_i \partial z_i}\right\} + c\left\{d\frac{\partial^2 u_i}{\partial z_i^2} - \frac{\partial G}{\partial Q}\frac{\partial^2 u_i}{\partial G \partial z_i}\right\}$, $N = d\left\{d\frac{\partial^2 u_i}{\partial z_i^2} - 2\frac{\partial G}{\partial Q}\frac{\partial^2 u_i}{\partial G \partial z_i}\right\} + \left(\frac{\partial G}{\partial Q}\right)^2\frac{\partial^2 u_i}{\partial G^2} + \frac{\partial u_i}{\partial G}\frac{\partial^2 G}{\partial Q^2}$, $\Delta = KN - LM$,
$\pi = \frac{\partial x_i}{\partial A_i}\left\{\frac{\partial x_i}{\partial a_i}\frac{\partial^2 u_i}{\partial x_i^2} - c\frac{\partial^2 u_i}{\partial x_i \partial z_i}\right\} + \frac{\partial u_i}{\partial x_i}\frac{\partial^2 x_i}{\partial A_i \partial a_i} - \rho\frac{\partial Q_{-i}}{\partial A_i}$, $\theta = \frac{\partial x_i}{\partial A_i}\left\{\frac{\partial G}{\partial Q}\frac{\partial^2 u_i}{\partial x_i \partial G} - d\frac{\partial^2 u_i}{\partial x_i \partial z_i}\right\} - \varphi\frac{\partial Q_{-i}}{\partial A_i}$, $\rho = \frac{\partial x_i}{\partial a_i}\left\{d\frac{\partial^2 u_i}{\partial z_i \partial x_i} - \frac{\partial G}{\partial Q}\frac{\partial^2 u_i}{\partial x_i \partial G}\right\} - c\left\{d\frac{\partial^2 u_i}{\partial z_i^2} - \frac{\partial G}{\partial Q}\frac{\partial^2 u_i}{\partial G \partial z_i}\right\}$ and $\varphi = \frac{\partial G}{\partial Q}\left\{d\frac{\partial^2 u_i}{\partial z_i \partial x_i} - \left(\frac{\partial G}{\partial Q}\right)^2\frac{\partial^2 u_i}{\partial G^2} + d\frac{\partial^2 u_i}{\partial G \partial z_i}\right\} - \frac{\partial u_i}{\partial G}\frac{\partial^2 G}{\partial Q^2} - d\frac{\partial^2 u_i}{\partial z_i^2}$.

Proof: See the appendix. □

From the assumptions of the model it follows that $K < 0$, $L = M < 0$, $N < 0$, $\rho > 0$ and $\varphi > 0$. The second order condition for the welfare maximization problem of nation n requires that $K < 0$ and $\Delta > 0$.[13] We will assume in our model $\Delta > 0$ so that the second order condition for nation i's welfare maximization problem $\forall i = 1, \ldots \ldots, n$ holds.

13 Solving equations (10) and (11), which are the first order conditions of the ith nation's welfare maximization problem, we obtain the welfare maximizing values of $\{a_i > 0, q_i > 0\}$. The second order condition of the maximization problem requires satisfaction of the following condtions:

The assumptions of the model also imply that the signs of π and θ are uncertain.

Notice from equation (10) that as A_i changes the choice of a_i is affected through two different ways. One is the direct effect, given $\frac{\partial^2 u_i}{\partial x_i^2} < 0$ and $\frac{\partial^2 u_i}{\partial x_i \partial z_i} > 0$, that works through a change in the adaptation output x_i of the nation; also, the change in A_i influences $\frac{\partial x_i}{\partial a_i}$ and Q_{-i}. The other is the indirect effect that works through a change in q_i from equation (11). In lemma 3, in the expression of $\frac{\partial a_i}{\partial A_i}$, both the direct effect, captured through $\frac{-N\pi}{\Delta}$, and the indirect effect, captured through $\frac{L\theta}{\Delta}$, have uncertain signs.

Similarly, from equation (11), as A_i changes the choice of q_i gets affected through two different ways. One is the direct effect, given $\frac{\partial^2 u_i}{\partial x_i \partial z_i} > 0$, $\frac{\partial G}{\partial Q} > 0$ and $\frac{\partial^2 u_i}{\partial x_i \partial G} < 0$, that works through a change in the adaptation output x_i of the nation; also, given $\frac{\partial G}{\partial Q} > 0$ as the change in A_i affects Q. The other is the indirect effect that works through a change in a_i from equation (10). In lemma 3, in the expression of $\frac{\partial q_i}{\partial A_i}$, both the direct effect that is captured through $\frac{-K\theta}{\Delta}$ and the indirect effect that is captured through $\frac{M\pi}{\Delta}$ have uncertain signs.

Let us call $\rho \frac{\partial Q_{-i}}{\partial A_i}$ the strategic effect of a change in A_i on the choice of a_i and $\varphi \frac{\partial Q_{-i}}{\partial A_i}$ the strategic effect of a change in A_i on the choice of q_i.

Observation 1:(i) If the strategic effect $\varphi \frac{\partial Q_{-i}}{\partial A_i}$ is negative or weakly positive, the direct effect of a change in adaptation aid on the choice of mitigation expenditure is negative, and
- a) if it dominates the indirect effect of the change, $\frac{\partial q_i}{\partial A_i} < 0$;
- b) if $\frac{\partial^2 x_i}{\partial A_i \partial a_i}$ is strongly positive so that the direct effect of a change in adaptation aid on the choice of adaptation expenditure is positive, then $\frac{\partial q_i}{\partial A_i} < 0$.

(ii) If the strategic effect $\varphi \frac{\partial Q_{-i}}{\partial A_i}$ is strongly positive so that the direct effect of a change in adaptation aid on the choice of mitigation expenditure is positive, and
- a) if it dominates the indirect effect of the change, $\frac{\partial q_i}{\partial A_i} > 0$;
- b) if $\frac{\partial^2 x_i}{\partial A_i \partial a_i}$ is negative or weakly positive, the direct effect of a change in adaptation aid on the choice of adaptation expenditure is negative and in

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$\frac{\partial^2 u_i}{\partial a_i^2} < 0, \frac{\partial^2 u_i}{\partial a_i^2} \frac{\partial^2 u_i}{\partial q_i^2} - \left(\frac{\partial^2 u_i}{\partial a_i \partial q_i}\right)^2 > 0$. Since $K < 0$, $\frac{\partial^2 u_i}{\partial a_i^2} < 0$. The assumption $\Delta > 0$ ensures that $\frac{\partial^2 u_i}{\partial a_i^2} \frac{\partial^2 u_i}{\partial q_i^2} - \left(\frac{\partial^2 u_i}{\partial a_i \partial q_i}\right)^2 > 0$.

the choice of mitigation expenditure, if the indirect effect of adaptation expenditure dominates, then $\frac{\partial q_i}{\partial A_i} > 0$.

Proof:

(i)

a) From the expression of $\frac{\partial q_i}{\partial A_i}$ in lemma 3, the statement follows since the direct effect captured through $\frac{-K\theta}{\Delta} < 0$ dominates the indirect effect captured through $\frac{M\pi}{\Delta}$ with uncertain sign.

b) If $\frac{\partial^2 x_i}{\partial A_i \partial a_i}$ is strongly positive such that $\pi > 0$, the indirect effect $\frac{M\pi}{\Delta}$ in the expression of $\frac{\partial q_i}{\partial A_i}$ becomes negative. The statement follows.

(ii)

a) From the expression of $\frac{\partial q_i}{\partial A_i}$ in lemma 3, the statement follows since the direct effect captured through $\frac{-K\theta}{\Delta} > 0$ dominates the indirect effect captured through $\frac{M\pi}{\Delta}$ with uncertain sign.

b) If $\frac{\partial^2 x_i}{\partial A_i \partial a_i}$ is negative or weakly positive, then $\pi < 0$. Therefore, in the expression of $\frac{\partial q_i}{\partial A_i}$ the indirect effect $\frac{M\pi}{\Delta} > 0$ and the statement follows. □

Observation 1 shows that the mitigation expenditure undertaken by a recipient of the adaptation aid may be positively or negatively affected by the adaptation aid flow. If the adaptation aid and the domestic adaptation input are strong complements of each other, the higher aid flow raises the domestic adaptation expenditure which forces the recipient nation to reduce its mitigation expenditure. The mitigation expenditure by the recipient nation can rise if the complementarity between the adaptation aid and domestic adaptation input is weak or they are substitutes of each other. Other than this, as the adaptation aid rises since the marginal utility the recipient nation receives from improvement in global climate falls, it tends to contribute less to mitigation. While deciding about the allocation of adaptation aid, the donor takes the mitigation reaction of the recipient nation into consideration because if it leads to deterioration of the global climate, climate change being a global public bad adversely affects the donor too.

Now, the indirect welfare function of nation i is written as:

$$v_i(A_0) = u_i(z_i(A_0), x_i(a_i(A_0), A_i), G(Q(A_0))). \tag{14}$$

While deciding about the allocation of aid, the donor would like to use equation (14) to check for all $i = 1, \dots \dots .., n$ the way recipient nation i's welfare changes on receipt of A_i.

For all $i, j = 1, \dots \dots .., n$ from equation (14) by implications of lemma 3 and equation (8), it follows:

$$\frac{\partial v_i}{\partial A_i} = \frac{\partial u_i}{\partial x_i}\frac{\partial x_i}{\partial A_i} + \frac{\partial u_i}{\partial G}\frac{\partial G}{\partial Q}\frac{\partial Q_{-i}}{\partial A_i}; \tag{15}$$

$$\frac{\partial v_j}{\partial A_i} = \frac{\partial u_j}{\partial G}\frac{\partial G}{\partial Q}\frac{\partial Q_{-j}}{\partial A_i}. \tag{16}$$

Here the flow of adaptation aid from the donor to the recipient nation i affects the welfare of the recipient nation in three alternative ways. The first one is the direct effect which works through a rise in adaptation output x_i as described in equation (1) and is positive. The second is the indirect effects which work through a change in the equilibrium behavior of the nation as its choices of a_i, q_i and z_i change. Because of the budget constraint (12), the indirect effects work in opposite directions and cancel each other. The third is the climate change effect that works through the change in global climate consequent on the change in mitigation behavior of the other nations.

The donor's aid allocation problem, in this case, is written as: maximization of

$$W(v_1(A_0), \ldots \ldots v_i(A_0), \ldots \ldots \ldots.., v_n(A_0), G(Q(A_0)))$$

by choice of $\{A_1, \ldots\ldots\ldots, A_i, \ldots\ldots\ldots, A_n\}$ subject to the constraint given in equation (5) above.

We assume that an interior solution to the donor's problem exists. The solution $\{A_1 > 0, \ldots\ldots\ldots, A_i > 0, \ldots\ldots\ldots, A_n > 0\}$ satisfies the following set of n equations given by

$$\sum_{j=1}^{n} \frac{\partial W}{\partial v_j}\frac{\partial v_j}{\partial A_i} + \frac{\partial G}{\partial Q}\frac{\partial Q}{\partial A_i} = \lambda\ \forall i = 1, 2, \ldots \ldots \ldots., n; \tag{17}$$

and the equation of the budget constraint (5).[14]

Because of the budget constraint of the donor, more aid given to nation i reduces the amount available for the others. This affects both the mitigation and adaptation effort undertaken by the all the recipient nations and the global mitigation effort Q. The LHS of equation (17) shows that the donor, while choosing the amount of aid for nation i, takes care not only of the marginal impact of the aid on nation i's welfare, but also of its marginal impact on the welfare of other nations. It also takes care of the marginal impact it would have on the global mitigation effort since it affects the quality of global climate, which directly affects the donor's utility along with all the recipient countries' welfare. At the optimum allocation, the marginal benefits from giving an additional amount of adaptation aid to each of the nations are equalized to the marginal cost of giving such aids, represented by λ.

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14 Equation (17) is derived from the following first order condition of the donor's welfare maximization problem for choice of $A_i > 0$,
$\frac{\partial W}{\partial v_1}\frac{\partial v_1}{\partial A_i} + \ldots \ldots . + \frac{\partial W}{\partial v_i}\frac{\partial v_i}{\partial A_i} + \ldots \ldots .. + \frac{\partial W}{\partial v_n}\frac{\partial v_n}{\partial A_i} + \frac{\partial G}{\partial Q}\frac{\partial Q}{\partial A_i} = 0.\ \forall i = 1, 2, \ldots \ldots \ldots., n.$

Using equations (15) and (16), equation (17) can be rewritten as:

$$\frac{\partial W}{\partial v_i}\left(\frac{\partial u_i}{\partial x_i}\frac{\partial x_i}{\partial A_i}\right)+\left[\frac{\partial G}{\partial Q}\frac{\partial Q}{\partial A_i}+\frac{\partial W}{\partial v_i}\left(\frac{\partial u_i}{\partial G}\frac{\partial G}{\partial Q}\frac{\partial Q_{-i}}{\partial A_i}\right)+\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{\partial W}{\partial v_j}\left(\frac{\partial u_j}{\partial G}\frac{\partial G}{\partial Q}\frac{\partial Q_{-j}}{\partial A_i}\right)\right]=\lambda$$

$$\forall i,j=1,2,\ldots\ldots\ldots,n. \qquad (18)$$

Like equation (7) in the basic model, equation (18) may be used as a guiding principle by the donor for allocation of adaptation aid among the recipient nations in the extended model.

Like equation (7), equation (18) also balances the marginal utility of the donor from giving 1 extra \$ of aid to nation i to the marginal utility from keeping the extra \$ to itself. But now the marginal utility depends not only on the three factors defined earlier *viz.* the weight the nation i's welfare enjoys in the donor's utility function (i.e. $\frac{\partial W}{\partial v_i}$), the vulnerability of nation i (i.e. $\frac{\partial u_i}{\partial x_i}$) and the physical productivity of the aid in nation i (i.e. $\frac{\partial x_i}{\partial A_i}$) as before, but unlike equation (7) it also depends on three additional factors included in the third bracket on the LHS of equation (18) above. To understand these additional factors, we must keep in mind that an allocation of adaptation aid A_i to nation i (for all $i = 1, 2, \ldots \ldots \ldots, n$) affects not only the ith nation's own mitigation effort (as discussed in Observation 1), but also the mitigation effort of all other nations through their strategic interaction in the mitigation game described above. The global mitigation level decided as the outcome of the game affects the welfare of all the recipient nations and the donor. Therefore, the factors are the following. The first one is the way the ith nation's mitigation effort reacts to the adaptation aid and therefore affects global climate (i.e. $\frac{\partial G}{\partial Q}\ \frac{\partial Q}{\partial A_i}$) which has a direct impact on the donor's welfare. The second factor is the way the change in mitigation effort of the ith nation affects the ith nation's welfare and therefore the donor's welfare through a change in global climate because of the corresponding strategic mitigation behavior (in aggregate) of the other nations (i.e. $\frac{\partial W}{\partial v_i}\left(\frac{\partial u_i}{\partial G}\ \frac{\partial G}{\partial Q}\ \frac{\partial Q_{-i}}{\partial A_i}\right)$). The third is the way the mitigation behavior of the ith nation affects the jth nation's welfare and therefore the donor's welfare as described above (i.e. $\sum_{\substack{j=1\\ j\neq i}}^{n}\ \frac{\partial W}{\partial v_j}\left(\frac{\partial u_j}{\partial G}\ \frac{\partial G}{\partial Q}\ \frac{\partial Q_{-j}}{\partial A_i}\right)$). In other words, during allocation of adaptation aid, in the extended model, the donor not only takes care of the way the adaptation aid affects the welfare of the recipient country through implementation of the adaptation project, it also takes care of the way the allocation decision affects its own welfare, both directly through the effect of the decision on global climate change and indirectly through the effect of global climate change on the welfare of the recipient countries.

On the LHS of equation (18) notice that from the assumptions of the model, $\frac{\partial G}{\partial Q} > 0$, $\frac{\partial W}{\partial v_i} > 0$ and $\frac{\partial u_i}{\partial G} > 0$ for all $i = 1, 2, \ldots \ldots \ldots, n$. However, the sign of the 'global climate change effect' $\left[\frac{\partial G}{\partial Q}\frac{\partial Q}{\partial A_i} + \frac{\partial W}{\partial v_i}\left(\frac{\partial u_i}{\partial G}\frac{\partial G}{\partial Q}\frac{\partial Q_{-i}}{\partial A_i}\right) + \sum_{\substack{j=1\\ j\neq i}}^{n}\frac{\partial W}{\partial v_j}\left(\frac{\partial u_j}{\partial G}\frac{\partial G}{\partial Q}\frac{\partial Q_{-j}}{\partial A_i}\right)\right]$ is uncertain. The uncertainty arises from two different sources. First, the budget constraint of the donor (equation (5)) suggests an increase in aid for one nation means reduction in aid for some other nations; and second, as Observation 1 implies, in nation *i* the effect of adaptation aid on its mitigation effort depends on the nature of substitute/complement relations between a_i and A_i in the adaptation aid production function $x_i(a_i, A_i)$.

Proposition 2: *A nation receives higher adaptation aid from the donor if:*
(i) *it enjoys a higher weight in the donor's utility function;*
(ii) *it is more vulnerable to climate change;*
(iii) *the physical productivity of adaptation aid is higher in that nation;*
(iv) *the greater is the 'global climate change effect' represented by*

$$\left[\frac{\partial G}{\partial Q}\frac{\partial Q}{\partial A_i} + \frac{\partial W}{\partial v_i}\left(\frac{\partial u_i}{\partial G}\frac{\partial G}{\partial Q}\frac{\partial Q_{-i}}{\partial A_i}\right) + \sum_{\substack{j=1\\ j\neq i}}^{n}\frac{\partial W}{\partial v_j}\left(\frac{\partial u_j}{\partial G}\frac{\partial G}{\partial Q}\frac{\partial Q_{-j}}{\partial A_i}\right)\right].$$

Proof: It follows from equation (18) that higher values of $\frac{\partial W}{\partial v_i}$ or $\frac{\partial u_i}{\partial x_i}$ or $\frac{\partial x_i}{\partial A_i}$ or $\left[\frac{\partial G}{\partial Q}\frac{\partial Q}{\partial A_i} + \frac{\partial W}{\partial v_i}\left(\frac{\partial u_i}{\partial G}\frac{\partial G}{\partial Q}\frac{\partial Q_{-i}}{\partial A_i}\right) + \sum_{\substack{j=1\\ j\neq i}}^{n}\frac{\partial W}{\partial v_j}\left(\frac{\partial u_j}{\partial G}\frac{\partial G}{\partial Q}\frac{\partial Q_{-j}}{\partial A_i}\right)\right]$ imply that at the initial equilibrium $\frac{\partial W}{\partial A_i} > 0$. Since $\frac{\partial^2 W}{\partial A_i^2} < 0$, the statement of the proposition follows. □

Notice from equation (20) that the contingency (iv) mentioned in Proposition 2 stands independent of the other three contingencies for disbursing aid to nation *i*. If a nation scores zero in either of the first three counts, it can still receive adaptation aid from the donor if the aid complements global mitigation effort: the donor in such a case transfers aid to nation *n* solely based on its consideration that the aid improves the quality of global climate, which being a global public good, would positively influence its own welfare and the welfare of the recipient nations. Proposition 2 also implies that if the 'global climate change effect' in contingency (iv) is negative, the amount of adaptation aid given to nation *i* based on contingencies (i)–(iii) should be adjusted in downward direction. The opposite should be the case in presence of a positive 'global climate change effect': there should be a revision in the upward direction.

It is possible to gain additional insight (a rule of thumb) for the allocation rule described in equation (20) if we assume $\frac{\partial Q_{-i}}{\partial A_j} \approx 0$ for all $i, j = 1, \ldots \ldots \ldots, n$, i.e. the change in adaptation aid to nation i keeps the aggregate level of mitigation by all other nations, decided at the Nash equilibrium of the mitigation game, roughly unchanged. The assumption is plausible since the budget constraint of the donor suggests an increase in aid for one nation means a reduction in aid for some other nations; and as Observation 1 implies, in nation i the effect of adaptation aid on its mitigation effort depends on the nature of substitute/complement relations between a_i and A_i in the adaptation aid production function $x_i(a_i, A_i)$.

Rule of Thumb: when $\frac{\partial \boldsymbol{Q}_{-i}}{\partial \boldsymbol{A}_j} \approx \mathbf{0}$ for all $\boldsymbol{i}, \boldsymbol{j} = \mathbf{1}, \ldots \ldots .., \boldsymbol{n}$.

In this case, equations (17) and (18) imply: $\frac{\partial v_i}{\partial A_i} \approx \frac{\partial u_i}{\partial x_i} \frac{\partial x_i}{\partial A_i} > 0$ for all $i = 1, \ldots \ldots .., n$; and $\frac{\partial v_j}{\partial A_i} \approx 0$ for all $i, j = 1, \ldots \ldots .., n$, $i \neq j$ respectively. As a consequence, the 'global climate change effect' $\left[\frac{\partial G}{\partial Q} \frac{\partial Q}{\partial A_i} + \frac{\partial W}{\partial v_i}\left(\frac{\partial u_i}{\partial G} \frac{\partial G}{\partial Q} \frac{\partial Q_{-i}}{\partial A_i}\right) + \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{\partial W}{\partial v_j}\left(\frac{\partial u_j}{\partial G} \frac{\partial G}{\partial Q} \frac{\partial Q_{-j}}{\partial A_i}\right)\right] \approx \frac{\partial G}{\partial Q} \frac{\partial q_i}{\partial A_i}$.

If the adaptation aid is expected to substitute the mitigation expenditure in nation i (i.e. $\frac{\partial q_i}{\partial A_i} < 0$), the 'global climate change effect' turns negative and equation (20) suggests that given contingencies (i)–(iii) mentioned in Proposition 2, the adaptation aid to nation i should adjust in downward direction. If the adaptation aid is expected to complement the mitigation expenditure in nation i (i.e. $\frac{\partial q_i}{\partial A_i} > 0$), the opposite happens. Observation 1 suggests that both substitutability and complementarity is possible.

If the adaptation aid project is chosen in such a way that $\frac{\partial^2 x_i}{\partial A_i \partial a_i}$ is strongly positive, then Observation 1 suggests that $\frac{\partial q_i}{\partial A_i} < 0$. Proposition 2 implies that in such a situation the amount of adaptation aid given to nation i based on contingencies (i)–(iii) should adjust in downward direction owing to contingency (iv). The result therefore may appear to be counterintuitive since now a nation that qualifies in contingencies (i)–(iii) may not receive the aid at all. This goes against the expectation of the conventional literature on foreign aid reviewed in Section 2 of the paper. The counter-intuitiveness is explained by the hard budget constraint (9) faced by the recipient nation. If the selection of the aid project requires the recipient nation to spend a lot on adaptation because of the complementarity, its mitigation expenditure would fall.

If, on the other hand, the adaptation aid project is run independently of the domestic adaptation input or saves domestic adaptation input, we have $\frac{\partial^2 x_i}{\partial A_i \partial a_i} \leq 0$. Then the aid has weak complementarity/substitutability with the domestic adaptation input and under the condition specified by Observation 1, there is a chance that $\frac{\partial q_i}{\partial A_i} > 0$. In such a situation, Proposition 2 suggests that the nation i must receive additional aid from the donor from mitigation considerations, above the amount it receives on the first three counts.

5 Practical Example Based on Data

Proposition 1 of the paper suggests that a donor can use equation (7) for the purpose of allocation of its adaptation aid. The available indices on the poverty adjusted by historical proximity/trade relations, vulnerability to climate change (physical vulnerability weighted by world population share) and quality of governance adjusted by technical ability/education can be, first, mapped on a common scale. Second, they can be multiplied with each other to form a composite score of the recipient nations which helps the donor to decide about appropriateness of disbursement of adaptation aid to the nations, representing the LHS of equation (7). A nation having a higher composite score receives higher adaptation aid.

For construction of the indices one can use the available data. A good description of the available datasets has been given in Weiler et al. (2018). Let us mention some of them here, which may be useful for the construction of the index proposed in Proposition 1. One can use the cross-country poverty data from the World Population Review.[15] For bi-lateral trade data between the donor country and the recipient country one can use the UN Comtrade database (United Nations 2022, available at https://comtrade.un.org, accessed on 24.04.2022). The data for historical ties between the nations can be retrieved from the Quality of Government Institute. The scores on sensitivity of countries to climate change and their readiness to adapt to the situation both are provided by the University of Notre Dame Global Adaptation Index (ND-GAIN 2019, available at https://gain.nd.edu, accessed on 24.04.2022). In ND-GAIN the sensitivity to climate change measures the proportion of population of the country susceptible to climate change hazard. The readiness of a country is judged from three different aspects: economic readiness, governance readiness and social readiness. The ‘economic readiness’ captures the readiness of a country’s business environment to accept investment that could be applied to adaptation in the form of business formation and maintenance. A simple multi-factor index, ‘Doing Business’ Index from the World Bank is the measure of economic readiness. The ‘governance readiness’ captures the institutional factors that enhance application of investment for adaptation. The indicators include: political stability and non-violence, control of corruption, regulatory quality, and rule of law, all of which come from the World Governance

15 Poverty Rate by Country 2022 (worldpopulationreview.com), accessed on 24.04.2022.

Indicators (WGI). The 'social readiness' captures the social factors that enhance the mobility of investment to be converted to adaptation actions. Indicators include: social inequality, ICT infrastructure, education and innovation. A higher score in this index reflects greater readiness of the country.

For illustrating the use of the aid index proposed in Proposition 1, we will use the example of Germany as a donor of adaptation aid. From OECD, DAC tracker (available at http://www.oecd.org/dac/stats/rioconventions.htm) we found that during 2019, Germany provided maximum adaptation aid to India, followed by Brazil, Macedonia, Albania, Tunisia and Mexico. We develop an index of these countries following Proposition 1 and examine if the observed allocation pattern is the optimum one. For our purpose, we collect the scores of these countries for sensitivity to climate change and their readiness to adapt from ND-GAIN (2019). For capturing bilateral relations between Germany and each of these countries, we rely on the data on German exports to these countries. The data (in billion US$) on exports is available in the UN Comtrade database. We have taken the flow data for the year 2019 and developed an index for these countries. The index takes values between 0 and 1. The higher the value of the index, the more important is the country as a German export destination. We multiply the three scores to generate the aid-score of the countries as suggested in Proposition 1 and rank them according to the index. Table 1 below compares the present ranking with the optimum ranking.

From Table 1, it is clear that according to Proposition 1 the present allocation of German adaptation aid is the not the optimum one. The proposition suggests that in the optimum allocation, the maximum aid should go to India as it was happening in 2019, but it should have been followed by Mexico, Brazil, Tunisia, North Macedonia and Albania.

Let us now consider an illustration of the application of the rule of thumb derived from Proposition 2, which suggests an adjustment of the aid index derived above in terms of the mode in which aid is given (technical assistance vs. monetary

Table 1: Comparison between the present rank of the adaptation-aid receiving countries from Germany and their optimum rank suggested by Proposition 1.

Present rank	Sensitivity	Readiness	Trade	Aid index	Optimum rank (from Proposition 1)
India	0.353118	0.363206	0.102273	0.013117	1
Brazil	0.268577	0.349400	0.085822	0.008054	3
North Macedonia	0.268552	0.466404	0.009991	0.001251	5
Albania	0.312652	0.413464	0.002271	0.000294	6
Tunisia	0.373020	0.431302	0.011374	0.001830	4
Mexico	0.245672	0.372623	0.107942	0.009881	2

transfer) and the elasticity of the mitigation effort of the recipient with respect to the aid. In absence of estimates of the elasticity of the mitigation effort of the recipient countries in our example with respect to the aid, for the illustration purpose, we assume the elasticity value is 1/1000th of their CO_2 emission intensity in kg per PPP $ of GDP. While for technical assistance the elasticity takes a negative value, for monetary assistance it takes a positive value. We collect the data on CO_2 emission intensity of the recipient countries in kg per PPP $ of GDP from the World Bank (available on http://data.wouldbank.org/indicator/EN.ATM.CO@E.PP.GD, accessed on 26.04.2022). The aid-index adjusted for climate change as suggested by Proposition 2 is presented in Table 2 below.

Table 2 assumes that for every aid-receiving country the elasticity of mitigation with respect to aid maintains a constant relation with their emission intensity, which implies a country with higher (lower) emission intensity has higher (lower) mitigation elasticity with respect to aid. The assumption, although reasonable, may not be true. Then depending on the relative values of elasticity the ranking mentioned in Table 2 can change. Consider an example, where India has mitigation elasticity with respect to aid that is 1/10th of its CO_2 emission intensity in kg per PPP $ of GDP. All other countries in our example have their elasticity value at 1/1000th of their CO_2 emission intensity in kg per PPP $ of GDP as assumed before. Then going by our theory, Table 2 transforms into:

Notice from Table 3 that if India has high mitigation elasticity with respect to adaptation aid, our theory suggests that it should receive the lowest technical

Table 2: Comparison of present rank of adaptation-aid receiving countries from Germany and of their optimum rank suggested by Proposition 2.

Present rank of adaptation aid	Aid index	Emission intensity (kg per PPP $ of GDP)	Aid index adjusted for climate change (technical assistance)	Optimum rank for technical assistance (from Proposition 2)	Aid index adjusted for climate change (monetary assistance)	Optimum rank for monetary assistance (from Proposition 2)
India	0.01312	0.27000	0.01285	1	0.01339	1
Brazil	0.00805	0.14000	0.00791	3	0.00819	3
North Macedonia	0.00125	0.21000	0.00104	5	0.00146	5
Albania	0.00029	0.14000	0.00015	6	0.00043	6
Tunisia	0.00183	0.22000	0.00161	4	0.00205	4
Mexico	0.00988	0.19000	0.00969	2	0.01007	2

Table 3: Comparison of present rank of adaptation-aid receiving countries from Germany and of their optimum rank suggested by Proposition 2 when India has mitigation elasticity with respect to aid that is 1/10th of its CO_2 emission intensity in kg per PPP $ of GDP, while the other countries have their elasticity value at 1/1000th of their CO_2 emission intensity in kg per PPP $ of GDP.

Present rank of adaptation aid	Aid index	Emission intensity (kg per PPP $ of GDP)	Aid index adjusted for climate change (technical assistance)	Optimum rank for technical assistance (from Proposition 2)	Aid index adjusted for climate change (monetary assistance)	Optimum rank for monetary assistance (from Proposition 2)
India	0.01312	0.27000	−0.01388	6	0.04012	1
Brazil	0.00805	0.14000	0.00791	2	0.00819	3
North Macedonia	0.00125	0.21000	0.00104	4	0.00146	5
Albania	0.00029	0.14000	0.00015	5	0.00043	6
Tunisia	0.00183	0.22000	0.00161	3	0.00205	4
Mexico	0.00988	0.19000	0.00969	1	0.01007	2

assistance, but the highest monetary assistance among the six countries considered in the example.

6 Conclusions

The aim of the analysis was to find criteria to allocate transfers from industrialized to developing countries for adaptation to climate change. In the first part, the basic model without endogenous mitigation was presented. In the second part of the analysis, we extended the basic model by including the effects that adaptation aid may exert on the recipient countries' mitigation efforts. In doing so, we take care of the global mitigation game that the recipient country plays vis-à-vis the other nations.

Our analysis suggests that the allocation criteria must be based primarily on the scores obtained by the recipient nations in an index constructed by incorporation of three factors: poverty of the nations along with their historical proximity/trade relations with the donor, vulnerability of the nations to climate change (physical vulnerability weighted by world population share); quality of governance along with technical ability/education level of the nations that helps efficient use of the aid. Finally, the score needs to be adjusted for the impact of the adaptation aid on the quality of global climate which works through a change in mitigation behavior of the nation. If the ‘global climate change effect’ is negative, the adaptation aid hurts the global climate and the score is adjusted in the downward direction. The opposite happens if the effect is positive. A nation with a higher score receives a higher allocation of aid.

Since the calculation of the ‘global climate change effect’ is difficult, the paper solves the problem by designing a rule of thumb for accounting for this effect. If it is the case that the adaptation aid is strongly complementary with the adaptation effort of the recipient nation, its mitigation effort falls, which leads to deterioration of global climate. Then, there is a case for downward adjustment of the score. The opposite happens if adaptation aid is a weak complement or substitute of domestic adaptation effort. While the former is the more likely to be the case if the aid takes the form of technical assistance, the latter is more likely to be the case for monetary transfers. Therefore, the paper argues for an upward adjustment in allocation of the aid if it is given in monetary terms; and for a downward adjustment if it is given in terms of technical assistance. The amount of adjustment must depend on the estimated complementarity of the aid with the domestic adaptation input in the adaptation output production function at the recipient nations.

The contribution of the paper to the literature on adaptation aid allocation is the following. First, it attempts to replace the traditional development aid theory which is being used in this field with a normative theory of its own, by introducing three factors: (i) the optimum reaction of the recipient nations in terms of their choice of adaptation effort and mitigation effort in response to adaptation aid; (ii) an aggregative game to determine Nash equilibrium behavior of the nations in the mitigation game in response to adaptation aid; (iii) vulnerability of nations and the ‘global climate change effect’ as criteria for the allocation of adaptation aid. Second, it defines a rule of thumb for the allocation of adaptation aid that can be implemented by using available data. Third, it suggests some modifications in the empirical approach used in this field which follows the traditional development aid literature. Specifically, it argues for modification of indices used for such studies and introduction of the ‘global climate change effect’ as a factor for studying the flow of adaptation funding from the North to the Global South. In this paper the donor does not care about whether the mitigation effort put by the recipient is optimum or not. It cares only about the direction and magnitude of the change. This is in accordance with the donor’s idea about putting the recipient on the sustainable development path.

The analysis presented in the paper has important limitations. First, the index based on the ‘global climate change effect’ crucially depends on the estimate of the adaptation-aid elasticity of the mitigation output of the aid-receiving nations. However, no such estimate is currently available. For illustration purpose a proxy has been used in Section 5 of the paper, which is clearly inadequate. The estimation remains as our future research agenda. Second, the paper assumes that the recipient nations have welfare maximizing governments which is seldom true. In other words, the analysis abstracts away from the distribution of adaptation aid within the recipient nation which has various types of inequalities within it. The suggested scores are based on the position of the nations in the aggregate indices. It remains an issue whether the donor should base its allocation criteria on the internal politics of the recipient nations. However, this requires a separate analysis which also remains as our future research agenda.

Appendix

Proof of Lemma 1: Let α_i be the Lagrange multiplier in the constrained maximization problem of nation i. Then the first order conditions for an interior solution are:

$$\frac{\partial u_i}{\partial z_i} - \alpha_i = 0; \tag{A.1}$$

$$\frac{\partial u_i}{\partial x_i}\frac{\partial x_i}{\partial a_i} - \alpha_i c = 0; \tag{A.2}$$

$$I_i - z_i - c a_i = 0. \tag{A.3}$$

By application of (A.1) and (A.2) from equation (4):

$$\frac{\partial v_i}{\partial A_i} = \frac{\partial u_i}{\partial x_i}\frac{\partial x_i}{\partial A_i} + \alpha_i \frac{\partial z_i}{\partial A_i} + \alpha_i c \frac{\partial a_i}{\partial A_i}. \tag{A.4}$$

Since equation (A.3) implies: $\frac{\partial z_i}{\partial A_i} = -c\frac{\partial a_i}{\partial A_i}$, from (A.4) the statement of the lemma follows.

Proof of Lemma 2: A unique solution of equation (13) in terms of Q exists under two alternative conditions:

(i) $R'(Q) < 0$ and as $Q \to 0, R_i(Q) \to ve$;
(ii) $R'(Q) > 1$ and as $Q \to 0, R_i(Q) \to -ve$.

Since $R_i(Q)$ cannot be negative as demanded by (ii), the condition is infeasible. So (i) provides the condition for existence of a unique solution of equation (13).

The sufficient condition under which (i) holds is $R_i'(Q) = \frac{\partial q_i}{\partial Q} < 0$ for all $i = 1, \ldots\ldots, n$.

From equation (11):

$$R_i'(Q) = \frac{\partial q_i}{\partial Q} = \frac{\frac{\partial u_i}{\partial G}\frac{\partial^2 G}{\partial Q^2} + \frac{\partial x_i}{\partial Q}\left\{\frac{\partial G}{\partial Q}\frac{\partial^2 u_i}{\partial x_i \partial G} - d\frac{\partial^2 u_i}{\partial x_i \partial z_i}\right\} + \left(\frac{\partial G}{\partial Q}\right)^2 \frac{\partial^2 u_i}{\partial G^2}}{d\left[\frac{\partial G}{\partial Q}\frac{\partial^2 u_i}{\partial z_i \partial G} - d\frac{\partial^2 u_i}{\partial z_i^2}\right]}. \tag{A.5}$$

In equation (A.5) the denominator on the RHS is positive since $\frac{\partial G}{\partial Q} > 0$, $\frac{\partial^2 u_i}{\partial z_i \partial G} > 0$ and $\frac{\partial^2 u_i}{\partial z_i^2} < 0$ from the assumptions of the model. However, the numerator has an ambiguous sign since $\frac{\partial u_i}{\partial G}\frac{\partial^2 G}{\partial Q^2} \le 0$, $\left(\frac{\partial G}{\partial Q}\right)^2 \frac{\partial^2 u_i}{\partial G^2} < 0$ and $\frac{\partial x_i}{\partial Q}\left\{\frac{\partial G}{\partial Q}\frac{\partial^2 u_i}{\partial x_i \partial G} - d\frac{\partial^2 u_i}{\partial x_i \partial z_i}\right\} > 0$ from the assumptions of the model. Therefore, from (A.5), $R_i'(Q) = \frac{\partial q_i}{\partial Q} < 0$ if and only if $\frac{\partial u_i}{\partial G}\frac{\partial^2 G}{\partial Q^2} + \left(\frac{\partial G}{\partial Q}\right)^2 \frac{\partial^2 u_i}{\partial G^2} + \frac{\partial x_i}{\partial Q}\left\{\frac{\partial G}{\partial Q}\frac{\partial^2 u_i}{\partial x_i \partial G} - d\frac{\partial^2 u_i}{\partial x_i \partial z_i}\right\} < 0$.

Proof of Lemma 3: Taking the total differential from equation (10) yields:

$$\frac{\partial x_i}{\partial a_i}\left[\frac{\partial^2 u_i}{\partial x_i^2}\left\{\frac{\partial x_i}{\partial a_i}\frac{\partial a_i}{\partial A_i} + \frac{\partial x_i}{\partial A_i}\right\} + \frac{\partial^2 u_i}{\partial z_i \partial x_i}\frac{\partial z_i}{\partial A_i} + \frac{\partial^2 u_i}{\partial G \partial x_i}\frac{\partial G}{\partial Q}\frac{\partial Q}{\partial A_i}\right] + \frac{\partial u_i}{\partial x_i}\left[\frac{\partial^2 x_i}{\partial a_i^2}\frac{\partial a_i}{\partial A_i} + \frac{\partial^2 x_i}{\partial A_i \partial a_i}\right]$$
$$= c\left[\frac{\partial^2 u_i}{\partial z_i^2}\frac{\partial z_i}{\partial A_i} + \frac{\partial^2 u_i}{\partial x_i \partial z_i}\left\{\frac{\partial x_i}{\partial a_i}\frac{\partial a_i}{\partial A_i} + \frac{\partial x_i}{\partial A_i}\right\} + \frac{\partial^2 u_i}{\partial G \partial x_i}\frac{\partial G}{\partial Q}\frac{\partial Q}{\partial A_i}\right],$$

Substituting $Q = q_i + Q_{-i}$ and collecting the terms, we rewrite the above equation as:

$$K\frac{\partial a_i}{\partial A_i} + L\frac{\partial q_i}{\partial A_i} = -\frac{\partial x_i}{\partial A_i}\left[\frac{\partial x_i}{\partial a_i}\frac{\partial^2 u_i}{\partial x_i^2} - c\frac{\partial^2 u_i}{\partial x_i \partial z_i}\right] - \frac{\partial u_i}{\partial x_i}\frac{\partial^2 x_i}{\partial A_i \partial a_i} + \rho\frac{\partial Q_{-i}}{\partial A_i} \tag{A.6}$$

where $K = \frac{\partial x_i}{\partial a_i}\left\{\frac{\partial x_i}{\partial a_i}\frac{\partial^2 u_i}{\partial x_i^2} - 2c\frac{\partial^2 u_i}{\partial z_i \partial x_i}\right\} + \frac{\partial u_i}{\partial x_i}\frac{\partial^2 x_i}{\partial a_i^2} + c^2\frac{\partial^2 u_i}{\partial z_i^2}$, $L = \frac{\partial x_i}{\partial a_i}\left\{\frac{\partial G}{\partial Q}\frac{\partial^2 u_i}{\partial x_i \partial G} - d\frac{\partial^2 u_i}{\partial x_i \partial z_i}\right\} + c\left\{d\frac{\partial^2 u_i}{\partial z_i^2} - \frac{\partial G}{\partial Q}\frac{\partial^2 u_i}{\partial G \partial z_i}\right\}$, $\rho = \frac{\partial x_i}{\partial a_i}\left\{d\frac{\partial^2 u_i}{\partial z_i \partial x_i} - \frac{\partial G}{\partial Q}\frac{\partial^2 u_i}{\partial x_i \partial G}\right\} - cd\left\{\frac{\partial^2 u_i}{\partial z_i^2} - \frac{\partial G}{\partial Q}\frac{\partial^2 u_i}{\partial G \partial z_i}\right\}$.

Similarly, taking the total differential from equation (11) yields:

$$\frac{\partial G}{\partial Q}\left[\frac{\partial^2 u_i}{\partial z_i \partial G}\frac{\partial z_i}{\partial A_i} + \frac{\partial^2 u_i}{\partial x_i \partial G}\left\{\frac{\partial x_i}{\partial a_i}\frac{\partial a_i}{\partial A_i} + \frac{\partial x_i}{\partial A_i}\right\} + \frac{\partial^2 u_i}{\partial G^2}\frac{\partial G}{\partial Q}\frac{\partial Q}{\partial A_i}\right] + \frac{\partial u_i}{\partial G}\frac{\partial^2 G}{\partial Q^2}\frac{\partial Q}{\partial A_i}$$
$$= d\left[\frac{\partial^2 u_i}{\partial z_i^2}\frac{\partial z_i}{\partial A_i} + \frac{\partial^2 u_i}{\partial x_i \partial z_i}\left\{\frac{\partial x_i}{\partial a_i}\frac{\partial a_i}{\partial A_i} + \frac{\partial x_i}{\partial A_i}\right\} + \frac{\partial^2 u_i}{\partial G \partial z_i}\frac{\partial G}{\partial Q}\frac{\partial Q}{\partial A_i}\right],$$

Substituting $Q = q_i + Q_{-i}$ and collecting the terms, we rewrite the above equation as:

$$M\frac{\partial a_i}{\partial A_i}+N\frac{\partial q_i}{\partial A_i}=-\frac{\partial x_i}{\partial A_i}\left[\frac{\partial G}{\partial Q}\frac{\partial^2 u_i}{\partial x_i\,\partial G}-d\frac{\partial^2 u_i}{\partial x_i\,\partial z_i}\right]+\varphi\frac{\partial Q_{-i}}{\partial A_i} \tag{A.7}$$

where $M=\frac{\partial x_i}{\partial a_i}\left\{\frac{\partial G}{\partial Q}\frac{\partial^2 u_i}{\partial x_i\partial G}-d\frac{\partial^2 u_i}{\partial x_i\partial z_i}\right\}+c\left\{d\frac{\partial^2 u_i}{\partial z_i^2}-\frac{\partial G}{\partial Q}\frac{\partial^2 u_i}{\partial G\partial z_i}\right\}$, $N=d\left\{d\frac{\partial^2 u_i}{\partial z_i^2}-2\frac{\partial G}{\partial Q}\frac{\partial^2 u_i}{\partial G\partial z_i}\right\}+\left(\frac{\partial G}{\partial Q}\right)^2\frac{\partial^2 u_i}{\partial G^2}+\frac{\partial u_i}{\partial G}\frac{\partial^2 G}{\partial Q^2}$, $\varphi=\frac{\partial G}{\partial Q}\left\{d\frac{\partial^2 u_i}{\partial z_i\partial x_i}-\left(\frac{\partial G}{\partial Q}\right)^2\frac{\partial^2 u_i}{\partial G^2}+d\frac{\partial^2 u_i}{\partial G\partial z_i}\right\}-\frac{\partial u_i}{\partial G}\frac{\partial^2 G}{\partial Q^2}-d\frac{\partial^2 u_i}{\partial z_i^2}$.

Solving (A.6) and (A.7) for $\frac{\partial a_i}{\partial A_i}$ and $\frac{\partial q_i}{\partial A_i}$, the statement of Lemma 3 follows. □

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