S.V. Aruna Kumar* and B.S. Harish*

A Modified Intuitionistic Fuzzy Clustering Algorithm for Medical Image Segmentation

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Abstract: This paper presents a modified intuitionistic fuzzy clustering (IFCM) algorithm for medical image segmentation. IFCM is a variant of the conventional fuzzy C-means (FCM) based on intuitionistic fuzzy set (IFS) theory. Unlike FCM, IFCM considers both membership and nonmembership values. The existing IFCM method uses Sugeno's and Yager's IFS generators to compute nonmembership value. But for certain parameters, IFS constructed using above complement generators does not satisfy the elementary condition of intuitionism. To overcome this problem, this paper adopts a new IFS generator. Further, Hausdorff distance is used as distance metric to calculate the distance between cluster center and pixel. Extensive experimentations are carried out on standard datasets like brain, lungs, liver and breast images. This paper compares the proposed method with other IFS based methods. The proposed algorithm satisfies the elementary condition of intuitionism. Further, this algorithm outperforms other methods with the use of various cluster validity functions.

Keywords: Intuitionistic fuzzy set, medical image segmentation, cluster validity, Hausdorff distance.

2010 Mathematics Subject Classification: 68U10.

1 Introduction

In clinical diagnosis, medical imaging plays a crucial role which helps in understanding and analysis of the organs. Medical imaging is a technique to create an internal image of a human body for clinical or medical purpose. Medical images are obtained by using different modalities that includes X-ray, computed tomography (CT), positron emission tomography and magnetic resonance imaging (MRI). Segmentation is an indispensable step in medical image analysis which helps in identifying appropriate therapy for abnormal changes in tissues and organs. In the literature, there are many classical image processing techniques like thresholding, region growing and merging applied for medical image segmentation. However, these methods fail to give good segmentation results because the techniques work only on pixel attributes. Many researchers have applied clustering technique to solve medical image segmentation problems.

Cluster analysis involves classifying a collection of objects into homogeneous groups, such that objects in the same group are more similar when compared to objects present in other groups. Clustering algorithms are broadly classified into hierarchical and partitional clustering [20]. Hierarchical clustering algorithm generates a hierarchical tree of clusters called dendrogram which can be either divisive or agglomerative. The partitional clustering algorithm gives a single C partition of objects, with a predefined C number of clusters. There are two well-known partition clustering approaches: k -means and fuzzy C-means (FCM). In k -means clustering, data are divided into a number of clusters where data elements belong to exactly one cluster. The k -means clustering technique works well when data elements are not overlapped. To overcome the problem of overlapping, Bezdek [5] proposed FCM clustering algorithm. In FCM clustering, data elements are clustered based on the membership values assigned to each of them. In this technique, the data elements can belong to

*Corresponding authors: S.V. Aruna Kumar and B.S. Harish, Department of Information Science and Engineering, Sri Jayachamarajendra College of Engineering, Mysuru, India, e-mail: arunkumarsv55@gmail.com (S.V. Aruna Kumar), bsharish@sjce.ac.in (B.S. Harish)

more than one cluster. In medical images, object definition is not always crisp and knowledge about objects in the image may be vague. To deal with vagueness, fuzzy logic, and fuzzy theory are ideally suited. Many researchers have developed fuzzy set theory based clustering methods to solve image segmentation problem [1, 11, 12, 21, 30, 33, 40]. Maji and Roy [27] proposed a texture based brain MRI image segmentation method. Texture features are selected based on maximum relevance and maximum significance. Further, rough fuzzy clustering has been applied for segmentation. Feng et al. [15] developed a modified FCM algorithm for MR image segmentation. This method incorporated nonlocal pixel information via Hausdroff distance metric. The objective and membership function of traditional FCM is modified with addition of Hausdroff distance function.

In medical images, uncertainty occurs in terms of vagueness in imprecise gray levels, object boundary and so on. While defining membership function, hesitation arises due to the presence of uncertainty in gray levels. Unfortunately, classical fuzzy clustering techniques fail to handle this hesitation. To handle this hesitation, Atanassov in [4] proposed another higher order fuzzy set called intuitionistic fuzzy set (IFS). IFS considers both membership (μ) and nonmembership (ν) values. In traditional fuzzy set, the nonmembership degree is equal to the complement of the membership degree, but in IFS nonmembership the degree is less than or equal to the complement of the membership degree due to the index of intuitionism or hesitation degree. IFS found various applications in areas such as logic programming, medical image processing, decision making, etc. Recently, IFS based methods have been receiving great attention from researchers, and a lot of IFS based clustering techniques have been developed [8, 9, 22, 47, 53]. Chaira in [8] has developed a novel medical image segmentation method based on intuitionistic fuzzy *C*-means algorithm. This method incorporates intuitionistic fuzzy entropy based objective function to the traditional FCM algorithm. Zhang et al. [53] proposed an IFS clustering method. This method creates intuitionistic fuzzy similarity matrix using similarity degree between two IFSs. Xu et al. [47] developed a clustering algorithm for IFS based on the concept of association matrix and equivalent association matrix. This method calculates association coefficient by considering hesitation degree. Chaira and Anand [9] developed a novel IFS approach for tumor detection in medical images. This method uses histogram thresholding to eliminate unwanted regions from clustered image. Further, the edge of the tumor is extracted. Zhao et al. [54] investigated the minimum spanning tree based intuitionistic fuzzy clustering (IFCM). Further, the same method was extended for interval-valued IFS (IVIFS). The IFCM is based on intuitionistic fuzzy equivalence matrices. As a result they need high computation time and they may fall into local optimal solution. To overcome these drawbacks, many researchers proposed variants of IFCM [42, 43, 48, 49, 55].

Xu et al. [49] developed an intuitionistic spectral clustering method. This method is based on eigen value analysis which takes less computation time and is not easy to fall into the local optimal solution. Wang et al. [42] developed an IFCM based on netting technique. The aforementioned method uses a new intuitionistic fuzzy similarity metric to construct an intuitionistic fuzzy similarity matrix. Experimentation results on numerical example reveals that this method takes less computation time than traditional IFCM. Wang et al. [43] developed a direct method based IFCM. This method clusters input data based on the intuitionistic fuzzy triangular product and square product. Zhao et al. [55] developed a new IFCM using λ cutting matrix of a similarity matrix. Further, the same method was extended for IVIFS. Lin [22] proposed a novel evolutionary kernel intuitionistic fuzzy *C*-means (EKIFCM) clustering algorithm. EKIFCM combines IFS with kernel fuzzy *C*-means and genetic algorithm. A genetic algorithm is used to select the parameters of EKIFCM. Chaudhuri [10] proposed an intuitionistic fuzzy possibilistic *C*-means clustering algorithm. This algorithm combines fuzzy possibilistic *C*-means with the IFS. Further, this method is extended to cluster interval valued IFSs. Huang et al. [17] proposed a novel hybrid model by combining neighborhood intuitionistic fuzzy *C*-means and genetic algorithm. This method uses neighborhood membership to reduce the noise/outlier influence. The optimal parameters are selected using genetic algorithm.

In the past decade, granular computing has emerged as a new research topic and has been widely applied for clustering, data mining and other fields. Granular computing is motivated by human problem solving strategies, and it deals with the information granules derived from underlying complex data. Recently, many researchers developed various clustering methods based on granular computing theory [2, 3, 13, 14, 23–26, 28,

29, 32, 34, 35, 44, 46, 51]. Medical images are not equally and well illuminated. IFS deals with the uncertainty in the form of hesitation degree. So the results obtained using IFS based methods are better than the fuzzy methods. In recent days, IFS is being explored for more accurate segmentation results.

The existing IFS based method uses Sugeno and Terano's [36] and Yager's [50] complement generators to construct the IFS. If $A = \{x_1, x_2, \dots, x_n\}$ is a finite set, then IFS A in a universe of discourse X is represented mathematically as $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$. The complement generators are defined as follows:

(i) Sugeno and Terano's negation [36]

$$c_\lambda(x) = \frac{1 - \mu_A(x)}{1 + \lambda \mu_A(x)}; \quad \lambda > 0 \quad (1)$$

(ii) Yager's negation [50]

$$c_\lambda(x) = (1 - \mu_A(x)^\lambda)^{\frac{1}{\lambda}}; \quad \lambda > 0 \quad (2)$$

where λ is constant. Ref. [4] defines that IFS should satisfy the elementary condition of intuitionism, i.e. the index of intuitionism or hesitation degree which is obtained by subtracting the sum of the membership and nonmembership from 1 should be positive and less than or equal to 1. However, while considering the above complement generators, for certain values of λ , the index of intuitionism or hesitation degree becomes negative. Therefore, these complement generators fail to satisfy the elementary condition given by [4]. To overcome this problem, Bustince et al. [7] developed a new intuitionistic fuzzy generator which satisfies the elementary condition.

In soft clustering methods, the membership value is computed based on a distance function. So distance metric plays an important role. In the literature, many distance metrics are proposed for IFS [37–39]. Hung and Yang [18] developed similarity measures of IFSs based on Hausdorff distance. Experimental results show that Hausdorff distance is simple and works better than other distance metrics. Hence, in this paper, our motivation is to take advantage of IFS and Hausdorff distance to increase the segmentation accuracy. In this paper, we are proposing a medical image segmentation algorithm using modified intuitionistic fuzzy C -means.

In summary, the main contributions of this paper are the following:

- We presented a modified IFCM algorithm for medical image segmentation.
- To overcome the drawback of existing IFS generators, we used Bustince IFS generator.
- We used modified Hausdorff distance measure to compute the distance between cluster center and pixel value.
- We conducted experiments on standard MRI brain image dataset and our own created dataset of CT scan of lung tumor images, CT scan of liver images and MRI breast images.
- We validated the proposed method using four cluster validity functions.

The rest of the paper is organized as follows: Section 2 gives background information about IFS and Hausdorff distance. Section 3 presents the proposed method. The datasets used for experimentation and experimental results are presented in Section 4. Conclusion and future scope are presented in Section 5.

2 Background

2.1 Intuitionistic Fuzzy Set

Fuzzy set theory was introduced by Zadeh in 1965 [52]. The fuzzy set was designed to mathematically represent uncertainty and vagueness. A fuzzy set is mainly characterized by membership values of elements.

The values of membership vary between 0 and 1. The nonmembership value is calculated as $1 - \text{membership value}$. However, in reality because of uncertainty, the nonmembership is not always equal to $1 - \text{membership value}$. To deal with this uncertainty, Atanassov proposed another higher order fuzzy set called IFS [4]. An IFS $\tilde{A}$ in X is given by

$$\tilde{A} = \{ \langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) \rangle \mid x \in X \} \quad (3)$$

where X is universe of discourse and $\mu_{\tilde{A}}(x): X \rightarrow [0, 1]$, $\nu_{\tilde{A}}(x): X \rightarrow [0, 1]$ with the condition $0 \leq \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1$; $\forall x \in X$ and $\mu_{\tilde{A}}(x)$, $\nu_{\tilde{A}}(x)$ denote membership and nonmembership degree, respectively.

For each IFS $\tilde{A}$ in X , the hesitation degree should be considered. The hesitation degree of an element $x \in X$ is defined as $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x)$, where $\pi_{\tilde{A}}(x)$ is hesitation degree and should satisfy the elementary condition of intuitionism, i.e. $0 \leq \pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x) \leq 1$. In the literature, two fuzzy complements or IFS generators are used to construct intuitionistic fuzzy set: Sugeno's [shown in equation (1)] and Yager's [shown in equation (2)]. The fuzzy complement function is defined as

$$N(\mu(x)) = g^{-1}(g(1) - g(\mu(x))) \quad (4)$$

where $g(\cdot)$ is an increasing function and $g: [0, 1] \rightarrow [0, 1]$.

Yager's class can be generated by using the following function:

$$g(x) = x^\lambda \quad (5)$$

So Yager's intuitionistic fuzzy complement is written as

$$N(x) = (1 - x^\lambda)^{\frac{1}{\lambda}}; \quad \lambda > 0 \quad (6)$$

Nonmembership values are calculated from Yager's intuitionistic fuzzy complement $N(x)$. The IFSs using Yager's intuitionistic fuzzy complement become

$$\tilde{A}_\lambda^{\text{IFS}} = \left\{ x, \mu_{\tilde{A}}(x), (1 - \mu_{\tilde{A}}(x)^\lambda)^{\frac{1}{\lambda}} \mid x \in X \right\} \quad (7)$$

Sugeno's class can be generated by using the following function:

$$g(x) = \frac{x(1 + \lambda)}{1 + \lambda x} \quad (8)$$

So Sugeno's intuitionistic fuzzy complement is written as

$$N(x) = \frac{1 - x}{1 + \lambda x}; \quad \lambda > 0 \quad (9)$$

Nonmembership values are calculated from Sugeno's intuitionistic fuzzy complement $N(x)$. IFS constructed using Sugeno's fuzzy complement is as follows:

$$\tilde{A}_\lambda^{\text{IFS}} = \left\{ x, \mu_{\tilde{A}}(x), \frac{1 - \mu_{\tilde{A}}(x)}{1 + \lambda \mu_{\tilde{A}}(x)} \mid x \in X \right\} \quad (10)$$

2.2 Hausdorff Distance

Hausdorff distance [18] is a measure of the maximum of the minimum distance between two sets of objects. The problem of determining MAX – MIN distance over two sets may arise in spatial problems that require a similarity measure between two point sets. Hausdorff distance measures how much two nonempty compact

sets X and Y in a metric space U resemble each other with respect to their positions. Let us consider a real space $\Re$. For any two intervals $X=[x_1, x_2]$ and $Y=[y_1, y_2]$, Hausdorff distance $H(X, Y)$ is given by

$$H(X, Y) = \max\{|x_1 - y_1|, |x_2 - y_2|\} \quad (11)$$

Let $\tilde{A}$ and $\tilde{B}$ be two IFSs in X and let $I_{\tilde{A}}(x_i)$ and $I_{\tilde{B}}(x_i)$ be sub-intervals on $[0, 1]$ denoted by

$$\begin{aligned} I_{\tilde{A}}(x_i) &= [\mu_{\tilde{A}}(x_i), 1 - v_{\tilde{A}}(x_i)] \\ I_{\tilde{B}}(x_i) &= [\mu_{\tilde{B}}(x_i), 1 - v_{\tilde{B}}(x_i)], \quad i = 1, 2, \dots, n. \end{aligned}$$

The Hausdorff distance between $\tilde{A}$ and $\tilde{B}$ is defined as

$$d_H(\tilde{A}, \tilde{B}) = \frac{1}{n} \sum_{i=1}^n H(I_{\tilde{A}}(x_i), I_{\tilde{B}}(x_i)) \quad (12)$$

3 Proposed Method

The proposed method consists of two steps: intuitionistic representation of image and IFCM.

3.1 Intuitionistic Representation of Image

In images, uncertainty arises in terms of vagueness due to imprecise pixel intensity values. To handle uncertainty, we converted input image into IFS representation using IFS complement generator. If we consider an image $X = \langle x_1, x_2, \dots, x_N \rangle$ consisting of N pixels having an intensity level between 0 and $L - 1$, IFS representation of the image can be given as

$$I = \{(x_{ij}, \mu_I(x_{ij}), v_I(x_{ij}), \pi_I(x_{ij}))\} \quad (13)$$

where $\mu_I(x_{ij})$ is membership value, $v_I(x_{ij})$ is nonmembership value and $\pi_I(x_{ij})$ is hesitation degree of the pixel x_{ij} . In an image, each pixel is associated with intensity value. To convert intensity values into membership values, we calculate normalized intensity level for each pixel, i.e.

$$\mu_I(x_{ij}) = \frac{x_{ij}}{L-1} \quad (14)$$

Generally, IFS is constructed using an intuitionistic complement generator. In the literature, researchers employed two intuitionistic complement generators i.e. Sugeno's and Yager's. Unfortunately, these generators fail to satisfy the elementary condition of intuitionism. To address this problem, in this paper, we have used intuitionistic complement generator which was proposed by Bustince et al. [7]. The Bustince IFS generator is defined as

$$\phi(I) = \{\langle x_{ij}, \phi(1 - v_I(x_{ij})), 1 - \phi(\mu_I(x_{ij})) \rangle\} \quad (15)$$

where, $\phi(x)$ denotes Sugeno's or Yager's generator. The Bustince fuzzy complement is written as

$$v_I(x_{ij}) = 1 - \phi(\mu_I(x_{ij})) \quad (16)$$

Since, compared to Sugeno's generator Yager's generator gives better results [8], in this paper we used Yager's generator to construct IFS. Therefore, the fuzzy complement becomes

$$v_I(x_{ij}) = 1 - ((1 - \mu_I(x_{ij}))^\lambda)^{\frac{1}{\lambda}}, \quad \lambda > 0 \quad (17)$$

Thus, after applying Bustince IFS generator, IFS image becomes

$$I_{\lambda}^{IFS} = \{(x_{ij}, \mu_I(x_{ij}), 1 - ((1 - \mu_I(x_{ij})^{\lambda})^{\frac{1}{\lambda}}), \pi_I(x_{ij}))\} \quad (18)$$

The hesitation degree of pixel is calculated as

$$\pi_I(x_{ij}) = 1 - \mu_I(x_{ij}) - (1 - ((1 - \mu_I(x_{ij})^{\lambda})^{\frac{1}{\lambda}})) \quad (19)$$

After converting input image into IFS, each pixel is associated with three values, i.e. membership value $\mu_I(x_{ij})$, nonmembership value $\nu_I(x_{ij})$ and hesitation degree $\pi_I(x_{ij})$. Further, the IFS converted image is clustered by applying IFCM algorithm.

3.2 Modified Intuitionistic Fuzzy Clustering

A modified IFCM algorithm for segmenting medical images is used in the present study. The proposed method clusters feature vectors by searching for local minima with the following objective function:

$$\min J_m = \sum_{j=1}^p \sum_{i=1}^C \mu_{ij}^m d(A_j, V_i) \quad (20)$$

where $A = \{A_1, A_2, \dots, A_p\}$ are p IFSs each with n elements, C is the number of clusters and $V = \{V_1, V_2, \dots, V_C\}$ are cluster centers. m is fuzzy coefficient value. μ_{ij} is membership value of the j^{th} pixel to i^{th} cluster. d is distance between cluster center (V_i) and IFS of pixel values (A). This membership value μ_{ij} satisfies the following condition:

$$\sum_{i=1}^C \mu_{ij} = 1, 1 \leq j \leq p \quad (21)$$

$$\mu_{ij} \geq 0, 1 \leq i \leq C, 1 \leq j \leq p \quad (22)$$

$$\sum_{j=1}^p \mu_{ij} > 0, 1 \leq i \leq C \quad (23)$$

If we solve optimization problem in equation (20) by employing Lagrange multiplier method [19], membership value becomes

$$\mu_{ij} = \frac{1}{\sum_{k=1}^c \left(\frac{d(A_j, V_i)}{d(A_j, V_k)} \right)^{\frac{2}{m-1}}} \quad (24)$$

Cluster center V_i which is associated with three values ($\mu_{V_i}(x_k)$), $\nu_{V_i}(x_k)$ and $\pi_{V_i}(x_k)$) is calculated using the following equations:

$$\mu_{V_i}(x_k) = \frac{\sum_{j=1}^p u_{ij}^m \mu_{A_j}(x_k)}{\sum_{j=1}^p u_{ij}^m}, 1 \leq i \leq c, 1 \leq k \leq n \quad (25)$$

$$v_{V_i}(x_k) = \frac{\sum_{j=1}^p u_{ij}^m v_{A_j}(x_k)}{\sum_{j=1}^p u_{ij}^m}, \quad 1 \leq i \leq c, \quad 1 \leq k \leq n \quad (26)$$

$$\pi_{V_i}(x_k) = \frac{\sum_{j=1}^p u_{ij}^m \pi_{A_j}(x_k)}{\sum_{j=1}^p u_{ij}^m}, \quad 1 \leq i \leq c, \quad 1 \leq k \leq n \quad (27)$$

The performance of clustering algorithm directly depends on distance metric. The existing IFCM methods use euclidean distance metric. This metric fails to give good results for nonspherical input data. To overcome this drawback, we used modified Hausdorff distance. Unlike existing Hausdorff distance metric, modified Hausdorff distance considers hesitation degree along with membership and nonmembership value. Modified Hausdorff distance is defined as follows:

$$d_H(A_j, V_i) = \max\{|\mu_{A_j} - \mu_{V_i}|, |v_{A_j} - v_{V_i}|, |\pi_{A_j} - \pi_{V_i}|\} \quad (28)$$

In this paper, we used modified Hausdorff distance function to calculate the distance between cluster center and data points. We replace the distance function d in equations (20) and (24) by equation (28). After converting input image into intuitionistic fuzzy set representation, we initialize the cluster centers randomly and perform clustering algorithm. At each iteration, the cluster centers and membership value are updated using equations (24)–(27). Algorithm stops when $\{J(i) - J(i-1)\} < \text{stopping criteria}$. To obtain the segmented image, pixels are assigned to a cluster based on maximum membership value. The individual steps of the proposed method are given in Algorithm 1.

Algorithm 1: Proposed Method

Data: Input image, initial cluster centers V_0 , m, λ , stopping criteria

Result: Membership function μ , cluster centers V , objective function J

Step 1: Convert input image into IFS image

Step 2: Perform modified intuitionistic fuzzy clustering on IFS image:

Repeat

- Compute membership values μ according to equation (24);
- Calculate objective function J value according to equation (20);
- Update cluster center value according to equations (25)–(27);

until $\{J(i) - J(i-1)\} < \text{stopping criteria}$;

Step 3: Assign pixel to a cluster for which it has maximum membership value.

4 Experimental Results

To demonstrate the effectiveness of proposed method, we performed experiments on medical images from different modalities including standard MRI Brain images and our own created datasets that include CT scan of lung tumor, CT scan of liver and MRI breast images.

We compared our proposed method (Alg6) with all five algorithms listed in Table 1. The algorithm uses two IFS generators and two distance metrics. For example, Alg1 uses Sugeno's IFS generator and euclidean distance metric. In this paper, for all algorithms in comparison, we set fuzzy coefficient m to widely used value 2. All the cluster centers are initialized randomly. We set stopping criterion ε to 0.0001. All algorithms were simulated using matlab2013.

Table 1: Algorithms for Comparison.

Algorithm	IFS generator	Distance metric
Alg1	Sugeno's	Euclidean
Alg2	Yager's	Euclidean
Alg3	Bustince	Euclidean
Alg4	Sugeno's	Hausdorff
Alg5	Yager's	Hausdorff
Alg6	Bustince's	Hausdorff

The bold values indicate the results obtained using proposed method.

Performance of the proposed method was evaluated using cluster validity indices. Wide varieties of cluster validity indices are present in the literature [41]. In this study, we have used four well-known cluster validity functions: partition coefficient (V_{pc}), partition entropy (V_{pe}), Fukuyama-Sugeno function (V_{fs}) and Xie-Beni function (V_{xb}). Bezdek [5] proposed partition coefficient (V_{pc}) and partition entropy (V_{pe}), which uses only membership values to calculate the cluster validity value. Equations (29) and (30) show the V_{pc} and V_{pe} :

$$V_{pc}(U) = \frac{\sum_{j=1}^n \sum_{i=1}^C \mu_{ij}^m}{n} \quad (29)$$

$$V_{pe}(U) = \frac{1}{n} \sum_{j=1}^n \sum_{i=1}^C \mu_{ij}^m \log \mu_{ij} \quad (30)$$

The value of V_{pc} varies between $\left[\frac{1}{C}, 1\right]$ where C indicates the number of clusters. The value of V_{pe} ranges between $[0, \log_a C]$ where C is the number of cluster and a is the base of the logarithm. When V_{pc} is maximum or V_{pe} is minimum, optimal clusters are achieved. The third cluster validity function used is Fukuyama-Sugeno (V_{fs}) [16] which is given by

$$V_{fs}(U, V; X) = \sum_{i=1}^C \sum_{j=1}^n \mu_{ij}^m (\|x_j - v_i\|^2 - \|v_i - \bar{v}\|^2) \quad (31)$$

where $\bar{v} = \frac{1}{C} \sum_{i=1}^C v_i$. V_{fs} uses both the membership information and input data. When V_{fs} value is minimum, better clustering results are achieved. The fourth function is Xie-Beni (V_{xb}) which was initially proposed by Xie and Beni (XB) [45] and modified by Pal and Bezdek [31]. It is defined as

$$V_{xb}(U) = \frac{\sum_{i=1}^C \sum_{j=1}^n \mu_{ij}^m \|x_j - v_i\|^2}{n(\min_{i \neq k} \{\|v_i - v_k\|^2\})} \quad (32)$$

In V_{xb} , the numerator indicates compactness of the fuzzy partition and the denominator indicates strength of the separation between clusters. When V_{xb} is minimum, better clustering results are achieved.

4.1 Illustration

To demonstrate the effectiveness of the proposed method, first we conducted experiments on a real-world dataset (i.e. car dataset). Car dataset contains information of 10 new cars to be classified in Guangzhou car market in Guangdong, China [47]. Each car is described by six attributes: (1) K_1 : fuel economy; (2) K_2 : aerod degree; (3) K_3 : price; (4) K_4 : comfort; (5) K_5 : design; and (6) K_6 : safety. The membership values associated with characteristics of 10 new cars under six attributes are presented in Table 2.

Table 2: Attributes Used to Describe Car Dataset.

	K_1	K_2	K_3	K_4	K_5	K_6
car_1	0.30	0.20	0.40	0.80	0.40	0.20
car_2	0.40	0.500	0.600	0.200	0.300	0.700
car_3	0.40	0.60	0.80	0.20	0.30	0.50
car_4	0.30	0.90	0.80	0.70	0.10	0.20
car_5	0.80	0.70	0.70	0.40	0.80	0.40
car_6	0.40	0.30	0.20	0.70	0.50	0.30
car_7	0.60	0.40	0.70	0.30	0.30	0.60
car_8	0.90	0.70	0.70	0.40	0.40	0.80
car_9	0.40	1	0.90	0.60	0.20	0.10
car_{10}	0.90	0.80	0.60	0.50	0.80	0.60

We compute nonmembership value of each attribute using Sugeno's, Yager's and Bustince IFS generators and perform the clustering task. We have varied the λ values between 0.2 and 10 with a uniform increment of 0.2. From experimentation, we observed that variations in the results are found only when $\lambda = 0.7$ and $\lambda = 2$. Table 3 presents the values of validity indices for all six algorithms with $\lambda = 0.7$ and $\lambda = 2$. From Table 3, we can observe that when λ value is set to 2, our proposed method outperforms other methods.

4.2 Experiments on Medical Images

From illustrations presented in the previous section, we can observe that our proposed method outperforms the existing methods. In this section, we conducted experiments specifically on medical images from different modalities including MRI brain, CT scan of lung, CT scan of liver and MRI breast images. The MRI image of brain chosen for the experiment is available in three bands: T1-weighted, proton density (pd)-weighted and T2-weighted. Normal brain images are obtained from Brain-web database [6]. In this paper, we use transversal slice map with slice thickness of 1 mm and size of 217×181 pixels. Further, we have created our own dataset of lung, liver and breast images. Our dataset consists of 50 different lung images, 30 different liver images and 50 MRI breast images.

Since Alg1 and Alg4, Alg2 and Alg5, and Alg3 and Alg6 use the same IFS generator, human perception fails to notice the difference in segmentation results. So we presented the results only for Alg1, Alg2 and Alg6. Figures 1–3 show the segmentation results of the Alg1, Alg2 and Alg6, respectively.

We varied λ value from 0.1 to 30 with a uniform increment of 0.1. Since results are not varied in between values, we recorded the results only for those values which give different results. From experiments, we

Table 3: Values of Validity Indices for Car Dataset.

λ Value	Method	V_{pc}	V_{pe}	$V_{fs} [-1 \times 10^4]$	V_{xb}
0.7	Alg1	0.921	0.183	1.443	0.137
	Alg2	0.912	0.196	2.130	0.352
	Alg3	0.931	0.171	2.617	0.201
	Alg4	0.892	0.181	1.920	0.131
	Alg5	0.901	0.123	1.210	0.182
	Alg6	0.931	0.171	3.681	0.096
2	Alg1	0.710	0.382	0.617	0.162
	Alg2	0.612	0.582	2.784	0.312
	Alg3	0.893	0.182	1.013	0.121
	Alg4	0.623	0.521	0.431	0.234
	Alg5	0.640	0.451	1.384	0.156
	Alg6	0.946	0.163	2.562	0.118

The bold values indicate the results obtained using proposed method.

Figure 1: Segmentation Result of Alg1 With Different λ Values on Brain Image With Cluster Number = 4 (A) Input Image (B) $\lambda = 0.5$, (C) $\lambda = 1$, (D) $\lambda = 5$.

Figure 2: Segmentation Result of Alg2 With Different λ Values on Brain Image With Cluster Number = 4 (A) Input Image (B) $\lambda = 0.5$, (C) $\lambda = 1$, (D) $\lambda = 5$.

observe that there are no variations in the results when $\lambda > 30$. Table 4 presents a performance comparison of the proposed method with other methods in terms of cluster validity functions for brain image. The proposed method achieves good results when $\lambda = 30$. Table 5 presents a performance comparison of lung images. For lung images, when $\lambda = 10$ the proposed method outperforms other methods. Table 6 presents a performance comparison of liver images. The proposed method outperforms other methods when $\lambda = 25$. Table 7 presents performance comparison of breast images. The proposed method outperforms other methods when $\lambda = 5$.

4.3 Discussion

In Figures 1 and 2 we can observe that Alg1 and Alg2 segment the input image properly only when $\lambda < 1$. This is because when $\lambda > 1$, Alg1 and Alg2 fail to satisfy the elementary condition of intuitionism. By observing Figures 1–3, we can say that our proposed method outperforms other methods. When $\lambda = 1$ the proposed method turns into traditional fuzzy clustering. In Tables 4–7, Alg1 and Alg2 have optimal values than the proposed method when $\lambda > 1$. However, these methods fail to satisfy the elementary condition of intuitionism when $\lambda > 1$. From experiments, we observed that when λ value > 1 , our proposed method outperforms other methods and satisfies the elementary condition of intuitionism.

Figure 3: Segmentation Result of Alg6 With Different λ Values on Brain Image With Cluster Number=4 (A) Input Image (B) $\lambda=0.5$, (C) $\lambda=1$, (D) $\lambda=5$.

Table 4: Performance Comparison for Brain Images.

Brain	Method	λ										
		0.2	0.5	0.9	1	2	5	10	15	20	25	30
V_{pc}	Alg1	0.860	0.854	0.863	0.865	0.862	0.849	0.906	0.919	0.932	0.962	0.972
	Alg2	0.860	0.865	0.868	0.868	0.870	0.932	0.950	0.955	0.958	0.96	0.983
	Alg3	0.870	0.861	0.859	0.855	0.835	0.846	0.853	0.856	0.859	0.866	0.869
	Alg4	0.866	0.854	0.863	0.865	0.862	0.849	0.906	0.919	0.932	0.962	0.972
	Alg5	0.860	0.865	0.868	0.865	0.870	0.932	0.950	0.955	0.961	0.980	0.981
	Alg6	0.870	0.861	0.859	0.855	0.835	0.846	0.856	0.889	0.887	0.892	0.894
V_{pe}	Alg1	0.206	0.186	0.241	0.240	0.255	0.277	0.174	0.145	0.121	0.064	0.047
	Alg2	0.265	0.255	0.247	0.246	0.239	0.137	0.100	0.088	0.082	0.078	0.037
	Alg3	0.242	0.259	0.268	0.277	0.314	0.290	0.279	0.272	0.267	0.254	0.246
	Alg4	0.206	0.186	0.241	0.240	0.255	0.277	0.174	0.145	0.121	0.064	0.047
	Alg5	0.265	0.255	0.247	0.246	0.229	0.137	0.100	0.088	0.082	0.043	0.076
	Alg6	0.242	0.259	0.260	0.270	0.314	0.292	0.279	0.272	0.116	0.124	0.101
$V_{fs} [-1 \times 10^4]$	Alg1	0.097	1.822	2.273	2.009	0.567	0.046	0.003	0.002	0.004	2.147	2.167
	Alg2	2.235	2.420	2.596	2.631	2.875	3.779	3.873	3.857	3.837	3.822	4.516
	Alg3	2.932	2.717	2.214	2.07	1.301	1.038	1.031	1.052	1.067	1.123	1.168
	Alg4	2.048	1.910	1.136	1.004	0.283	0.023	0.002	0.003	0.003	1.073	1.088
	Alg5	1.117	1.210	1.298	1.315	1.437	1.889	1.936	1.928	1.918	2.274	1.905
	Alg6	1.467	3.358	1.107	3.036	2.650	1.519	3.517	2.524	4.530	1.561	4.584
V_{xb}	Alg1	0.004	0.081	0.306	0.278	0.160	0.215	0.277	1.114	4.12	0.158	0.160
	Alg2	0.218	0.238	0.266	0.273	0.356	0.093	0.079	0.084	0.096	0.118	0.032
	Alg3	0.148	0.167	0.192	0.208	0.237	0.127	0.106	0.102	0.099	0.088	0.081
	Alg4	0.002	0.044	0.153	0.139	0.080	0.108	0.138	0.559	2.105	0.079	0.080
	Alg5	0.109	0.119	0.133	0.136	0.178	0.046	0.039	0.042	0.048	0.026	0.065
	Alg6	0.074	0.083	0.096	0.104	0.118	0.063	0.053	0.052	0.049	0.044	0.040

The bold values indicate the results obtained using proposed method.

5 Conclusion and Future Scope

Fuzzy C-means is a well-known clustering algorithm which is widely applied for segmenting medical images. FCM is based on fuzzy set theory. But sometimes uncertainty arises due to manual error in defining membership function. IFS deals with this uncertainty by using hesitation degree. In this paper, we proposed a

Table 5: Performance Comparison for Lung Images.

Lung	Method	λ										
		0.2	0.5	0.9	1	2	5	10	15	20	25	30
V_{pc}	Alg1	0.881	0.871	0.884	0.876	0.873	0.880	0.917	0.948	0.963	0.971	0.976
	Alg2	0.888	0.887	0.874	0.873	0.873	0.875	0.876	0.877	0.878	0.894	0.870
	Alg3	0.876	0.864	0.875	0.876	0.861	0.916	0.941	0.840	0.835	0.923	0.924
	Alg4	0.881	0.881	0.884	0.876	0.873	0.880	0.917	0.948	0.971	0.977	0.963
	Alg5	0.887	0.877	0.874	0.873	0.8736	0.8750	0.875	0.876	0.877	0.877	0.878
	Alg6	0.876	0.864	0.890	0.876	0.861	0.916	0.941	0.840	0.835	0.923	0.924
V_{pe}	Alg1	0.208	0.212	0.220	0.229	0.230	0.200	0.143	0.090	0.065	0.050	0.041
	Alg2	0.265	0.255	0.247	0.246	0.239	0.137	0.100	0.088	0.082	0.078	0.037
	Alg3	0.229	0.252	0.231	0.229	0.2520	0.1700	0.125	0.287	0.299	0.154	0.147
	Alg4	0.209	0.212	0.220	0.229	0.230	0.207	0.143	0.092	0.050	0.040	0.065
	Alg5	0.217	0.229	0.231	0.231	0.229	0.225	0.223	0.224	0.226	0.222	0.221
	Alg6	0.229	0.252	0.213	0.229	0.252	0.170	0.125	0.287	0.299	0.154	0.147
$V_{fs} [-1 \times 10^4]$	Alg1	0.060	0.377	1.685	0.187	1.675	2.197	2.454	2.513	2.510	2.496	2.481
	Alg2	1.717	1.633	1.484	1.446	1.118	0.586	0.274	0.159	0.104	0.073	0.054
	Alg3	0.939	1.086	1.769	1.879	1.723	1.910	2.126	0.836	0.815	1.341	1.520
	Alg4	0.033	0.188	0.842	0.939	0.834	1.098	1.227	1.256	1.248	1.240	1.255
	Alg5	0.869	0.816	0.742	0.723	0.594	0.293	0.374	0.796	0.519	0.365	0.271
	Alg6	1.469	1.542	1.857	1.939	1.861	1.955	2.363	1.418	1.407	1.670	1.760
V_{xb}	Alg1	2.202	0.281	0.317	0.478	0.220	0.245	0.165	0.094	0.064	0.048	0.039
	Alg2	0.257	0.395	0.420	0.418	0.379	0.317	0.289	0.281	0.792	0.279	0.281
	Alg3	0.239	0.429	0.405	0.478	1.010	1.250	0.900	1.170	1.110	0.251	0.152
	Alg4	2.835	0.140	0.158	0.239	0.110	0.122	0.082	0.032	0.024	0.019	0.032
	Alg5	0.129	0.198	0.210	0.209	0.189	0.158	0.144	0.140	0.139	0.139	0.140
	Alg6	0.119	0.113	0.107	0.139	0.150	0.162	0.094	0.158	0.055	0.102	0.076

The bold values indicate the results obtained using proposed method.

Table 6: Performance Comparison for Liver Images.

Liver	Method	λ										
		0.2	0.5	0.9	1	2	5	10	15	20	25	30
V_{pc}	Alg1	0.896	0.915	0.918	0.915	0.893	0.892	0.937	0.990	0.980	0.970	0.960
	Alg2	0.927	0.929	0.930	0.931	0.9320	0.930	0.934	0.935	0.934	0.932	0.933
	Alg3	0.930	0.930	0.927	0.925	0.920	0.924	0.925	0.923	0.954	0.926	0.924
	Alg4	0.996	0.935	0.928	0.925	0.893	0.893	0.936	0.998	0.999	0.945	0.956
	Alg5	0.874	0.877	0.870	0.873	0.873	0.8750	0.876	0.877	0.877	0.878	0.873
	Alg6	0.930	0.930	0.927	0.925	0.920	0.924	0.925	0.925	0.925	0.938	0.926
V_{pe}	Alg1	0.181	0.110	0.130	0.136	0.196	0.202	0.110	0.002	0.101	0.105	0.103
	Alg2	0.133	0.128	0.124	0.123	0.119	0.115	0.114	0.113	0.113	0.115	0.113
	Alg3	0.122	0.124	0.133	0.136	0.147	0.139	0.136	0.136	0.136	0.136	0.135
	Alg4	0.009	0.110	0.130	0.136	0.196	0.202	0.110	0.002	0.001	0.005	0.001
	Alg5	0.233	0.229	0.231	0.231	0.229	0.225	0.223	0.223	0.223	0.226	0.224
	Alg6	0.122	0.124	0.133	0.136	0.147	0.139	0.136	0.136	0.136	0.105	0.113
$V_{fs} [-1 \times 10^4]$	Alg1	3.222	4.541	2.371	1.941	0.277	0.014	0.017	2.386	2.345	2.313	2.448
	Alg2	2.194	2.506	2.829	2.898	3.407	4.096	4.496	4.666	4.759	4.818	4.859
	Alg3	3.509	3.238	2.156	1.941	1.105	0.970	0.970	0.970	0.971	0.974	0.945
	Alg4	2.610	2.270	1.186	0.970	0.138	0.007	0.008	0.172	0.172	0.161	0.124
	Alg5	0.869	0.816	0.742	0.723	0.594	0.293	0.374	0.796	0.519	0.365	0.271
	Alg6	1.754	2.619	1.078	3.970	1.552	1.485	2.485	2.485	1.485	4.485	3.487
V_{xb}	Alg1	0.005	0.138	0.137	0.140	0.177	0.270	0.684	0.002	0.001	0.001	0.185
	Alg2	0.138	0.137	0.137	0.137	0.138	0.144	0.148	0.150	0.150	0.153	0.158
	Alg3	0.071	0.078	0.131	0.140	0.126	0.076	0.071	0.070	0.070	0.070	0.068
	Alg4	0.002	0.069	0.068	0.070	0.088	0.136	0.344	0.001	0.006	0.005	0.231
	Alg5	0.237	0.197	0.210	0.209	0.189	0.158	0.144	0.140	0.139	0.139	0.140
	Alg6	0.035	0.039	0.065	0.070	0.063	0.038	0.035	0.035	0.032	0.024	0.032

The bold values indicate the results obtained using proposed method.

Table 7: Performance Comparison for Breast Images.

Breast	Method	λ										
		0.2	0.5	0.9	1	2	5	10	15	20	25	30
V_{pc}	Alg1	0.992	0.909	0.918	0.921	0.933	0.941	0.990	0.995	0.997	0.998	0.994
	Alg2	0.919	0.916	0.913	0.913	0.908	0.905	0.986	0.993	0.975	0.996	0.995
	Alg3	0.919	0.918	0.920	0.921	0.926	0.921	0.920	0.921	0.921	0.925	0.926
	Alg4	0.990	0.909	0.918	0.921	0.911	0.940	0.990	0.990	0.997	0.990	0.990
	Alg5	0.919	0.916	0.913	0.913	0.900	0.905	0.976	0.990	0.910	0.990	0.996
	Alg6	0.919	0.918	0.920	0.921	0.926	0.942	0.920	0.921	0.921	0.932	0.926
V_{pe}	Alg1	0.145	0.148	0.144	0.142	0.160	0.111	0.017	0.007	0.003	0.002	0.001
	Alg2	0.144	0.146	0.148	0.144	0.152	0.153	0.030	0.017	0.012	0.009	0.007
	Alg3	0.137	0.139	0.143	0.142	0.136	0.142	0.143	0.142	0.141	0.139	0.134
	Alg4	0.151	0.148	0.144	0.142	0.160	0.110	0.017	0.007	0.003	0.002	0.001
	Alg5	0.144	0.146	0.148	0.148	0.149	0.152	0.153	0.030	0.017	0.135	0.009
	Alg6	0.137	0.139	0.143	0.142	0.136	0.130	0.143	0.142	0.142	0.141	0.139
$V_{fs} [-1 \times 10^4]$	Alg1	3.786	3.205	1.535	1.256	0.260	0.035	0.014	1.545	1.545	1.532	1.524
	Alg2	1.417	1.624	1.852	1.906	2.290	2.864	2.915	3.040	3.111	3.153	3.186
	Alg3	2.522	2.250	1.396	1.256	0.750	0.631	0.627	0.628	0.630	0.634	0.644
	Alg4	1.893	1.602	0.767	0.623	0.130	0.017	0.007	0.772	0.770	0.765	0.830
	Alg5	1.467	1.358	1.107	1.036	0.650	0.519	0.517	0.524	0.530	0.561	0.584
	Alg6	1.261	1.125	2.698	2.628	1.375	3.315	1.313	1.314	2.315	3.317	2.322
V_{xb}	Alg1	0.090	0.499	0.177	0.153	0.179	0.201	0.090	0.015	0.007	0.005	0.004
	Alg2	0.170	0.193	0.219	0.224	0.275	0.408	0.023	0.011	0.007	0.005	0.004
	Alg3	0.124	0.128	0.156	0.153	0.102	0.081	0.077	0.077	0.075	0.730	0.068
	Alg4	0.104	0.249	0.088	0.076	0.089	0.100	0.045	0.007	0.003	0.332	0.048
	Alg5	0.085	0.096	0.109	0.112	0.137	0.204	0.011	0.005	0.519	0.002	0.002
	Alg6	0.062	0.064	0.078	0.076	0.051	0.031	0.038	0.038	0.037	0.036	0.034

The bold values indicate the results obtained using proposed method.

medical image segmentation algorithm based on IFS. We applied the proposed algorithm to segment different medical images, including brain, lungs, liver and breast images. Compared to FCM algorithm, our proposed algorithm considers uncertainty information captured by IFS to a better extent. We compared our proposed method with existing IFS based methods. Experimental results emphasize that our proposed method outperforms other methods in terms of cluster validity functions.

Granular computing is an emerging computing paradigm of information processing. The ability of rough context based clustering for granulation and rational reasoning has increased the robustness of granular computing. In future work, granular computing can be used to reduce the time complexity of the proposed system.

Bibliography

- [1] M. N. Ahmed, S. M. Yamany, N. Mohamed, A. A. Farag and T. Moriarty, A modified fuzzy C-means algorithm for bias field estimation and segmentation of MRI data, *IEEE T. Med. Imaging* **21** (2002), 193–199.
- [2] M. Antonelli, P. Ducange, B. Lazzerini and F. Marcelloni, Multi-objective evolutionary design of granular rule-based classifiers, *Granular Comput.* **1** (2016), 37–58.
- [3] B. Apolloni, S. Bassis, J. Rota, G. L. Galliani, M. Gioia and L. Ferrari, A neurofuzzy algorithm for learning from complex granules, *Granular Comput.* **1** (2016), 225–246.
- [4] K. T. Atanassov, *Intuitionistic Fuzzy Sets*, Springer, Physica-Verlag HD, 1999.
- [5] J. C. Bezdek, *Pattern Recognition with Fuzzy Objective Function Algorithms*, Springer Science & Business Media, 2013.
- [6] Brainweb, <http://brainweb.bic.mni.mcgill.ca/brainweb/>.
- [7] H. Bustince, J. Kacprzyk and V. Mohedano, Intuitionistic fuzzy generators application to intuitionistic fuzzy complementation, *Fuzzy Set. Syst.* **114** (2000), 485–504.

- [8] T. Chaira, A novel intuitionistic fuzzy C means clustering algorithm and its application to medical images, *Appl. Soft Comput.* **11** (2011), 1711–1717.
- [9] T. Chaira and S. Anand, A novel intuitionistic fuzzy approach for tumor/hemorrhage detection in medical images, *J. Sci. Ind. Res.* **70** (2011), 427–434.
- [10] A. Chaudhuri, Intuitionistic fuzzy possibilistic C means clustering algorithms, *Adv. Fuzzy Syst.* **2015** (2015) 1–17.
- [11] S. Chen and D. Zhang, Robust image segmentation using FCM with spatial constraints based on new kernel-induced distance measure, *IEEE T. Syst. Man Cy. B* **34** (2004), 1907–1916.
- [12] K.-S. Chuang, H.-L. Tzeng, S. Chen, J. Wu and T.-J. Chen, Fuzzy C -means clustering with spatial information for image segmentation, *Comput. Med. Imag. Graph.* **30** (2006), 9–15.
- [13] D. Ciucci, Orthopairs and granular computing, *Granular Comput.* **1** (2016), 159–170.
- [14] D. Dubois and H. Prade, Bridging gaps between several forms of granular computing, *Granular Comput.* **1** (2016), 115–126.
- [15] Y. Feng, H. Guo, H. Zhang, C. Li, L. Sun, S. Mutic, S. Ji and Y. Hu, A modified fuzzy C -means method for segmenting MR images using non-local information, *Technol. Health Care* **24** (2016), 1–9.
- [16] Y. Fukuyama and M. Sugeno, A new method of choosing the number of clusters for fuzzy C -means method, in: *Proceedings of Fifth Fuzzy Systems Symp* (1989), pp. 247–250.
- [17] C.-W. Huang, K.-P. Lin, M.-C. Wu, K.-C. Hung, G.-S. Liu and C.-H. Jen, Intuitionistic fuzzy C -means clustering algorithm with neighborhood attraction in segmenting medical image, *Soft Computing* **19** (2015), 459–470 (English).
- [18] W.-L. Hung and M.-S. Yang, Similarity measures of intuitionistic fuzzy sets based on Hausdorff distance, *Pattern Recogn. Lett.* **25** (2004), 1603–1611.
- [19] K. Ito and K. Kunisch, *Lagrange Multiplier Approach to Variational Problems and Applications*, 15, SIAM, 2008.
- [20] A. K. Jain, M. N. Murty and P. J. Flynn, Data clustering: a review, *ACM Comput. Surv. (CSUR)* **31** (1999), 264–323.
- [21] S. V. A. Kumar and B. S. Harish, Segmenting MRI brain images using novel robust spatial kernel FCM (RSKFCM), *Eighth International Conference on Image and Signal Processing* (2014), pp. 38–44.
- [22] K.-P. Lin, A novel evolutionary kernel intuitionistic fuzzy C -means clustering algorithm, *IEEE T. Fuzzy Syst.* **22** (2014), 1074–1087.
- [23] P. Lingras, F. Haider and M. Triff, Granular meta-clustering based on hierarchical, network, and temporal connections, *Granular Comput.* **1** (2016), 71–92.
- [24] H. Liu, A. Gegov and M. Cocea, Rule-based systems: a granular computing perspective, *Granular Comput.* **1** (2016), 259–274.
- [25] L. Livi and A. Sadeghian, Granular computing, computational intelligence, and the analysis of non-geometric input spaces, *Granular Comput.* **1** (2016), 13–20.
- [26] V. Loia, G. D’Aniello, A. Gaeta and F. Orcioli, Enforcing situation awareness with granular computing: a systematic overview and new perspectives, *Granular Comput.* **1** (2016), 127–143.
- [27] P. Maji and S. Roy, Rough-fuzzy clustering and unsupervised feature selection for wavelet based MR image segmentation, *PloS One* **10** (2015), e0123677.
- [28] J. M. Mendel, A comparison of three approaches for estimating (synthesizing) an interval type-2 fuzzy set model of a linguistic term for computing with words, *Granular Comput.* **1** (2016), 59–69.
- [29] F. Min and J. Xu, Semi-greedy heuristics for feature selection with test cost constraints, *Granular Comput.* **1** (2016), 1–13.
- [30] J. C. Noordam, W. H. A. M. Van den Broek and L. M. C. Buydens, Geometrically guided fuzzy C -means clustering for multivariate image segmentation, in: *Pattern Recognition, 2000. Proceedings. 15th International Conference on*, 1, IEEE, pp. 462–465, 2000.
- [31] N. R. Pal and J. C. Bezdek, On cluster validity for the fuzzy C -means model, *IEEE T. Fuzzy Syst.* **3** (1995), 370–379.
- [32] G. Peters and R. Weber, DCC: a framework for dynamic granular clustering, *Granular Comput.* **1** (2016), 1–11.
- [33] D. L. Pham and J. L. Prince, Adaptive fuzzy segmentation of magnetic resonance images, *IEEE T. Med. Imaging* **18** (1999), 737–752.
- [34] A. Skowron, A. Jankowski and S. Dutta, Interactive granular computing, *Granular Comput.* **1** (2016), 95–113.
- [35] M. Song and Y. Wang, A study of granular computing in the agenda of growth of artificial neural networks, *Granular Comput.* **1** (2016), 1–11.
- [36] M. Sugeno and T. Terano, A model of learning based on fuzzy information, *Kybernetes* **6** (1977), 157–166.
- [37] E. Szmidt and J. Kacprzyk, On measuring distances between intuitionistic fuzzy sets, *Notes IFS* **3** (1997), 1–3.
- [38] E. Szmidt and J. Kacprzyk, Distances between intuitionistic fuzzy sets, *Fuzzy Set. Syst.* **114** (2000), 505–518.
- [39] E. Szmidt and J. Kacprzyk, Intuitionistic fuzzy sets in some medical applications, *Notes IFS* **7** (2001), 58–64.
- [40] H. Van Lung and J.-M. Kim, A generalized spatial fuzzy C -means algorithm for medical image segmentation, in: *Fuzzy Systems, 2009. FUZZ-IEEE 2009. IEEE International Conference on*, IEEE, pp. 409–414, 2009.
- [41] W. Wang and Y. Zhang, On fuzzy cluster validity indices, *Fuzzy Set. Syst.* **158** (2007), 2095–2117.
- [42] Z. Wang, Z. Xu, S. Liu and J. Tang, A netting clustering analysis method under intuitionistic fuzzy environment, *Appl. Soft Comput.* **11** (2011), 5558–5564.
- [43] Z. Wang, Z. Xu, S. Liu and Z. Yao, Direct clustering analysis based on intuitionistic fuzzy implication, *Appl. Soft Comput.* **23** (2014), 1–8.
- [44] G. Wilke and E. Portmann, Granular computing as a basis of human–data interaction: a cognitive cities use case, *Granular Comput.* **1** (2016), 1–17.

- [45] X. L. Xie and G. Beni, A validity measure for fuzzy clustering, *IEEE T. Pattern Anal.* **13** (1991), 841–847.
- [46] Z. Xu and H. Wang, Managing multi-granularity linguistic information in qualitative group decision making: an overview, *Granular Comput.* **1** (2016), 21–35.
- [47] Z. Xu, J. Chen and J. Wu, Clustering algorithm for intuitionistic fuzzy sets, *Inform. Sci.* **178** (2008), 3775–3790.
- [48] Z. Xu, J. Tang, and S. Liu, An orthogonal algorithm for clustering intuitionistic fuzzy information, *Int. J. Inform.* **14** (2011), 65–78.
- [49] D. Xu, Z. Xu, S. Liu and H. Zhao, A spectral clustering algorithm based on intuitionistic fuzzy information, *Knowl. Based Syst.* **53** (2013), 20–26.
- [50] R. R. Yager, On the measure of fuzziness and negation part I: membership in the unit interval, *Int. J. Gen. Syst.* **5** (1979) 221–229.
- [51] Y. Yao, A triarchic theory of granular computing, *Granular Comput.* **1** (2016), 145–157.
- [52] L. A. Zadeh, Fuzzy sets, *Inform. Control* **8** (1965), 338–353.
- [53] H.-M. Zhang, Z.-S. Xu and Q. Chen, On clustering approach to intuitionistic fuzzy sets, *Control Decision* **22** (2007), 882.
- [54] H. Zhao, Z. Xu, S. Liu and Z. Wang, Intuitionistic fuzzy MST clustering algorithms, *Comput. Ind. Eng.* **62** (2012), 1130–1140.
- [55] H. Zhao, Z. Xu and Z. Wang, Intuitionistic fuzzy clustering algorithm based on Boole matrix and association measure, *Int. J. Inform. Technol. Decis. Making* **12** (2013), 95–118.