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Abstract: The coexistence of fiat money (cash) and digital monies constitutes a system of parallel currencies as media of exchange. This paper asks whether a new (digital) currency is essential: Does a new currency allow for a better resource allocation even if a fully accepted currency is in circulation and remains in circulation? Using the dual currency search model of Kiyotaki and Wright (1993. A search-theoretic approach to monetary economics. Am. Econ. Rev. 83: 63–77), we show how the introduction of a secondary currency affects average utility. There is some scope for a welfare improvement as the welfare effect depends on differences in returns and costs, and, in particular, on the proportion of cash traders who will be replaced by digital money traders.

Keywords: digital money, dual currency regime, welfare comparison

JEL Classification: E41, E42, E51

1 Introduction

The process of digitalization accelerates the emergence of new currencies such as cryptocurrencies, corporate currencies and central bank digital currencies. These currencies may serve as an additional medium of exchange and are new competitors on the markets for liquidity services. Kiyotaki and Wright (1993) have shown that fiat money is essential, i.e. compared to a barter economy, fiat money allows for a better resource allocation. In this paper, we put forward a similar question: Is the secondary currency essential too? Does the introduction of a new currency allow for a welfare improvement even if a fully accepted currency, i.e. cash, is in circulation and remains in circulation? To tackle this question, we use the dual currency search framework of Kiyotaki and Wright (1993). The answer we find is a conditional “yes”.

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Not surprisingly, the scope for a welfare improvement depends on differences in returns and costs. But in addition, the sign of the welfare effect very much depends on the proportion of cash traders who will be replaced by digital money traders, or, equivalently, the degree of substitution between the traditional and the new currency.

The focus of our model is an advanced economy with a well-functioning payment system. We have in mind the Eurozone and/or the United States, where cash is an established medium of exchange and where now a cryptocurrency such as Bitcoin is emerging. Another example is Switzerland, where the euro is accepted in most parts of the country despite the universal acceptance of the Swiss franc. We do not believe in a cashless society; our framework therefore assumes that cash as traditional currency remains in circulation even if the new currency is fully accepted. We do not model the process of currency substitution with the use of the new currency instead of cash. Such a full crowding out of government fiat money is more relevant for high-inflation countries and countries with eroding economic and political institutions; see, for instance, Jácome (2004), Noko (1993), and Rivera-Solis (2012) for Ecuador, Zimbabwe and El Salvador, respectively. The use of multiple currencies during turbulent times, studied and surveyed in, e.g. Giovannini and Turtelboom (1994) and Airudo (2014), is no equilibrium phenomenon, so that the Kiyotaki–Wright framework is not appropriate. Note, however, the different view of Colacelli and Blackburn (2009), who employ the dual currency approach to investigate the multiple currency usage during the Great Depression in the United States and the 2002 recession in Argentina. The coexistence of both the traditional and the new currency has to be an equilibrium outcome. The Kiyotaki–Wright framework shows this desirable feature. Moreover, this framework allows a distinction to be made between partial and full acceptance of the secondary currency. For different modelling approaches, we refer to the overlapping generation model of Lippi (2021), the currency competition model of Schilling and Uhlig (2019) and the New Keynesian framework of Uhlig and Xie (2020).

The economics of dual currency regimes is the topic of a wide body of theoretical and empirical literature. An excellent overview of the search-theoretic foundations of the use of multiple currencies is presented by Craig and Waller (2000). Aiyagari et al. (1996) study the coexistence of money and interest-bearing securities, Camera et al. (2004) distinguish between safe and risky fiat monies, Curtis and Waller (2000) focus on the simultaneous use of legal and illegal currencies, while Lotz (2004) addresses the question how to regulate a new currency. Ding and Puzello (2020) use laboratory experiments to explore how governmental interventions such as legal restrictions on the use of a foreign currency or a change in using costs affect the circulation of the domestic currency. Also using a laboratory experimental design, Rietz (2019) analyzes the determinants of the acceptance of a secondary currency. Surprisingly, all these studies say very little about the scale of a welfare improvement of a secondary currency. This paper aims to fill this gap. The remainder of the paper
is organized as follows. Section 2 describes the model setup of our analysis. Section 3 presents the single currency regime as benchmark economy. Section 4 discusses two switching scenarios in which we distinguish between partial and full acceptance of the new currency. Section 5 concludes.

2 Framework

Our setup borrows heavily from the dual currency framework of Kiyotaki and Wright (1993). Referring to yield differences and differences in the liquidity value, Kiyotaki and Wright (1993) show that equilibria exist with both currencies in circulation. However, they do not discuss the transition from a single currency to a dual currency regime. Yet neglecting the impact of a new currency on the supply of the traditional currency turns out to be decisive for the welfare effect of a new currency. We thus modify the Kiyotaki–Wright framework in two ways: First, we take into account the interaction between the traditional currency and the new one, and second, we use an economy with a fully accepted traditional currency as initial equilibrium (benchmark).

The economy consists of a continuum of infinitely lived agents with population size normalized to unity. We follow Matsuyama et al. (1993) and assume that agents of type \( i \in \{1, \ldots, I\} \) with \( I \geq 3 \), consume only goods of type \( i \), but produce goods of type \( i + 1 \) (modulo \( I \)). As a consequence, there is no double coincidence of wants and no pure barter in the economy. Money is necessary for trading.1

In accordance with Kiyotaki and Wright (1993) and, again, Matsuyama et al. (1993), we assume that goods production requires a consumption good as input and that agents cannot produce until they have consumed. An agent produces one unit of output according to a Poisson process with constant arrival rate, \( \alpha \), where \( \alpha \) measures output per unit of time. We will focus on the limiting case, \( \alpha \to \infty \), so that production is instantaneous. The share of the population who are producers degenerates to zero, all agents are traders (see Appendix A for the dynamic structure of the model).

In addition to the commodities, the economy is endowed with two types of money, a traditional and a new secondary currency. The traditional currency is fiat money (cash) issued by the government. Since we primarily have a cryptocurrency or a central bank digital currency in mind, we call the new currency digital money. However, the secondary currency may also of course be fiat money issued by a foreign government. We distinguish between three trading states: Agents are cash traders C, digital money traders D or commodity traders (sellers S). We use the

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1 Not allowing for pure barter simplifies the algebra tremendously. From an economic point of view, this assumption may be justified by the large degree of specialization in the production of goods, which implies an almost zero probability of a double coincidence of wants.
following notation: In a dual currency regime, let $\mu_C$ and $\mu_D$ be the share of agents endowed with one unit of cash and digital money, respectively. The share of commodity traders, $\mu_S$, then is $\mu_S = 1 - \mu_C - \mu_D$. In a single currency regime, there is no digital money, $\mu_D = 0$, the share of cash traders is $\mu_C$, the share of commodity traders is $\mu_S = 1 - \mu_C$. The superscript $s$ stands for single currency regime.

Meetings are pairwise and occur according to a Poisson process with constant arrival rate, $\beta$, with $\frac{\beta}{\gamma} = 1$. This assumption ensures that a money trader meets exactly one seller within one period who is able to produce the preferred good of the money trader. The probability of a seller meeting a cash (digital money) trader is $\mu_C$ ($\mu_D$). Given such a meeting, the seller decides whether to accept cash (digital money) and to switch the status from a commodity trader to a cash (digital money) trader. The decision is captured by the probabilities $\pi_C$ and $\pi_D$. The return of switching the state is given by $V_C - V_S$ respective $V_D - V_S$, where $V_j, j = S, C, D$, are the value functions for a trader of type $j$. If $r > 0$ denotes the rate of time preference, the sellers’ expected return to search is given by the Bellman equation

$$rv = \mu_C \max_{\pi_C} [\pi_C (V_C - V_S)] + \mu_D \max_{\pi_D} [\pi_D (V_D - V_S)]$$

(1)

If the return of switching the state is positive (negative), a seller always accepts (rejects) the currencies and sets the optimal response, $\pi_C$ respective $\pi_D$, to unity (zero). If sellers are indifferent between states, they flip a coin with $0 < \pi_C, \pi_D < 1$, a currency is partially accepted. Since, by assumption, there is no pure barter and no

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2 The assumption of a uniform random matching process, where the matching of any pair of agents is equally likely, is common in the literature. Due to the randomness of meetings, agents cannot commit to a long-term agreement, credit arrangements cannot be enforced. Corbae et al. (2003) relax the assumption of random meetings and develop a model of monetary exchange with directed search. Matsuyama et al. (1993) stick to the assumption of random meetings, but they assume a non-uniform matching process. In a two-country, two-currency model, agents are randomly paired, but the probability of meeting a domestic agent with the domestic currency exceeds the probability of meeting a foreign agent with the foreign currency. Most interesting is the case of (choice of the currency in) international pairings. The authors discuss the conditions under which either both currencies circulate or an international currency emerges.

3 There are some alternatives to model partial acceptance of a currency. For instance, assume two types of sellers, A and B. Seller A always accepts a currency, while seller B always rejects the currency. The overall acceptance rate depends on the distribution across the two types. This approach may be seen as more intuitive, but needs the assumption of a fourth trading state. Based on some calculations of our own, we conclude that the additional insights do not warrant the additional algebra, we give precedence to simplicity.

4 Neither the Trejos and Wright (1995) nor the Lagos and Wright (2005) framework allows for the modelling of a partially accepted currency. In the model of Trejos and Wright (1995), a buyer and a seller bargain over the quantity of goods the buyers gets for one unit of money. Partial acceptance of a currency requires that the seller is indifferent between trading and non-trading. In the bargain, the
consumption of the own production, a positive flow return to a seller requires a switch of status from a commodity to a money trader.

The flow return to a cash trader is given by the probability of meeting a seller, \( \mu_S \), times the probability that a random commodity trader accepts cash (overall acceptance rate), \( \Pi_C \), times the utility from consumption, \( U \), minus a transaction fee, \( \eta_C \), minus the loss of switching from C to S, \( (V_S - V_C) \). Apart from that, we assume that cash has some nonuse value (monetary benefit), \( \gamma_C \). Cash has some pleasing aesthetics, holding financial assets in the form of cash has the advantage of anonymity, cash may serve as safe haven. If there are some storage and/or transportation costs, we have \( \gamma_C < 0 \). For a digital money trader, the line of argument is very much the same. Then the Bellman equations are

\[
\begin{align*}
\text{r}V_C &= \gamma_C + \mu_S \Pi_C(U - \eta_C + V_S - V_C) \\
\text{r}V_D &= \gamma_D + \mu_S \Pi_D(U - \eta_D + V_S - V_D).
\end{align*}
\]

Note that although we call the secondary currency digital money, we do not model any specific feature of digital currencies. Transaction fees, a high rate of return or degree of volatility, a more speedy settlement of payments etc. are subsumed under \( \gamma_D \) and \( \eta_D \). Moreover, a trade between cash and digital money trader does not make both agents better off. In case of such a meeting, both agents continue with their own money. To put it another way, we rule out side payments, see also Aiyagari et al. (1996).

Our focus will be on symmetric equilibria with \( \pi_C = \Pi_C \) and \( \pi_D = \Pi_D \). In accordance with Kiyotaki and Wright (1993), welfare is defined by the expected utility of all agents before the initial endowment of money and commodities is randomly distributed among them. In terms of expected flow returns, the welfare criterion can be expressed as (see Appendix A):

\[
\text{r}W = \mu_S rV_S + \mu_C rV_C + \mu_D rV_D.
\]

### 3 Single Currency Regime

Despite the truism that the welfare effect of a new currency depends to a large extent on the starting point (or initial equilibrium), the literature has neglected this issue. Since we are primarily interested in developed economies with a well-functioning buyer can always ensure that the seller is not indifferent by offering to take an infinitesimally smaller amount of goods, or equivalently, by offering an infinitesimally higher price, see also Craig and Waller (2000). The seller always accepts, a trade always occurs, but this is the scenario of a fully accepted currency. The same line of reasoning holds for the Lagos and Wright (2005) framework.
payment system, our starting point will be a single currency regime in which only cash is in circulation, and cash is fully accepted. In the initial equilibrium, there is no digital money, $\mu_D = 0$. Full acceptance of cash, $\pi_C = \Pi_C = 1$, requires that the gain of accepting cash and switching the state from $S$ to $C$ must be positive, $V^*_C - V^*_S > 0$. For the single currency regime the Bellman equations simplify to

$$rV^*_S = \mu^*_C (V^*_C - V^*_S)$$  \hspace{1cm} (5)

$$rV^*_C = \gamma_C + \mu^*_S (U - \eta_C + V^*_S - V^*_C)$$  \hspace{1cm} (6)

By combining these equations it is easy to show that the condition $V^*_C - V^*_S > 0$ is equivalent to

$$\rho^*_C \equiv \gamma_C + \mu^*_S (U - \eta_C) > 0.$$  \hspace{1cm} (7)

Here, $\rho^*_C$ is the expected per period return of cash. If the sum of the expected net utility from buying and consuming a good minus the storage costs (or plus the monetary benefit) is positive, cash will be universally accepted. Inserting (5) and (6) into (4), and observing $\mu_D = 0$, delivers the level of welfare in the single currency regime:

$$rW^*_S = \mu^*_S \mu^*_C (V^*_C - V^*_S) + \mu^*_C \mu^*_S (U - \eta_C + V^*_S - V^*_C) + \mu^*_C \gamma_C = \mu^*_C \rho^*_C.$$  \hspace{1cm} (8)

Note that the welfare effects of switching the status (from sellers to cash traders and from cash traders to sellers) add up to zero. By switching from S to C, the group of sellers improve their welfare by $\mu^*_S \mu^*_C (V^*_C - V^*_S)$. By switching from C to S, the group of cash traders face a loss of $\mu^*_C \mu^*_S (V^*_S - V^*_C)$. These effects add up to zero. For the cash traders, the loss of switching is overcompensated by the increase in welfare due to consumption. On aggregate, welfare is thus positive, see (8).

An increase in the money supply, in our model captured by an increase in the share of cash traders, has two (well-known) effects on welfare. A higher $\mu_C^*$ facilitates trade and sellers find a trading partner more easily (liquidity effect). But a higher $\mu_C^*$ means a lower $\mu_S^*$, the number of commodities (sellers) declines. The welfare-maximizing share of cash traders, $(\mu^*_C)^*$, balances these effects. Observing (7) as well as $\mu^*_S = 1 - \mu^*_C$, the derivation of (8) with respect to $\mu^*_C$ yields

$$(\mu^*_C)^* = \frac{1}{2} + \frac{\gamma_C}{2(U - \eta_C)}.$$  \hspace{1cm} (9)

Equation (9) extends Kiyotaki and Wright (1993), who focus on the special case $\gamma_C = 0$ with $(\mu^*_C)^* = 1/2$. Depending on the sign of $\gamma_C$ (monetary benefit versus storage costs), $(\mu^*_C)^*$ exceeds or falls short of 1/2.
4 Two Switching Scenarios

Besides the initial equilibrium, the welfare effect also depends on the acceptance of the new currency. We distinguish between two scenarios. First, cash is fully accepted and digital money is partially accepted (Section 4.1), and second, both currencies are fully accepted (Section 4.2).

4.1 Cash Fully Accepted, Digital Money Partially Accepted

The introduction of a new currency means that digital money is part of the initial endowment, $\mu_D > 0$. As mentioned above, partial acceptance of digital money requires that sellers are indifferent between state S and state D, $V_S = V_D$. Sellers flip a coin with $0 < \pi_D = \Pi_D < 1$. Denoting partial acceptance of digital money with the superscript $p$, the Bellman equations are now:

\[ rV^p_S = \mu^p_C (V^p_C - V^p_S) \]
\[ rV^p_C = \gamma_C + \mu^p_s (U - \eta_C + V^p_S - V^p_C) \]
\[ rV^p_D = \gamma_D + \mu^p_s \Pi_D (U - \eta_D). \]

Any comparative statics analysis needs a hypothesis on the replacement of sellers and cash traders by the digital money traders. This is done by

\[ \mu^p_s = \mu^s_s - \lambda \mu_D \]
\[ \mu^p_C = \mu^s_C - (1 - \lambda) \mu_D, \]

where $\lambda \in [0, 1]$ denotes the replacement parameter. For $\lambda = 0$, digital money traders do not replace any seller, the economy’s endowment with goods remains the same, the digital money traders replace cash traders one to one. The new currency does not change the endowment of the economy with money, but the money supply is now made up of two currencies. For $\lambda = 1$, digital money traders replace only sellers. Since

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5 We do not model the way to becoming a cashless society. Cash will retain the status of legal tender, and more importantly, central banks will not be powerless witnesses of the decline in the demand for their product. We agree with Rogoff (2017): “..., it is hard to see what would stop central banks from creating their own digital currencies and using regulation to tilt the playing field until they win. The long history of currency tells us that what the private sector innovates, the state eventually regulates and appropriates”. An interesting case study is Sweden, where the usage of cash has dramatically declined. But the decline is not the result of cryptocurrencies or corporate currencies, but primarily the result of the app “Swish”, which allows for payments avoiding the central bank clearing system, see Sveriges Riksbank (2021).
the proportion of cash traders remains constant, the new currency implies an increase in the economy’s money supply. The replacement parameter serves as a measure of the degree of substitution between digital money and cash. For low values \(\lambda < 0.5\), digital money and cash are close substitutes, whereas for large values \(\lambda > 0.5\), these currencies are bad substitutes.

The equilibrium acceptance rate turns out to be

\[
V^p_S = V^p_D \quad \iff \quad \Pi_D = \frac{\mu^p D\rho^p D - Y_D}{\mu^p S(U - \eta_D)}
\]

with \(\nu^p C \equiv \frac{1}{1 + r - \mu_D}\) and \(\rho^p C \equiv \nu^p C + \mu^p S(U - \eta_C) > 0\). Only if the acceptance rate for digital money is given by (15), is digital money partially accepted. Note that \(\Pi_D\) is decreasing in the monetary benefit of digital money, \(\gamma_D\), and increasing in the using costs, \(\eta_D\). If the monetary benefit goes up, digital money will become more attractive, the expected flow return of digital money increases and exceeds the flow return to a seller. To restore indifference between being a seller and a digital money trader requires a lower acceptance rate for digital money.\(^6\) In a similar vein, when the expected per period return of cash, \(\rho^p C\), increases, the seller’s gain of switching from S to C increases, \(V^p_S > V^p_D\). Again, to restore indifference, the equilibrium acceptance rate must be higher.

Let us consider welfare. We use the Bellman Eqs. (10)–(12) to compute the new expected returns to search and insert the results into (4). We yield

\[
r_W^p = (1 + \mu_D \nu^p C)\mu^p C \rho^p C.
\]

The comparison of (16) with (8) starts with the polar case, \(\lambda = 0\), digital money traders replace only cash traders. Then we can show that \(rW^p - rW^s > 0\) requires \(0 > r + \mu^p S\). This condition is never fulfilled. Therefore, for \(\lambda = 0\), the introduction of a new partially accepted currency unambiguously lowers welfare. The cash traders, who are replaced by digital money traders, switch from a currency with full acceptance to a currency with partial acceptance. The aggregate money supply does not change, but the probability of a successful match and thus the liquidity value declines. For \(\lambda = 1\), where digital money traders replace only sellers, we get

\[
r_W^p - rW^s > 0 \quad \iff \quad \frac{Y_C}{r + \mu^p C} > U - \eta_C.
\]

We distinguish between three effects on welfare. First, the economy is less well endowed with goods. Second, exchange is made easier by the increase in the money

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6 Economic intuition may suggest that an increase in the monetary benefit leads to a higher acceptance rate. But because of \(V^p_S < V^p_D\), the new equilibrium acceptance rate would be unity, and we would switch from partial to full acceptance. To remain in the scenario of partial acceptance, a lower acceptance rate is necessary.
supply (liquidity). And third, from the cash traders point of view, the number of trades decreases, so that the expected holding period of cash goes up. For $\gamma_C \neq 0$, this matters for welfare.

For $\gamma_C = 0$, condition (17) is not fulfilled, the new currency lowers welfare. Since a fully accepted currency already in place, the liquidity effect is positive but small. The negative endowment effect unambiguously dominates. If cash has some storage costs, $\gamma_C < 0$, the prolongation of the holding period amplifies the decline in welfare. A monetary benefit of the traditional currency and thus a positive prolongation effect, $\gamma_C > 0$, turns out to be a necessary condition for a positive welfare effect of the new currency. Note that the prolongation effect declines in both the discount rate, $r$, and the share of cash traders, $\mu_C^*$. The higher $\mu_C^*$, the longer the holding period in the initial equilibrium, and the lower the marginal welfare effect.

The welfare-maximizing share of cash traders is also affected by the introduction of a new partially accepted currency. Maximizing (16) with respect to $\mu_C^C$ yields

$$
(\mu_C^C)^* = \frac{1 - \mu_D}{2} + \frac{\gamma_C}{2(U - \eta_C)} = (\mu_C^C)^* - \frac{\mu_D}{2}.
$$

The optimal share of cash traders is decreasing in the share of digital money traders. The optimal response to an increase in liquidity supplied by digital money traders is a reduction in liquidity supplied by the cash traders. Note that this result does not depend on the replacement parameter, $\lambda$, and thus on the question whether digital money and cash are good or bad substitutes. The replacement parameter comes into play, if the optimal response to the new currency, given by (18), differs from the actual response assumed in (14). The optimal response to the introduction of digital money is a decline of $\mu_C^C$ by $0.5\mu_D$, the (assumed) actual response of $\mu_C^C$ is a decline by $(1 - \lambda)\mu_D$. If digital money and cash are close substitutes ($\lambda < 0.5$), the actual decline exceeds the optimal decline, and to close the gap, it is optimal to increase the cash money supply. If digital money and cash are bad substitutes ($\lambda > 0.5$), on the other hand, the actual decline of $\mu_C^C$ falls short of the optimal decline, and now it is optimal to lower the cash money supply. Proposition 1 summarizes.

**Proposition 1:** Suppose that cash is fully accepted and the new currency (digital money) is partially accepted. (i) If digital money and cash are very close substitutes ($\lambda \to 0$), digital money lowers welfare. (ii) If digital money and cash are very bad substitutes ($\lambda \to 1$), a positive welfare effect requires a “strong” monetary benefit of cash. (iii) Digital money lowers the welfare-maximizing supply of cash. (iv) If digital money primarily replaces cash (goods), the welfare-maximizing response to the new currency is an increase (a decrease) in the cash money supply.
4.2 Both Currencies Fully Accepted

Our second switching scenario assumes \( \Pi_C = \Pi_D = 1 \). Full acceptance of cash requires \( V_C > V_S \), full acceptance of the digital money requires \( V_D > V_S \). Rearranging the Bellman Eqs. (1)–(3) shows that these constraints are fulfilled if and only if

\[
\begin{align*}
V_C^f > V_S^f & \Rightarrow \rho_C^f > \mu_D v_D^f \rho_D^f \\
V_D^f > V_S^f & \Rightarrow \rho_D^f > \mu_C v_C^f \rho_C^f
\end{align*}
\]  

(19) (20)

hold. Here, \( \rho_D^f \equiv \gamma_D + \mu_S^f (U - \eta_D) \) is the expected per period return of digital money, and \( v_D^f \equiv 1/r + \mu_D^f + \mu_D \). The superscript \( f \) denotes the dual currency regime with full acceptance of the new currency. If the expected per period return of cash does not exceed threshold (19), cash will no longer be fully accepted. Similarly, if the expected per period return of the digital money does not exceed threshold (20), the digital money will not be fully accepted. In other words, the existence of an equilibrium requires that

\[
\begin{align*}
\mu_C^f v_C^f - 1 < \frac{\rho_D^f - \rho_C^f}{\rho_C^f} < \frac{1}{\mu_D v_D^f} - 1
\end{align*}
\]  

(21)

holds. The relative spread between \( \rho_D^f \) and \( \rho_C^f \) must not be too big, otherwise either digital money or cash is no longer fully accepted. Kiyotaki and Wright (1993: 75) report a similar result, but they do not specify the interval.

Welfare in the regime of two fully accepted currencies can be computed as

\[
rW^f = \mu_C^f \rho_C^f + \mu_D^f \rho_D^f.
\]  

(22)

To sign the net welfare effect of the introduction of a universally accepted new currency, we have to compare (22) with (8). Again, we need a hypothesis on the replacement of sellers and cash traders by the digital money traders. We adapt Eqs. (13) and (14) by assuming \( \mu_C^f = \mu_S^f - \lambda \mu_D \) and \( \mu_C^f = \mu_C^f - (1 - \lambda) \mu_D \). The condition for a positive net welfare effect is

\[
rW^f - rW^i > 0 \quad \Rightarrow \quad -\Gamma_1 \lambda^2 + \Gamma_2 \lambda + \Gamma_3 > 0
\]  

(23)

with \( \Gamma_1 = \mu_D (U - \eta_C) \), \( \Gamma_2 = y_C + (\mu_D + \mu_S^f - \mu_C^f) (U - \eta_C) - \mu_D (U - \eta_D) \) and \( \Gamma_3 = y_D + \mu_S^f (U - \eta_D) - [y_C + \mu_S^f (U - \eta_C)]. \) Suppose digital money and cash are very close substitutes, so that digital money traders replace only cash traders, while the number of sellers remains constant, \( \lambda = 0 \). In this case, (23) boils down to \( \Gamma_3 > 0 \). The cash traders who switch status from \( C \) to \( D \) switch to a currency with the same liquidity value (acceptance rate), they gain \( y_D + \mu_S^f (U - \eta_D) \), they lose \( y_C + \mu_S^f (U - \eta_C) \). If the former exceeds the latter, the economy yields a payoff. If
digital money and cash are bad substitutes, digital money traders replace only sellers, $\lambda = 1$, and condition (23) simplifies to $\rho_D^f > \mu_C^s (U - \eta_C)$. The sellers, who switch status from S to D, gain $\rho_D^f$. But the cash traders face a loss. Since there is a smaller number of sellers, the probability of exchange and consumption declines.

Net welfare is a quadratic function in $\lambda$. Depending on $\lambda$, the sign of the net welfare effect may change. Figure 1 illustrates this, we assume $\Gamma_2 > 0$ and $\Gamma_3 = 0$. For $\lambda = 0$, digital money is neutral with respect to welfare. As $\lambda$ increases, so does the sum of cash and digital money (aggregate money supply). Therefore, an increase in $\lambda$ very much resembles an increase in money supply in the Kiyotaki and Wright (1993) framework. Endowing more agents with money facilitates exchange and improves welfare; the net welfare effect becomes positive. But endowing more agents with money is equivalent to endowing fewer agents with commodities, and consumption and welfare go down. If the replacement parameter, $\lambda$, exceeds a critical value, $\lambda_{\text{crit}} = \Gamma_2/\Gamma_1$, the net welfare effect switches the sign and turns into negative.

Two remarks are in order: First, the higher the share of cash traders in the initial equilibrium, $\mu_C$, the lower is the welfare-enhancing liquidity effect of a new currency, and the more important is the negative effect of the lower number of commodities, $\lambda_{\text{crit}}$ declines, the probability of a negative net welfare goes up. Second, $\lambda_{\text{crit}}$ may be larger than one, see the dashed line in Figure (1). In this case, we observe a net welfare gain for all $\lambda \in (0, 1]$. The welfare effects of a relaxation of the assumption $\Gamma_3 = 0$ are straightforward, in Figure 1 the net welfare curve shifts up ($\Gamma_3 > 0$) or down ($\Gamma_3 < 0$). Since there are no novel and crucial insights, we skip the discussion.

In a world with two fully accepted currencies, the welfare-maximizing share of cash traders is given by

$$\left(\mu_C^s\right)^* = \frac{1 - \mu_D}{2} + \frac{\gamma_C^s}{2(U - \eta_C)} - \frac{\mu_D(U - \eta_D)}{2(U - \eta_C)} = \left(\mu_C^s\right)^* - \frac{\mu_D(U - \eta_D)}{2(U - \eta_C)}.$$

As shown in (24), the optimal response to the introduction of a partially accepted currency is a reduction in the supply of cash (share of cash traders) by $0.5\mu_D$. If
instead digital money is fully accepted, its liquidity value is even higher, so that the decline in the optimal supply of cash is even stronger, \((\mu^* C)^* < (\mu^* C)^*\). We get

**Proposition 2:** Suppose that both cash and the new currency (digital money) are fully accepted. (i) The existence of an equilibrium requires that the relative spread between \(\rho_D\) and \(\rho_C\) fulfills (21). (ii) If digital money and cash are very close substitutes \((\lambda \rightarrow 0)\), a positive spread ensures a net welfare gain. (iii) The lower the degree of substitution between digital money and cash \((\lambda)\), the higher the probability of a negative net welfare effect. (iv) A new fully accepted currency lowers the welfare-maximizing supply of cash more than the introduction of a partially accepted currency.

5 Conclusion

Money is essential. The use of fiat money relaxes information constraints and thus promotes trade and allows for a better resource allocation. This paper shows that, for plausible parameter constellations, a secondary currency is essential too, even if the traditional currency remains in circulation. Given the fact that digital monies are on the rise, this result is noteworthy.

How should the government and/or monetary policymakers respond to this development? Most important from our point of view: Policymakers should not obstruct digital monies by, for instance, a legal ban. Such a ban probably hinders welfare improvements. Similarly, the monetary authority should accept the emergence of private providers of liquidity. The best policy response is a decline in the supply of the traditional currency. Moreover, the government should pay more attention to regulations concerning the market for payment systems. The new competitors on this market must not be a threat to payment security. Our framework is too simple to draw more far-reaching policy conclusions, and therefore extensions are necessary. However, we are at the starting point of a fruitful discussion of the economic consequences of digital monies. Two promising lines of research are the impact on financial intermediation, for an overview see Thakor (2020), and the macroeconomic consequences of a central bank digital currency, see, e.g. Barrdear and Kumhof (2022) or Fegatelli (2022).

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**Appendix A: Dynamic Structure of the Model**

The dynamic structure of our model is visualized in Figure 2.

![Figure 2: Dynamic structure.](image)

Here, $N_P$, $N_S$, $N_C$ and $N_D$ denote the proportions of the population who are producers, commodity traders (sellers), cash traders and digital money traders. Producers are no traders; we thus denote $\mu_S$, $\mu_C$ and $\mu_D$ as proportions of traders who are commodity traders (sellers), cash traders and digital money traders. A steady state (flow equilibrium) requires an equal flow out of and into a knot. For producers, cash traders and digital money traders we get:

$$a N_P = \mu_S \Pi_C N_C + \mu_S \Pi_D N_D$$  \hspace{1cm} (A1)

$$\mu_S \Pi_C N_C = \mu_C \Pi_C N_S$$  \hspace{1cm} (A2)

$$\mu_S \Pi_D N_D = \mu_D \Pi_D N_S,$$  \hspace{1cm} (A3)

where $\Pi_C$ ($\Pi_D$) is the overall acceptance of cash (digital money). Thus, the inflow to commodity traders, $a N_P$, has to be equal to the outflow, $\mu_S \Pi_C N_C + \mu_S \Pi_D N_D$, see (A1). Things are analogous for cash traders and digital money holders, see (A2) and (A3). Observing $N_P + N_S + N_C + N_D = 1$ as well as $\mu_S + \mu_C + \mu_D = 1$, Eqs. (A1)–(A3) deliver

$$N_P = 1 - \frac{a}{a + \mu_S \mu_C \Pi_C + \mu_S \mu_D \Pi_D}.$$  \hspace{1cm} (A4)
As mentioned in the text, we focus on the limiting case, \( \alpha \to \infty \), so that production is instantaneous, and the equilibrium number of producers approaches zero, \( N_P = 0 \). It immediately follows that \( N_S = \mu_S \), \( N_C = \mu_C \) and \( N_D = \mu_D \).

As also mentioned in the text, we define welfare by the expected utility of all agents before the initial endowment is randomly distributed among them:

\[
W = N_P V_P + N_S V_S + N_C V_C + N_D V_D.
\]

Inserting our results leads to Eq. (4), where the welfare criterion is expressed in terms of expected flow returns.

In order to compare these shares with their analogues in a single currency regime, we redo the analysis with \( N_D = \mu_D = 0 \). This delivers \( N_S^s = 0 \), \( N_S^s = \mu_S^s \) and \( N_C^s = \mu_C^s \), where the superscript \( s \) stands for single currency regime. The link between the shares in the single and the dual currency regime (with partial acceptance of digital money) is given by (13) and (14).

References


