Research Article

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Long Memory and Asymmetry for Matrix-Exponential Dynamic Correlation Processes

Abstract: We propose a fractionally integrated matrix-exponential dynamic conditional correlation (FIEDCC) model to capture the asymmetric effects and long- and short-range dependence of a correlation process. We also propose employing an inverse Wishart distribution for the disturbance of a covariance structure, which gives an alternative interpretation for a multivariate t conditional distribution. Using the inverse Wishart distribution, we present a three-step procedure to obtain initial values for estimating a high-dimensional conditional covariance model with a multivariate t distribution. We investigated the finite-sample properties of the ML estimator. Empirical results for nine assets from chemical firms, banks, and oil and gas producers in the US indicate that the new FIEDCC model outperforms the other dynamic correlation models for the AIC and BIC and for forecasting value-at-risk thresholds. Furthermore, the new FIEDCC model captures the stronger connection among the nine assets for the period right after the global financial crisis.

Keywords: matrix-exponential, long memory, asymmetric effects, dynamic correlation, inverse Wishart distribution, heavy tails

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1 Introduction

The multivariate generalized autoregressive conditional heteroskedasticity (GARCH) class of models has been used to model the co-movements of volatilities

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in financial assets. One approach is modeling conditional covariances directly as in the BEKK (Baba, Engle, Kraft, and Kroner) model of Engle and Kroner (1995), while the other approach is based on specifying correlation dynamics as in Engle (2002), Tse and Tsui (2002), McAleer et al. (2008), and Silvennoinen and Teräsvirta (2009). Leverage effects and fat tails are typical characteristics of financial time series, and several approaches have been developed to incorporate them. With respect to asymmetric effects, multivariate specifications are most commonly based on either the GJR model of Glosten, Jagannathan, and Runkle (1992) or the exponential GARCH (EGARCH) model of Nelson (1991), whereby positive and negative shocks of equal magnitude have different effects on conditional volatility. Based on the GJR model, McAleer, Hoti, and Chan (2009) suggested the vector autoregressive moving average process with asymmetric GARCH (VARMA-AGARCH) model, while Kroner and Ng (1998) proposed the asymmetric BEKK model. For the EGARCH model, Kawakatsu (2006) suggested the matrix-exponential GARCH model. Among others, Cappiello, Engle, and Sheppard (2006) suggested an approach for considering asymmetric effects on volatility and correlation separately, i.e. the asymmetric DCC (ADCC) model with the GJR, EGARCH, or other asymmetric volatility specifications. Regarding heavy-tailed conditional distributions, Harvey, Ruiz, and Sentana (1992) employed the multivariate t distribution. In examining asymmetry, Vlaar and Palm (1993) considered mixtures of multivariate normal densities, while Bauwens and Laurent (2005) developed a multivariate skew-Student density. Although the quasi-maximum likelihood estimation method investigated by Bollerslev and Wooldridge (1992) and Engle (2002) is robust to assumptions on distributions, assuming fatter tails has the benefit of improving forecasts for value-at-risk (VaR) thresholds.

The purpose of this paper is to develop a long-memory dynamic correlation model with asymmetric effects and a heavy-tailed conditional distribution. For long-range dependences in financial volatility, Baillie, Bollerslev, and Mikkelsen (1996) developed the fractionally integrated GARCH model, while Bollerslev and Mikkelsen (1996) suggested the fractionally integrated EGARCH model. In addition to GARCH specifications, Breidt, Crato, and de Lima (1998), Harvey (1998), Pérez and Ruiz (2001), So (2002), and So and Kwok (2006) studied a long-memory stochastic volatility model. As a multivariate extension, we consider a matrix-exponential dynamic conditional correlation (EDCC) specification and extend it to a fractionally integrated process. Regarding asymmetric effects on volatility and dynamic correlations, we work with a hyperbola function and its extension, rather than with the absolute value function originally considered by Nelson (1991), to model multivariate asymmetric effects. In considering heavy tails, we suggest employing an inverse Wishart distribution to create the error term for dynamic conditional covariance. The specification we propose gives an

alternative interpretation for multivariate t conditional distributions. We also propose a three-step procedure to obtain initial values for estimating a high-dimensional conditional covariance model with a multivariate t distribution. This procedure incorporates “singular” multivariate Beta type II distribution density function derived by Srivastava (2003).

The reminder of this paper is organized as follows. Section 2 develops the fractionally integrated EDCC model with asymmetric effects and a stochastic covariance structure. Section 3 discusses likelihood inference procedures for parameter estimation, proposes the three-step procedure for obtaining initial values, and describes Monte Carlo experiments conducted to study finite-sample properties of estimators. Section 4 provides empirical example using daily stock returns for nine assets from chemical firms, banks, and oil and gas producers in the US. Section 5 sets out our conclusions.

2 New dynamic correlation models

2.1 Matrix-exponential dynamic correlations

Let y_t be an m -dimensional vector of asset returns, defined by

$$y_t = \mu_t + \Omega_t^{1/2} u_t,$$

where u_t is an m -dimensional random vector which has independent and identical distribution, $u_t \sim \text{i.i.d.}(0, I_m)$, μ_t is the predetermined m -dimensional mean vector of y_t , and Ω_t is the predetermined m -dimensional covariance matrix of y_t . We may write $\mu_t = E[y_t | \mathfrak{S}_{t-1}]$ and $\Omega_t = \text{Var}[y_t | \mathfrak{S}_{t-1}]$, where $\mathfrak{S}_t$ is the information set up to t . To avoid the problem of simultaneous equation bias, we exclude the exogenous variables for period t from the conditional covariance matrix. On the other hand, the conditional mean can be extended to include such exogenous variables.¹

Following Engle (2002), we decompose the conditional covariance matrix $\Omega_t = V_t^{1/2} P_t V_t^{1/2}$, where P_t is the conditional correlation matrix and V_t is the diagonal matrix of conditional variances. We can write $V_t = (I_m \circ \Omega_t)$, where “ $\circ$ ” denotes the Hadamard (element-by-element) product. We also consider a positive definite matrix, Q_t , in order to describe the correlation dynamics with

¹ Ren and Polasek (2000) considered the VAR-GARCH in mean model. Although we do not deal with such models here, our new specification includes conditional covariance models with feedback effects in the mean as a special case.

$P_t = Q_t^{*-1/2} Q_t Q_t^{*-1/2}$, where $Q_t^* = (I_m \circ Q_t)$. We also define the vector of conditional volatilities as $v_t = \text{vecd}(V_t)$, where the operator $\text{vecd}(A)$ creates a vector from the diagonal elements of the square matrix A . As in Engle (2002), we specify the process of $\{Q_t, v_t\}$ in the following. Furthermore, we define $z_t = V_t^{-1/2}(y_t - \mu_t)$ instead of $u_t = \Omega_t^{-1/2}(y_t - \mu_t)$, so that we have $E[z_t z_{t'}' | \mathfrak{S}_{t-1}] = P_t$.

To model dynamic correlations, we consider the matrix-logarithmic transformation of Q_t . Chiu, Leonard, and Tsui (1996) proposed the idea of specifying the time-varying covariance matrix, Kawakatsu (2006) considered the matrix-exponential GARCH model, while Ishihara, Omori, and Asai (2012) developed the matrix-exponential SV model. Here, we apply the idea to model the correlation process. For any square matrix A , the matrix-exponential transformation is defined by $\text{Exp}(A) = \sum_{i=0}^{\infty} (1/i!) A^i$ with $A^0 = I$. The same result is obtained by working with the singular value decomposition, as we have $\text{Exp}(A)$ by replacing the eigenvalues by their exponential transformation. Note that $\text{Exp}(A)$ is positive definite, whenever A is symmetric. In the same manner, $\text{Log}(B)$ is defined by its spectral decomposition of a positive definite matrix, B , with replacement of the logarithmic transformation of eigenvalues. We also denote $\log(x)$ ($\exp(x)$) for vector x as the element-wise logarithmic (exponential) transformation of x .

We will consider stochastic processes for $\text{Log}(Q_t)$ and $\log(v_t)$ instead of Q_t and v_t , respectively. Noting that $Q_t = \text{Exp}(\text{Log}(Q_t))$ and $v_t = \exp(\log(v_t))$, these transformations guarantee that $\{Q_t\}$ are positive definite matrices and that all elements of $\{v_t\}$ are positive, corresponding to any finite values for $\{\text{Log}(Q_t)\}$ and $\{\log(v_t)\}$. Consider the following processes for $\text{Log}(Q_t)$ and $\log(v_t)$:

$$A(L) \circ \{\text{Log}(Q_t) - M\} = B(L) \circ G_c(z_{t-1}), \quad [1]$$

$$\Phi(L) \{\log(v_t) - \kappa\} = \Psi(L) g_v(z_{t-1}), \quad [2]$$

where L is the lag operator, $G_c(z_{t-1})$ is the $m \times m$ symmetric error matrix, and $g_v(z_{t-1})$ is the m -dimensional error vector. For the error components, we assume (i) $E[G_c(z_t)] = O$; (ii) all the elements of $E[\text{vec}\{G_c(z_t)\} \text{vec}\{G_c(z_t)\}']$ are constant; (iii) $E[\text{vec}\{G_c(z_t)\} \text{vec}\{G_c(z_s)\}'] = O$ for $t \neq s$; (iv) $E[g_v(z_t)] = \mathbf{0}$; (v) all the elements of $E[g_v(z_t) g_v(z_t)']$ are constant; and (vi) $E[g_v(z_t) g_v(z_s)'] = O$ for $t \neq s$. These conditions guarantee that the error terms are independently and identically distributed (IID) if z_t is an IID process. With respect to the autoregressive and moving average components, we consider

$$A(L) = \iota_m \iota_{m'}' - A_1 L - \cdots - A_p L^p,$$

$$B(L) = \iota_m \iota_{m'}' + B_2 L + \cdots + B_q L^{q-1},$$

$$\Phi(L) = I_m - \Phi_1 L - \dots - \Phi_r L^r,$$

$$\Psi(L) = I_m + \Psi_2 L - \dots - \Psi_s L^{s-1},$$

where A_i , B_i , Φ_i , and Ψ_i are $m \times m$ parameter matrices and ι_m is the m -dimensional vector of ones. The weak (covariance) stationary condition of the EDCC model requires that $\{Q_t, v_t\}$ are weak stationary processes. We assume that $\Phi(L)$ and the elements of $A(L)$ satisfy the stationary condition and that $\Psi(L)$ and the elements of $B(L)$ satisfy the invertibility condition. Later we will extend eqs [1] and [2] for the cases of fractional integration in the processes of $\text{Log}(Q_t)$ and $\log(v_t)$. We should note that we need to impose no restrictions on $A(L)$, $B(L)$, and $G_c(z_t)$ for the purpose of guaranteeing the positive definiteness of Q_t . This is one of the features of the matrix-exponential transformation. In addition, M is the $m \times m$ matrix of parameters and κ is the m vector of parameters enabling $E(\text{Log}(Q_t)) = M$ and $E(\log(v_t)) = \kappa$. We call the new model as the matrix-exponential dynamic conditional correlation (EDCC) model.

To illustrate the new model, we consider the case of two-dimensional model of y_t with first-order autoregressive specification for $\text{Log}(Q_t)$. For convenience, we denote $M_t = \text{Log}(Q_t)$. Then, the correlation dynamics for y_t becomes

$$P_t = \begin{pmatrix} 1 & \rho_t \\ \rho_t & 1 \end{pmatrix}, \quad \rho_t = \frac{q_{12,t}}{\sqrt{q_{11,t}q_{22,t}}}, \quad \begin{pmatrix} q_{11,t} & q_{12,t} \\ q_{12,t} & q_{22,t} \end{pmatrix} = \sum_{i=0}^{\infty} \frac{1}{i!} \begin{pmatrix} m_{11,t} & m_{12,t} \\ m_{12,t} & m_{22,t} \end{pmatrix}^i,$$

$$\begin{pmatrix} m_{11,t} \\ m_{12,t} \\ m_{22,t} \end{pmatrix} = \begin{pmatrix} m_{11} \\ m_{12} \\ m_{22} \end{pmatrix} + \begin{pmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{12} & 0 \\ 0 & 0 & \alpha_{22} \end{pmatrix} \begin{pmatrix} m_{11,t-1} - m_{11} \\ m_{12,t-1} - m_{12} \\ m_{22,t-1} - m_{22} \end{pmatrix} + \begin{pmatrix} g_{c,11,t} \\ g_{c,12,t} \\ g_{c,22,t} \end{pmatrix}.$$

In this case, $(m_{11,t}, m_{12,t}, m_{22,t})$ follows the VAR process with the diagonal restriction on the coefficient matrix of the first-order term. Note that the EDCC specification indicates that elements of $(g_{c,11,t}, g_{c,12,t}, g_{c,22,t})$ are mutually correlated. By definition, $q_{ij,t} \neq \exp(m_{ij,t})$ and $q_{ij,t}$ are determined by interactions of $(m_{11,t}, m_{12,t}, m_{22,t})$. As shown later in empirical analysis, one of the merits of the matrix-exponential transformation is that it can capture large fluctuation which conventional DCC cannot. We may use the news impact curves defined by Engle and Ng (1993) for examining the relationship between Q_t and y_{t-1} .

We can relax the definition by considering an additional stochastic term η_t such that $G_c(z_{t-1}, \eta_t)$ are $g_v(z_{t-1}, \eta_t)$, which indicates that the model belongs to the class of multivariate SV models with stochastic correlations. Although we go no further in this direction, we note that So and Kwok (2006) developed the symmetric multivariate long-memory SV model with constant correlation.

In the next subsection, we extend the new EDCC model in order to accommodate long memory and asymmetric effects.

2.2 Long memory and asymmetry

We develop the fractionally integrated matrix-exponential dynamic conditional correlation (FIEDCC) model, based on the following long-memory processes for $\text{Log}(Q_t)$ and $\log(v_t)$:

$$A(L) \circ D_c(L) \circ \{\text{Log}(Q_t) - M\} = B(L) \circ G_c(z_{t-1}), \quad [3]$$

$$\Phi(L)D_v(L)\{\log(v_t) - \kappa\} = \Psi(L)g_v(z_{t-1}), \quad [4]$$

where

$$D_c(L) = \begin{pmatrix} (1-L)^{d_{11}^c} & \cdots & (1-L)^{d_{1m}^c} \\ \vdots & \ddots & \vdots \\ (1-L)^{d_{m1}^c} & \cdots & (1-L)^{d_{mm}^c} \end{pmatrix},$$

$$D_v(L) = \text{diag}\left((1-L)^{d_1^v}, \dots, (1-L)^{d_m^v}\right),$$

with stationary conditions $|d_{ij}^c| < 1/2$ ($i, j = 1, \dots, m$) and $|d_i^v| < 1/2$ ($i = 1, \dots, m$). While $D_c(L)$ and $D_v(L)$ describe the long-memory properties, $G_c(z_{t-1})$ and $g_v(z_{t-1})$ will be defined below to capture asymmetric effects.

The two popular approaches for introducing asymmetry for volatility dynamics are employing threshold function, as in the GJR model of Glosten, Jagannathan, and Runkle (1992), and working with an absolute value function, as in the specification of Nelson (1991). Kroner, and Ng (1998) developed a multivariate extension of the GJR model, while Kawakatsu (2006) suggested an extension for the EGARCH model. Unlike these approaches, we consider the second-order Hermit polynomial for the error term. Hansen, Huang, and Shek (2012) suggested this approach for a univariate model. In this case, we have

$$g_{vi}(z_{it}) = \gamma_{1i}z_{it} + \gamma_{2i}(z_{it}^2 - E(z_{it}^2)).$$

Noting that $E(z_{it}^2) = 1$, we also have $E(g_{vi}(z_{it})) = 0$, $\text{Var}(g_{vi}(z_{it})) = \gamma_{1i}^2 + 2\gamma_{2i}^2$, and $E[z_{it}g_{vi}(z_{it})] = \gamma_{1i}$, as we assume that each element of z_t has zero mean and unite variance. Hence, if γ_{1i} is negative, the model describes the leverage effect. On the

other hand, γ_{2i} is expected to be positive, as in the conventional GARCH families. Here, we suggest the following multivariate extension of the approach:

$$G_c(z_t) = \Lambda_1 \circ (z_t \iota_{m'} + \iota_m z_t') + \Lambda_2 \circ (z_t z_t' - E(z_t z_t' | \mathfrak{S}_{t-1})).$$

Noting that $E(z_t z_t' | \mathfrak{S}_{t-1}) = P_t$, it is straightforward to show that $E(G_c(z_t) | \mathfrak{S}_{t-1}) = O$ and that each element has finite variance. Unlike the leverage effects, there are no conditions on Λ_1 and Λ_2 , except that either $\lambda_{1,ij}$ or $\lambda_{2,ij}$ is not equal to zero. It should be noted that the specifications of $G_c(z_{t-1})$ and $g_v(z_{t-1})$ satisfy the conditions in the previous subsection.

2.3 Stochastic covariance structure

In this subsection, we introduce the stochastic covariance structure to accommodate heavy-tailed distributions. Let C_t ($m \times m$) be an unobservable stochastic covariance matrix for y_t , defined by

$$\begin{aligned} y_t &= \mu_t + C_t^{1/2} \varepsilon_t, \quad \varepsilon_t \sim N(0, I_m), \\ C_t &= \Omega_t^{1/2} \{(\nu - m - 1) \Xi_t\} \Omega_t^{1/2}, \quad \Xi_t^{-1} \sim W(I_m, \nu), \end{aligned} \tag{5}$$

where $W(A, p)$ denotes the Wishart distribution with the scale matrix A and the degrees-of-freedom parameter p .

The expression has the following alternative form:

$$\begin{aligned} y_t &| \mathfrak{S}_{t-1}, C_t \sim N(\mu_t, C_t), \\ C_t &| \mathfrak{S}_{t-1} \sim IW((\nu - m - 1) \Omega_t, \nu), \end{aligned} \tag{6}$$

where $IW(B, p)$ denotes the inverse Wishart distribution with the scale matrix B and the degrees-of-freedom parameter p . We have $\mu_t = E[y_t | \mathfrak{S}_{t-1}]$ and $\Omega_t = E[C_t | \mathfrak{S}_{t-1}]$ by the definitions.² We investigate statistical properties of the model structure in the next section, including (i) $\Omega_t = \text{Var}[y_t | \mathfrak{S}_{t-1}]$ and (ii) the conditional distribution of y_t is the multivariate t distribution. Instead of assuming the multivariate t distribution directly, we start from the stochastic covariance structure to develop a procedure to estimate dynamic correlations separately, which is one of the tasks for the next section.

² It is possible to extend μ_t to depend on C_t , as in the stochastic volatility in mean model suggested by Koopman and Uspensky (2002).

2.4 Parsimonious specification

In this subsection, we consider parsimonious specification for the new EDCC model. In general, multivariate GARCH models have a parameter number of order $O(m^2)$, which causes a problem on estimating model with large number of parameters. As in the class of DCC model of Engle, we assume $A_i = \alpha_i \iota_m \iota_{m'}$, $B_i = \beta_i \iota_m \iota_{m'}$, and Φ_i and Ψ_i are diagonal. Especially, the assumption of diagonal matrices of Φ_i and Ψ_i implies that the conditional volatility of y_{it} , which is given by v_{it} in eq. [2], has the EGARCH process.

We may consider more flexible specification for $\text{Log}(Q_t)$, by assuming $A_i = \alpha_i \iota_m \iota_{m'}$ with flexible B_i . It should be noted that the number of parameters added is $m^2 - 1$, which is 80 when $m = 9$. In the same manner, we set $\Lambda_1 = \lambda_1 \iota_m \iota_{m'}$ and $\Lambda_2 = \lambda_2 \iota_m \iota_{m'}$, in order to reduce the number of parameters in $G_c(z_t)$. We also restrict the parameters on M , where $M = E(\text{Log}(Q_t))$, to avoid the over-parameterization caused by the transformation $P_t = (I_m \circ Q_t)^{-1/2} Q_t (I_m \circ Q_t)^{-1/2}$. In simple terms, we may set $M = \text{Log}(S)$, where S is a positive definite symmetric matrix with unit diagonal elements. We can impose such a restriction on K using the decomposition via the triangular matrix of Jaekel and Rebonato (2000).

For the FIEDCC model, we use a single parameter for the long memory in correlation dynamics, i.e. $d_{ij}^c = d^c$. As can be seen from the illustrative example in Section 2.1, $q_{ij,t}$ depends on all the elements of $\text{Log}(Q_t)$. Even in the case of different memory parameters in the elements of $\text{Log}(Q_t)$, the elements in $\text{Exp}(\text{Log}(Q_t))$ will have common memory corresponding to the maximum value of the memory parameters in $\text{Log}(Q_t)$. Thus it is natural to consider a common parameter for d_{ij}^c .

We now detail some important special cases for the specification of the new EDCC model. The first model is the EDCC model written as

$$\text{Log}(Q_t) = M + \alpha(\text{Log}(Q_{t-1}) - M) + G_c(z_{t-1}), \quad [7]$$

with the EGARCH(1,1) model of Nelson (1991) written as

$$\log(v_{it}) = \kappa_i + \phi_i(\log(v_{i,t-1}) - \kappa_i) + g_{vi}(z_{i,t-1}). \quad [8]$$

The second model is the FIEDCC model given by

$$(1 - \alpha L)(1 - L)^{d^c} (\text{Log}(Q_t) - M) = G_c(z_{t-1}), \quad [9]$$

with the FIEGARCH(1, d ,1) model of Bollerslev and Mikkelsen (1996), defined by,

$$(1 - \phi_i L)(1 - L)^{d_i^{d^c}} (\log(v_{it}) - \kappa_i) = g_{vi}(z_{i,t-1}). \quad [10]$$

We employ these two models for the empirical analysis.

3 Estimation

3.1 Some properties of stochastic covariance structure

Our new EDCC model is based on the framework of the stochastic covariance structure [6]. Here, we develop three interesting properties: (i) the connection between y_t and conditional covariance; (ii) the distribution of $z_t z_t'$; and (iii) the distribution of portfolio returns. We derive (i) via an approach similar to Bayesian modeling, while we apply recent developments in multivariate Beta distribution research for (ii). Deriving property (iii) is a straightforward extension of (i).

First, we define $\theta = (\theta_1', v, \theta_2')'$ as the parameter vector, where θ_1 is the vector of parameters for μ_t and θ_2 consists of the parameters for Ω_t . The density functions of y_t and C_t are denoted as $f(y_t|\mathfrak{S}_{t-1}, C_t, \theta_1)$ and $f(C_t|\mathfrak{S}_{t-1}, v, \theta_2)$, respectively. We have the distribution of $y_t|\mathfrak{S}_{t-1}$ as

$$\begin{aligned} f(y_t|\mathfrak{S}_{t-1}, \theta) &= \int f(y_t|\mathfrak{S}_{t-1}, C_t, \theta_1) f(C_t|\mathfrak{S}_{t-1}, v, \theta_2) dC_t \\ &= (\pi k)^{-\frac{m}{2}} \frac{\Gamma(\frac{k+m}{2})}{\Gamma(\frac{k}{2})} |H_t|^{-\frac{1}{2}} \left[1 + \frac{(y_t - \mu_t)' H_t^{-1} (y_t - \mu_t)}{k} \right]^{-\frac{k+m}{2}}, \end{aligned} \quad [11]$$

where $k = v - m + 1$ and $H_t = k^{-1}(v - m - 1)\Omega_t$. This defines $y_t|\mathfrak{S}_{t-1} \sim MT(\mu_t, H_t, k)$, a multivariate t distribution with the location parameter μ_t , the scale parameter H_t , and k degrees of freedom. Because each component of y_t follows a univariate conditional t distribution with k degrees of freedom, it has a finite fourth moment only when $k > 4$. The conditional mean and variance of y_t are

$$\begin{aligned} E(y_t|\mathfrak{S}_{t-1}) &= \mu_t, \\ \text{Var}(y_t|\mathfrak{S}_{t-1}) &= \frac{k}{k-2} H_t = \Omega_t. \end{aligned}$$

Hence, the specification in eq. [6] makes the conditional mean of stochastic covariance, C_t , the conditional covariance matrix of y_t . In addition, each element of z_t has a standardized t distribution. Another important implication of this specification is as follows. The property of the inverse Wishart distribution requires that $v > m + 1$ to guarantee the positive definiteness of C_t . However, it does not limit the fat-tailed distribution of y_t , as tail heaviness depends only on the difference $v - m$.

We now turn to the distribution of $z_t z_t'$. Given C_t , it is straightforward to show that $z_t z_t' | C_t, \mathfrak{S}_{t-1} \sim W(V_t^{-1/2} C_t V_t^{-1/2}, 1)$. As C_t follows the inverse Wishart

distribution, we can also show that $V_t^{-1/2}C_tV_t^{-1/2}|\mathfrak{S}_{t-1} \sim IW((v-m-1)P_t, v)$. In general, if $W \sim W(U, v_1)$ and $U \sim IW(A, v_2)$ where $v_1, v_2 > m-1$, then W follows the multivariate Beta type II distribution (see Olkin and Rubin (1964) and Tan (1969)). Srivastava (2003) recently derived the density function of a “singular” multivariate Beta type II distribution for the case $v_1 < m$. By developing the result of Srivastava (2003) with a simple extension, the distribution of $W_t = z_t z_t'$ is given by

$$f(W_t|\mathfrak{S}_{t-1}, v, \theta_2) = \frac{\pi^{(1-m)/2} \Gamma_m\left(\frac{v+1}{2}\right) |A_t|^{m/2}}{\Gamma_m\left(\frac{v}{2}\right) \Gamma\left(\frac{1}{2}\right)} |W_{11,t}|^{-m/2} |A_t + W_t|^{-(v+1)/2}, \quad [12]$$

where $A_t = (v-m-1)P_t$, $\Gamma_m(x)$ is the multivariate gamma function defined by $\Gamma_m(x) = \pi^{\frac{1}{2}m(m-1)} \prod_{j=1}^m \Gamma(x - \frac{1}{2}(j-1))$ and $W_{11,t}$ is the $(1,1)$ -element of W_t .

As shown later in this section, we may use these two properties to estimate the various dynamic conditional covariance models.

The next property is used for portfolio analysis. Let y_t be the vector of returns of financial assets following the model in eq. [5]. The return of the portfolio consisting of m assets is denoted by

$$y_{pt} = w_t' y_t = w_t' \mu_t + w_t' C_t^{1/2} \varepsilon_t, \quad [13]$$

where $\varepsilon_t \sim N(0, I_m)$ and $w_t = (w_{1t}, \dots, w_{mt})'$ are the portfolio weights, such that $\sum_{i=1}^m w_{it} = 1$. The weight vector w_t is predetermined. For notational convenience, assume that $w_t = w$ (constant) in this section, though it is straightforward to consider time-varying weights, and we do so in the empirical analysis. Note that the stochastic variance of the portfolio return is given by $c_{pt} = \text{Var}(y_{pt}|\mathfrak{S}_{t-1}, C_t) = w' C_t w$. Deriving from eq. [5], we have

$$y_{pt}|C_t, \mathfrak{S}_{t-1} \sim N(\mu_{pt}, c_{pt}),$$

where $\mu_{pt} = E(y_{pt}|\mathfrak{S}_{t-1}) = w' \mu_t$ is the conditional mean of the portfolio return. According to eq. [11], we also have the conditional mean of c_{pt} and the conditional variance of y_{pt} given $\mathfrak{S}_{t-1}$ as follows:

$$\omega_{pt} = \text{Var}(y_{pt}|\mathfrak{S}_{t-1}) = w' E(C_t|\mathfrak{S}_{t-1}) w = w' \Omega_t w.$$

By the property of the multivariate t distribution in eq. [11], it is straightforward to show that conditional on $\mathfrak{S}_{t-1}$,

$$y_{pt} = \mu_{pt} + \omega_{pt}^{1/2} \eta_{pt}, \quad [14]$$

where η_{pt} follows the standardized t distribution with k degrees of freedom. The conditional distribution of y_{pt} given $\mathfrak{S}_{t-1}$ is the t distribution, with the

conditional mean and variance given by μ_{pt} and ω_{pt} , respectively. When $m = 1$, our formulation reduces to the portfolio index GARCH specification of Asai and McAleer (2008). By setting the portfolio weight as $w_i = 1$ and $w_j = 0$ ($j \neq i$) in eq. [14], we have that $y_{it} = \mu_{it} + \omega_{iit}^{1/2} \eta_{it}$, where η_{it} follows the standardized t distribution with k degrees of freedom.

3.2 Maximum likelihood estimation

This subsection explains the method used to conduct the maximum likelihood estimation of the new FIEDCC model in eqs [3], [4], and [6]. We will suggest a procedure in order to obtain initial values for ML estimation, using the results in previous subsection.

In the DCC class of models, Engle (2002) developed an approach to decompose the likelihood function into two parts: one consists of the likelihood functions of individual mean and volatility equations, while the other is the remaining part that captures dynamic correlations (see also Song, Fan, and Kalbfleisch (2005)). Because the parameter ν in the new model plays an important role in both volatility and dynamic correlations, the approach of Engle (2002) is inappropriate. Instead, we suggest the following three-step procedure for obtaining initial value of the ML estimator. We denote θ_{li} and θ_{vi} as the parameter vectors corresponding to the parameters of μ_{it} and v_{it} , respectively. In addition, we denote θ_r as the parameter vector for Q_t . Hence, we have $\theta_1 = (\theta'_{11}, \dots, \theta'_{1m})'$ and $\theta_2 = (\theta'_v, \theta'_r)'$, where $\theta_v = (\theta'_{v1}, \dots, \theta'_{vm})'$.

Step 1. For each y_{it} , estimate ν , θ_{li} , and θ_{vi} using the standardized t distribution to obtain $\tilde{\nu}^{(i)}$.

Step 2. Obtain the harmonic mean of $\tilde{\nu}^{(1)}, \dots, \tilde{\nu}^{(m)}$ as $\nu^* = m / (1/\tilde{\nu}^{(1)} + \dots + 1/\tilde{\nu}^{(m)})$.

Fixing $\nu = \nu^*$, estimate θ_{li} and θ_{vi} based on the standardized t distribution to obtain $\tilde{\theta}_{li}$ and $\tilde{\theta}_{vi}$, standardized residuals $\{\tilde{z}_{it}\}_{t=1}^T$.

Step 3. Fixing $\nu = \nu^*$ and $\theta_v = \tilde{\theta}_v$ and working with $\{\tilde{z}_{it}\}_{t=1}^T$, estimate θ_r via the singular multivariate Beta type II distribution [12] to obtain $\tilde{\theta}_r$. In this step, we may use $T^{-1} \sum \tilde{z}_t \tilde{z}_t'$ for the initial value of $\text{Exp}(M)$.

We can use $\tilde{\theta}_{li}$, ν^* , $\tilde{\theta}_{vi}$, and $\tilde{\theta}_r$ as the initial values of parameters to obtain the maximum likelihood estimator of θ , $\hat{\theta}$ based on the multivariate t distribution.

Turning to the implementation issue, as we consider the ARFIMA(p, d, q) process for volatilities and dynamic correlations, it is not straightforward to incorporate it into the likelihood function. Hence, we employ an MA(J) approximation of the MA(∞) representation of the ARFIMA part as suggested by

Bollerslev and Mikkelsen (1996), in the sense that the coefficient of the J th lagged term is almost zero and is negligible for large J values such as $J = 1,000$.³

Although Engle's (2002) DCC model is widely used in the literature, only Engle and Sheppard (2001) attempted to provide some statistical properties and finite-sample performance results for the (quasi-)ML estimator. Similar to Engle's (2002) DCC model, it is not easy task to derive the asymptotic theories of the new FIEDCC models. Hence, we investigate the finite-sample properties of the ML estimator in the next subsection. Further research is required in this field.

3.3 Monte Carlo experiments

We now investigate finite-sample properties of the ML estimator for the new FIEGARCH-FIEDCC specification comprising eqs [9] and [10]. Note that the EGARCH-EDCC model is a special case of the FIEGARCH-FIEDCC model. As our focus is on modeling Ω_t , we impose the restriction that $\mu_t = 0$ in the Monte Carlo experiments and empirical applications.

Let us specify the parameters for the conditional correlations and the degrees of freedom as

$$(d_c, \alpha, \lambda_1, \lambda_2, s_{12}, \nu) = \begin{cases} (0.3, 0.194, -0.0334, 0.10, 0.4, 5) & \text{for DGP1,} \\ (0.1, 0.494, -0.5650, 0.08, -0.4, 10) & \text{for DGP2,} \end{cases}$$

in the dynamic correlations and set the parameters for the conditional variances as

$$(\kappa_1, d_1^\nu, \phi_1, \gamma_{11}, \gamma_{21}) = (0, 0.48, 0.164, -0.255, 0.1),$$

$$(\kappa_2, d_2^\nu, \phi_2, \gamma_{12}, \gamma_{22}) = (0, 0.40, 0.638, -0.119, 0.2).$$

Here, s_{12} is the (1,2)th element of S . These parameters are chosen in the following manner. With respect to the dynamic correlations, first-order autocorrelation for each element of $\text{Log}(Q_t)$ is given by

$$\frac{(1 + \alpha^2)F(1, d_c; 1 - d_c; \alpha) - 1}{\alpha\{2F(1, d_c; 1 - d_c; \alpha) - 1\}},$$

³ We can also consider the truncation based on the $\text{AR}(\infty)$ representation. For the purpose of establishing asymptotic theories, we need to consider a "moving truncation" with J_T , which depends on the sample size, using either $\text{MA}(\infty)$ or $\text{AR}(\infty)$ representation. As it is beyond the scope of this paper, we will not go into details here.

where $F(a, b; c; z)$ is the hypergeometric function. In this case, DGP1 and DGP2 give the same value of 0.6. The variance of the diagonal element of $\text{Log}(Q_t)$ is given by

$$(4\lambda_1^2 + 2\lambda_2^2) \frac{\Gamma(1 - 2d_c)}{\{\Gamma(1 - d_c)\}^2} \frac{F(1, 1 + d_c; 1 - d_c; \alpha)}{(1 + \alpha)},$$

and the standard deviations for DGP1 and DGP2 are 0.2. Hence, DGP1 shows longer persistence than DGP2, and the leverage effect of DGP1 is stronger than that of DGP2. DGP1 considers positive correlation, while DGP2 considers negative correlation. The degrees-of-freedom parameter of the Wishart distribution of DGP1 is smaller than that of DGP2. Similarly, we specify the first-order auto-correlations of log-volatilities as 0.95, while we set the standard deviations as one, where the variance is given by

$$(\gamma_{1i}^2 + 2\gamma_{2i}^2) \frac{\Gamma(1 - 2d_i^v)}{\{\Gamma(1 - d_i^v)\}^2} \frac{F(1, 1 + d_i^v; 1 - d_i^v; \varphi_i)}{(1 + \varphi_i)}.$$

The first log-volatility, v_{1t} , exhibits longer persistence than the second, v_{2t} . The leverage effect of v_{2t} is weaker than that of v_{1t} . We consider two sample sizes, $T = 1,500$ and $3,000$, with $1,000$ replications. We apply the Levinson-Durbin algorithm for generating long-memory process [9] and [10], setting $Q_0 = \text{Exp}(M)$ and $v_0 = \text{exp}(\kappa)$.

Table 1 reports the sample means, standard deviations, and root mean squared errors of the ML estimators for the FIEGARCH-FIEDCC model. For the case of $T = 1,500$, the estimates of long-memory parameters d_c and d_i^v have a downward bias. When their values increase, the bias becomes larger. The bias on d_i^v affects the estimates of ϕ_i , γ_{1i} , and γ_{2i} . On the other hand, the bias on d_c influences on α , but does not affect λ_1 and λ_2 . This is because the values of d_c are relatively small. The finite-sample biases on the other parameters are negligible, as their standard deviations are close to their RMSE. As shown in the results for $T = 3,000$, all biases tend to zero when the sample size increases. The finite-sample properties for $d \geq 0.5$ are still open question. For these cases, we need to increase the sample size T , in order to decrease biases.

4 Empirical analysis

In this section, we estimate the FIEGARCH-FIEDCC and EGARCH-EDCC models and compare their out-of-sample forecasts. We selected nine assets from

Table 1: Finite sample performance of the ML estimator for the FIEGARCH-FIEDCC model

Parameters	True	$T = 1,500$			$T = 3,000$		
		Mean	Std Dev.	RMSE	Mean	Std Dev.	RMSE
(a) DGP1							
K_1	0	-0.0114	0.2141	0.2142	-0.0092	0.1510	0.1511
K_2	0	-0.0098	0.2215	0.2215	-0.0138	0.1555	0.1560
d_1^y	0.48	0.4077	0.1140	0.1349	0.4514	0.0671	0.0728
d_2^y	0.40	0.3517	0.1180	0.1274	0.3767	0.0784	0.0817
ϕ_1	0.164	0.2177	0.2260	0.2321	0.1828	0.1505	0.1515
ϕ_2	0.638	0.6524	0.1327	0.1334	0.6469	0.0867	0.0870
γ_{11}	-0.255	-0.3262	0.6674	0.6705	-0.2851	0.4126	0.4133
γ_{12}	-0.119	-0.1720	0.6638	0.6652	-0.1249	0.4637	0.4633
γ_{21}	0.1	0.2438	0.4440	0.4663	0.1494	0.2750	0.2792
γ_{22}	0.2	0.3402	0.4038	0.4271	0.2501	0.2349	0.2400
$1/\nu$	0.16	0.1674	0.0165	0.0365	0.1695	0.0118	0.0327
s_{12}	0.4	0.3360	0.2540	0.2617	0.3122	0.2163	0.2332
d_c	0.3	0.2437	0.0983	0.1132	0.2698	0.0502	0.0585
α	0.194	0.2434	0.2254	0.2305	0.2281	0.1324	0.1366
λ_1	-0.0334	-0.0348	0.0121	0.0122	-0.0345	0.0080	0.0080
λ_2	0.10	0.1004	0.0105	0.0105	0.1003	0.0059	0.0059
(b) DGP2							
K_1	0	0.0038	0.1528	0.1527	0.0020	0.1161	0.1161
K_2	0	0.0061	0.1421	0.1421	0.0021	0.1103	0.1103
d_1^y	0.48	0.4084	0.1352	0.1529	0.4378	0.0934	0.1028
d_2^y	0.40	0.3540	0.1247	0.1328	0.3707	0.0992	0.1033
ϕ_1	0.164	0.1793	0.2580	0.2582	0.1773	0.1985	0.1988
ϕ_2	0.638	0.6401	0.1575	0.1574	0.6435	0.1192	0.1192
γ_{11}	-0.255	-0.3152	0.5787	0.5812	-0.2784	0.3610	0.3614
γ_{12}	-0.119	-0.1351	0.4858	0.4856	-0.1349	0.3322	0.3322
γ_{21}	0.1	0.2298	0.4181	0.4374	0.1843	0.2511	0.2646
γ_{22}	0.2	0.2904	0.3453	0.3566	0.2626	0.2572	0.2644
$1/\nu$	0.1	0.0783	0.0154	0.0266	0.0800	0.0111	0.0229
s_{12}	-0.4	-0.3995	0.0713	0.0713	-0.4027	0.0218	0.0220
d_c	0.1	0.0729	0.1020	0.1054	0.0776	0.0810	0.0840
α	0.494	0.4747	0.1891	0.1899	0.4928	0.1304	0.1304
λ_1	-0.0565	-0.0590	0.0130	0.0132	-0.0581	0.0094	0.0095
λ_2	0.08	0.0791	0.0130	0.0130	0.0797	0.0090	0.0090

chemical firms, banks, and oil and gas producers. The companies are: Air Products and Chemicals (APD), Eastman Chemicals (EMN), and Ashland (ASH) for the chemical sector; Bank of America (BAC), JP Morgan Chase (JPM), and Wells Fargo (WFC) for the banking sector; Chevron (CVX), Exxon Mobil (XOM),

and ConocoPhillips (COP) for the oil and gas sector. The sample period is from May 28, 2002 to April 27, 2012, giving 2,500 observations. We employ the open-to-close return instead of the close-to-close return as suggested by Hansen, Huang, and Shek (2012). Their empirical results indicate that the open-to-close return is appropriate for estimating daily volatility. We reserve the last 500 observations for the out-of-sample forecast and set the sample size of the estimation as $T = 1,000$. The first 1,000 observations are used as the initial values for estimation. Rather than considering a longer sample period for estimating under the fractionally integrated process, we prefer to compare the EGARCH-EDCC model on as fair a basis as possible. Table 2 shows summary statistics for all observations. Among the nine series, the distributions of JPM and WFC from the financial sector are right-skewed, unlike typical results for financial assets. Note that the sample period covers the global financial crisis (GFC) and that the period before the GFC causes the right-skewed distributions.

Table 2: Summary statistics for returns

Variable	Mean	SD	Min.	Max.	Skew.	Kurt.
APD	0.0226	1.6065	−11.2076	9.9283	−0.3053	8.9602
EMN	0.0229	1.8327	−12.2617	13.4317	−0.0088	8.5500
ASH	−0.0255	2.3760	−26.8374	12.2024	−1.1517	15.5724
BAC	−0.1359	2.8997	−23.9480	19.4727	−0.5576	19.6623
JPM	−0.0109	2.4695	−17.0994	20.3488	0.4177	14.6228
WFC	−0.0108	2.4277	−19.3271	18.9660	0.2611	17.3108
CVX	0.0159	1.4745	−14.1798	14.9749	−0.0186	15.0038
XOM	0.0497	1.4255	−13.2060	11.8707	−0.0325	13.6689
COP	−0.0285	1.6748	−16.1106	10.2126	−0.4925	9.4751

In order to examine the in-sample and out-of-sample performances of the new FIEGARCH-FIEDCC and EGARCH-EDCC models, we work with two kinds of models. The first model is the modified version of the ADCC model of Cappiello, Engle, and Sheppard (2006). The approach for ADCC model to accommodate asymmetric effects in volatility and correlation dynamics is different from that of the EGARCH-EDCC model. For this purpose, we also employ the stochastic covariance framework and work with the consistent version of DCC suggested by Aielli (2006), since Aielli (2006) showed that the DCC of Engle (2002) does not provide consistent estimator of the correlation matrix. See Engle, Shepard, and Sheppard (2009) for its application. Our ADCC model is given by eq. [5] and

$$u_t = y_t - \mu_t,$$

$$v_t = \omega + \delta \circ u_{t-1} \circ u_{t-1} + \eta \circ v_{t-1} + \xi \circ u_{t-1}^- \circ u_{t-1}^-, \quad [15]$$

$$Q_t = (\bar{P} - a\bar{P} - b\bar{P} - g\bar{N}) + a(u_{t-1}u'_{t-1}) + g(u_{t-1}^-u_{t-1}^{-'}) + bQ_{t-1},$$

where ω , δ , η , and ξ are vectors of parameters, a , b , and g are scalar parameters, $u_t^- = (u_{1t}^-, \dots, u_{mt}^-)'$, $u_{it}^- = u_{it}I(u_{it} < 0)$, $I(\cdot)$ is an indicator function which takes the value of 1 if the argument is true and 0 otherwise, $\bar{P} = E(u_t u_t')$, and $\bar{N} = E(u_t^- u_t^{-'})$. Note that u_{t-1} is used instead of z_{t-1} because of the consistent version. We work with the GJR model for the volatility process due to its status as another popular asymmetric model in the ARCH family. As proposed by Cappiello, Engle, and Sheppard (2006), we replace $\bar{P}$ and $\bar{N}$ by the sample covariance matrices. We estimate the ADCC model using the approach explained in Section 3.2.

The second model is an EDCC type extension of the heterogeneous ARCH model of Müller et al. (1997) and the heterogeneous AR model of Corsi (2009). Following Corsi (2009), we denote the h horizon average of past x_t as

$$(x_{t-1})_h = \frac{1}{h}(x_{t-1} + \dots + x_{t-h}).$$

By the notations, we can define the heterogeneous exponential ARCH (HEARCH) model as

$$\log(v_{it}) = \kappa_i + \psi_i^d g_{vi}(z_{i,t-1}) + \psi_i^w g_{vi}((z_{i,t-1})_5) + \psi_i^m g_{vi}((z_{i,t-1})_{22}), \quad [16]$$

where $\psi_i^d = 1 - \psi_i^w/5 - \psi_i^m/22$. The purpose of the restriction on ψ_i^d is to set the coefficient of $g_{vi}(z_{i,t-1})$ to unity based on the specification that $g_{vi}((z_{i,t-1})_5)$ and $g_{vi}((z_{i,t-1})_{22})$ contain $g_{vi}(z_{i,t-1})$. Note that we may develop a heterogeneous exponential GARCH model, but the simple HEARCH specification is successful enough in our empirical studies, as in the heterogeneous ARCH model of Müller et al. (1997). Similarly, we consider the heterogeneous EDCC (HEDCC) model

$$\text{Log}(Q_t) = M + \alpha(\text{Log}(Q_{t-1}) - M) + \beta^d G_c(z_{t-1}) + \beta^w G_c((z_{t-1})_5) + \beta^m G_c((z_{t-1})_{22}), \quad [17]$$

where $\beta^d = 1 - \beta^w/5 - \beta^m/22$. Using the HEARCH-HEDCC model, we may examine the daily, weekly, and monthly (asymmetric) effects on log-volatility and dynamic correlations. These effects via heterogeneous time horizons can be alternative to the specification of the EGARCH-EDCC model.

Table 3 presents the ML estimates for four kinds of DCC model. The upper panel summarizes the estimates for corresponding parameters, while the lower panel shows the estimates themselves. For the EGARCH-EDCC model, the

estimates of ϕ_i vary from 0.9878 to 0.9962, with a median of 0.9928. This indicates the possibility of long-range dependence in log-volatility. The estimates of γ_{i1} are negative and significant other than the two cases yielding insignificantly positive values. The estimates of s_{ij} range from 0.3103 to 0.8342, showing positive unconditional correlations among variables. With respect to the EDCC part, the estimate of α is close to one, indicating the possibility of a long-memory process. The estimate of λ_1 is insignificant. The estimate of ν is 17.285, showing the Wishart disturbance in the covariance matrix is not negligible.

Regarding the estimates of the FIEGARCH-FIEDCC model shown in Table 3, the median of the estimates of d_i^v is 0.3153, but the minimum is almost zero. Here, two assets follow the EGARCH specification more close than they do the FIEGARCH model. All the estimates of γ_{i1} are negative, while those of γ_{i2} are positive. The estimates of s_{ij} are similar to those produced by the EDCC model. The estimate of d_c is significant at 0.13, showing the long memory in dynamic correlations. The estimates of λ_1 and λ_2 are positive and significant. The estimate of ν is 16.666, similar to the value derived by the EDCC specification. In comparison with the EDCC model, the FIEDCC model provides better fit for AIC, while the EDCC model provides better fit for BIC. The likelihood ratio test rejects the EDCC specification.

With respect to the HEDCC model, we expect the model to capture not only daily effect but also weekly and monthly effects. Table 3 shows that the estimates of ψ_i^w and ψ_i^m are positive. All the estimates of γ_{i1} are negative, indicating the existence of leverage effects. The estimates of β_i^w and β_i^m are positive and significant. The estimates of λ_1 are positive and significant, unlike those for the previous two models. The estimate of ν is 12.675. The HEDCC model has the largest AIC and BIC values among the three models.

Table 3 also gives the estimates of the ADCC model. The median of the estimates of δ_i , ξ_i , and η_i are 0.0226, 0.0235, and 0.9499, respectively. These values are typical for the GJR specification. The estimates of ρ_{ij} vary from 0.3836 to 0.8742. The estimate of a is 0.0226, while the estimate of g is 0.9773. Although the estimate of b is relatively small, it is significant. The estimate of ν is close to the estimate of the EDCC specification. The ADCC specification has the second largest AIC and BIC values of the four models considered.

Figure 1 shows the news impact curve from the standardized return, z_{it} , to log-volatility, $\ln v_{i,t+1}$, for each asset. In all panels, the centers of the parabola are positive, showing the existence of leverage effects. The banks (BAC, JPM, and WFC) have stronger asymmetric effects, while APD and CVX have weaker asymmetric effects.

Figure 2 presents the estimated dynamic correlations obtained by the FIEDCC and ADCC models for the cases of (APD, EMN) and (JPM, XOM).

Table 3: ML estimates for DCC models

Param.	EDCC	FIEDCC	HEDCC	Param.	ADCC
κ_i	−0.7508 [−0.1774, 1.2081]	0.7319 [0.1250, 2.1352]	1.2355 [1.0539, 1.7401]	ω_i	0.0312 [0.0190, 0.0553]
d_i^v		0.3153 [0.0000, 0.4999]		δ_i	0.0226 [0.0079, 0.0332]
ϕ_i	0.9928 [0.9878, 0.9962]	0.8910 [0.7491, 0.9878]		ξ_i	0.0235 [0.0095, 0.0673]
ψ_i^w			1.1115 [0.5521, 1.5138]	η_i	0.9499 [0.9279, 0.9707]
ψ_i^m			11.926 [9.5258, 18.227]		
γ_{i1}	−0.0057 [−0.0380, 0.0046]	−0.0488 [−0.0784, −0.0133]	−0.1296 [−0.2221, −0.0812]		
γ_{i2}	0.0364 [0.0275, 0.0453]	0.0583 [0.0396, 0.0793]	0.5300 [0.26010, 0.9410]		
s_{ij}	0.4075 [0.3103, 0.8342]	0.3965 [0.2791, 0.8143]	0.4020 [0.2945, 0.8173]	ρ_{ij}	0.4975 [0.3836, 0.8742]
d_c		0.1297 (0.0552)		a	0.0226 (0.0010)
α	0.9875 (0.0039)	0.9516 (0.0113)		b	0.5711×10^{-4} (0.2204×10^{-4})
β_w			0.5018 (0.0264)	g	0.9773 (0.00096)
β_m			0.1007 (0.0475)		
λ_1	0.0011 (0.0006)	0.0035 (0.0004)	−0.0030 (0.0003)		
λ_2	0.0037 (0.0008)	0.0011 (0.0001)	0.0000 (0.0039)		
$1/\nu$	0.0579 (0.0092)	0.0600 (0.0037)	0.0789 (0.0149)	$1/\nu$	0.0568 (0.0083)
LogLike	−14,943.4	−14,896.8	−15,922.8	LogLike	−15,009.1
AIC	30,038.9	29,965.7	32,017.6	AIC	30,170.2
BIC	30,936.9	30,981.8	33,033.8	BIC	31,068.2

Notes: The upper panel summarizes the estimates for corresponding parameters. The entry presents the median, while minimum and maximum values are given in brackets. The lower panel shows the estimates themselves and reports standard errors in parentheses.

Figure 1: News impact curves from z_{it} to $\ln v_{i,t+1}$

Note: In each panel, the x-axis indicates z_{it} , while the y-axis measures $\ln v_{i,t+1}$.

Clearly, the fluctuations of the FIEDCC model are larger than those of the ADCC model. As shown in Panel (a), the dynamic correlations of the ADCC model fluctuate between the values of 0.55 and 0.7, indicating non-negligible dynamic movement. However, the FIEDCC model captures the movement more sensitively, especially due to the matrix-exponential transformation. A comparison of Panels (a) and (b) indicates that the dynamic correlations of the same model follow similar patterns, which are caused by the parsimonious specifications. With respect to the GFC, Lehman Brothers filed for bankruptcy on September 15, 2008, which corresponds to $t = 1,586$. Both plots of the FIEDCC model show a stronger correlation between the two stocks for the period after the crisis than in the period before the crisis. In addition, the bear market rally began on March 10, 2009 ($t = 1,708$), the impact of which is described by the FIEDCC model.

We now examine the out-of-sample forecasts of the covariance matrices of the three exponential dynamic correlations models – the EDCC, FIEDCC, and

Figure 2: Dynamic correlations for FIEDCC and ADCC models

Note: Panel (a) shows the dynamic correlations between APD and EMN, while Panel (b) gives the dynamic correlations between JPM and XOM.

HEDCC models – with respect to the minimum variance portfolio (MVP) with the weights given by $w_t = \hat{\Omega}_t^{-1} \iota / (\iota' \hat{\Omega}_t^{-1} \iota)$, where $\hat{\Omega}_t$ is the one-step ahead forecast of the conditional covariance matrix constructed at time $t - 1$ and ι is an m vector of ones. We calculated one-step-ahead forecasts, $\{\hat{\Omega}_{T+h}\}_{h=1}^{500}$, with the rolling window of the size of 1,000 observations. VaR thresholds are computed for the MVP at the 1% and 5% levels. As we explained in Section 3, the portfolio return, y_{pt} , has a t distribution with mean μ_{pt} and variance ω_{pt} , thus enabling us to obtain the 1% and 5% quantiles of the standardized t distribution directly in each step. We compare the performance of the exponential dynamic correlations models with that of the ADCC model as before. Note that we may improve VaR thresholds forecasts by employing the filtered historical simulation of Barone-Adesi, Giannopoulos, and Vosper (1999).

The unconditional coverage and independence tests developed by Christoffersen (1998) are widely used to assess the estimated VaR thresholds.

One drawback of the Christoffersen (1998) test for independence is that it tests against a particular alternative of first-order dependence. The duration-based approach developed by Christoffersen and Pelletier (2004) allows for testing against more general forms of dependence, but still requires a specific alternative. Candelon et al. (2011) have recently developed a more robust procedure which does not need a specific distributional assumption for durations under the alternative. Consider the “hit sequence” of VaR violations, which takes a value of one if the loss is greater than the VaR threshold and the value of zero if the VaR threshold is not violated. If we could predict VaR violations, then that information may help to construct a better model. Hence, the hit sequence of violations should be unpredictable and should follow an independent Bernoulli distribution with parameter p , indicating that the duration of the hit sequence should follow a geometric distribution. The GMM duration-based test developed by Candelon et al. (2011) works with the J-statistic based on the moments defined by the orthonormal polynomials associated with the geometric distribution. The conditional coverage (CC) test and independence (IND) test based on q orthonormal polynomials have asymptotic χ^2_q and χ^2_{q-1} distributions under their respective null distributions. The Monte Carlo simulation results reported by Candelon et al. (2011) suggest that setting $q = 3$ ($q = 5$) is enough for the 1% (5%) VaR threshold analysis. In what follows, IND and CC tests work with a conservative setting of $q = 5$. The unconditional coverage (UC) test is given as a special case of the CC test with $q = 1$.

Table 4 presents the percentage of days on which returns are below the VaR threshold, as well as the results for the GMM duration-based tests for UC, IND,

Table 4: Backtesting VaR thresholds

Model	VaR 5%				VaR 1%			
	% Violation	UC	ID	CC	% Violation	UC	ID	CC
EDCC	0.062	2.7482 [0.0974]	20.658* [0.0004]	18.197* [0.0027]	0.018	1.9008 [0.1680]	4.4989 [0.3427]	4.9207 [0.4256]
FIEDCC	0.058	2.4665 [0.1163]	7.8693 [0.0965]	8.8291 [0.1161]	0.014	2.3674 [0.1239]	2.3746 [0.6672]	4.3279 [0.5032]
HEDCC	0.046	0.0938 [0.7594]	17.218* [0.0017]	23.213* [0.0003]	0.014	2.3674 [0.1239]	1.7357 [0.7842]	3.3211 [0.6506]
ADCC	0.066	3.8725* [0.0491]	28.046* [0.0000]	21.954* [0.0005]	0.014	2.3674 [0.1239]	2.4178 [0.6594]	4.3401 [0.5016]

Notes: % Violation is the percentage of days on which returns are below the VaR threshold. UC, IND, and CC are the GMM duration-based tests for unconditional coverage, independence, and conditional coverage, respectively, developed by Candelon et al. (2011). The number of orthonormal polynomials is set to 5. p -Values are in brackets. “**” indicates significance at the 5% level.

and CC at the 5% and 1% thresholds. The percentage of violations for the EDCC model is similar to that of the ADCC model, while those of the FIEDCC and HEDCC models are close to each other. The FIEDCC model passes the UC, IND, and CC tests at the 5% and 1% thresholds. With respect to the other models, they all pass the tests at the 1% threshold, while the IND and CC tests reject all three models at the 5%. The results show the superiority of the FIEDCC model for the last 500 observations before April 27, 2012.

Figure 3 shows the estimated values of d_c in each step for obtaining the one-step-ahead forecasts with the rolling window, along with the portfolio returns based on the MVP forecasts. Surprisingly, some of the d_c estimates are close to zero, while others are even negative. With respect to the insignificant d_c values, note that the estimates of d_c are biased toward zero in a finite sample, as shown in the Monte Carlo simulations. As it can be difficult to distinguish a long-memory process from an AR model with a high degree of persistence, we may expect d_c to take values close to zero. A comparison of the movement of portfolio returns with d_c values shows negative d_c values seem to appear after the size of fluctuations in the return process increases. However, rigorous

Figure 3: Estimated long-memory parameter for the FIEDCC model

Note: The figure shows the estimates of d_c for each step of the out-of-sample forecasting process.

theoretical and empirical investigations of this dynamic correlation process are required using the realized covariance and correlation measures such as those defined by Barndorff-Nielsen et al. (2011) and Tao et al. (2011). We also need to examine the appropriateness of considering single d_c for the correlation dynamics. At this stage, we may state that the FIEDCC model is one of the more parsimonious specifications for accurately describing this dynamic correlation process.

5 Conclusions

In this paper, we propose a fractionally integrated matrix-exponential DCC (FIEDCC) model within the stochastic covariance structure framework to capture asymmetry and long memory among dynamic correlations. In estimating these models, we suggested a three-step procedure for obtaining initial values, working with the “singular” multivariate Beta type II distribution. Monte Carlo simulation results show that the ML estimator yields satisfactory results in finite samples. Our empirical analysis of nine assets from three sectors in the US demonstrates that the new FIEDCC model outperforms the other dynamic correlation models examined in estimating AIC and BIC and forecasting VaR thresholds and that this new model captures the stronger connection among these assets in the period immediately following the GFC.

This paper has made certain contributions, but several extensions are still possible. Firstly, we can consider the multivariate skewed t distribution of Bauwens and Laurent (2005), using the stochastic covariance structure with the multivariate skewed normal density. Secondly, we may relax the assumption of single degrees-of-freedom parameter for the Wishart distribution, by assuming Bellman Type I and II distributions for stochastic covariance process. See Gupta and Nagar (2000) for these distributions. Thirdly, we may improve the ML estimation technique of FIEGARCH model suggested by Bollerslev and Mikkelsen (1996), using the $AR(\infty)$ representation with the moving truncation. Such tasks await future researches.

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