Research Article

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Numerically statistical investigation of the partly super-exponential growth rate in the COVID-19 pandemic (throughout the world)

<https://doi.org/10.1515/jiip-2020-0043>

Received April 13, 2020; accepted June 21, 2020

Abstract: A number of particles in a multiplying medium under rather general conditions is asymptotically exponential with respect to time t with the parameter λ , i.e., with the index of power λt . If the medium is random, then the parameter λ is the random variable. To estimate the temporal asymptotics of the mean particles number (via the medium realizations), it is possible to average the exponential function via the corresponding distribution. Assuming that this distribution is Gaussian, the super-exponential estimate of the mean particle number could be obtained and expressed by the exponent with the index of power $tE\lambda + t^2 D\frac{\lambda}{2}$. The application of this new formula to investigation of the COVID-19 pandemic is performed.

Keywords: The novel Coronavirus pandemic, COVID-19, statistical investigation

MSC 2010: 65C05

In [1] the multiplication process of the particles in a random medium σ with asymptotically exponential (via the time t) number $n(t, \sigma) \approx C(\sigma) \exp(\lambda(\sigma)t)$ was considered. It was assumed that the distribution of random variable $\lambda(\sigma)$ is Gaussian, the limiting process $n(t, \sigma) \xrightarrow{t \rightarrow \infty} C(\sigma) \exp(\lambda(\sigma)t)$ is homogeneous over σ and the quantities $C(\sigma)$, $\exp(\lambda(\sigma)t)$ are asymptotically weakly correlated. On this basis the following estimate of the function $En(t, \sigma) = n(t)$ was obtained

$$n(t) \approx \frac{C}{\sqrt{2\pi d}} \int_{-\infty}^{\infty} \exp(tx) \exp\left(-\frac{(x-a)^2}{2d^2}\right) dx,$$

where $a = E\lambda(\sigma)$ is the mathematical expectation and $d^2 = D\lambda(\sigma)$ is the variance of random λ . By the use of [2, integral formula (2.3.15), No. 11] the following expression was obtained:

$$n(t) \approx C \exp\left(\frac{a^2}{2d^2}\right) \exp\left(\frac{d^2}{2}t^2 + at\right). \quad (1)$$

Note that formula (1) may be the basis for numerical investigations of concrete problems. The law of increase, corresponding to formula (1), may be naturally called “super-exponential”. More total and practically convenient definition of super-exponentiality is related to the growth of the *increase coefficient* $\alpha(t) = \frac{n(t+\Delta t)}{n(t)}$ while t rises.

In real problems the relation $-\infty < \lambda_1 < \lambda(\sigma) < \lambda_2 < +\infty$ holds. Therefore, the obtained asymptotics possibly approximates $n(t)$ only in the interval $0 < T_1 \leq t \leq T_2 < +\infty$. So, when solving real problems, it is expedient to perform additional numerical investigations. Similar investigations were fulfilled in [1] for the special stochastic model of particles multiplication process.

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Date (2020 year)	9.03	12.03	15.03	18.03	21.03
WHO data	109577	125260	153517	191127	266073
Approximation (2)	109577	125757	152239	194400	261845

Table 1: WHO statistical data and approximation (2).

Figure 1: The number of confirmed cases of COVID-19 in the world (circles) and approximation by the formula (2) (solid curve)

The above results show that growth rate of mean number of the arbitrary nature particles (for example, microorganisms) may be super-exponential. This behavior corresponds to the rise of the increase coefficient α of the particle numbers $\{n_i\}$, which are fixed at the equidistant moments $\{t_i\}$, i.e., the ratio $\alpha_i = \frac{n_{i+1}}{n_i}$ grows. On the other hand if this rise is observed, then it is possible that the distribution of breeding grounds is related with it.

In particular, the novel coronavirus pandemic in the world behaved like this according to the WHO (World Health Organization) statistics [3] in the time interval from 9.03.2020 to 21.03.2020. The corresponding number of confirmed cases (via days) is approximated as in formula (1) with the error not exceeding 2 % as

$$n_i \approx 109577 \cdot \exp\{0.002965(i-9)^2 + 0.037(i-9)\} \quad (2)$$

for $i = 9, 10, \dots, 21$. Some comparative results are given in Table 1.

Approximation (2) was obtained as follows. Let

$$n_i = n(i) = n(9) \exp(d(i-9)^2 + a(i-9))$$

and therefore

$$\ln \frac{n_{i+1}}{n_i} = d(2i-17) + a, \quad i = 9, \dots, 20.$$

Solving these equations by the least-squares method (i.e., by the linear regression method) gives us coefficients in formula (2). Comparison of the results obtained by this formula with statistical data is given in Figure 1 in logarithmic scale. Comparatively with formula (2) the rise of WHO statistics weakens over time, in particular, due to the medical activities. When $t > 21$, this growth can be exponentially approximated. This possibly happens because fluctuations of λ diminish and therefore the quantity $D\lambda$ decreases.

It is remarkable that accordingly to expression (1) formula (2) may be obtained by averaging the random number $n(t, \sigma)$, for which λ has Gaussian distribution with parameters

$$E\lambda = 0.037, \quad D\lambda = 2 \cdot 0.002965 \approx (0.077)^2.$$

So, if the estimates of the quantities $E\lambda$, $D\lambda$ for $t = t_0$ are known, then it is possible to extrapolate $n(t)$ by formula (2) significantly more accurate than those made by $n(t) = n(t_0) \exp(E\lambda(t - t_0))$.

Funding: This work was supported in part by the Russian Foundation for Basic Research, project numbers 18-01-00356 and 18-01-00599.

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