Research Article

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Significance of Entropy Generation and the Coriolis Force on the Three-Dimensional Non-Darcy Flow of Ethylene-Glycol Conveying Carbon Nanotubes (SWCNTs and MWCNTs)

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Abstract: Nanofluids based on CNTs/ethylene glycol have a potential role in contributing to industrial applications like heat exchangers, domestic refrigerator, electronics cooling, etc. The aim and novelty of the present research is to communicate the significance of the Coriolis force and Darcy-Forchheimer stretched flow of ethylene glycol (EG) conveying carbon nanotubes (CNTs) in a rotating frame. Furthermore, entropy analysis is the main focus in this study. Two types of CNTs known as multiwalled (MWCNT) and single-walled (SWCNT) carbon nanotubes are considered. Ethylene glycol (EG) is treated as the base liquid. Xue's model is utilized for the physical aspects of specific heat, density and thermal conductivity. The heat transfer mechanism is modeled through nonlinear thermal radiation, viscous dissipation and convective condition. The governing flow problems have been computed numerically via the NDSolve method. Outcomes for single-walled and multi-walled CNTs are arranged and compared. Our findings reveal that entropy generation is accompanied by an increasing trend in the Brinkman number and temperature ratio parameter. Temperature increases with the intensification of radiative and convective variables. Moreover, the temperature gradient has marginally larger values in the case of SWCNT, when compared with MWCNT.

Keywords: entropy generation, CNTs, Darcy-Forchheimer porous medium, rotating frame, numerical scheme

1 Introduction

The Coriolis force is basically a fictitious or inertial force that functions on bodies that are moving within the reference frame having rotation with respect to an inertial frame. The importance of the Coriolis force in the analysis of nanofluid in porous space plays a key factor in various industries and engineering fields, for instance, electronic devices, computer storage devices, rotating machinery, medical gadgets, spinning process, etc. In the basic flow expression, the Coriolis force is very important. The Coriolis effect depends on the rotation of the earth. Importantly, the earth rotates slower at the poles than it does at the Equator. Whenever objects move on a global scale in the Northern Hemisphere, e. g., balls, they tend to curve to the right. This apparent diversion is the Coriolis effect. Atmospheric fluids, such as air currents, that span large regions act in the same way as the balls. They seem to swerve to one side on the northern side of the Equator. The impact of Coriolis depends on velocity, the velocity of the earth and velocity of the fluid or body that is being deviated by the Coriolis force. The effect of this force is more significant over a long distance or at high speed. The weather affecting speedy moving bodies, such as rockets and airplanes, is influenced by the Coriolis effect. The direc-

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tion of prevailing winds are mostly determined by Coriolis effect. Most significantly, the climatic variation is greatly influenced by the Coriolis force. Mathematical expression for the impact of planetary rotation was formulated by Wang [1]. The combined features of rotation and the Coriolis effect on unsteady MHD flow was explored by Mitteilungen [2]. Hasan and Sanghi [3] theoretically examined the impact of the Coriolis force in 2D flow in rotating enclosures having arbitrary cross-sections. Their experiments showed no temperature and velocity distributions variations due to the Coriolis effect. Some current examples of research on the Coriolis effect are [4–8].

Generally, in any thermal process the amount of energy and its quality are the two main factors. The quantity of energy during a heat exchange process can be calculated by a value known as entropy (which was given by second law of thermodynamics). As stated by the second law, an amount of energy will be lost during the conversion of energy that consequently reduces the efficiency of any thermal system. Such a loss of energy grows with the entropy production. In order to raise the capability of the system, we have to lower the production of energy. Therefore, minimization of entropy production is important. This concept is useful for the optimization of engineering tools to achieve a high energy capability [9–11]. Entropy generation demonstrates the importance of irreversible factors concerned with friction, heat transfer and other non-ideal processes inside a system. It is utilized to find out the upper limits for the efficiency of several engineering systems like combustion engines, refrigerators and chemical reactors. A few recent uses of entropy are solar energy collectors, cooling of new electronic systems and nuclear fuel rods, slurry systems, solar heat exchangers in pseudo-optimization process, heat waste from steam pipes and nuclear swirl electromagnetic propulsion, etc. Keeping in mind the significance of entropy generation, many theoretical as well as experimental attempts for entropy have been accomplished by numerous researchers. The problem of entropy in fluid flow was explored by Baytas and Reveillere [12]. Rashidi et al. [13] studied entropy analysis in MHD flow. Entropy generation analysis in hydromagnetic slip flows with injection/suction was discussed by Ibanez [14]. The obtained outcomes show that degradation of entropy can be accomplished by proper physical and geometrical variables. Jian [15] explored the entropy generation and hydromagnetic flow for micro-parallel channel. Mustafa et al. [16] investigated the radiation impact on Bödewadt flow with entropy analysis. Ullah et al. [17] analyzed entropy generation in flow nanomaterial with melting effect. Seth et al. [18] examined irreversibility analysis in dissipative flows of CNT nanomaterials through porous space. Seth et al. [19] numerically treated the problem of slip by considering irreversibility analysis. Alsaadi et al. [20] explored the entropy optimization in porous space.

Nanomaterials [21–27] have attracted worldwide consideration in ongoing technology since the exploration of carbon nanotubes by Iijima in 1991. CNTs (carbon nanotubes) are allotropes of pure carbon atoms that are thin, long and hexagonal nanostructure (that have been shaped into cylinder). CNTs can be subdivided into two classes, i. e., SWCNTs and MWCNTs. SWCNTs are a typical category of carbon materials term as one-dimensional materials. They comprise of graphene sheets, moved up to shape empty cylinders with walls one particle thick. MWCNTs are a peculiar type of carbon nanotubes wherein different SWCNTs are settled inside each other. For the nanomaterials the shape of the nanoparticles is very important. In view of heat transport, cylindrical shaped nanoparticles (nanotubes) is very efficient than the other particles like blade, bricks, spherical etc. The potential for using ethylene-glycol conveying CNT nanofluids remains high because they possess high thermophysical properties that are useful many applications. These materials have a variety of uses in heat exchangers, domestic refrigerators, electronics cooling, optics, atomic force microscope, gas storing, extra strong fibers, basic composite materials, field emission displays, nuclear power magnifying lenses, flat-panel displays, antifouling shade, conductive plastics and many more. CNTs are additionally utilized as electrical contacts, warming sources, high-temperature refractories and biosensors in medical gadgets. In daily life CNTs can be used as antennas for radios and some other electromagnetic gadgets. Moreover, CNTs don't contribute any risk to the environment due to the carbon chain. In this regard Xue [28] presented the model for the transportation of nanomaterials based on thermal conductivity. Wang et al. [29] examined the phenomenon of the pressure drop in nanoliquids consisting of CNTs. Hayat et al. [30] studied Newtonian heating impact on stagnation point flow by considering CNTs. MHD flows of nanomaterials with CNTs dispersed in a saltwater mixture was analyzed by Ellahi et al. [31]. Imtiaz et al. [32] analytically treated the problem of convective and radiative flow of a nanoliquid by adding CNTs. Mahanthesh et al. [33] discussed

the CNTs liquid model, describing the thermal and exponential heat source. Kumar et al. [34] investigated water-based CNTs nanoliquid flow with quartic chemical reactions. Bhattacharyya et al. [35] explored the CNTs flow between rotating disks.

On analyzing the literature just cited, it is clear that the nonlinear radiative flow of ethylene-glycol based CNTs suspended in a Darcy-Forchheimer porous medium has not been examined yet. Thus, we primarily focus this article on five aspects. First, we consider CNTs nanoliquids based on ethylene-glycol (ethylene glycol is a clear, sweet, slightly viscous liquid). Second, we explain the flow in porous space using the Darcy-Forchheimer porous model. Third, we explore the consequences of the Coriolis force in porous space. Fourth, we analyze non-linear thermal radiation, viscous dissipation and convective condition. Fifth, we address a useful study about the entropy in a system. Furthermore, we employ the Xue [28] model for the transportation of nanomaterials. A concise depiction of the entropy of a system is presented. Implementation of suitable variables yields dimensionless system. A numerical scheme is employed to solve the nonlinear systems. The behaviors of many regulatory flow variables are depicted through plots. In addition, the variation of some valuable engineering quantities are interpreted via tabular values. The present article is designed to respond to these research questions:

- What is the effect of skin friction and the Nusselt number in relation to interesting embedded parameters?
- How does rotation and the porosity of the system effect the nanofluid motion?
- What does the comparative study of MWCNT and SWCNT in ethylene glycol reveal?
- How is the temperature of nanofluids effected by the variation of the Biot number, nanoparticles volume fraction and the radiation parameter?
- How can the production of entropy be optimized?

2 Formulation of the model

Here, entropy generation in 3D rotating ethylene-glycol CNTs nanoliquid flow by stretching the surface is scrutinized. The following assumptions are considered:

- Darcy-Forchheimer model is implemented to explain the flow in porous space.
- The system rotates with angular velocity ω (see Fig. 1).
- Let U_w denote the stretching velocity.
- It is assumed that the temperature at the surface is controlled by means of convection that is portrayed by a hot liquid at temperature T_f below the sheet and the heat-transport coefficient h_f .
- Irreversibility of nonlinear thermal radiation is addressed.

The governing expressions in view of the aforementioned considerations are [36–38]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - 2\omega v = \nu_{nf} \left(\frac{\partial^2 u}{\partial z^2} \right) - \frac{\nu_{nf}}{K_p} u - F u^2, \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + 2\omega u = \nu_{nf} \left(\frac{\partial^2 v}{\partial z^2} \right) - \frac{\nu_{nf}}{K_p} v - F v^2, \quad (3)$$

$$\left. \begin{aligned} u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} &= \alpha_{nf} \frac{\partial^2 T}{\partial z^2} + \frac{\nu_{nf}}{(c_p)_{nf}} \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right] \\ &+ \frac{1}{(\rho c_p)_{nf}} \frac{\partial}{\partial z} \left(\frac{16\sigma^*}{3m^*} T^3 \frac{\partial T}{\partial z} \right), \end{aligned} \right\}, \quad (4)$$

where (u, v, w) show the respective velocity components in (x, y, z) directions, $F = \frac{C_b}{x\sqrt{K_p}}$ the non-uniform inertia coefficient (due to porous space), ν_{nf} the nanofluid kinematic viscosity, K_p the porous space permeability, C_b the drag coefficient, α_{nf} the thermal diffusivity of the nanofluid, (T, T_∞) the nanomaterials and ambient temperatures, σ^* the Stefan-Boltzmann constant and m^* the coefficient of mean absorption.

Figure 1: Flow configuration.

2.1 Thermophysical features of carbon nanotubes and ethylene glycol

The model of a nanofluid accorded by Xue [28] is

$$\left. \begin{aligned} \frac{k_{nf}}{k_f} &= \frac{1 - \phi + 2\phi \frac{k_{CNT}}{k_{CNT} - k_f} \ln \frac{k_{CNT} + k_f}{2k_f}}{1 - \phi + 2\phi \frac{k_f}{k_{CNT} - k_f} \ln \frac{k_{CNT} + k_f}{2k_f}}, \rho_{nf} = \rho_f(1 - \phi) + \rho_{CNT}\phi, \nu_{nf} = \frac{\mu_{nf}}{\rho_{nf}}, \\ \alpha_{nf} &= \frac{k_{nf}}{(\rho c_p)_{nf}}, \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5/10}}, (\rho c_p)_{nf} = (\rho c_p)_f(1 - \phi) + (\rho c_p)_{CNT}\phi, \end{aligned} \right\}, \quad (5)$$

where nf denotes a nanofluid, ϕ the solid-volume fraction of CNTs, $(\rho_f, \rho_{CNT}, \rho_{nf})$ the base fluid, CNTs and nanomaterials densities, (μ_f, μ_{nf}) the nanofluid effective and base liquids dynamic viscosity, $(\rho c_p)_{nf}$ the nanofluid effective heat capacity and (k_{nf}, k_f) the thermal conductivities of carbon nanotubes and the base liquid.

2.2 Boundary conditions

The related conditions are

$$\left. \begin{aligned} u &= U_w(x) = ax, v = 0, w = 0, \\ k_{nf} \frac{\partial T}{\partial z} &= h_f(T_f - T) \text{ at } z = 0, \\ u &\rightarrow 0, v \rightarrow 0, \\ T &\rightarrow T_\infty \text{ as } y \rightarrow \infty. \end{aligned} \right\} \quad (6)$$

2.3 Transformations

The nondimensional variables for the considered problem can be expressed by

$$\left. \begin{aligned} v &= axg(\eta), u = axf'(\eta), w = -(av_f)^{1/2}f(\eta), \\ \theta(\eta) &= (T - T_\infty)/(T_f - T_\infty), \eta = \left(\frac{a}{\nu_f}\right)^{1/2} z. \end{aligned} \right\}. \quad (7)$$

Insertion of these expressions fulfills expression (1) automatically.

2.4 Transformed systems

Now, using eq. (8) in eqs. (2)–(3), one obtains

$$\frac{1}{(1-\phi)^{2.5} \left(1-\phi + \frac{\rho_{CNT}}{\rho_f} \phi\right)} (f'''' - \lambda f') + f f'' + 2\Omega g - (1 + F_r) f'^2 = 0, \quad (8)$$

$$\frac{1}{(1-\phi)^{2.5} \left(1-\phi + \frac{\rho_{CNT}}{\rho_f} \phi\right)} (g'' - \lambda g) + f g' - f' g - 2\Omega f' - F_r g^2 = 0, \quad (9)$$

$$\left. \begin{aligned} & \frac{1}{Pr} \left(\frac{k_{nf}}{k_f} \right) \theta'' + \frac{Rd}{Pr} (\theta(\theta_w - 1))^2 (3\theta'^2(\theta_w - 1) + (\theta(\theta_w - 1) + 1)\theta'') \\ & + \frac{Ec}{(1-\phi)^{2.5}} (f'^2 + g'^2) + \left(1 - \phi + \frac{(\rho c_p)_{CNT}}{(\rho c_p)_f} \phi \right) f \theta' = 0, \end{aligned} \right\} \quad (10)$$

$$\left. \begin{aligned} & f'(0) = 1, f(0) = 0, g(0) = 0, \theta'(0) = -\frac{k_f}{k_{nf}} \gamma (1 - \theta(0)), \\ & f'(\infty) \rightarrow 0, g(\infty) \rightarrow 0, \theta(\infty) \rightarrow 0, \end{aligned} \right\}, \quad (11)$$

where a prime expresses differentiation via η , $F_r = \frac{C_b}{K_p^{1/2}}$ the inertia coefficient, $\lambda = \frac{\nu_f}{aK_p}$ the porosity parameter and $Rd = \frac{16\sigma^* T_\infty^3}{3m^* k_f}$ the radiation parameter; $\Omega = \frac{\omega}{a}$ refers to rotation parameter, $Pr = \frac{(\mu c_p)_f}{k_f}$ the Prandtl number, $\gamma = \frac{h_f}{k_f} \sqrt{\frac{\nu_f}{a}}$ the Biot number, $Ec = \frac{U_w^2}{(c_p)_f (T_f - T_\infty)}$ denotes the Eckert number and $\theta_w = \frac{T_f}{T_\infty}$ the temperature-ratio variable.

2.5 Physical quantities (C_{fx} and Nu_x)

To satisfy our engineering curiosity, we are concerned with the drag force (C_{fx}) and temperature gradient (Nu_x) expressed by

$$\left. \begin{aligned} C_{fx} &= \frac{2\tau_{wx}}{\rho_f U_w^2}, \\ Nu_x &= \frac{x q_w}{k_f (T_f - T_\infty)}, \end{aligned} \right\}, \quad (12)$$

where (τ_{wx}) and (q_w) denote the wall shear stress and heat flux, respectively,

$$\left. \begin{aligned} \tau_{wx} &= \mu_{nf} \left(\frac{\partial u}{\partial z} \right) \Big|_{z=0}, \\ q_w &= -\frac{k_f}{k_{nf}} \left(\frac{\partial T}{\partial z} \right) \Big|_{z=0} + \frac{4\sigma^*}{3m^*} \frac{\partial T^4}{\partial z} \Big|_{z=0}. \end{aligned} \right\} \quad (13)$$

Invoking eq. (13) in eq. (12) and then using eq. (7), we have

$$\left. \begin{aligned} Re_x^{0.5} C_{fx} &= \frac{1}{(1-\phi)^{2.5/10}} f''(0), \\ Nu_x &= -\left(\frac{K_{nf}}{k_f} + Rd(\theta(0)(\theta_w - 1) + 1)^3 \right) \theta'(0), \end{aligned} \right\} \quad (14)$$

where the local Reynolds number is expressed as $Re_x = \frac{x U_w}{\nu_f}$. It is important to highlight that the conventional liquid situation is recovered when $\phi = 0$.

3 Entropy

An analysis of entropy yields significant insights about irreversibility of thermal energy of a certain system. Following Bejan [9], the volumetric rate of entropy production can be given as

$$E_G = \frac{k_f}{T_\infty^2} \left[\frac{k_{nf}}{k_f} + \frac{16\sigma^*}{3m^* k_f} \right] \left(\frac{\partial T}{\partial z} \right)^2 + \frac{\mu_{nf}}{T_\infty} \Psi, \quad (15)$$

where Ψ represents viscous dissipation, i. e.,

$$\Psi = \left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2. \quad (16)$$

- The term $\frac{k_f}{T_\infty^2} \left[\frac{k_{nf}}{k_f} + \frac{16\sigma^*}{3m^*k_f} \right] \left(\frac{\partial T}{\partial z} \right)^2$ stands for entropy due to irreversibility of heat transfer.
- The term $\frac{\mu_{nf}}{T_\infty} \Psi$ denotes the entropy due to energy viscous dissipation.

Using eq. (17) in eq. (16), one obtains

$$E_G = \frac{k_f}{T_\infty^2} \left[\frac{k_{nf}}{k_f} + \frac{16\sigma^*}{3m^*k_f} \right] \left(\frac{\partial T}{\partial z} \right)^2 + \frac{\mu_{nf}}{T_\infty} \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right]. \quad (17)$$

Let us define entropy generation number (N_G) that is equal to the ratio of the generation rate of volumetric entropy (E_G) to the generation rate of characteristic entropy ($E_0 = \frac{k_{nf}(T_f - T_\infty)}{T_\infty(\frac{\eta}{z})^2}$). Mathematically, this relationship can be expressed as

$$N_G = \frac{E_G}{E_0}. \quad (18)$$

By utilizing transformation (8), eqs. (18) and (19) take the form

$$N_G = \left[\frac{k_{nf}}{k_f} + \text{Rd}(\theta(\theta_w - 1) + 1)^3 \right] \theta'^2 \alpha_1 + \frac{\text{Br}}{(1 - \phi)^{25/10}} (f''^2 + g'^2). \quad (19)$$

For the sake of the relative significance of entropy generation by heat transfer, we define another essential parameter known as Bejan number, i. e.,

$$\text{Be} = \frac{\text{Entropy due to heat transfer}}{\text{Total entropy}}, \quad (20)$$

or

$$\text{Be} = \frac{\left[\frac{k_{nf}}{k_f} + \text{Rd}(\theta(\theta_w - 1) + 1)^3 \right] \theta'^2 \alpha_1}{\left[\frac{k_{nf}}{k_f} + \text{Rd}(\theta(\theta_w - 1) + 1)^3 \right] \theta'^2 \alpha_1 + \frac{\text{Br}}{(1 - \phi)^{25/10}} (f''^2 + g'^2)}, \quad (21)$$

where $\text{Br} = \frac{U_w^2 \mu_f}{k_f \Delta T}$ means the Brinkman number and $\alpha_1 = \frac{(T_f - T_\infty)}{T_\infty}$ the temperature-difference variable due to the production of entropy.

Further, note that for $\text{Be} = 1$ the heat-transfer irreversibility effects become dominant, whereas for the case of $\text{Be} = 0$ the entropy due to liquid friction dominates. Furthermore, $\text{Be} = 0.5$ represents that production of entropy because the liquid friction and heat transfer are similar.

4 Research methodology

Numerical solutions to the proposed nonlinear coupled problems (9)–(11) subjected to boundary conditions (12) are obtained by employing the bvp4c technique (based on FDM). To achieve this, we reduced eqs. (9)–(12) to first ODEs. The mesh and error control selection is develop on the progressive computations residual. The boundary layer region is attained for each collection of parameters values. The iterative error is set 10^{-6} . The

main algorithm can be summarized as follows:

$$y_1 = f, y_2 = f', y_3 = f'', y_4 = g, y_5 = g', y_6 = \theta, y_7 = \theta', \quad (22)$$

$$\begin{bmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \\ y_5' \\ y_6' \\ y_7' \end{bmatrix} = \begin{bmatrix} y_2 \\ y_3 \\ \frac{((1 + F_r)y_2^2 - y_1y_3 - 2\Omega y_4) + \lambda y_2}{(1 - \phi)^{2.5} \left(1 - \phi + \frac{\rho_{CNT}}{\rho_f} \phi\right)} \\ y_5 \\ \frac{(F_r y_4^2 + y_2 y_4 + 2\Omega y_2 - y_1 y_5) + \lambda y_4}{(1 - \phi)^{2.5} \left(1 - \phi + \frac{\rho_{CNT}}{\rho_f} \phi\right)} \\ y_7 \\ \frac{\left[-\left(1 - \phi + \frac{\rho_{CNT}}{\rho_f} \phi\right) y_1 y_7 + \frac{\text{Pr Ec}}{(1 - \phi)^{2.5}} (y_3^2 + y_5^2) - \text{Rd} (y_6 (\theta_w - 1) + 1)^2 (3(\theta_w - 1)) y_7^2\right]}{\left[\frac{k_{nf}}{k_f} + \text{Rd} [y_6 (\theta_w - 1) + 1]\right]} \end{bmatrix} \quad (23)$$

with initial conditions

$$\begin{bmatrix} y_1(0) \\ y_2(0) \\ y_2(\infty) \\ y_4(0) \\ y_4(\infty) \\ y_6(\infty) \\ y_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{k_{nf}}{k_f} y_6(0) \end{bmatrix}. \quad (24)$$

5 Analysis of results and discussion

This portion is dedicated to the interpretation of the physical features of embedding variables on quantities, namely velocities ($f'(\eta)$ and $g'(\eta)$), temperature $\theta(\eta)$, entropy generation N_G and the Bejan number Be in relation to SWCNTs and MWCNTs. These outcomes are well explained and presented graphically (see Figs. 2–11). In these graphs the dotted lines present the features of SWCNTs, and the solid lines show the behavior of MWCNTs. Skin frictions ($(\text{Re}_x^{0.5} f_x)$ and $(\text{Re}_x^{0.5} f_y)$) and Nusselt numbers ($\text{Re}^{-0.5} \text{Nu}_x$ [MWCNTs] and $\text{Re}^{-0.5} \text{Nu}_x$ [SWCNTs]) are also computed and analyzed (see Tables 2 and 3). Further, the physical and thermal features of carbon nanotubes (SWCNTs and MWCNTs) and ethylene glycol ($\text{C}_2\text{H}_6\text{O}_2$) are shown in Table 1. The values assigned to the physical variables in our entire analysis are $\text{Fr} = \lambda = \theta_w = \text{Br} = 0.1$, $\Omega = 0.3$, $\text{Pr} = 40.36$, $\gamma = 5.0$, $\text{Rd} = 2.5$ and $\alpha_1 = 0.5$.

5.1 Velocity

Figures (2–5) depict the variations of interesting physical variables on velocity distributions ($f'(\eta)$ and $g'(\eta)$). The effect of ϕ in the range of $0.1 \leq \phi \leq 0.5$, for both SWCNTs and MWCNTs, on velocity distributions ($f'(\eta)$ and $g'(\eta)$) are shown in Fig. 2 (a) and (b). It should be noted that larger estimations of nanoparticles' volume fraction lead to enhanced ($f'(\eta)$ and $g'(\eta)$) in both cases of CNTs. Figure 3 (a and b) shed light on the behavior of Fr on ($f'(\eta)$ and $g'(\eta)$). Here, a higher Fr decreases the fluid velocities ($f'(\eta)$ and $g'(\eta)$). Physically, for a higher Fr the internal force increases and thus velocity decreases. Plots of λ for ($f'(\eta)$ and $g'(\eta)$) are depicted in Figs. 4 (a and b). It is clear from these Figs. that velocities ($f'(\eta)$ and $g'(\eta)$) increase for a higher λ . In fact, resistive force increases in the nanoliquid motion due to the porous medium. Thus, velocities ($f'(\eta)$ and $g'(\eta)$) are reduced. Figure 5 (a and b) portray the aspect of rotation parameter Ω on velocities ($f'(\eta)$ and $g'(\eta)$).

Figure 2: Change in $f'(\eta)$ and $g'(\eta)$ for varying ϕ when $Fr = \lambda = \theta_w = Br = 0.1$, $\Omega = 0.3$, $Pr = 40.36$, $\gamma = 5.0$, $Rd = 2.5$ and $\alpha_1 = 0.5$.

Figure 3: Change in $f'(\eta)$ and $g'(\eta)$ for varying Fr when $\lambda = \theta_w = Br = 0.1$, $\Omega = 0.3$, $Pr = 40.36$, $\gamma = 5.0$, $Rd = 2.5$ and $\alpha_1 = 0.5$.

Figure 4: Change in $f'(\eta)$ and $g'(\eta)$ for varying λ when $Fr = \theta_w = Br = 0.1$, $\Omega = 0.3$, $Pr = 40.36$, $\gamma = 5.0$, $Rd = 2.5$ and $\alpha_1 = 0.5$.

These figures point out that intensified values of Ω decreased the velocity distribution in both directions [39]. Higher estimations of Ω results in a higher rotation rate when compared with the stretching rate. Therefore, the influence of higher rotation results in a downward trend in the liquid velocity. Furthermore, Figs. (2–5) also indicate that SWCNTs have a lower velocity when compared with MWCNTs. In fact SWCNTs has higher density than MWCNTs.

Figure 5: Change in $f'(\eta)$ and $g'(\eta)$ for varying Ω when $F_r = \lambda = \theta_w = Br = 0.1$, $\Omega = 0.3$, $Pr = 40.36$, $\gamma = 5.0$, $Rd = 2.5$ and $\alpha_1 = 0.5$.

Figure 6: Change in $\theta(\eta)$ against (a) ϕ (b) θ_w when $F_r = \lambda = Br = 0.1$, $\Omega = 0.3$, $Pr = 40.36$, $\gamma = 5.0$, $Rd = 2.5$ and $\alpha_1 = 0.5$.

Figure 7: Change in $\theta(\eta)$ against (a) Rd (b) γ when $F_r = \lambda = \theta_w = Br = 0.1$, $\Omega = 0.3$, $Pr = 40.36$ and $\alpha_1 = 0.5$.

5.2 Variation in temperature

To understand the behavior of dimensionless temperature versus ϕ , θ_w , Rd and γ , Figs. (6 and 7) are useful. Figure 6(a) shows the effect of ϕ on the thermal field. Here, temperature significantly decays for higher ϕ . Also, thermal field $\theta(\eta)$ is larger in the case of MWCNTs when compared with SWCNTs. The behavior of

Figure 8: Change in N_G and Be for varying Br when $F_r = \theta_w = \lambda = 0.1$, $\Omega = 0.3$, $Pr = 40.36$, $\gamma = 5.0$, $Rd = 2.5$ and $\alpha_1 = 0.5$.

Figure 9: Change in N_G and Be for varying α_1 when $F_r = \theta_w = \lambda = Br = 0.1$, $\Omega = 0.3$, $Pr = 40.36$, $\gamma = 5.0$, and $Rd = 2.5$.

thermal field $\theta(\eta)$ for larger θ_w is disclosed in Fig. 6(b). Here, both the thermal field and layer thickness are enhanced for both CNTs. Moreover, $\theta(\eta)$ is less in the case of SWCNTs than MWCNTs. Figure 7(a) shows how the thermal field is effected by Rd . An increment in temperature for Rd is visible. In a physical sense this phenomenon is anticipated since the temperature rises as the radiation variable supplies more heat to the nanoliquid or the rise in strength of thermal layer is due to the transport of radiant energy to liquid particles by the higher radiation parameter. Figure 7(b) visualizes the variation of γ versus $\theta(\eta)$. Temperature increases abruptly by Biot number γ . This behavior is due to the higher convection that enhanced the surface temperature.

5.3 Behavior of various parameters on entropy generation and Bejan numbers

Figures (8–11) illustrate the effects of various embedding variables (Br , α , Rd and γ) in the range of $0 \leq \eta \leq 5.0$ on N_G and Be for both MWCNTs and SWCNTs. The consequences of Br on N_G and Be are depicted in Fig. 8 (a and b). Higher Brinkman numbers significantly increment the thermal energy irreversibility. This behavior is seen in Fig. 8(a). From Fig. 8(b) it can be seen that the entropy generation rate N_G has a crossover point at $\eta = 3.5$. Before this variation, the entropy increases, and then it starts to fall [40]. The behavior of the temperature- difference variable due to entropy α_1 on Be and N_G are reported in Fig. 9 (a and b). Here, we observed that entropy and Bejan number are intensified for both (SWCNTs) and (MWCNTs). Furthermore, Be is higher in the case of (MWCNTs) than (SWCNTs). Figure 10 (a and b) shows how Rd effects Be and N_G . Here, we see that both Be and N_G are increasing functions of Rd . A physically larger Rd raises the internal heat

Figure 10: Change in N_G and Be for varying R_d when $F_r = \theta_w = \lambda = Br = 0.1$, $\Omega = 0.3$, $Pr = 40.36$, $\gamma = 5.0$, and $\alpha_1 = 0.5$.

Figure 11: Change in N_G and Be for varying γ when $F_r = \theta_w = \lambda = Br = 0.1$, $\Omega = 0.3$, $Pr = 40.36$, and $\alpha_1 = 5.0$.

Table 1: Physical and thermal features of carbon nanotubes (SWCNTs and MWCNTs) and ethylene glycol ($C_2H_6O_2$) [39, 40].

Constituents	ρ (kg/m ³)	c_p (J/kgK)	k (W/mk)
MWCNT	1600	796	3000
SWCNT	2600	425	6600
$C_2H_6O_2$	1115	2430	0.253

generation in a moving liquid, which consequently enhances Be and N_G . Figure 11 shows the outcome for Be and N_G in response to γ . It is found that larger estimations of γ enhances both Be and N_G . In a physical sense, an enhancement in γ corresponds to a rise the stretching-surface thermal-energy irreversibility. Thus Be and N_G decreases.

5.4 Variations of nondimensional drag forces and the Nusselt number

Tables 2 and 3 are organized numerically to analyze the variation of physical quantities (skin frictions and Nusselt number) for both SWCNTs and MWCNTs. Table 2 indicates that skin friction is increased via ϕ , λ and Ω . Also, higher estimations of F_r increase the primary skin friction ($-\text{Re}_x^{0.5} C_{fx}$), whereas a slow reduction occurs in secondary skin friction ($-\text{Re}_x^{0.5} C_{fy}$). Table 3 manifests that the Nusselt number increases for higher ϕ , θ_w , γ and R_d . Furthermore, the Nusselt number has marginally higher values in the case of SWCNTs when

Table 2: Variations of $(\text{Re}_x^{0.5} C_{fx})$ and $(\text{Re}_x^{0.5} C_{fy})$ via ϕ , F_r , λ and Ω for both MWCNTs and SWCNTs.

Parameters (fixed)	variables		MWCNTs		SWCNTs	
			$-\text{Re}_x^{0.5} C_{fx}$	$-\text{Re}_x^{0.5} C_{fy}$	$-\text{Re}_x^{0.5} C_{fx}$	$-\text{Re}_x^{0.5} C_{fy}$
$\text{Fr} = \lambda = \theta_w = \text{Br} = 0.1, \text{Pr} = 6.7, \Omega = 0.3,$ $\gamma = 0.5, \text{Rd} = 2.5, \alpha_1 = 0.5$	ϕ	0.1	2.3676	0.1888	2.3557	0.2050
		0.2	3.0515	0.2092	3.0670	0.2423
		0.3	4.1012	0.2361	4.1334	0.2870
$\phi = \lambda = \theta_w = \text{Br} = 0.1, \text{Pr} = 6.7, \Omega = 0.3,$ $\gamma = 0.5, \text{Rd} = 2.5, \alpha_1 = 0.5$	Fr	0.1	1.9827	0.1888	2.0179	0.2050
		0.2	2.0012	0.1884	2.0380	0.2046
		0.3	2.0195	0.1881	2.0580	0.2042
$\text{Fr} = \phi = \theta_w = \text{Br} = 0.1, \text{Pr} = 6.7, \Omega = 0.3,$ $\gamma = 0.5, \text{Rd} = 2.5, \alpha_1 = 0.5$	λ	0.1	2.1267	0.1888	2.1756	0.2050
		0.2	2.1576	0.2204	2.2062	0.2364
		0.3	2.1882	0.2518	2.2365	0.2675
$\text{Fr} = \lambda = \theta_w = \text{Br} = 0.1, \text{Pr} = 6.7, \phi = 0.1,$ $\gamma = 0.5, \text{Rd} = 2.5, \alpha_1 = 0.5$	Ω	0.1	1.9743	0.0846	2.0077	0.0899
		0.2	1.9754	0.1367	2.0091	0.1475
		0.3	1.9772	0.1888	2.0113	0.2050

Table 3: Variation of $\text{Re}^{-0.5} \text{Nu}_x$ [MWCNTs] and $\text{Re}^{-0.5} \text{Nu}_x$ [SWCNTs] via ϕ , F_r , λ and Ω .

Parameters (fixed)	variables		MWCNTs	SWCNTs
			$\text{Re}_x^{-0.5} \text{Nu}_x$	$\text{Re}_x^{-0.5} \text{Nu}_x$
$\text{Fr} = \lambda = \theta_w = \text{Br} = 0.1, \text{Pr} = 6.7, \Omega = 0.3,$ $\gamma = 0.5, \text{Rd} = 2.5, \alpha_1 = 0.5$	ϕ	0.1	0.9061	0.9126
		0.2	2.8954	2.9213
		0.3	7.1033	7.1730
$\phi = \lambda = \theta_w = \text{Br} = 0.1, \text{Pr} = 6.7, \Omega = 0.3,$ $\text{Fr} = 0.1, \text{Rd} = 2.5, \alpha_1 = 0.5$	γ	0.1	0.9061	0.9126
		0.2	1.7678	1.7799
		0.3	2.5881	2.6051
$\text{Fr} = \phi = \text{Br} = \lambda = 0.1, \text{Pr} = 6.7, \Omega = 0.3,$ $\gamma = 0.5, \text{Rd} = 2.5, \alpha_1 = 0.5$	θ_w	0.1	0.9061	0.9126
		0.2	0.9064	0.9128
		0.3	0.9066	0.9130
$\text{Fr} = \lambda = \theta_w = \text{Br} = 0.1, \text{Pr} = 6.7, \phi = 0.1,$ $\gamma = 0.5, \text{Rd} = 2.5, \alpha_1 = 0.5, \Omega = 0.3$	Rd	0.1	0.9061	0.9126
		0.2	0.9975	1.0045
		0.3	1.0947	1.1025

compared with MWCNTs. The Nusselt number basically describes the rate of heat transfer from the surface to the fluid. Here, the Nusselt number is higher for SWCNT nanofluid when compared with MWCNT-based nanofluids. This is because MWCNTs have a higher thermal conductivity than SWCNTs. Therefore, a lower amount of heat is transferred in the case of the MWCNT nanofluid.

6 Conclusion

Our intention here is to explore the Coriolis impact on nonlinear radiative flows of CNT-based nanomaterials. Here, the second law of thermodynamics is utilized to calculate the total entropy production. The governing expressions are modeled including the Darcy-Forchheimer model, viscous dissipation, ethylene glycol CNTs and nonlinear thermal radiation. The simplified mathematical model is numerically approximated. We summarize our main findings here:

- Higher Ω , λ and F_r lead to decay of the liquid velocities for both CNTs.
- Appearance of θ_w , Rd and γ strengthens the thermal field, while the opposite trend is noticed for ϕ .
- Ω , λ and ϕ decrease the skin frictions for both CNTs.

- Temperature gradient is enhanced via θ_w , Rd, ϕ and γ .
- Larger α and γ boost the Bejan number Be.
- Generation rate of entropy shows the increasing impact of Br, γ and α .

Nomenclature

C_{fx}	Local skin-friction coefficient	Br	Brinkman number
MWCNT	Multiwalled carbon nanotubes	a	Stretching dimensional constant
Ω	Rotation parameter	T_∞	Ambient fluid temperature
Rd	Radiation variable	q_w	Surface heat flux
F	Porous medium nonuniform inertia coefficient	T	Temperature
K_p	Permeability of porous medium	τ_w	Surface shear stress
C_b	Drag coefficient	h_f	Heat transfer coefficient
Nu_x	Local Nusselt number	c_p	Specific heat
SWCNT	Single-walled carbon nanotube	U_w	Wall stretching velocity
Re_x	Local Reynolds number	T_f	Heated fluid temperature below the surface
CNT	Carbon nanotubes	σ^*	Stefan-Boltzmann constant
nf	Nanofluid	f'	Dimensionless velocity
m^*	Mean absorption coefficient	ω	Angular velocity
f	Base fluid	α	Thermal diffusivity
F_r	Inertia coefficient	(u, v, w)	Velocity components
Pr	Prandtl number	ν	Kinematic viscosity
θ_w	Temperature-difference ratio parameter	ρ	Density
N_G	Entropy generation number	μ	Dynamic viscosity
E_G	Volumetric entropy rate	λ	Porosity parameter
E_0	Characteristics entropy rate	γ	Biot number
Ec	Eckert number	Ψ	Viscous dissipation
		α_1	Temperature-difference parameter
		θ	Dimensionless temperature

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