Research Article

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Forecasting Volatility and the Risk–Return Tradeoff: An Application on the Fama–French Benchmark Market Return

Abstract: The paper uses the daily stock market index returns of Fama–French to attempt a comparative forecasting analysis of different volatility models. The comparison naturally pre-requests the specification of the competing volatility frameworks and therefore the paper among other issues deals with dilemmas about whether volatility–return relations hold. As expected the analysis focuses on FIEGARCH-M models that extend the basic long memory volatility framework of Bollerslev and Mikkelsen (1996. “Modeling and Pricing Long Memory in Stock Market Volatility.” *Journal of Econometrics* 73:151–84) with the introduction of a volatility in mean effect. Taking also into consideration the work of Christensen, Nielsen, and Zhu (2007. “The Effect of Long Memory in Volatility of Stock Market Fluctuations.” *Review of Economics and Statistics* 89:684–700) for the existence of spillover effects when conditional in mean equations hold a stationary and a long memory component the analysis estimates the filter long memory volatility models FIEGARCH-MG and FIEGARCH-MH presented in Christensen, Nielsen, and Zhu (2010. “Long Memory in Stock Market Volatility and the Volatility-in-Mean Effect: The FIEGARCH-M Model.” *Journal of Econometrics* 155:170–87) in order to test whether such filter adjustments can improve volatility forecasting. Although there is no particular reason to assume that the stationary inputs in the return equations will necessarily follow the normal distribution that Christensen, Nielsen, and Zhu (2010. “Long Memory in Stock Market Volatility and the Volatility-in-Mean Effect: The FIEGARCH-M Model.” *Journal of Econometrics* 155:170–87) assume, the paper follows this path but nevertheless enriches this aspect of the analysis by introducing alternative distributional assumptions. The results indicate the existence of a statistically significant mean effect when both filter models are estimated under the assumption of t-student distribution, although as far as volatility forecasting is concerned both

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filtered models cannot outperform in terms of forecasting criteria the parsimonious FIEGARCH version that dominates filter and non-filter volatility models under various forecasting horizons.

Keywords: FIEGARCH, financial leverage, GARCH, long memory, risk–return tradeoff, stock returns, volatility feedback

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1 Introduction

The main goal of the paper targets the comparison of different volatility models in terms of their volatility forecasting potentials. Of course the competing models must be specified first, since modeling volatility often implicates various dilemmas about (a) the ideal number of ARCH and GARCH terms employed in every volatility specification, (b) the distributional assumptions applied and upon which the corresponding likelihood functions are formed, (c) the incorporation or not in the estimated frameworks of volatility–return relations, and finally (d) dilemmas related to long memory features and spillover effects when both phenomena are present.

The specification process addresses simultaneously the possibility that certain frameworks may acknowledge risk–return tradeoffs and therefore decides whether these should be enriched with such relations.¹

There are three theoretical approaches that justify the existence of such a risk–return connection: (a) the risk–return tradeoff assumption, (b) the financial leverage effect, and (c) the volatility feedback effect mechanism. The first underlines a positive relation among volatility and returns² assuming of course rational investors that will take on additional risk whenever they expect higher returns. The second approach is attributed to Black (1976) and is broadly known as financial leverage effect. The theory states that although bad news decreases the price of stocks at the same time increasing financial leverage or the debt/equity ratio for the corresponding firms, and this situation by setting stocks riskier after the price drop increases future expected volatility. So leverage effect

¹ These volatility models are denoted with the M designation. See for example GARCH-M, EGARCH-M, FIEGARCH-M, FIGARCH-M.

² The idea triggered Engle (1982) to introduce the GARCH-M model in volatility analysis. This sets volatility as an explanatory variable in the corresponding conditional mean equation.

sequence introduces a negative relation between volatility and returns. Finally, the last approach is attributed to Cambell and Hentschel (1992) and is known as the volatility feedback effect mechanism. The last resumes the following sequence of events: an increase in volatility increases further the risk premium and the discount rate in the economy and with the stream of dividends unchanged this lowers the price of stocks and therefore produces a negative volatility–return relation.³

Another issue extensively explored in the present analysis is the presence of long memory in volatility. Although it is questionable if long memory exists in stock market returns, however there seems to be consensus among empirical evidence about the existence of long memory in volatility and how this should be modeled.⁴

These characteristics which are common findings in a number of empirical studies set naturally FIEGARCH model of Bollerslev and Mikkelsen (1996) at the core of the present analysis. The last not only incorporates the asymmetric volatility reactions to positive and negative innovations as the exponential EGARCH model of Nelson (1991) does, but also accounts for long memory features in volatility through the famous FIGARCH modeling of Baillie, Bollerslev, and Mikkelsen (1996). Note that a natural extension of the last model is the FIEGARCH-M framework that incorporates the mean relation among the FIEGARCH volatility specification and the data return series.⁵

An interesting insight on the properties of the FIEGARCH-M model has recently been demonstrated in Christensen, Nielsen, and Zhu (2010). The analysis states that the introduction of volatility in the return equation generates long memory in returns and the avoidance of this so-called spillover effect through long memory filters proves crucial for both the forecasting properties of FIEGARCH and the acknowledgment of the risk–return tradeoff. So these filter volatility models of Christensen, Nielsen, and Zhu (2010) set an interesting empirical challenge that is taken on by the present analysis. Specifically the paper estimates two filter FIEGARCH-M models, the FIEGARCH-MG and FIEGARCH-MH. Both consider the possibility that changes in volatility enter the conditional in mean equation rather than volatility levels themselves.

3 Christensen, Nielsen, and Zhu (2010) state that volatility feedback effect mechanism will be strongest at the market level, whereas the leverage effect is something to be seen mainly in individual stocks time series.

4 See for example Crato and de Lima (1994), Baillie et al. (1996), Robinson (1991), Baillie and Morana (2009), Barkoulas and Baum (1996), Barkoulas, Baum, and Travlos (2000), Sadique and Silvapulle (2001), Henry (2002), Tolvi (2003), Gil-Alana (2006), Cheung and Lai (1995), Crato (1994).

5 This is the basic operation of any volatility-M model.

In terms of volatility forecasting FIEGARCH is initially compared to GARCH, IGARCH, FIGARCH, EGARCH, and GJR. All these models are estimated for the daily stock market index returns of Fama–French for a period of 37 years from 01.07.1963 to 31.06.2010.⁶ The specification settings applied on every volatility model are decided upon standard information criteria that reward both the goodness-of-fit and parsimony and the out of sample forecasting.⁷ For example although all the above models are estimated initially under the normality assumption, a trialing process of applying alternative distributions such as the t-student, the general error, and the skewed t-student distribution of Fernandez and Steel (1998) indicates that the fit of the above models is significantly enhanced whenever the skewed t-student option⁸ is assumed, and so the majority of the estimated models are estimated under this latter distributional assumption.

At this initial stage and controlling for possible autocorrelations in data returns the comparisons verify three things. First, FIEGARCH is the best forecasting volatility model. Second, although normal distribution favors the statistical presence of the latter relation however the last is not supported by the selection criteria and so is excluded as a setup option. Third, the volatility in mean effect irrespective of the estimated volatility framework turns out being not statistically significant, and so there seems to be no risk–return tradeoffs.

Finally and as far as the filter long memory volatility models is concerned, there is no obvious reason why innovations in the corresponding conditional in mean equations must follow the standard normal distribution that Christensen, Nielsen, and Zhu (2010) assume and therefore the paper changes these estimation settings by introducing various possibilities over the distributional assumptions. The t-student distribution which is finally chosen for both filter volatility models acknowledges a statistically significant mean effect, although both filter models cannot outperform in terms of forecasting the parsimonious FIEGARCH model according to standard forecasting criteria.

The rest of the paper is organized as follows. Section 2 presents briefly the mathematical equations of the M volatility models and the quasi-maximum

⁶ The data are available on <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/>.

⁷ The specification term here refers to certain characteristics of the estimated volatility frameworks. Such characteristics refer to (a) the hypothesized distribution of innovations, (b) the number of GARCH-ARCH terms employed, (c) the inclusion or not in the estimated frameworks of possible M features, and (d) the number of statistically significant autoregressive terms introduced in the conditional in mean equation in order to control existence of possible autocorrelations in the daily stock market returns.

⁸ The main drawback of t-student distribution and GED is that even if account for fat – tails densities they are both symmetric and so miss skewness and kurtosis characteristics that are extremely important in financial empirical applications.

likelihood (QML) method introduced in the analysis. Section 3 estimates all competing volatility frameworks. Specifically, Section 3.1 estimates the non-filter EGARCH, FIEGARCH, FIGARCH, GARCH, GJR, and IGRACH volatility models, and Section 3.2 estimates the filter models, FIEGARCH-MH and FIEGARCH-MG. Section 4 presents a comparative forecasting analysis of all estimated volatility frameworks, and finally Section 5 concludes.

2 The volatility-M models and the quasi-maximum likelihood estimation

The basic idea of any volatility-M model is the introduction of volatility in the return equation. Denoting r_t as the daily return of a stock or stock market index at time t , F_{t-1} as the available set of information up to $t - 1$ moment, z_t as a white noise process at time t , the general representation of any volatility-M model is given in equation system [1], with eqs [1a]–[1c] stating, respectively, the conditional in mean specification, the conditional variance of the residuals, and the general ARCH representation.

$$r_t = \mu + \lambda \sigma_t^2 + \varepsilon_t \quad [1a]$$

$$\sigma_t^2 = E(\varepsilon_t^2 / F_{t-1}) \quad [1b]$$

$$\varepsilon_t = z_t(v_t a)^{1/2} \quad [1c]$$

$$v_t = \left(1, \varepsilon_{t-1}^2, \dots, \varepsilon_{t-q}^2, \sigma_{t-1}^2, \dots, \sigma_{t-p}^2\right) \quad [1d]$$

Equation system [1] is estimated with QML.⁹ Although ordinary least squares (OLS) deliver consistent estimates, however, the maximum likelihood approach is more efficient since the estimated parameters will converge to their population counterparts at a much faster rate.¹⁰ However, in this case the problem is that log-likelihood functions will depend upon the assumed distribution of

⁹ If normality is assumed but the true conditional distribution is not normal the maximum likelihood estimations are known to be QML. Weiss (1986) and Bollerslev and Wooldridge (1992) show that these estimations are consistent if equations of conditional mean and conditional variance are correctly specified.

¹⁰ Engle (1982) shows that estimating ARCH (1) model with maximum likelihood method provides gains in efficiency that are quite large.

innovations and specifically upon the assumed conditional distributions of ε_t and r_t .¹¹

Although Engle (1982) states that conditional normality may not be as restrictive choice as it initially appears,¹² however, a common strategy is to let alternative assumptions form the candidate conditional distributions. Indeed Palm (1996), Pagan (1996), and Bollerslev, Chou, and Kroner (1992) underline the widespread use of fat-tailed distributions in volatility literature¹³ and particularly state that the symmetric t-student distribution, the generalized error distribution (GED), and the skewed student distribution of Fernandez and Steel (1998) are among the most interesting alternatives. In fact since distribution of asset returns in most cases is negatively skewed,¹⁴ the skewed t-student distribution of Fernandez and Steel (1998) which adjusts by default to phenomena of kurtosis and skewness is an extremely useful choice.

Although equation system [1] describes almost any volatility-M model, however the paper of Christensen, Nielsen, and Zhu (2010) proposes an alternative system that introduces volatility changes in the conditional in mean equation rather than volatility levels themselves. The approach in the present analysis is applied solely for the FIEGARCH-M model, and further details will be given as soon as there is presentation of the basic FIEGARCH framework.

As it is known FIEGARCH combines both the FIGARCH model of Baillie et al. (1996) and the asymmetric EGARCH model of Nelson (1991). The model is specified in the next equation

$$\varphi(L)(1-L)^d(\ln \sigma_t^2 - \omega) = \psi(L)g(z_{t-1}) \quad [2]$$

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(d+1)}{\Gamma(k+1)\Gamma(d-k+1)} L^k \quad [3]$$

where ω denotes the mean of the logarithmic conditional variance, $\varphi(L)$ and $\psi(L)$ are polynomials in the lag operator, $(1-L)^d$ stands for the fractional

11 Z_t term in eq. [1c] is usually named innovation process. The reader must keep in mind that the term innovation in the present context refers either to the previous definition or to the one stated by Christensen, Nielsen, and Zhu (2010). The last will be clarified as soon as there is presentation of the filter volatility models.

12 Engle (1982) states that if conditional distribution of returns is normal, the unconditional will not be normal since its shape will strongly display a leptokurtic shape.

13 Bollerslev (1987), Hsieh (1989), Baillie and Bollerslev (1989), Palm and Vlaar (1997) among others show that these distributions perform better in capturing the higher observed Kurtosis.

14 Note that this is the case for the Fama–French stock market index. For more information on the use of non-normal distributions when estimating GARCH models see Laurent and Peters (2002).

difference operator defined in eq. [3] with Γ in the same equation referring to the Gamma function, z_t denotes the normalized innovation at time t defined as $z_t = \varepsilon_t / \sigma_t$ and $g(z_{t-1})$ introduces the news impact function defined as in eq. [4]

$$g(z_t) = \theta z_t + \gamma(|z_t| - E|z_t|) \quad [4]$$

The fractional difference operator allows a persistence of shocks to volatility that is stronger than the one designating the short memory volatility models of GARCH, ARCH, EGARCH, etc. On the other hand the incorporation of asymmetries or leverage effects in the estimated conditional variance is ensured by the presence of the news impact function $g(z_{t-1})$.¹⁵ As Nelson (1991) states the leverage effect should be modeled as a function of both magnitude and sign and coefficients θ and γ presented in eq. [4] manage exactly that. Specifically γ coefficient displays the rate at which innovations affect the current levels of volatility, while θ coefficient manages the way the sign of normalized innovations affect volatility.

Christensen, Nielsen, and Zhu (2010) introduce in the conditional in mean equation of FIEGARCH-M the following expression that abstracts the long memory features of volatility:

$$h_t = (1 - L)^d (\ln \sigma_t^2 - \omega) \quad [5]$$

An analogous stationary expression is found in Ang et al. (2006) and uses instead the news impact function of the previous period. The last resumes the most recent innovation to volatility and is expressed by the $g(z_{t-1})$ function. Both stationary inputs h_t and $g(z_{t-1})$ are applied in eq. [1a] and generate the following two expressions:

$$r_t = \mu + \lambda g(z_{t-1}) + \varepsilon_t \quad [6]$$

$$r_t = \mu + \lambda h_t + \varepsilon_t \quad [7]$$

These conditional in mean equations alongside FIEGARCH modeling of conditional variance premise the filtered long memory volatility models presented in Christensen, Nielsen, and Zhu (2010). Specifically eqs [6] and [7] alongside FIEGARCH modeling correspond to FIEGARCH-MG and FIEGARCH-MH models, respectively.

¹⁵ FIEGARCH uses the natural log of conditional variance as the reliant variable. This guarantees that conditional variance will always be positive even in the presence of negative estimated coefficients. The transformation eliminates the necessity of imposing non-negativity parameter restrictions, a strategy that is most common for GARCH and ARCH models.

3 Application to the Fama–Frech stock market index, 1963–2010

3.1 Estimating volatility models with no filtering: GARCH, EGARCH, FIGARCH, GJR, FIEGARCH, IGARCH

The first part of the analysis uses the daily returns of the Fama–French stock market index for a period of 37 years from 01.07.1963 to 31.06.2010 to estimate the non-filter volatility models GARCH-M, EGARCH-M, FIGARCH-M, FIEGARCH-M, IGARCH-M and GJR-M.

Following Bollerslev and Mikkelsen (1996) the analysis also adds on each conditional mean equation the autoregressive terms of an AR(3) process and therefore models presented here account also for possible autocorrelations in stock market returns. The adjustment theoretically stems from the arguments of Scholes and Williams (1977) and Lo and MacKinlay (1990) and is very useful if discontinuous trading among stocks that make up the market index exists. The last according to Bollerslev and Mikkelsen (1996) results in significant serial dependence over the index returns and leads to misspecified conditional in mean equation. So in order to adjust for such misspecifications eq. [1] in all models is changing into the following expression:

$$r_t = \mu + \mu_1 r_{t-1} + \mu_2 r_{t-2} + \mu_3 r_{t-3} + \lambda \sigma_t^2 + \varepsilon_t \quad [8]$$

Table 1 concentrates all estimations on the above volatility-M models. The estimation procedure here assumes standard normal distribution for the disturbance terms in eq. [1c], while all volatility frameworks are estimated under a common set of specifications for the GARCH and ARCH terms, which uses one ARCH and one GARCH term each time. So estimated models in Table 1 are the GARCH(1,1)-M-[1,2,3], EGARCH-(1,1)-M-[1,2,3], FIGARCH-(1,1)-M-[1,2,3], FIEGARCH-(1,1)-M-[1,2,3], IGARCH-(1,1)-M-[1,2,3], and GJR (1,1)-M-[1,2,3].¹⁶

16 Estimation of the above models is carried out with the G@rch 6 program. For calculation of standard errors the option of sandwich formula is consistently used. Numbers x and y in parentheses (x, y) denote, respectively, the number of GARCH (x) and ARCH (y) terms employed, while numbers in brackets indicate the autoregressive variables enriching the conditional in mean equations. For example FIEGARCH (1,1)-M-[1] denotes an FIEGARCH-M model that uses one GARCH and ARCH term for the volatility specification, while adds beside the log σ^2 the first lagged return variable, this is r_{t-1} , as an additional explanatory variable in the conditional in mean equation. If the model under consideration does not belong to the M family of models then letter M is missing from the above notation. For example FIEGARCH (2,1)-[1,3] states that model under consideration is FIEGARCH with two GARCH and one ARCH terms specifying the

The statistical significance of λ coefficients in results reported in Table 1 indicates that in four out of six models¹⁷ the return–volatility relation is present, while this implies a positive risk–return tradeoff. Furthermore θ and γ parameters in both news impact functions of EGARCH-M and FIEGARCH-M models are statistically significant and have the expected signs,¹⁸ while the fractionally differenced parameter although positive and strongly significant in both long memory volatility models differs significantly between the two and create speculations about the goodness of fit. Specifically for FIGARCH and FIEGARCH the estimated long memory parameters are 0.525 and 0.173, respectively.

For the serial correlation of residuals, the non-statistically significant Ljung–Box Q statistics¹⁹ that are reported for all models irrespective of the lag selection indicate an absence of such misspecifications from the corresponding conditional in mean equations, while this outcome is likely attributed to the presence of autoregressive terms.²⁰ The same is also true for the Ljung–Box Q^* statistics of the squared residuals, even though results for the FIEGARCH case indicate a possible misspecification in the corresponding volatility equation. Note that the previous results although hold useful information for further model comparison, however they must be weighted with the arguments reported in Harris and Sollis (2003) that in GARCH type models p -values drawn from X^2 distributions cannot be considered reliable indicators of whether statistical significance exists or not.

Beyond these notations clearly FIEGARCH-M reports the best Akaike and log-likelihood values among all estimated volatility models, with EGARCH-M practically matching those standings and in the case of Schwarz information criteria outperforming FIEGARCH-M. Finally note that the last row of Table 1 estimates the mean square error (MSE) over the last 100 days of the sample period using one-day ahead out of sample volatility forecasts. Oddly FIEGARCH-M turns

conditional variance equation, and first and third lagged return variables, those are r_{t-1} and r_{t-3} , added in conditional mean equation. Note that the same notations are consistently used throughout the analysis.

17 These models are IGARCH, FIGARCH, FIEGARCH, and GARCH.

18 The news impact function is used solely in EGARCH and FIEGARH models. The successive incorporation of asymmetries induced by the sign and size of innovations implies that the estimated coefficients θ and γ must fulfil certain characteristics. These are basically two. First, both coefficients should be statistically significant at conventional levels, and second, both should verify the expected sign of the theory. Specifically, θ coefficient is supposed to be negative, while γ should be positive. Note that GJR incorporates only asymmetries induced by the sign of innovations and in this case by default the expected sign of θ is positive. So this model does not use the news impact function presented in eq. [4].

19 The level of significance is 5%.

20 Estimating the same conditional in mean equations without the autoregressive terms of eq. [8] turns over statistically significant Ljung–Box Q^* statistics at both choices of lags.

over the highest value and hence the worst forecasting property among all volatility models, while the best forecasting performance is delivered by FIGARCH-M.

Since models in Table 1 are estimated under a common set of applied specifications for (a) the number of ARCH and GARCH terms, (b) the assumption of conditional distribution, and (c) the orders of autoregressive variables introduced in the corresponding conditional in mean equations, estimates in Table 1 may embed biases and therefore can mislead the quest for the best forecasting volatility model.

Therefore specifications applied in Table 1 should not be considered final and contrary wise they should be regarded as initial milestones for evolving further the fit of the presented volatility models.

A versatile and perhaps a more realistic approach would let volatility specifications change every time a different volatility model is estimated. Such an analysis finds auspicious ground in many empirical applications²¹ and in the present analysis is carried over in estimations presented in Table 2. The specifications and settings chosen here for every volatility model are decided upon standard information criteria rewarding both the goodness of fit and parsimony as well as the out of sample forecasting.²²

For example all models in Table 2, except EGARCH which is estimated under the GED, are estimated presuming the skewed student distribution for the corresponding innovations. The last not only improves the fit but in many occasions enhances the forecasting properties of the corresponding volatility models. Indeed comparison of the log-likelihood values and the Akaike-Schwarz information criteria in Tables 1 and 2 unveils that all models of Table 2 achieve considerably higher and smaller values, respectively.

Implementing however a non-normal distribution not only improves the fit but in many occasions alters the statistical significance of certain coefficients. Take for example the GARCH model and watch out how coefficients λ and μ_3 are missing from the estimations presented in Table 2. Precluding those variables is

21 Nelson (1991) for example suggests that EGARCH model should be estimated under the GED, while French, Schwert, and Stambaugh (1987) propose the GARCH (2,1) specification every time a GARCH model is estimated.

22 The analysis re-estimates the volatility frameworks presented in Table 1 although this time assumes non-normal distributions. Such alternatives are (a) the t-student, (b) the generalized error, and (c) the skewed t-student distribution. All cases are reported in the appendix, while Table 2 resumes the final specifications chosen for each volatility model. This process consisted of two parts. The first by comparing the log-likelihood values and the Akaike/Schwarz information criteria corresponding each distributional alternative decides the distributional forms upon which the volatility models are going to be estimated. The second stage tracks in those latter estimates the non-statistically significant coefficients and using the likelihood ratio test decides the restricted volatility forms. Relative results are reported in the appendix.

Table 1: Estimations of GARCH(1,1)-M-[1,2,3], EGARCH(1,1)-M-[1,2,3], IGARCH(1,1)-M-[1,2,3], GJR(1,1)-M-[1,2,3], FIGARCH(1,1)-M-[1,2,3], FIEGARCH(1,1)-M-[1,2,3] assuming standard normal distribution.

	IGARCH	EGARCH	GJR	FIGARCH	FIEGARCH	GARCH
μ	0.022 [0.005]**	0.009 [0.341]	0.015 [0.048]*	0.021 [0.012]*	−0.004 [0.527]	0.022 [0.008]**
μ_1	0.153 [0.000]**	0.157 [0.000]**	0.161 [0.000]**	0.155 [0.000]**	0.1614 [0.000]**	0.153 [0.000]**
μ_2	−0.015 [0.122]	−0.005 [0.535]	−0.008 [0.390]	−0.016 [0.125]	−0.003 [0.706]	−0.016 [0.126]
μ_3	0.011 [0.249]	0.021 [0.012]	0.018 [0.072]	0.009 [0.372]	0.022 [0.028]	0.010 [0.271]
ω	0.005 [0.000]**	0.000 [1.000]	0.007 [0.000]**	0.015 [0.000]**	−6.265 [0.000]**	0.006 [0.000]**
λ	0.028 [0.004]**	0.002 [0.874]	0.005 [0.633]	0.033 [0.004]**	0.042 [0.001]**	0.030 [0.006]**
α_1	0.090 [0.000]**	−0.363 [0.000]**	0.024 [0.000]**	0.225 [0.000]**	−0.517 [0.000]**	0.087 [0.000]**
β_1	0.909 [0.000]**	0.989 [0.000]**	0.915 [0.000]**	0.651 [0.000]**	0.955 [0.000]**	0.908 [0.000]**
θ	—	−0.125 [0.000]**	0.104 [0.000]**	—	−0.142 [0.000]**	—
γ	—	0.194 [0.000]**	—	—	0.193 [0.000]**	—
d	—	—	—	0.525 [0.000]**	0.173 [0.000]**	—
Logl	−13,729.9	−13,565.7	−13,610.8	−13,710.2	−13,543.6	−13,727.707
Akaike	2.342	2.315	2.322	2.339	2.311	2.342
Schwarz	2.346	2.315	2.322	2.339	2.318	2.342
$Q(100)$	101.689 [0.434]	101.123 [0.449]	97.181 [0.561]	106.808 [0.302]	106.494 [0.309]	101.954 [0.426]
$Q(200)$	219.196 [0.167]	218.316 [0.178]	217.383 [0.189]	224.067 [0.116]	220.992 [0.147]	219.744 [0.161]
$Q^*(100)$	95.9477 [0.539]	101.123 [0.449]	99.173 [0.447]	91.327 [0.670]	161.449 [0.000]**	94.659 [0.576]
$Q^*(200)$	186.545 [0.710]	222.698 [0.110]	210.752 [0.254]	173.752 [0.892]	282.624 [0.000]**	189.056 [0.663]
MSE	12.76	13.57	13.45	12.96	16.39	13.1

Notes: QML estimates are reported. Also reported are the Logl, this is the value of the maximized log-likelihood function, the Akaike and Schwarz information criteria, the values of the Ljung–Box portmanteau statistic for testing the m th order serial dependence of standardized and absolute standardized residuals denoted respectively as $Q(m)$ and $Q^*(m)$. Finally, the table reports the MSE for 100 one-day-ahead out of sample forecasts. The values in parentheses stand for p -values. * denotes rejection at 5% significant level, while ** denotes rejection at 1% significant level.

Table 2: Estimations of IGARCH (1,1)-[1], GARCH(1,1)-[1], FIGARCH(1,1)-[1], FIEGARCH(1,1)-[1], and GJR(1,1)-[1] under the skewed distribution. Estimation EGARCH (2,1)-M-[1] using the GED

	IGARCH	EGARCH	GJR	FIGARCH	FIEGARCH	GARCH
μ	0.034 [0.000]**	0.040 [0.000]**	0.021 [0.000]**	0.034 [0.000]**	0.019 [0.000]**	0.034 [0.000]**
μ_1	0.137 [0.000]**	0.138 [0.000]**	0.144 [0.000]**	0.137 [0.000]**	0.145 [0.000]**	0.137 [0.000]**
ω	0.003 [0.000]**	2.257 [0.000]**	0.005 [0.000]**	0.009 [0.000]**	-2.294 [0.000]**	0.004 [0.000]**
λ	—	-0.026 [0.003]**	—	—	—	—
α_1	0.079 [0.000]**	-0.980 [0.000]**	0.026 [0.000]**	0.196 [0.000]**	-0.468 [0.000]**	0.078 [0.000]**
β_1	0.920 [0.000]**	1.928 [0.000]**	0.920 [0.000]**	0.697 [0.000]**	0.759 [0.000]**	0.919 [0.000]**
μ_2	—	—	—	—	—	-0.928 [0.000]**
μ_3	—	—	—	—	—	—
θ	—	-0.114 [0.000]**	0.097 [0.000]**	—	-0.121 [0.000]**	—
γ	—	0.156 [0.000]**	—	—	0.158 [0.000]**	—
d	—	—	—	0.573 [0.000]	0.565 0.000	—
Logl	-13,466.2	-13,348.2	-13,381.1	-13,453.9	-13,296.2	-13,465.8
Akaike	2.297	2.277	2.282	2.295	2.268	2.297
Schwarz	2.300	2.283	2.287	2.300	2.275	2.301
$Q(100)$	110.010 [0.231]	112.954 [0.177]	102.255 [0.418]	114.201 [0.157]	108.880 [0.255]	109.821 [0.235]
$Q(200)$	228.858 [0.079]	230.429 [0.068]	222.613 [0.130]	173.619 [0.893]	226.944 [0.077]	228.809 [0.079]
$Q^*(100)$	97.545 [0.493]	123.147 [0.037]	94.132 [0.591]	95.1898 [0.561]	127.563 [0.024]	96.846 [0.513]
$Q^*(200)$	228.858 [0.079]	230.429 [0.068]	196.698 [0.512]	173.619 [0.893]	226.944 [0.077]	184.429 [0.746]
MSE	12.8	13.25	13.04	12.93	13.97	12.98
Asymmetry	-0.062 [0.000]**	—	-0.064 [0.000]**	-0.061 [0.000]**	-0.062 [0.000]**	-0.062 [0.000]**

Notes: QML estimates are reported. Also reported are the Logl, the value of the maximized log-likelihood function, the Akaike and Schwarz information criteria, the values of the Ljung–Box portmanteau statistic for testing the up to m th order serial dependence in the standardized and absolute standardized residuals denoted respectively as $Q(m)$ and $Q^*(m)$. Finally, the table reports the MSE for 100 one-day-ahead out of sample forecasts and the estimated asymmetry coefficient of skewed student distribution. The values in parentheses stand for p -values. *denotes rejection at 5% significance level, while ** denotes rejection at 1% significance level.

a decision made in a two-stage process. The first stage estimates the unrestricted GARCH version and proclaims the variables that are individually non-statistically significant and therefore may be dropped, while the second stage estimates the likelihood ratio test and draws conclusions on the reception of the restricted volatility forms.

For example estimation of the unrestricted GARCH²³ model in Table 2 indicates that λ_1 , μ_2 and μ_3 coefficients at 5% significance level are not statistically significant, while the likelihood ratio statistic that tests the joint exclusion of those coefficients accepts the corresponding null hypothesis and so GARCH (1,1)-[1,2] is the final restricted framework estimated in Table 2. Obviously applying this two stage process in all models of Table 1 delivers the estimations reported in Table 2.

The main features reported in Table 2 are the following: (a) Except EGARCH which acknowledges a negative statistically significant volatility–return relation all other models in Table 2 reject the validity of the risk–return tradeoff.²⁴ (b) Although all models in Table 2 are estimated using one ARCH and one GARCH term, however the EGARCH model of Nelson (1991) is estimated under a different combination that constitutes of two GARCH and one ARCH terms. (c) Even though all models strengthen the interpretive properties of the corresponding conditional in mean equations by introducing r_{t-1} in the relative equations, the GARCH model additionally uses r_{t-2} . (d) The majority of the estimated Ljung–Box Q and Q^* statistics are not statistically significant at 5% significant level,²⁵ while coefficients θ and γ of both news impact functions acknowledge the presence of a leverage sign and size effect. This fact along with the presence of a statistically significant long memory parameter probably justify the competitive fit succeeded by FIEGARCH. Note that further evidence on the superiority of FIEGARCH is reported in Table 3 where the Engle and Ng (1993) sign and size bias misspecification tests are reported.²⁶ As expected IGARCH, FIGARCH, and GARCH models reject the null hypothesis in all Engle and Ng (1993) tests, while the rest models although account for asymmetries do not always manage successfully both leverage and size effects. Specifically the results in Table 3 indicate that only FIEGARCH incorporates efficiently all three asymmetries,

23 This is GARCH(1,1)-M-[1,2,3] estimated under the skewed student distribution.

24 Using t-student distribution, GED, and skewed student distribution results in non-statistically significant risk–return coefficients. On the other hand applying the normality assumption ends up in the acknowledgment of the risk–return trade off in four of six models. So the risk–return relation is sensitive to the density assumed.

25 Only $Q^*(100)$ statistics of EGARCH and FIGARCH are statistically significant at 5%.

26 Engle and Ng (1993) propose three tests: (a) the sign bias test (SBT), (b) the negative sign bias test (NSBT), and (c) the positive sign bias test (PSBT).

while the EGARCH and GJR models although both accept the corresponding null hypothesis in the sign bias (SB) and negative size bias test (NSB), however both indicate that positive size effects may well exist.

Results in Table 3 are considered somewhat unexpected. The acceptance of the null hypothesis in the NSB test for the GJR model is completely unjustified since the model by default accounts solely for the sign asymmetries. On the other hand although FIEGARCH seems to adjust well on the asymmetries induced by the sign and size of innovations, however the model unexpectedly rejects the null in the joint Engle and Ng (1993) test at 5% significant level. This contradiction naturally surprises but similar outcomes have reported in other empirical studies.²⁷

Table 3: Engle and Ng (1993) sign and size bias test

	IGARCH	EGARCH	GJR	FIGARCH	FIEGARCH	GARCH
SB	2.187 [0.028]*	1.242 [0.213]	2.882 [0.003]**	2.006 [0.044]*	1.558 [0.119]	2.148 [0.031]*
NSB	2.452 [0.014]*	0.120 [0.904]	0.303 [0.761]	3.091 [0.001]**	0.100 [0.920]	2.631 [0.008]**
PSB	3.000 [0.002]**	2.417 [0.015]*	2.287 [0.022]*	2.798 [0.005]**	1.716 [0.085]	2.912 [0.003]**
Joint	55.730 [0.000]**	16.404 [0.000]**	31.505 [0.000]**	58.505 [0.000]**	12.648 [0.049]**	56.176 [0.000]**

Notes: (SB) stands for sign bias, (NSB) for negative size bias test, (PSB) for positive size bias test, (Joint) for the joint statistic. Number in parentheses report *p*-values.* and (**) denote rejection at 5% and 1% significant levels, respectively.

The above analysis indicates FIEGARCH (1,1)-[1] as the best volatility model, although outcomes on the MSE in Table 2 make this vantage less obvious since FIEGARCH actually delivers the worst forecast error measurement. Seeking an explanation for this contradiction the analysis results in Tables 20 and 21 in the appendix which report different error measurements on one-day-ahead out of sample forecasts for 100 and 200 days of forecasts, respectively. The results reverse the last impression entirely.²⁸ Specifically outcomes affirm that

²⁷ See for example Harris and Solis (2003).

²⁸ The forecasts error measurements reported are the (MSE) Mean Squared Error, the (MedSE) Median Squared Error, the (MAE) Mean Absolute Error, the (RMSE) Root Mean Squared Error, the (MAPE) Mean Absolute Percentage Error, the (AMAPE) Adjusted Mean Absolute Percentage Error, the (TIC) Theil Inequality Coefficient, the (LL) Logarithmic Loss Function.

although FIEGARCH(1,1)-[1] in terms of MSE does not stamp its forecasting superiority, however the model does provide competitive forecasts in terms of the majority of forecast error measurements.²⁹ Furthermore results in Table 4 where the 300 one-day-ahead out of sample forecasts are reported consolidate the superiority of FIEGARCH (1,1)-[1] and impressively underline the fact that the model exceeds any other alternative in 7 out of 8 forecasting error measurements.³⁰ So these outcomes provide even greater endorsement for conclusions drawn from Table 2 and further provide support on the decision to consider FIEGARCH (1,1)-[1] as the best volatility model among all non-filter volatility alternatives.

Table 4: Forecasts errors for 300 one-day-ahead out of sample forecasts

	IGARCH	EGARCH	GJR	FIGARCH	FIEGARCH	GARCH
MSE	46.21	10.52	23.41	25.96	9.858*	26.99
MedSE	52.61	2.706	25.43	26.68	1.693*	27.28
MAE	6.508	2.320	4.526	4.781	2.052*	4.882
RMSE	6.797	3.243	4.838	5.095	3.14*	5.196
MAPE	1,498	400.2	1,023	1,066	318.3*	1,081
AMAPE	0.748	0.608	0.705	0.712	0.594*	0.714
TIC	0.593	0.504*	0.529	0.537	0.534	0.540
LL	15.16	8.956	13.06	13.34	8.162*	13.44

Notes: (MSE) Mean Squared Error, (MedSE) Median Squared Error, (MAE) Mean Absolute Error, (RMSE) Root Mean Squared Error, (MAPE) Mean Absolute Percentage Error, (AMAPE) Adjusted Mean Absolute Percentage Error, (TIC) Theil Inequality Coefficient, (LL) Logarithmic Loss Function, * denotes the best forecasting model.

3.2 Estimating volatility models with filters: FIEGARCH-MH and FIEGARCH-MG

The previous analysis indicated FIEGARCH (1,1)-[1] as the best volatility model according to information criteria that reward both the goodness of fit and parsimony and the out of sample forecasting. Furthermore, the results revealed that the inclusion of long memory feature alone as in the FIGARCH case does not

²⁹ Such measurements are the median squared error, the mean absolute error, the mean absolute percentage error, and the logarithmic loss function.

³⁰ The only exception concerns Theil’s Inequality coefficient.

imperatively improve the fit, while the statistical significance of the risk–return relation although conceded under the assumption of normality it is however rejected under the rest distributional assumptions.

This section follows the approach of Christensen, Nielsen, and Zhu (2010) and estimates the filtered long memory volatility models FIEGARCH-MH and FIEGARCH-MG. Note that the analysis here diverges slightly in terms of the conditional in mean equation since the last is somewhat altered. Specifically the corresponding equations not only include the autoregressive terms of an AR(3) process but also account for potential lagged volatility in mean effects.

So the conditional in mean equation for FIEGARCH-MH model is defined as in the next equation

$$r_t = \mu + \mu_1 r_{t-1} + \mu_2 r_{t-2} + \mu_3 r_{t-3} + \lambda_1 h_t + \lambda_2 h_{t-1} + \lambda_3 h_{t-2} \quad [9]$$

while for FIEGARCH-MG the following specification is applied

$$r_t = \mu + \mu_1 r_{t-1} + \mu_2 r_{t-2} + \mu_3 r_{t-3} + \lambda_1 g(z_{t-1}) + \lambda_2 g(z_{t-2}) + \lambda_3 g(z_{t-3}) \quad [10]$$

Assuming innovations following the normal distribution Tables 5–9 concentrate estimations on both filter long memory volatility models.³¹ Although both cases aim the filter of conditional variance, however the models turn over quite different results as far as the risk–return relation is concerned. For example λ_1 coefficient for FIEGARCH-MH indicates a significant positive risk–return trade-off, while acceptance of null hypothesis in all lagged volatility variables of FIEGARCH-MG indicates a complete absence of such relations.

However, the models still present a number of common characteristics. For example both models acknowledge the statistical significance of the first autoregressive component, while both estimate well-specified conditional in mean equations.³² Another common feature is found in terms of the Ljung–Box Q^* statistics and refers to the rejection of the null hypothesis irrespective of the lag selection. The outcome clearly conjectures the existence of misspecifications in both volatility equations, although similar results are not extracted when the LM statistics are computed.³³

Note that Table 8 concentrates different error measurements on 100 one-day-ahead out of sample forecasts. Comparing these results with the ones reported in Table 20 in the appendix, where the same forecasts error

³¹ The visual representations of both regressors appears in Figure 1.

³² This is seen in the results reported for the Ljung–Box Q statistic.

³³ Except of ARCH (1–100) test that rejects LM's null hypothesis at 5% significant level all other statistics accept null hypothesis.

measurements are reported for various non-filter volatility models, reveals the poor forecasting potentials of FIEGARCH-MG and FIEGARCH-MH. Both deliver forecasts that never challenge the best values reported for each forecast error measurement.

The analysis so far aimed the filter of long memory in volatility. However, the a priori use of normality as in Christensen, Nielsen, and Zhu (2010) is not entirely warranted by the present data set since the last is characterized by a strong leptokurtic, negative skewed shape.

Figure 1: Graphical representation of h_t and $g(z_{t-1})$ variables. The initial estimation procedure assumes innovations that follow the standard normal distribution

The possible misuse of normality can also be seen in Table 9 where outcomes on the adjusted Pearson goodness-of-fit test are reported. The test compares the empirical distribution of innovations with its theoretical shape. The rejection of the null in all sections of cells indicates a possible mismatch, while this holds true for both filter long memory volatility models.

The next part of the analysis uses different distributional assumptions in order to estimate FIEGARCH-MH and FIEGARCH-MG.³⁴ Obviously introducing

³⁴ The appropriateness of applied distributions is judged according to standard criteria in the volatility literature. These criteria are parsimony, goodness of fit, and out of sample forecasting.

assumptions besides normality requires re-estimation of the news impact function $g(z_t)$ and the stationary variable h_t . Once this process is completed the estimates of both filter long memory volatility models are concentrated in Table 10 and then models are tested for the joint exclusion of the non-statistically significant variables. The estimated restricted frameworks that come out of these processes and are denoted as FIEGARCH-MH*and FIEGARCH-MG* are both presented in Table 11.

Table 5: Estimated coefficients for FIEGARCH-MH, FIEGARCH-MG assuming standard normal distribution

FIEGARCH-MH				FIEGARCH-MG			
μ	0.022 [0.000]**	d	0.549 [0.000]**	μ	0.020 [0.000]	d	0.534 [0.000]**
μ_1	0.187 [0.000]**	β_1	0.777 [0.000]**	μ_1	0.154 [0.000]	β_1	0.818 [0.000]**
μ_2	0.008 [0.304]	Logl	-13,518.3	μ_2	0.001 [0.922]	Logl	-13,533.4
μ_3	0.026 [0.098]	Akaike	2.307	μ_3	0.019 [0.114]	Akaike	2.310
λ_1	0.262 [0.000]**	Schwarz	2.315	λ_1	-0.039 [0.437]	Schwarz	2.318
λ_2	0.107 [0.069]	$Q(100)$	106.832 [0.301]	λ_2	0.051 [0.310]	$Q(100)$	104.485 [0.359]
λ_3	0.014 [0.837]	$Q(200)$	223.073 [0.126]	λ_3	0.018 [0.721]	$Q(200)$	222.575 [0.130]
ω	0.000 [1.000]	$Q^*(100)$	133.828 [0.009]**	ω	0.000 [1.000]	$Q^*(100)$	134.152 [0.008]**
α_1	-0.535 [0.000]**	$Q^*(200)$	240.772 [0.020]*	α_1	-0.588 [0.000]**	$Q^*(200)$	238.715 [0.025]*
θ	-0.131 [0.000]**	γ	0.176 [0.000]**	θ	-0.134 [0.000]**	γ	0.186 [0.000]**

Table 6: Engle and Ng tests (1993)

SBT	1.352 [0.176]	NSB	0.791 [0.428]	SBT	0.987 [0.323]	NSB	0.638 [0.522]
PSB	1.593 [0.110]	Joint	8.761 [0.032]	PSB	1.953 [0.050]	Joint	8.876 [0.030]

Table 7: Engle’s LM ARCH test (1982) for FIEGARCH-MG, FIEGARCH-MH

ARCH 1– 100 test	1.2939 [0.026]*	ARCH 1– 200 test	1.1231 [0.114]	ARCH 1– 100 test	1.2845 [0.029]	ARCH 1– 200 test	1.1052 [0.149]
ARCH 1– 300 test	1.0573 [0.240]	–	–	ARCH 1– 300 test	1.0370 [0.3202]	–	–

Notes: QML estimates are reported. Also reported are the Logl, the value of the maximized log-likelihood function, the Akaike and Schwarz information criteria, the values of the Ljung–Box portmanteau statistic for testing up to m th order serial dependence of the standardized and absolute standardized residuals denoted, respectively, as $Q(m)$ and $Q^*(m)$. (SBT) stands for the Sign Bias Test, (NSBT) for the Negative Size Bias test, (PSBT) for the Positive Size Bias Test, and (Joint) for the Joint Statistic. The table also reports Engle’s (1982) LM ARCH test. The values in parentheses stand for p -values.* denotes rejection at 5% significance level, while ** denotes rejection at 1% significance level.

Table 8: Error measurements for 100 one-day-ahead out of sample forecasts.

	MSE	MedSE	MAE	RMSE	MAPE	AMAPE	TIC	LL
FIEGARCH-MG	13.21	2.851	2.242	3.635	388.8	0.640	0.611	11.13
FIEGARCH-MH	13.18	2.648	2.201	3.63	375.7	0.637	0.617	10.97

Notes: (MSE) mean squared error, (MedSE) median squared error, (MAE) mean absolute error, (RMSE) root mean squared error, (MAPE) mean absolute percentage error, (AMAPE) Adjusted mean absolute percentage error, (TIC) Theil inequality coefficient, (LL) Logarithmic loss function.

Table 9: The adjusted Pearson goodness-of-fit test for FIEGARCH-MH and FIEGARCH-MG under the normality assumption

Cells	FIEGARCH-MH			FIEGARCH-MG		
	Statistic	p -Value ($g-1$)	p -Value ($g-k-1$)	Statistic	p -Value ($g-1$)	p -Value ($g-k-1$)
300	436.6828	[0.000]**	[0.000]**	481.1917	[0.000]**	[0.000]**
400	561.2224	[0.000]**	[0.000]**	589.8035	[0.000]**	[0.000]**
600	742.4638	[0.000]**	[0.000]**	800.5812	[0.000]**	[0.000]**

Notes: *denotes rejection at 5% significance level, while **denotes rejection at 1% significance level.

The objective is to estimate both filtered volatility models under the best distributional assumptions. For FIEGARCH-MH this process results among either estimating the model under the t-student assumption or estimating it under the GED. This dilemma arises because the asymmetry coefficient of the skewed

student choice reported in Table 10 is not statistically significant and naturally this result excludes the corresponding model from the rest part of the analysis.³⁵

However, making a choice among the remaining two restricted³⁶ options is no easy task since both assumptions achieve almost equal fit and identical estimations in the majority of implicated coefficients. Specifically both models acknowledge the same set³⁷ of coefficients as being statistically significant at conventional levels, while both estimate a positive λ_1 coefficient and therefore conclude the presence of a positive return–risk tradeoff. Furthermore the Akaike and Schwarz information criteria estimate identical values, while the Ljung–Box Q and Q^* statistics at 100 and 200 turn out being not statistically significant. Finally and as far as the sign and size bias tests of Engle and Ng (1993) is concerned both S.B.T and N.S.B.T statistics accept the null, while the joint and P.S.B tests both reject it.

Table 10: Estimation of the unrestricted FIEGARCH-MH-MG models assuming t-student distribution, GED, and skewed student distribution for the innovations

	FIEGARCH-MH			FIGARCH-MG		
	t-student	GED	Skewed	t-student	GED	Skewed
μ	0.014 [0.005]**	0.016 [0.005]**	0.015 [0.015]*	−0.011 [0.561]	0.264 [0.000]**	0.024 [0.551]
μ_1	0.176 [0.000]**	0.167 [0.000]**	0.170 [0.000]**	0.138 [0.000]**	−0.023 [0.009]**	0.137 [0.000]**
μ_2	−0.005 [0.635]	−0.008 [0.400]	−0.011 [0.212]	−0.010 [0.151]	−0.001 [0.899]	−0.011 [0.587]
μ_3	0.013 [0.235]	0.012 [0.289]	0.007 [0.458]	0.009 [0.252]	−0.015 [0.057]	0.008 [0.608]
λ_1	0.267 [0.000]**	0.244 [0.001]**	0.232 [0.001]**	−0.105 [0.002]**	1.819 [0.000]**	−0.115 [0.319]

(continued)

35 Note that G@RCH 6.0 which does not estimate ξ but $\log \xi$. So the skewed–student version equals the symmetric t-student distribution whenever when $\log(\xi)$ equals 0. The estimated value of $\log(\xi)$ is reported under the label asymmetry. See Lambert and Laurent (2001) and Bauwens and Laurent (2005) for more details on this outcome.

36 The full models are estimated in Table 10 while the restricted versions are reported in Table 11. Take for example the FIEGARCH-MH under the assumption of t-student. Looking at t-statistics and the corresponding p -values in Table 10 the individually non-statistical significant variables are the second- and third-order autoregressive term and the innovation h_{t-2} . These variables correspond respectively to the coefficients μ_2 , μ_3 and λ_2 and λ_3 . The likelihood ratio statistic that tests the joint exclusion of these variables accepts the null hypothesis and the restricted model is presented in Table 11.

37 The statistical significant coefficients are μ , μ_1 , λ_1 , d , θ , and γ .

Table 10: (Continued)

	FIEGARCH-MH			FIGARCH-MG		
	t-student	GED	Skewed	t-student	GED	Skewed
λ_2	0.117 [0.100]	0.085 [0.234]	0.079 [0.226]	0.061 [0.018]*	−0.214 [0.000]**	0.061 [0.583]
λ_3	−0.011 [0.861]	−0.022 [0.736]	−0.055 [0.369]	0.009 [0.252]	0.2621 [0.000]**	0.007 [0.961]
α_1	−0.439 [0.002]**	−0.524 [0.000]**	−0.467 [0.000]**	−0.494 [0.000]**	−0.552 [0.000]**	−0.496 [0.000]**
β_1	0.722 [0.000]**	0.760 [0.000]**	0.743 [0.000]**	0.753 [0.000]**	0.779 [0.000]**	0.755 [0.000]**
ω	0.000 [1.000]	0.000 [1.000]	0.000 [1.000]	0.000 [1.000]	0.000 [1.000]	0.000 [1.000]
θ	−0.116 [0.000]**	−0.124 [0.000]**	−0.113 [0.000]**	−0.120 [0.000]**	−0.127 [0.000]**	−0.119 [0.000]**
γ	0.161 [0.000]**	0.177 [0.000]**	0.164 [0.000]**	0.172 [0.000]**	0.179 [0.000]**	0.171 [0.000]**
d	0.605 [0.000]**	0.589 [0.000]**	0.602 [0.000]**	0.599 [0.000]**	0.590 [0.000]**	0.598 [0.000]**
Logl	−13,316.1	−13,355.7	−13,319.4	−13,324.3	−13,337.2	−13,323.6
Akaike	2.273	2.279	2.273	2.274	2.276	2.274
Schwarz	2.282	2.288	2.283	2.283	2.285	2.284
$Q(100)$	107.189 [0.293]	109.357 [0.245]	106.202 [0.316]	104.503 [0.359]	101.155 [0.448]	104.343 [0.363]
$Q(200)$	225.352 [0.105]	227.740 [0.086]	224.904 [0.109]	224.171 [0.115]	219.525 [0.163]	224.218 [0.115]
$Q^*(100)$	117.399 [0.088]	120.253 [0.063]	117.803 [0.084]	117.288 [0.089]	121.926 [0.051]	118.119 [0.081]
$Q^*(200)$	219.033 [0.145]	222.383 [0.112]	219.338 [0.142]	218.073 [0.156]	224.006 [0.099]	219.245 [0.143]
MSE	13.21	13.21	13.20	13.24	13.24	13.23
Asymmetry (log)	–	–	0.006 [0.712]	–	–	0.006 [0.702]
SB	1.339 [0.180]	1.271 [0.203]	1.530 [0.125]	1.500 [0.133]	1.458 [0.144]	1.331 [0.182]
NSB	0.397 [0.690]	0.708 [0.478]	0.436 [0.662]	0.547 [0.584]	0.945 [0.344]	0.415 [0.677]
PSB	2.141 [0.032]*	2.250 [0.024]*	2.139 [0.032]*	2.215 [0.026]*	2.408 [0.016]*	2.253 [0.024]*
Joint	13.380 [0.003]**	12.951 [0.004]**	14.711 [0.002]**	14.614 [0.002]**	15.163 [0.001]**	13.944 [0.002]**

Notes: QML estimates are reported. Also reported are the Logl, the value of the maximized log-likelihood function, the Akaike and Schwarz information criteria, the values of the Ljung–Box portmanteau statistic for testing the up to m th order serial dependence of the standardized and absolute standardized residuals, denoted respectively by $Q(m)$ and $Q^*(m)$, (SB) stands for Sign Bias, (NSB) for Negative Size Bias test, (PSB) for Positive Size Bias Test, (Joint) for the Joint Statistic. Numbers in parentheses report p -values. Finally, the table reports the MSE for 100 one-day-ahead out of sample. Forecast and the asymmetry coefficient estimated when the distribution skewed student is assumed. The values in parentheses stand for p -values.* denotes rejection at 5% significance level, while ** denotes rejection at 1% significance.

Table 11: Estimation of restricted FIEGARCH-MH and FIEGARCH-MG assuming t-student distribution and GED

	FIEGARCH-MH*		FIEGARCH-MG*	
	t-student	GED	t-student	GED
μ	0.014 [0.000]**	0.015 [0.001]**	—	0.232 [0.000]**
μ_1	0.175 [0.000]**	0.165 [0.000]**	0.136 [0.000]**	−0.025 [0.019]*
μ_2	—	—	—	—
μ_3	—	—	—	—
λ_1	0.274 [0.000]**	—	−0.106 [0.004]**	1.834 [0.000]**
λ_2	—	—	0.088 [0.016]*	−0.211 [0.000]**
λ_3	—	—	—	—
α_1	−0.428 [0.002]**	−0.512 [0.000]**	−0.491 [0.000]**	−0.552 [0.000]**
β_1	0.721 [0.000]**	0.757 [0.000]**	0.750 [0.000]**	0.778 [0.000]**
ω	—	—	—	—
θ	−0.117 [0.000]**	−0.123 [0.000]**	−0.120 [0.000]**	−0.127 [0.000]**
γ	0.161 [0.000]**	0.176 [0.000]**	0.172 [0.000]**	−0.127 [0.000]**
d	0.603 [0.000]**	0.588 [0.000]**	0.600 [0.000]**	0.589 [0.000]**
Logl	−13,335.3	−13,359.4	−13,325.1	−13,338.3
Akaike	2.273	2.279	2.273	2.276
Schwarz	2.278	2.285	2.279	2.282
$Q(100)$	111.326 [0.206]	113.526 [0.167]	107.000 [0.297]	103.981 [0.372]
$Q(200)$	228.332 [0.082]	230.501 [0.068]	226.332 [0.097]	222.389 [0.132]
$Q^*(100)$	117.338 [0.089]	119.949 [0.065]	117.088 [0.091]	121.093 [0.056]
$Q^*(200)$	217.060 [0.168]	220.242 [0.133]	217.800 [0.159]	222.389 [0.132]
MSE	14.29	13.24	13.25	13.24
SB	1.641 [0.100]	1.370 [0.170]	1.548 [0.121]	1.473 [0.140]
NSB	0.454 [0.649]	0.656 [0.511]	0.565 [0.571]	0.953 [0.340]
PSB	1.975 [0.048]*	2.143 [0.032]*	2.184 [0.028]*	2.437 [0.014]*
Joint	14.22 [0.002]**	12.828 [0.005]**	14.698 [0.002]**	15.490 [0.001]**

Notes: QML estimates are reported. Also reported are the Logl, the value of the maximized log-likelihood function, the Akaike and Schwarz information criteria, the values of the Ljung–Box portmanteau statistic for testing the up to m th order serial dependence of the standardized and absolute standardized residuals, denoted, respectively, by $Q(m)$ and $Q^*(m)$, (SB) stands for Sign Bias, (NSB) for Negative Size Bias test, (PSB) for Positive Size Bias test, (Joint) for the Joint statistic. Numbers in parentheses report p -values. Finally, the table reports the MSE for 100 one-day-ahead out of sample forecast and the asymmetry coefficient estimated when the distribution skewed student is assumed. The values in parentheses stand for p -values. * denotes rejection at 5% significance level, while ** denotes rejection at 1% significance.

Table 12 reports the log-likelihood values and the Akaike/Schwarz information criteria for all competing filter volatility frameworks of Table 10. In bold letters the table denotes the best estimates on each criterium. Note that the best ones belong to the t-student assumption.

Table 12: The log-likelihood values and the Akaike/Schwarz information criteria for all estimated filter volatility models

	FIEGARCH-MH			FIEGARCH-MG		
	Likelihood value	Akaike	Schwarz	Likelihood value	Akaike	Schwarz
Normal	−13,518.3	2.307	2.315	−13,533.4	2.1310	2.318
t-student-MH*-MG*	−13,335.3	2.273	2.278	−13,325.1	2.273	2.279
GED-MH*-MG*	−13,359.74	2.279	2.285	−13,338.3	2.276	2.282

4 Forecasting volatility: a comparative analysis

Estimating only one forecast error measurement or estimating various forecasts error measurements under the same forecasting horizon likely induces biases. In an attempt to overcome this drawback is worth using a criterion that assesses the overall MSEs of different forecasting horizons. This necessity leads to Clements and Hendry (1993) approach which introduces the generalized forecast error second moment statistic (GFESM). The last is given by the determinant of the complete forecast error second matrix which is presented in the following equation:

$$\text{GFESM} = |E[uu']| \tag{11}$$

Note that u stands for the vector of forecast errors.³⁸

Tables 13 and 14 use one step ahead out of sample forecasts for 20, 40, 60, 80, and 100 days in order to estimate the log-GFESM statistic of both restricted filter volatility models under the t-student distribution and GED. Obviously under the t-student assumption FIEGARCH-MH* constantly generates the smaller Log-GFESM values. This outcome along with the smaller Akaike and Schwarz values of FIEGARCH-MH* that are reported in Table 12 forces the analysis to estimate the filter FIEGARCH-MH model under the t-student assumption.

³⁸ In order to avoid large numbers the analysis reports results on the log transformation of GFESM. This is denoted as Log-GFESM.

For FIEGARCH-MG*, the results reported in Tables 10 and 11 are in fact very similar to the ones presented for FIEGARCH-MH*. The basic conclusions are briefly discussed in the following lines.

Under the assumption of skewed student distribution the FIEGARCH-MG³⁹ estimates an asymmetric coefficient that is strongly insignificant. The result naturally precludes the model from the rest part of the analysis and so the remaining competing volatility frameworks are the FIEGARCH-MG estimated under either the t-student distribution or the GED. Although the above models acknowledge different statistically significant sets on the estimated coefficients, however, the models appear identical according to standard information criteria.⁴⁰

The selection procedure continues next with the estimation of restricted versions for the above models, and these are presented in Table 11. The outcomes on Q and Q^* Ljung–Box statistics imply the correct specification of the mean and volatility equations, while the sign and size bias tests reiterate the same pattern as in the FIEGARCH-MH*case, with S.B.T and N.S.B.T statistics accepting the null hypothesis at 5% significant level and joint and P.S.B tests both reject it.

The similar results of the above models make impossible an evidence-based choice among them. Even comparing the mean squared errors (MSE) value reported in Table 11 does not really contribute any additive information in the selection procedure and so as before the weight for the selection falls entirely on forecasting properties and particular on Log-GFESM statistic.

Table 13: Estimation of the Log-GFESM statistic for FIEGARCH-MH* under the t-student distribution and GED

	FIEGARCH-MH*	
	t-student	GED
Log-GFESM		
20	0.067	0.466
40	0.196	0.948
60	1.166	1.473
80	2.333	2.571
100	3.488	3.692

³⁹ The analysis denotes this model as FIEGARCH-MG-skewed.

⁴⁰ As has been stated the selection procedure involves the restricted models. So here identical refers to the latter specification.

Table 14: Estimation of the Log-GFESM statistic for FIEGARCH-MG* under the t-student distribution and GED

	FIEGARCH-MH*	
	t-student	GED
Log-GFESM		
20	0.476	0.457
40	0.969	0.928
60	1.503	1.445
80	2.601	2.543
100	3.723	3.665

Table 14 indicates that the two models appear equally good performances when controlling for different forecasting horizons, although FIEGARCH-MG* under the GED is constantly generating smaller Log-GFESM values. So the decision of which model to choose falls entirely on Akaike and Schwarz criteria reported in Table 12. Considering the slightly smaller values of both when FIEGARCH-MG* is estimated under the t-student distribution makes a final decision reached.

Note that Tables 15 and 16 which estimate the adjusted Pearson goodness-of-fit test provide further support on the decision to estimate both filtered restricted models under the t-student assumption. Specifically results report that under the t-student assumption the null hypothesis is accepted in every section of cells, while the last is always rejected when the GED is chosen. So, results provide solid support over the decision to estimate both filter models under the assumption of t-student.

Table 15: Adjusted Pearson chi-square goodness-of-fit test for restricted FIEGARCH-MH*

Cells	FIEGARCH-MH*					
	t-student			GED		
	Stat	P	P*	Stat	P	P*
300	334.514	0.077	0.056	385.932	0.000	0.000
400	432.129	0.121	0.069	473.705	0.005	0.002
600	580.851	0.695	0.598	666.338	0.028	0.015

Notes: *P*-columns report *p*-values for each number of cells. The *P**-columns report the corresponding adjusted *p*-values.

Table 16: Adjusted Pearson chi-square goodness-of-fit test for restricted FIEGARCH-MG*

Cells	FIEGARCH-MG*					
	t-student			GED		
	Stat	P	P*	Stat	P	P*
300	335.642	0.071	0.053	416.065	0.000	0.000
400	434.346	0.107	0.059	473.023	0.006	0.002
600	597.579	0.508	0.405	695.090	0.003	0.001

Notes: *P*-column reports *p*-values corresponding *p*-each number of cells. The *P**-column reports the corresponding adjusted *p*-values.

Before proceeding with the final forecasting analysis there are two things that must be reported. First, estimates on the long memory parameter in all non-filter volatility models seem to be quite robust and vary between 0.525 and 0.573. The only exception is for FIEGARCH model in Table 1 which surprisingly estimates a value 0.173. On the other hand the long memory estimates for the restricted filter volatility models are approximately 0.60 and match the values found in Bollerslev and Mikkelsen (1996).

The second statement concerns the risk-volatility relation and the fact that FIEGARCH-MG* indicates the presence of two opposite in sign statistically significant relations. Moreover, λ_1 coefficient is found being negative while λ_2 is found being positive. The results suggest the dominance of a volatility feedback effect mechanism in the first lagged period and the existence of a positive volatility-return tradeoff in the second.

The combination of signs reported here is identical to the one reported in Christensen, Nielsen, and Zhu (2010) paper. Researchers in this case justify the findings by arguing that the negative sign of the first period is something to be seen first, since the volatility feedback effect mechanism induces an immediate price drop as soon as the discount rate in the economy reacts to an increase in volatility. So, negative effect is something to be seen first. On the other hand the positive sign of the second period is attributed to adjustments made in expectations, a phenomena that naturally requires time and which on this particular occasion appears on the second period.

Although the above explanations seem convincing and elegant however two things must be underlined when dealing with such phenomena. First, a possible non-linearity in either relation would make difficult the separation of the two results, while an omitted variable bias would produce a negative risk–return relation when in fact there is not any. So both of these statements offer logical

question marks and therefore undermine any certainty about the above statements.

Next, the paper compares the best volatility models of each category. Competing volatility frameworks are the FIEGARCH (1,1)-[1] estimated under the skewed student assumption and the restricted filtered FIEGARCH-MH* and FIEGARCH-MG* both estimated under the *t*-student distribution.

Obviously the estimations reported in Tables 2 and 11 indicate that the non-filtered FIEGARCH (1,1)-[1] outperforms both filtered FIEGARCH versions according to log-likelihood values and Akaike and Schwarz information criteria. This may be due to the fact that in FIEGARCH (1,1)-[1] case most null hypothesis in the relevant sign and size bias tests of Engle and Ng (1993) are been accepted, although this does not hold true for the joint test at 5% significant level since the corresponding *p*-value is 0.049 and hence rejects on the limit the relevant null hypothesis. Ignoring for a moment this limit rejection and having in mind the statement of Engle and Ng (1993) that the joint test is actually more powerful to the individual ones, then FIEGARCH probably sets the only framework efficiently managing all leverage effects.

The above statements along with the fact that FIEGARCH (1,1)-[1] under the skewed student distribution is endorsed by the results of the adjusted Pearson test⁴¹ presented in the appendix leave minimum doubts about the opportune incorporation of all asymmetrical effects.

Finally the last crucial criterium involves out of sample forecasting under different forecasting horizons. Earlier in the analysis the paper used 20, 40, 60, 80 and 100 one step ahead out of sample forecasts estimating on each occasion the corresponding log-GFESM statistic.

Table 17 re-estimates those statistics while Table 18 extends this analysis by assuming also 100, 150, 200 and 250 one step ahead out of sample forecasts. At the same time the table reports the values of eight forecasts error measurements in order to provide an overall assessment of the forecasting performances. The results of both tables verify the complete dominance of FIEGARCH(1,1)-[1] in short and long term forecasting horizons. The only exception is the 60 one step ahead out of sample forecasts where FIEGARCH(1,1)-[1] delivers the worst forecasts error measurement.

⁴¹ Another endorsement for estimating FIEGARCH under the skewed student distribution can be seen in the statistical significance of the estimated asymmetric coefficient reported in Table 3.

Table 17: One step ahead forecasts of FIEGARCH models considering different forecasting horizons for L-GFESM statistic

L-GFESM	FIEGARCH(1,1)-[1] Skewed student	FIEGARCH-MH*(1,1) t-student	FIEGARCH-MG*(1,1) t-student
20	0.480*	0.067	0.457
40	1.147*	0.196	0.928
60	1.686	1.166*	1.445
80	2.140*	2.333	2.543
100	2.673*	3.488	3.665

Table 18: One step ahead forecasts of FIEGARCH models considering different forecasting horizons

	FIEGARCH* Skewed student	FIEGARCH-MH* t-student	FIEGARCH-MG* t-student
L-GFESM			
100	0.708*	0.919	0.942
150	1.361*	1.844	1.886
200	1.926*	2.767	2.825
250	2.840*	3.810	3.878
MSE			
100	5.109*	8.309	8.762
150	4.504*	8.404	8.779
200	3.669*	8.378	8.690
250	8.209*	11.06	11.310
MEDSE			
100	3.003*	8.852	9.349
150	2.336*	9.253	9.813
200	1.776*	9.298	9.717
250	1.714*	8.852	9.135
MAE			
100	1.914*	2.694	2.773
150	1.715*	2.705	2.771
200	1.538*	2.725	2.780
250	1.814*	2.843	2.890
RMSE			
100	2.260*	2.883	2.960
150	2.122*	2.899	2.963
200	1.915*	2.894	2.948
250	2.865*	3.326	3.363

(continued)

Table 18: (Continued)

	FIEGARCH* Skewed student	FIEGARCH-MH* t-student	FIEGARCH-MG* t-student
MAPE			
100	432.2*	658.9	676.7
150	296.1*	456.6	468.7
200	348.8*	679.9	690
250	299.9*	603.5	612.3
AMAPE			
100	0.588*	0.639	0.643
150	0.588*	0.659	0.662
200	0.604*	0.689	0.692
250	0.604*	0.668	0.670
TIC			
100	0.466*	0.485	0.489
150	0.476*	0.505	0.508
200	0.477*	0.531	0.534
250	0.578*	0.514	0.515
LL			
100	8.209*	9.847	9.982
150	7.457*	9.582	9.694
200	8.628*	11.600	11.700
250	8.333*	11.190	11.270

Notes: *Denote the best forecasting model.

5 Conclusions

The present analysis uses the daily returns of the Fama–French stock market index and estimates various volatility models with the aim to compare their volatility forecasting potentials. The competing volatility frameworks are GARCH, FIGARCH, EGARCH, IGARCH, GJR, and FIEGARCH.

The pursue of the above objective naturally requires the exact specification of the competing volatility frameworks, and this obviously demands decisions upon dilemmas such as (a) the number of ARCH and GARCH terms used in every volatility model, (b) the distributional assumptions applied, and finally (c) decisions made about whether the estimated frameworks include volatility–return relations.

Furthermore, the analysis inspired by the work of Christensen, Nielsen, and Zhu (2010) focuses on FIEGARCH-filter models and applies a filter of long

memory in volatility in order to prevent possible spillover effects. Specifically, the analysis estimates the filter volatility models FIEGARCH-MH and FIEGARCH-MG presented in Christensen, Nielsen, and Zhu (2010) which introduce stationary volatility representations in the conditional in mean equations.

As far as the volatility–return relation is concerned the results suggest that this acknowledgement is generally sensitive to both distributional assumptions applied and to specific estimated models. On the other hand and as far as volatility forecasting is concerned the Fama–French stock market index provides strong support for models not incorporating such a risk–return relation, with FIEGARCH (1,1)–[1] truly outperforming any other model either according to standard information criteria or according to combined forecasting analysis which examines various forecasting error measurements under different forecasting horizons.

Appendix

ACF, spectral density, long memory tests of Fama–French stock market index

The estimated spectrum of the Fama–French stock market returns at zero frequency is a finite function, and so this is considered a stationary covariance process. The same results are also confirmed when the analysis in Table 19 estimates the log-periodogram regression method of Geweke and Porter-Hudak (1983) and the Gaussian semi-parametric method of Robinson and Henry (1998). Furthermore, the results in Table 19 confirm the negative skewed and leptokyrctic nature of the data set that justifies the selection of the skewed student t distribution in the majority of estimations.

Table 19: Descriptive statistics of Fama–French data set

	Mean	Std. dev.	Skewness	Kurtosis	J.B	GPH test	GSP test
r	0.019	0.983	−0.54655	17.655	1.5295e + 005	0.017	0.011
			4.5806e−129	[0.000]	[0.000]	[0.052]	[0.073]

Notes: The values in parentheses stand for p -values.*denotes rejection at 5% significance level, while **denotes rejection at 1% significance.

Forecasts error measurements for different volatility models

Table 20: Forecasts error measurements for 100 one-day-ahead out of sample forecasts

	IGARCH	EGARCH	GJR	FIGARCH	FIEGARCH	GARCH
MSE	12.80	14.19	13.04	12.93	13.97	12.98
MedSE	2.943	1.114	4.458	3.785	1.473	2.529
MAE	2.207	1.986	2.430	2.337	2.049	2.142
RMSE	3.577	3.766	3.612	3.596	3.738	3.602
MAPE	396.6	226.4	475.6	442.8	261.7	359.6
AMAPE	0.633	0.635	0.647	0.641	0.637	0.631
TIC	0.597	0.718	0.567	0.578	0.689	0.620
LL	11.120	9.57	11.92	11.61	9.96	10.78

Notes: (MSE) Mean Squared error, (MedSE) Median Squared Error, (MAE) Mean Absolute Error, (RMSE) Root Mean Squared error, (MAPE) Mean Absolute Percentage Error, (AMAPE) Adjusted Mean Absolute Percentage Error, (TIC) Theil Inequality Coefficient, (LL) Logarithmic Loss Function.

Table 21: Forecasts Error measurements for 200 one-day-ahead out of sample forecasts

	IGARCH	EGARCH	GJR	FIGARCH	FIEGARCH	GARCH
MSE	7.679	8.127	7.727	9.207	8.225	7.824
MedSE	1.636	0.783	1.222	5.747	0.981	1.262
MAE	1.658	1.46	1.567	2.362	1.527	1.558
RMSE	2.771	2.851	2.78	3.034	2.868	2.797
MAPE	266.5	168	230.4	430.8	191.7	222.8
AMAPE	0.580	0.570	0.572	0.635	0.579	0.574
TIC	0.583	0.682	0.611	0.515	0.661	0.622
LL	8.172	6.89	7.732	10.28	7.25	7.641

Notes: (MSE) Mean Squared Error, (MedSE) Median Squared Error, (MAE) Mean Absolute error, (RMSE) Root Mean Squared Error, (MAPE) Mean Absolute Percentage Error, (AMAPE) Adjusted Mean Absolute Percentage Error, (TIC) Theil Inequality Coefficient, (LL) Logarithmic Loss Function.

Table 22: Estimations of GARCH(1,1)-M-[1,2,3], EGARCH(1,1)-M-[1,2,3], IGARCH(1,1)-M-[1,2,3], GJR(1,1)-M-[1,2,3], FIGARCH(1,1)-M-[1,2,3], FIEGARCH(1,1)-M-[1,2,3] assuming t-student distribution

	IGARCH	EGARCH	GJR	FIGARCH	FIEGARCH	GARCH
μ	0.035 [0.000]**	0.023 [0.002]**	0.028 [0.000]**	0.035 [0.000]**	0.002 [0.775]	0.035 [0.000]**
μ_1	0.147 [0.000]**	0.147 [0.000]**	0.151 [0.000]**	0.147 [0.000]**	0.134 [0.000]**	0.147 [0.000]**
μ_2	-0.027 [0.003]**	-0.018 [0.023]*	-0.021 [0.025]*	-0.028 [0.003]**	-0.023 [0.036]*	-0.027 [0.003]**
μ_3	0.003 [0.692]	0.009 [0.231]	0.009 [0.335]	0.002 [0.775]	0.008 [0.544]	0.003 [0.689]
ω	0.003 [0.000]**	—	0.005 [0.000]**	0.010 [0.000]**	—	0.004 [0.000]**
λ	0.016 [0.118]	-0.016 [0.381]	0.002 [0.777]	0.018 [0.116]	0.007 [0.000]**	0.017 [0.120]
α_1	0.080 [0.000]**	-0.327 [0.000]**	0.026 [0.000]**	0.195 [0.000]**	-0.293 [0.673]	0.078 [0.000]**
β_1	0.919 [0.000]**	0.994 [0.000]**	0.919 [0.000]**	0.693 [0.000]**	0.922 [0.000]**	0.919 [0.000]**
θ	—	-0.109 [0.000]**	0.910 [0.000]**	—	-1.000 [0.151]	—
γ	—	0.181 [0.000]**	—	—	1.000 [0.160]	—
d	—	—	—	0.569 [0.000]**	0.217 [0.008]**	—
Logl	-13,470.7	-13,364.6	-13,389	-13,458	-13,345.6	-13,470.3
Akaike	2.298	2.280	2.285	2.296	2.367	2.453
Schwarz	2.303	2.287	2.291	2.303	2.374	2.302
$Q(100)$	106.903 [0.300]	99.838 [0.485]	98.029 [0.537]	111.305 [0.206]	114.273 [0.155]	112.151 [0.167]
$Q(200)$	226.916 [0.092]	219.202 [0.167]	219.489 [0.164]	231.070 [0.065]	231.490 [0.062]	223.430 [0.172]
$Q^*(100)$	97.428 [0.497]	100.486 [0.411]	94.036 [0.594]	94.7586 [0.573]	350.714 [0.000]**	264.372 [0.450]
$Q^*(200)$	185.233 [0.733]	203.100 [0.386]	197.540 [0.495]	174.781 [0.881]	507.979 [0.000]**	493.129 [0.000]**
MSE	13.25	13.01	12.98	13.47	12.98	13.90

Notes: QML estimates are reported. Also reported are the Logl, this is the value of the maximized log-likelihood function, the Akaike and Schwarz information criteria, the values of the Ljung-Box portmanteau statistic for testing the up to m th order serial dependence of standardized and absolute standardized residuals denoted respectively as $Q(m)$ and $Q^*(m)$. Finally, the table reports the MSE for 100 one-day-ahead out of sample forecasts. The values in parentheses stand for p -values. *denotes rejection at 5% significance, while **denotes rejection at 1% significant level.

Table 23: Estimations of GARCH(1,1)-M-[1,2,3], EGARCH(1,1)-M-[1,2,3], IGARCH(1,1)-M-[1,2,3], GJR (1,1)-M-[1,2,3], FIGARCH(1,1)-M-[1,2,3], FIEGARCH(1,1)-M-[1,2,3] assuming the GED

	IGARCH	EGARCH	GJR	FIGARCH	FIEGARCH	GARCH
μ	0.029 [0.001]**	0.024 [0.001]**	0.023 [0.002]**	0.029 [0.000]**	−0.033 [0.000]**	0.029 [0.000]**
μ_1	0.142 [0.000]**	0.148 [0.000]**	0.147 [0.000]**	0.142 [0.000]**	0.186 [0.000]**	0.142 [0.000]**
μ_2	−0.032 [0.000]**	−0.018 [0.006]**	−0.024 [0.010]*	−0.032 [0.000]**	0.024 [0.031]*	−0.032 [0.000]**
μ_3	0.001 [0.884]	0.010 [0.181]	0.008 [0.371]	0.000 [0.975]	0.042 [0.004]**	0.001 [0.881]
ω	0.003 [0.000]**	−	0.005 [0.000]**	0.010 [0.000]**	−	0.004 [0.000]**
λ	0.011 [0.284]		−0.001 [0.862]	0.014 [0.221]	−0.091 [0.000]**	0.012 [0.262]
α_1	0.079 [0.000]**	−0.316 [0.000]**	0.027 [0.000]**	0.196 [0.000]**	0.654 [0.011]*	0.078 [0.000]**
β_1	0.920	0.992 [0.000]**	0.920 [0.000]**	0.693 [0.000]**	−0.247 [0.322]	0.919 [0.000]**
θ	−	−0.107 [0.000]**		−	−0.239 [0.000]**	−
γ	−	0.179 [0.000]**	−	−	0.042 [0.000]**	−
d	−	−	−	0.568 [0.000]**	0.664 [0.000]**	−
Logl	−13,458.2	−13,362.7	−13,376.3	−13,445.4	−13,807.3	−13,457.7
Akaike	2.296	2.280	2.282	2.294	2.356	2.296
Schwarz	2.302	2.288	2.289	2.301	2.364	2.302
$Q(100)$	112.028 [0.193]	99.579 [0.493]	99.912 [0.483]	116.905 [0.118]	177.063 [0.000]**	111.872 [0.196]
$Q(200)$	232.636 [0.056]	218.749 [0.172]	221.664 [0.140]	237.306 [0.036]*	300.369 [0.000]**	232.578 [0.056]
$Q^*(100)$	97.354 [0.499]	101.654 [0.380]	93.479 [0.610]	94.557 [0.579]	2,180.330 [0.000]**	96.615 [0.520]
$Q^*(200)$	184.487 [0.745]	206.254 [0.329]	195.929 [0.528]	174.010 [0.889]	2,539.56 [0.000]**	184.964 [0.737]
MSE	13.23	12.93	14.56	13.8	13.46	13.58
Asymmetry	−0.065 [0.000]**	0.011 [0.513]	−0.066 [0.000]**	−0.065 [0.000]**	−0.145 [0.000]**	−0.065 [0.000]**

Notes: QML estimates are reported. Also reported are the Logl, this is the value of the maximized log-likelihood function, the Akaike and Schwarz information criteria, the values of the Ljung–Box portmanteau statistic for testing the up to m th order serial dependence of standardized and absolute standardized residuals denoted respectively as $Q(m)$ and $Q^*(m)$. Finally, the table reports the MSE for 100 one-day-ahead out of sample forecasts. The values in parentheses stand for p -values. *denotes rejection at 5% significance, while **denotes rejection at 1% significant level.

Table 24: Estimations of GARCH(1,1)-M-[1,2,3], EGARCH(1,1)-M-[1,2,3], IGARCH(1,1)-M-[1,2,3], GJR(1,1)-M-[1,2,3], FIGARCH(1,1)-M-[1,2,3], FIEGARCH(1,1)-M-[1,2,3] assuming the Generalized Error Distribution

	GARCH	EGARCH	GJR	FIGARCH	FIEGARCH	IGARCH
μ	0.038 [0.000]**	0.026 [0.001]**	0.031 [0.000]**	0.038 [0.000]**	-0.025 [0.000]**	0.038 [0.000]**
μ_1	0.137 [0.000]**	0.140 [0.000]**	0.143 [0.000]**	0.139 [0.000]**	0.192 [0.000]**	0.137 [0.000]**
μ_2	-0.028 [0.003]**	-0.019 [0.022]*	-0.022 [0.018]*	-0.029 [0.002]**	0.027 [0.004]**	-0.028 [0.002]**
μ_3	0.002 [0.785]	0.010 [0.259]	0.008 [0.396]	0.001 [0.869]	0.044 [0.000]**	0.002 [0.779]
ω	0.005 [0.000]**	-	0.005 [0.000]**	0.229 [0.000]**	-	0.004 [0.000]**
λ	0.014 [0.214]	-0.021 [0.177]	0.000 [0.929]	0.017 [0.209]	-0.098 [0.000]**	0.013 [0.211]
α_1	0.082 [0.000]**	-0.368 [0.000]**	0.026 [0.000]**	0.243 [0.000]**	0.687 [0.007]**	0.093 [0.230]
β_1	0.915 [0.000]**	0.993 [0.000]**	0.917 [0.000]**	0.571 [0.000]**	-0.334 [0.000]**	0.937 [0.000]**
θ	-	-0.118 [0.000]**	0.645 [0.000]**	-	-0.228 [0.000]**	-
γ	-	0.193 [0.000]**	-	-	0.034 [0.000]**	-
d	-	-	-	0.408 [0.000]**	0.672 [0.000]**	-
Logl	-13,498.2	-13,398.2	-13,472.4	-13,472.4	-13,921.5	-13,498.9
Akaike	2.303	2.286	2.290	2.299	2.376	2.303
Schwarz	2.309	2.293	2.296	2.305	2.383	2.308
$Q(100)$	111.782 [0.197]	101.269 [0.445]	100.754 [0.460]	119.852 [0.085]	195.649 [0.000]**	112.079 [0.192]
$Q(200)$	232.448 [0.057]	220.959 [0.147]	222.890 [0.127]	239.744 [0.028]*	320.131 [0.000]**	232.589 [0.056]
$Q^*(100)$	95.182 [0.561]	102.210 [0.365]	95.339 [0.557]	92.4645 [0.638]	2,427.05 [0.000]**	96.292 [0.529]
$Q^*(200)$	186.127 [0.717]	205.531 [0.341]	201.811 [0.411]	170.296 [0.923]	320.131 [0.000]**	185.208 [0.056]
MSE	14.00	12.97	14.01	13.47	13.8	13.76

Notes: QML estimates are reported. Also reported are the Logl, this is the value of the maximized log-likelihood function, the Akaike and Schwarz information criteria, the values of the Ljung–Box portmanteau statistic for testing the up to m th order serial dependence of standardized and absolute standardized residuals denoted respectively as $Q(m)$ and $Q^*(m)$. Finally, the table reports the MSE for 100 one-day-ahead out of sample forecasts. The values in parentheses stand for p -values. *denotes rejection at 5% significance, while **denotes rejection at 1% significant level.

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