Review Article

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GBAS: fundamentals and availability analysis according to $\sigma_{vig}$

Abstract: Ground-based augmentation system (GBAS) was developed to guide aircraft precision approach and landing, aiming to replace the instrument landing system (ILS), which is currently used in most airports worldwide. GBAS based on differential positioning with reference stations that provide differential corrections to the aircraft to improve its positioning accuracy and ensure other performance parameters such as integrity, continuity, and availability. However, using GBAS in low latitude regions such as Brazil, the occurrence of ionospheric irregularities can affect global navigation satellite system (GNSS) performance so that it does not meet the requirements for aviation. This article evaluates five vertical ionospheric gradient variability scenarios for a GNSS data set of four reference stations, one station simulating an aircraft with GBAS in a static model based on performance requirements for Category Approach Type – CAT I. The results showed that the increase in the variability of the ionosphere and the geometry of satellites used in positioning could affect the integrity and availability of GBAS. In the scenario of more significant variability of the ionosphere evaluated, there was a loss of 38.4% of the availability of GBAS for the CAT I approach.

Keywords: ionosphere gradient, positions errors, precision approach, protection levels, pseudorange corrections

1 Introduction

Ground-based augmentation system (GBAS) was developed to improve global navigation satellite system (GNSS) navigation used in precision approach and aircraft landing. In this system, a set of reference GNSS stations (usually 4) located at the airport provide differential corrections to the aircraft to apply in its positioning, improve accuracy and assist in flight operations. Differential corrections and other parameters are processed by a ground facility station (GFS) and sent to the aircraft through an antenna called very high frequency data broadcast (VDB).

GBAS emerged as an alternative to ILS in that it is a radiofrequency-transmitting equipment installed at the head of each runway that guides aircraft in precision approaching and landing, especially under low visibility conditions. GBAS makes it possible for a single station to offer approach routes for all runways at the airport (Pullen, 2017). In the literature, studies are highlighting the tendency to substitute the ILS for GBAS, as in the study by Felux et al. (2013) that compares flight performance using ILS and GBAS for a precision approach, in which the performance of GBAS was superior to ILS in terms of positional error in both components (horizontal and vertical).

However, in equatorial regions and low latitudes ($\pm 20^\circ$ to $20^\circ$), characterized by high instability and the occurrence of ionospheric events (plasma bubbles and ionospheric scintillation), the use of GBAS can be compromised by the degradation suffered by GNSS signals when propagating through the ionosphere, as pointed out by studies of Pereira et al. (2021), Yoon et al. (2019), and Pereira et al. (2017). One of the ways to minimize these effects would be to perform linear combinations of observations, for example, the ionosphere-free linear combination. However, initially, GBAS was authorized to operate using only GNSS data from the L1 frequency of the GPS (global position system) and Global’naya naviatsionnaya Sputnikovaya system constellations.

Works of authors such as Yoon et al. (2019) and Pereira et al. (2017) indicate limitations in the use of GBAS at international airports in Brazil, due to high ionospheric gradients (greater than 600 mm/km), at certain times of the day and seasons, depending on the occurrence of ionospheric events. These high ionospheric gradients and their variation indicate restrictions on the use
of GBAS as they result in higher levels of alert protection, including at the Tom Jobim International Airport, located in Rio de Janeiro – RJ. This is the only airport with GBAS infrastructure in Brazil, but it was not certified by aviation regulators due to its susceptibility to the ionosphere’s effects. This limitation of using only data on the L1 frequency may make it unfeasible in regions such as Brazil.

A tool to assess the conditions for installing a GBAS station is the PEGASUS software (Prototype EGNOS and GBAS Analysis System Using SAPPHIRE) developed by EUROCONTROL (European Organization for the Safety of Air Navigation). In the studies by Lipp et al. (2005); Gaglione and Vultaggio (2006), one can find performance analyses of the experimental GBAS at airports in Germany and Italy.

Among the potentialities available in PEGASUS, it uses for the operational validation of GNSS augmentation systems aimed at aviation, such as space based augmentation system and GBAS, forecasting capacities of future or experimental systems, and selection of future locations for the placement of ground subsystems. PEGASUS presents a set of modules implemented to perform different tasks in a flexible way of the main functionalities of GBAS, such as GNSS data conversion from reference stations, generation of the messages (types 1, 2, and 4), aircraft position calculation (including protection levels), data transmitted manipulation and simulation, in addition to the presentation of graphs and reports of the activities performed. Such functions can be applied individually using the modules in the standalone or through the creation of scenarios (EUROCONTROL, 2006). Thus, PEGASUS is interesting to analyze locations for GBAS implantation, evaluating potential errors that most affect the GNSS signals at that location.

Using GNSS data from static or mobile stations, it is possible to analyze the experimental performance of GBAS against the performance requirements for the approach categories. In this work, we show how the variation of the Sigma VIG (vertical ionosphere gradient – \( \sigma_{\text{vig}} \)), the main parameter that describes the variability of the ionosphere, affects the availability of GBAS for the CAT I precision approach category. It is different from previous work, such as those of Pereira et al. (2021), Yoon et al. (2019), Pereira et al. (2017), that analyze the estimates of ionospheric gradient values and their impacts on the GBAS, where only the gradients have been considered. We evaluated five scenarios from the implementations and simulations that can be performed by PEGASUS, including the variation of the \( \sigma_{\text{vig}} \) for GNSS data collected in receivers located in Pindamonhangaba and Guaratinguetá (São Paulo – Brazil). It consists of one of the first experiments carried out considering several ionosphere conditions for GBAS. Such investigation may support the future decision of the Brazilian aviation authorities to decide about the use of GBAS in Brazil.

2 GBAS: fundamentals

GBAS is a GNSS augmentation system developed by aviation to provide integrity, accuracy, continuity, and availability requirements for GNSS positioning that aircraft can use to support precision approach and aircraft landing. Using monitoring carried out by reference stations (usually 4) installed near the airport, GBAS (Figure 1) provides differential corrections based on the principle of correlation systematic ephemeris errors, tropospheric and ionospheric delays, that is, such errors that occur at ground stations are like those on the aircraft. This condition originating from the DGNSS (differential GNSS) is valid when the aircraft is close to the ground station and degraded when the aircraft is more distant. In addition to differential corrections, integrity parameters and information for precision approach and landing procedures are transmitted by a VDB antenna to the aircraft’s GNSS receiver (Pullen, 2017).

GBAS can be considered an alternative to traditional systems like ILS used for precision approach and landing. A single GBAS infrastructure can support multiple runways at the airport and make a curve approach, that is, without the need for alignment to the center of the runway to start the operation procedures. On the other hand, the ILS requires that the aircraft aligned with the runway’s center for each endpoint (Pullen, 2017).

2.1 Pseudorange corrections computations

As a GNSS augmentation system, the main function of GBAS is to calculate the differential corrections from the reference GNSS stations and transmit them so that an aircraft equipped with an onboard receiver can perform more accurate positioning. It allows the use of the GNSS with more reliability and safety in precision approach and landing operations, even with poor visibility (EUROCAE, 2013). GBAS use smoothed pseudorange observable (\( P_{\text{CSC}} \)) through the Hatch filter:

\[
P_{\text{CSC}}(t, t) = \frac{S}{r} P D \zeta + \left(1 - \frac{S}{r} \right) [P_{\text{CSC}}(t-1) + (\lambda \Phi(t) - \lambda \Phi(t-1))],
\]

(1)
where $S$ is the GNSS data sampling, $r$ is the smoothing time constant for the Hatch Filter (100 s in this article), $\text{PD}_t$ is raw pseudorange at time $t$ (C/A code modulated from L1 carrier phase), $P_{\text{CSC},t-1}$ is the previous smoothed pseudorange, $\lambda$ is the wavelength at L1 frequency, and $\Phi$ is the carrier phase on L1.

The pseudorange correction is obtained by the difference of the geometric distance calculated by the Euclidean distance between the reference station and a satellite $(i)$ with the pseudorange recorded by the reference station equation (2):

$$P_{\text{RC}}(i, j) = P_{\text{CSC}}(i, j) - (c \times \Delta_{\text{SV-GPS}}(i)), \quad (2)$$

where $P_{\text{CSC}}(i, j)$ is a pseudorange correction from satellite $(i)$ and reference station $(j)$; $R$ is a range between satellite $(i)$ and reference station $(j)$; $P_{\text{CSC}}$ is pseudorange smoothing for epoch $t$; $c$ is the light velocity; and $\Delta_{\text{SV-GPS}}$ is the correction satellite clock error.

Then, the $P_{\text{CSC}}$ is adjusted by removing the error from the receiver’s clock, which consists of an uncorrelated systematic effect, affecting each reference station differently (PULLEN, 2017). Equation (3) shows the removal of the receiver’s clock error in pseudorange correction:

$$P_{\text{SCA}}(i, j) = P_{\text{CSC}}(i, j) - \sum_{i \in S_i} k_i \cdot P_{\text{CSC}}(i, j), \quad (3)$$

where $P_{\text{SCA}}$ is the pseudorange correction adjusted with the remotion of the clock receiver error; $k_i$ is the weighting factor, with $\sum_{i \in S_i} k_i = 1$; $S_i$ is the set of satellites tracked by all reference stations. One of the ways to weight the value of $k$ can be by the sin function of the angle of elevation of the satellite ($\sin \theta$).

The broadcast pseudorange correction ($P_{\text{TX}}$) for the aircraft equation (4) is determined by the average of the $P_{\text{SCA}}$ obtained for satellite $(i)$ by the $M$ reference stations with corrections valid for this satellite:

$$P_{\text{TX}}(i) = \frac{1}{M(i)} \sum_{i \in S_i} P_{\text{SCA}}(i, j), \quad (4)$$

where $S_i$ is the set of receivers with valid measures for satellite $(i)$.

According to EUROCAE (2013), the rate of change of broadcast pseudorange corrections (range rate corrections – RRC), equation (5) can be calculated by dividing the difference of the $P_{\text{TX}}$ in the current time $(t)$ and the previous time $(t-1)$ by the time difference between the two epochs.

$$\text{RRC} = \frac{P_{\text{TX}}(t) - P_{\text{TX}}(t-1)}{t - (t-1)}. \quad (5)$$

### 2.2 Performance of the GFS parameters

Moreover, transmitting the pseudorange corrections and their variation rates, GFS also has some parameters that
indicate the quality and consistency of the corrections for each reference station.

Among these parameters, B-values equation (6) indicates the consistency of each receiver and satellite measurements that contribute to the generation of the pseudorange correction. They are calculated by the difference between the transmitted pseudorange corrections, and the average of the corrections determined in equation (6), excluding the corrections of the receiver j, whose consistency will be verified (EUROCAE, 2013).

\[
B(i, j) = \frac{1}{M(i) - 1} \sum_{k \neq j} PRC_{SCA}(i, k).
\]

According to Pullen (2017), Lee (2005), the B-value can be interpreted as an error that would occur in the pseudorange correction of satellite i if the measurement of the receiver M were failed. If the receiver j has faults, the correction will be given by the average of the measurements of other receivers, without faults, that track satellite i.

The errors that occur in pseudorange correction are given by the B-values. They occur because, in GBAS, not all systematic errors can be corrected. It is the case of the multipath and noise of GNSS observations. Thus, an uncertainty called \(\sigma_{pr, gnd}\) that describes the contribution of the GBAS Ground Subsystem to errors in the pseudorange correction transmitted to the aircraft. This parameter is calculated using the B-values equations (7 and 8) and transmitted to the aircraft, and is used to determine the position as a component of the uncertainty associated with the pseudorange (EUROCAE, 2013).

\[
\sigma_{pr, gnd}(b, j) = \sqrt{\frac{1}{N(b, j)} \sum_{k=1}^{N(b, j)} (B_k(b, j) - B(b, j))^2},
\]

\[
\sigma_{pr, gnd}(b, j) = \sqrt{\sigma_{pr}^2(b, j) \times (M - 1)},
\]

where b represents a satellite elevation interval to be considered and N is the number of B-values in the satellite elevation range.

As mentioned earlier, GBAS is based on the principle of spatiotemporal correlation errors, mainly atmospheric errors due to the ionosphere and troposphere. However, in regions where these layers are irregular, the correlation decreases, and there are many errors in the pseudorange corrections. These errors can be mitigated by two uncertainties related to the effects of the ionosphere (\(\sigma_{iono}\)) and troposphere (\(\sigma_{topo}\)) transmitted for the composition of errors associated with pseudorange. According to EUROCAE (2013), the uncertainty associated with the tropospheric error can be obtained by equation (9).

\[
\sigma_{topo}(i) = \sigma_N h_0 \frac{10^{-6}}{\sqrt{(0.002 + \sin^2(\theta_i))}} \left(1 - \frac{\Delta h}{h_0}\right),
\]

where \(\sigma_N\) is the refractivity uncertainty; \(h_0\) is the height in meters of the troposphere layer used to determine tropospheric error; \(\theta_i\) is the elevation angle of satellite i; \(\Delta h\) is the height difference of the aircraft (meters) and the GBAS reference point. As \(\sigma_N\) and \(h_0\) are parameters transmitted by GFS, they have to be evaluated prior to the operationalization of GBAS.

In the case of the ionosphere, \(\sigma_{v ig}\) is the parameter transmitted by GFS that describes the variability of the vertical ionospheric gradient. The \(\sigma_{v ig}\) is used to determine \(\sigma_{v ig}\) that represent uncertainty associated with the ionospheric error. Also, the \(\sigma_{v ig}\) parameter is crucial in the calculation of the protection levels and aircraft position.

\[
\sigma_{v ig}(i) = F_{pp}(i) \cdot \sigma_{v ig} (X_{aircraft} + 2 \cdot \tau \cdot V_{aircraft}),
\]

where \(F_{pp}\) is the slant factor for converting the standard deviation of the ionospheric error from the vertical to satellite–receiver direction, \(X_{aircraft}\) is the distance between aircraft and GBAS reference point (GFS), \(\tau\) is the time constant used in the smoothing filter (100 s for GBAS Approach Service Type [GAST] C), and \(V_{aircraft}\) is the horizontal speed of the aircraft, given in m/s.

In equatorial and low-latitude regions, the effects of the ionosphere on GNSS signals are one of the main obstacles to a GBAS certification since they represent a threat to accuracy and especially to integrity. According to Pereira et al. (2021), the ionospheric gradient and its uncertainty are parameters of the Ionospheric Threat Model that must be determined to implement GBAS. For such computation, a time series of GNSS data representing the ionospheric irregularity events have to be evaluated to identify its variability. It provides information on the uncertainty of the ionospheric gradient (Figure 2) for the GBAS coverage region.

The ionospheric gradient computation can be performed by two methods: Stations pairs and time step. In the studies by Lee et al. (2007); Pereira et al. (2017), details of these methods for determining the ionospheric gradient can be found.

The \(\sigma_{v ig}\) used results from the analysis of the probability density function of the vertical ionospheric gradient calculated for days with nominal ionospheric conditions and days with ionospheric anomalies. The transmitted \(\sigma_{v ig, inf}\) is inflated to contemplate the conditions of more significant variations in the ionospheric gradient, for which the mean \(\sigma_{v ig}\) value \((\mu_{v ig})\), the normalized \(\sigma_{v ig}\) value, and an inflation factor \((f)\) are considered.
An example of $\sigma_{\text{vig}}$ determination (Figure 3) can be found in the study by Lee et al. (2007), in which the sigma overbound method was developed based on the ionospheric gradient probability density function. This method was used to assess extreme or anomalous conditions from the CONUS (CONterminous United States) threat model.

### 2.3 Aircraft position computation

After receiving the corrections and integrity parameters transmitted by the GFS, the aircraft GNSS receiver will calculate the corrected pseudorange at the aircraft level. But, before being applied, some information is checked, such as the received signal strength, latency time valid, and the compatibility of the identical satellites with corrections. The accepted measures are applied in equation (12) of the aircraft’s pseudorange correction ($\text{PRC}_{\text{corr,air}}$) (Pullen, 2017):

$$
\text{PRC}_{\text{corr,air}}(i) = P_{\text{CSC,air}}(i)(t) + \text{PRC}_{\text{TX}}(i) + \text{RRC}(i)(t - t_{z,\text{count}}) - \text{TC} + c(\Delta t(i))_{\text{SV}},
$$

where $P_{\text{CSC,air}}(i)$ is the smoothed pseudorange recorded by satellite $i$ at the aircraft receiver at time $t$ before applying correction; $\text{PRC}_{\text{TX}}$ is the pseudorange correction transmitted by GFS, being valid for time $t_{z,\text{count}}$; $\text{RRC}$ is the rate of change of transmitted pseudorange corrections valid for time $t_{z,\text{count}}$; $\text{TC}$ is the differential correction of tropospheric delay; $c$ is the speed of light in a vacuum; ($\Delta t(i))_{\text{SV}}$ corresponds to the satellite clock error applied at time $t$.

Then, the three-dimensional components of aircraft position ($X$, $Y$, and $Z$) are estimated in the WGS84, as well as the receiver’s clock applying the weighted least squares method, according to EUROCAE (2013); Pullen (2017):

$$
\hat{x} = ((A^TW^{-1}A^T)^{-1}A^TW)\text{APRC}_{\text{corr,air}},
$$

where $\hat{x}$ is the vector to be estimated, with the three-dimensional components of position $X$, $Y$, and $Z$ and the clock error; $A$ is the design (Jacobian) satellite–receiver matrix, with a dimension of $N$ rows and four columns; $W$ is the weight matrix, with the variance of
each measurement by satellite. The Jacobian matrix $A$ is composed of the partial derivatives of the satellite-aircraft line-of-sight vector augmented by 1 for the clock.

$$A = \begin{pmatrix}
X_{s1} - X_a & Y_{s1} - Y_a & Z_{s1} - Z_a & 1 \\
X_{s2} - X_a & Y_{s2} - Y_a & Z_{s2} - Z_a & 1 \\
X_{s3} - X_a & Y_{s3} - Y_a & Z_{s3} - Z_a & 1 \\
\vdots & \vdots & \vdots & 1 \\
X_{sn} - X_a & Y_{sn} - Y_a & Z_{sn} - Z_a & 1
\end{pmatrix}, \quad (14)$$

where $X_{si}$, $Y_{si}$, and $Z_{si}$ are satellite positions; $X_a$, $Y_a$, and $Z_a$ are approximated aircraft position; $\rho_{si}$ is the computed geometric distance from the aircraft to the satellite $i$ (with $i = 1, 2, 3, \ldots, n$).

However, geocentric coordinates are not most adequate to evaluate the positioning accuracy. According to EUROCAE (2013), Jacobian matrix $A$ can be transformed to the Local Cartesian Frame, North, East, and Down, with origin fixed in the runway on landing threshold point/ fictitious threshold point using azimuth (Az) and elevation angle ($\theta$) of each satellite (ranging from 1 to $n$). In such a case, one has:

$$A = \begin{pmatrix}
\cos(\theta)\cos(Az)_1 & \cos(\theta)\sin(Az)_1 & \sin(\theta)_1 & 1 \\
\cos(\theta)\cos(Az)_2 & \cos(\theta)\sin(Az)_2 & \sin(\theta)_2 & 1 \\
\cos(\theta)\cos(Az)_3 & \cos(\theta)\sin(Az)_3 & \sin(\theta)_3 & 1 \\
\vdots & \vdots & \vdots & 1 \\
\cos(\theta)\cos(Az)_n & \cos(\theta)\sin(Az)_n & \sin(\theta)_n & 1
\end{pmatrix}. \quad (15)$$

The weight matrix $W$ of the satellite measurements consists of a diagonal matrix composed of the inverse of the fault-free variance ($\sigma^2_i$). This variance term contains all the elements of the ground and aircraft subsystems seen previously:

$$\sigma^2_i = \sigma^2_{pr,\text{grad}}(i) + \sigma^2_{pr,\text{air}}(i) + \sigma^2_{\text{tropo}}(i) + \sigma^2_{\text{iono}}(i), \quad (16)$$

where airborne error contribution ($\sigma_{pr,\text{air}}$) is defined in RTCA DO-245A (2004) as the combination of two error sources, receiver noise, and indifference and multipath:

$$\sigma_{pr,\text{air}}(i) = \sqrt{\sigma^2_{\text{multipath}}(i) + \sigma^2_{\text{noise}}(i)}, \quad (17)$$

where $\sigma_{\text{multipath}}(i) = 0.13 + 0.53*10^{-10}$, and $\sigma_{\text{noise}}(i) = a_0 + a_1*10^{-10}$. The $\sigma_{\text{noise}}$ term is calculated according to the performance defined by aircraft accuracy designator (AAD) for two levels A and B (Table 1).

The $\sigma_{\text{tropo}}$ and $\sigma_{\text{iono}}$ were presented, respectively, in equations (9 and 10). As aircraft was in static mode, the $\sigma_{\text{iono}}(i) = F_{\text{pr}}(i) \cdot \sigma_{\text{ig}} \cdot X_{\text{aircraft}}$.

Thus, one can then determine the matrix of $W$:

$$W = \text{diag}[\sigma^2_{\text{pr}(1)}, \sigma^2_{\text{pr}(2)}, \sigma^2_{\text{pr}(3)}, \sigma^2_{\text{pr}(4)}, \ldots, \sigma^2_{\text{pr}(N)}]. \quad (18)$$

According to EUROCAE (2013); Felux et al. (2017), the term $(A^T PA)^{-1}A^T P$ corresponds to the projection matrix, and making $S = (A^T PA)^{-1}A^T P$, it can determine the uncertainties of the position errors, according to equations (19 and 20). Another way to calculate standard deviations is to express them for errors in the horizontal and vertical components:

$$\sigma_{\text{error, horiz.}} = \sqrt{\sum_{i=1}^{N} S_{\text{horiz.}}^2(i) \cdot \sigma^2_i(i)}, \quad (19)$$

$$\sigma_{\text{error, vert.}} = \sqrt{\sum_{i=1}^{N} S_{\text{vert.}}^2(i) \cdot \sigma^2_i(i)}, \quad (20)$$

where $S_{\text{horiz.}}(i)$ and $S_{\text{vert.}}(i)$ are, respectively, the horizontal and vertical components in the projection matrix determined by equations (21 and 22) (Felux et al., 2017):

$$S_{\text{horiz.}}(i) = \sqrt{S_{11}^2(i) + S_{22}^2(i)}, \quad (21)$$

$$S_{\text{vert.}}(i) = S_{33}(i) + S_{44}(i) \cdot \tan(\theta)_{\text{GPA}}. \quad (22)$$

The elements $S_{1}(i)$, $S_{2}(i)$, and $S_{3}(i)$ represent, respectively, in the first, second, and third columns of row $i$ of the projection matrix $S$. The term $(\theta)_{\text{GPA}}$ is the angle of the approach path to the runway (3 degrees is generally used).

It can also be derived from the covariance propagation in equation (23), and one obtains the covariance matrix of the estimate parameters (coordinates and clock error):

$$\sum_{x} = (AP)^{-1}. \quad (23)$$

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**Table 1: Airborne thermal noise and interference error model for AAD parameters**

<table>
<thead>
<tr>
<th>AAD</th>
<th>$\theta_1$ (deg)</th>
<th>$\theta_0$ (deg)</th>
<th>$a_0$ (m)</th>
<th>$a_1$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$\geq 5$</td>
<td>6.9</td>
<td>0.15</td>
<td>0.43</td>
</tr>
<tr>
<td>B</td>
<td>$\geq 5$</td>
<td>4.0</td>
<td>0.11</td>
<td>0.13</td>
</tr>
</tbody>
</table>
2.4 Protection levels for precision approach

The protection levels (PL) represent limits for the position errors of each aircraft, based on extremely low probabilities of loss integrity that are required according to GBAS precision approach categories (Pullen, 2017) (Figure 4).

According to Pullen (2017), the protection levels are built by extrapolating probability distributions limiting the errors expected for the user (aircraft). PL calculation is determined in the vertical (VPL) and horizontal (HPL) components. For the precision approach (PA) service, HPL and VPL are calculated as the maximum value that position errors can achieve, considering hypothesis tests. For the first one, H0 is the nominal case, free of faults, while the alternative H1 assumes a single fault of the reference receiver (Dautermann et al., 2011).

For the nominal case, HPL and VPL are obtained from equations (24 and 25):

\[
\text{HPL}_{PA,H0} = (K_{\text{fmd}}) K_{\text{err}} \text{h}_{\text{horiz.}} + D_{\text{horiz}},
\]

\[
\text{VPL}_{PA,H0} = (K_{\text{fmd}}) K_{\text{err}} \text{v}_{\text{vert.}} + D_{\text{vert}},
\]

where \( K_{\text{fmd}} \) is a multiplication factor for fault detection, the value of which is defined according to the number of reference stations used to generate pseudorange corrections, in this case with 4 receivers \( K_{\text{fmd}} = 5.847 \text{ m} \); \( D_{\text{fmd}} \) and \( D_{\text{fmd}} \), are, respectively, the magnitudes of the horizontal and vertical projections, considering the difference between the smoothed position by the 30 and 100 second on smoothing pseudorange filter. For the GAST C, the values of \( D_{\text{horiz.}} \) and \( D_{\text{vert.}} \) are equal to zero, since only the 100 s smoothing filter is used (EUROCAE, 2013).

One additional protection level is defined to cover failures not detected by the ground system. In this case, \( K_{\text{fmd}} \) and \( D_{\text{fmd}} \) are obtained by equation (26 and 27, according to EUROCAE (2013); Pullen (2017):

\[
\text{HPL}_{PA,H1} = (K_{\text{fmd}}) (K_{\text{err}}) \sum \sum \sigma_{\text{err,horiz.}}(i),
\]

\[
\text{VPL}_{PA,H1} = (K_{\text{fmd}}) (K_{\text{err}}) \sum \sum \sigma_{\text{err,vert.}}(i),
\]

where \( K_{\text{fmd}} \) is the failure detection multiplier, which also depends on the number of reference receivers used: for 2 receivers \( K_{\text{fmd}} = 2.935 \text{ m} \); for 3 \( K_{\text{fmd}} \) receivers \( = 2.898 \text{ m} \); and using 4 \( K_{\text{fmd}} \) receivers \( = 2.878 \text{ m} \).

3 Methodology and experiments

To evaluate the performance and understanding of GBAS positioning and functionalities, we used PEGASUS software developed by EUROCONTROL. PEGASUS is composed of a set of tools arranged in modules, which make it possible to carry out several GBAS functionalities such as the conversion of GNSS data, generation of the content of messages transmitted by the VDB, calculation of the aircraft’s position and errors, as well as the protection levels, graphical representations of the results, and reports of the activities performed (EUROCONTROL, 2006).

GBAS experiments in the static mode used GNSS data from 4 reference stations in Pindamonhangaba – SP, as shown in Figure 5a, while the other GNSS receiver simulating aircraft was in Guaratinguetá – SP (Figure 5b). The distance between the reference receivers and the aircraft simulation is approximately 31 km (the coverage area of a GBAS is 23 NM or about 42 km).

The IACIT company (www.iacit.com.br) provided the GNSS data. All GNSS receivers used are NovAtel CMA 4048 model. Some results and analyzes will be presented according to the configurations carried out in this research.

The PEGASUS modules used for GBAS simulations were: GGC (generate gbas corrections), responsible for the content of messages generated by GFS, and GNSS Solution, which calculates the coordinates, position errors, and protection levels of the aircraft. Moreover, two other tools were used: the converter, which converts the file of the GNSS receivers to a specific program format, and the MFile Runner, which generates visualizations of the results of the simulations.
The $\sigma_{\text{vig}}$ indicates the variation of the ionospheric gradient and influences the estimate of HPL and VPL. The work analyzed the performance of 5 GBAS simulation scenarios varying the $\sigma_{\text{vig}}$ with the following values: 4, 8, 12, 16, and 20 mm/km. Such values were selected to characterize several ionosphere conditions. For example,
a 4 mm/km scenario characterizes low gradient ionospheric common in the United States according to CONUS Threat Model. The increase in \( \sigma_{\text{vig}} \) values may indicate the impact of high ionospheric gradients on the availability of GBAS, especially in Brazil, where studies such as Pereira et al. (2021), Pereira et al. (2017) showed that \( \sigma_{\text{vig}} \) calculated for March 2014, at the last high of the solar cycle reached values of 20 at 40 mm/km using pairs of reference stations up to 50 km.

### 3.1 Generation of GBAS messages

The GBAS simulation carried out met the category of precision approach CAT I or GAST C (GBAS Approach Service Type) that requires the generation and transmission to the aircraft GBAS Message Types 1, 2, and 4. The analysis period of the GBAS simulation was 9 h of data on 03/16/2019, starting at 05 h 08 min 25 s and ending at 14 h 08 min 24 s (Universal Time). All receivers tracked pseudorange (code C/A) and carrier phase on GPS L1 frequency data, with a sampling interval of 0.5 s. The smoothing filter initialization constant was 100 s to obtain smoothed pseudorange, as provided by Standard Recommended and Practices for CAT I or GAST C (GBAS Approach Service Type).

Part of the GGC module configuration is done with the insertion of text files with parameters referring to the geodetic coordinates of the stations; the satellite’s elevation weight function to correct the receiver’s clock error; and in this case, the \( \sin^2(\theta) \) function. Other relevant configurations were GAD 2 (ground accuracy designator); minimum elevation angle of the satellites equal to 5°; refractivity of 763; refractivity uncertainty of 20; GCID 1 (GBAS Continuity and Integrity Designator); 42 km for Dmax; kmPOSGPS of 6.0 m; kmCAT1GPS of 5.0 m; vertical alert limit of 10 m; horizontal alert limit of 40 m; GPA (glide path approach) equal to 3°; 49 m for threshold crossing height; and 105 m course width. A more detailed description of all these parameters can be found at EUROCAE (2013).

Another parameter configured in the GGC module is \( \sigma_{\text{vig}} \), which is part of the content of Type 2 messages. Although it is not a variable used in pseudorange correction generation algorithms and ground subsystem integrity parameters, \( \sigma_{\text{vig}} \) is transmitted as a parameter of the region covered by the airport. For this work, five scenarios with \( \sigma_{\text{vig}} \) values were evaluated: 4, 8, 12, 16, and 24 mm/km. As stated before, such selection aims to evaluate the impact of the ionospheric gradients on the availability of GBAS.

### 3.2 Positioning and integrity parameters

After the generation of GBAS Message content by the GGC module, the simulated aircraft data were used in the GNSS Solution module to obtain estimates of coordinates, protection levels, position errors, and other indicators inherent to GNSS positioning.

The GPS receiver representing the aircraft (simulated) was in Guaratinguetá – SP (Figure 5b) and had a similar setting to the receivers of the reference stations mentioned in Section 4.1. The AAD parameter was selected as level A. The AAD parameter is used to determine the pseudorange uncertainty calculated by the aircraft (\( \sigma_{\text{pr.ai}} \)) and is based on the multipath and noise at the aircraft’s receiver.

As the experiment was carried out in static mode and as the position of the simulated aircraft is known, they were inserted to analyze the horizontal and vertical positioning errors (HPE and VPE).

### 4 GBAS messages estimated and main results

One of the main related tasks of the GBAS ground sub-system is the generation of the pseudorange correction, resulting from the GNSS observations of the reference stations. Figure 6 shows the time series of the pseudorange corrections estimated for each visible satellite during the period considered.

The pseudorange corrections presented a behavior like a parabola with the concavity facing downwards. The highest correction values (in absolute) were recorded when the satellites were closer to the horizon. As satellites ascend in the zenith direction, corrections are reduced in value. For example, the correction on the PRN13 satellite reached almost –30 m in the interval from 1 pm to 2 pm. Considering that a 5° satellite elevation mask was used, the results of the PRCs are consistent with the those of the literature, which indicate that the GNSS observations carried out by satellites close to the horizon generally present a higher occurrence of systematic errors originating from multipath, ionosphere, troposphere, and others.

Figure 7 shows the satellites’ elevation angles from the GNSS data collected by the reference station Receiver 4. Considering that the distance between the reference stations is approximately 250 m in the other receivers, it is expected that the elevation angles of the satellite values are like those shown in Figure 7.

**B**: Values are integrity parameters used to check the consistency of the PRCs of each reference station and
satellite in relation to the value that was transmitted, in case of failure in one of the stations. Figure 8 shows the $B$-Values ($B_1$, $B_2$, $B_3$, and $B_4$), which represent what would be the difference in the PRC if there was a failure in each of the reference stations and the average of the PRCs of the other 3 were used. Therefore, $B_1$ indicates failure at station Receiver 1, $B_2$ at station Receiver 2, and so on.

The closer the $B$-Values are to zero, the more consistent are the PRCs transmitted values. This indicates a more homogeneous environment in terms of system errors and less variation in pseudorange corrections for each reference station.

From Figure 8, one can see that the $B$-values have an approximate variation within the range of $-0.3$ m to $0.3$ m for $B_1$, $B_2$, and $B_3$. As for $B_4$, the variation occurred between $-0.1$ m and $0.4$ m. In all situations, there were peaks of more significant variation in some satellites, of which the following stand out:
Figure 8: B-Values parameters for each satellite and reference station.
• For B1, the PRN2 differs by 0.45 m from the broadcast PRC.
• For B2, the PRN10 differs by 0.50 m from the broadcast PRC.
• For B3, the PRN13 differs by −0.45 m from the broadcast PRC.
• For B4, the PRN26 differs by 0.55 m from the broadcast PRC.

This analysis of the B-value parameter is relevant mainly because it is used to calculate protection levels, which are indicators of the integrity of GBAS. In Figure 9, these peaks of higher B-values generally coincide with the times of higher values in the PRC module. Therefore, they must be associated with the noisiest GNSS observation periods.

4.1 Results in the position domain

Now, GBAS simulations at the aircraft level will be carried out, with the estimates in the positions and their errors and the integrity indicators and performance analysis.

Figure 9: PRC and B-Values for PRN satellites 2, 10, 13, and 26.
Thus, Figure 10 shows the time series of the VPE and VPL calculated for the GAST C precision approach. In this scenario, a $\sigma_{\text{vig}}$ value of 4 mm/km was applied.

In the horizontal component, the HPE was lower than the HPL during the whole period analyzed for all scenarios of the $\sigma_{\text{vig}}$. The horizontal alert limit is 40 m and was not exceeded in any evaluated scenarios.

On the other hand, in the vertical component, 3.5% of the time, the VPE was higher than the VPL, indicating the occurrence of Misleading Information (MI). In this case, the system is declared available because VPE and VPL did not exceed the vertical alert limit. The protection level represents the maximum estimated error in positioning for the precision approach to occur safely.

The second scenario evaluated considered the $\sigma_{\text{vig}}$ value equal to 8 mm/km. In Figure 11, it is possible to observe the performance of the horizontal and vertical components for this scenario.

The VPE had only 9 epochs between 6 am and 7 am, where there was a slight superiority of the VPE over VPL. The $\sigma_{\text{vig}}$ of 8 mm/km was the scenario with the highest availability of 99.9% and the lowest occurrence of MI events. However, there was an increase in the protection levels compared to the previous scenario.

For the 12 mm/km $\sigma_{\text{vig}}$ scenario (Figure 12), HPE and VPE were lower than the HPL and VPL. However, there was an increase in the values of the protection levels, and 2.2% times, the VPL exceeded the alert limit that indicates loss of availability for the CAT-I approach.

This condition of the vertical protection levels exceeding the vertical alert limit can also be seen in the other tested scenarios with a $\sigma_{\text{vig}}$ of 16 mm/km and 20 mm/km, as

![Figure 10: Performance of positioning errors and protection levels for GBAS simulation considering $\sigma_{\text{vig}}$ of 4 mm/km.](image)

![Figure 11: Performance of positioning errors and protection levels for GBAS simulation considering $\sigma_{\text{vig}}$ of 8 mm/km.](image)
shown in Figures 13 and 14. For $\sigma_{\text{vig}}$ of 16 mm/km was 17.3% time and $\sigma_{\text{vig}}$ of 20 mm/km was 38.4%, the VPL was higher than VAL. Another interpretation of protection levels is that they represent the maximum estimated error that position errors can achieve. Therefore, Figures 13 and 14 show the reduced availability of GBAS due to the estimated error (VPL) being more extensive than the VAL. This reduction decreases with increasing $\sigma_{\text{vig}}$, although the position error is still below the alert limit.

A critical factor observed was the presence of peaks and discontinuities in the protection levels, increasing and decreasing the PL values. This occurrence coincides with the change in satellite geometry, as shown in Figure 15, which is related to PDOP (precision dilution of position) and the number of satellites visible (NSV) at each positioning epoch.

The exclusion of satellites can be due to several reasons such as no corrections, decorrelation of ephemeris, and the restart of the pseudorange smoothing process due to cycle slip. However, a detailed analysis was not carried out about each occurrence, only the identification of the behavior of the VPL with the variations of PDOP and NSV.

An anomalous ionospheric gradient occurrence may be one of the causes for the reduction of satellites observed in Figure 15, so Table 2 shows the impact of $\sigma_{\text{vig}}$ on the residuals errors uncertainty from the ionosphere ($\sigma_{\text{iono}}$) calculated according to equation 10 and the approximate distance of 31 km between the reference stations and simulated aircraft, as well as elevation angle: 5, 10, 15 and 20°. As the aircraft is in static mode, the speed is at zero, and $\sigma_{\text{iono}}$ is the result of the multiplication of the terms $F_{\text{pp}}$, $\sigma_{\text{vig}}$, and $X_{\text{aircraft}}$.

The increase in residual uncertainty due to the ionosphere can be observed as the $\sigma_{\text{vig}}$, and the elevation angle of the satellite increases, representing, therefore, a multiplicative factor of uncertainty as the pseudorange.

![Figure 12: Performance of positioning errors and protection levels for GBAS simulation considering $\sigma_{\text{vig}}$ of 12 mm/km.](image1)

![Figure 13: Performance of positioning errors and protection levels for GBAS simulation considering $\sigma_{\text{vig}}$ of 16 mm/km.](image2)
measure to be applied in the positioning of the aircraft and the calculation of the protection levels as shown in Figure 14.

It is important to emphasize that in the future, the tendency is for this degradation caused by the geometry and availability of satellites to be reduced, as there will be a more significant number of satellites, as well as the possibility of new GNSS signals that will provide linear combinations that aim to minimize effects systematic as well as the variation of the ionospheric gradient.

5 Conclusions

In this work, we presented a theoretical basis and results of GBAS simulations, evaluating five different scenarios of the $\sigma_{\text{vig}}$ parameter that describe the ionosphere’s variability, differently from previous works that concentrate on the computation of gradients.

In the five $\sigma_{\text{vig}}$ scenarios analyzed (4, 8, 12, 16, and 20 mm/km), the first two, with 4 and 8 mm/km, had, as expected, better performance, with the errors of the vertical position being lower than the alert limit. However, for $\sigma_{\text{vig}}$ 4 mm/km 3.5% of the time, the VPL was lower.
than the position errors, indicating a loss of integrity for the CAT I precision approach category in those epochs. In the 12, 16, and 20 mm/km scenarios, respectively, 2.2%, 17.3%, and 38.4% of the time, the VPL was higher than VAL, which expresses a loss of availability from GBAS to CAT I.

The results showed that an increase in $\sigma_{\text{vig}}$ works as an inflation factor of the protection levels. Further to this relationship, the changes in geometry resulting from the PDOP coincided with the increase or decrease in the protection levels. Lower values of PDOP minimize the protection levels, while higher values coincide with the increase of the protection levels.

The research carried out with a static GBAS station was essential to evaluate parameters such as accuracy, integrity, and local availability of GBAS in scenarios of ionosphere variations. The results obtained in the evaluated scenarios show that a $\sigma_{\text{vig}}$ from 4 to 8 mm/km may be able to maintain the quality parameters and requirements for approximation of GBAS CAT I accuracy. In contrast, a $\sigma_{\text{vig}}$ scenario higher than 8 mm/km implies in loss of availability for CAT I with the protection levels exceeding the alert limit.

However, the limitations involving the analyzes carried out in this work, which were based on simulations of the parameter $\sigma_{\text{vig}}$ must be considered. Therefore, an analysis with estimated values of $\sigma_{\text{vig}}$ in different years periods would make the analysis more realistic, comparing the performance in periods of low and high ionospheric scintillation, evaluating the behavior of $\sigma_{\text{vig}}$, position errors, and protection levels under these conditions.

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