Research Article

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On the separation method in stochastic reconstruction problem

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Abstract: The inverse problem of reconstruction second-order Ito stochastic differential equations by the given properties of motion is solved when control is included in the drift coefficient. By using the separation method, the form of control parameters is determined that provides sufficient conditions for the existence of a given integral manifold for the nonlinear, linear, and scalar cases.

Keywords: Ito stochastic equations, reconstruction problem, separation method

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1 Introduction

Methods for solving dynamics inverse problems have been quite fully developed for ordinary differential equations (ODEs) [1–6]. These problems go back to the fundamental work by Erugin [7], in which a set of ODEs is constructed given an integral curve. In [1,2], Galiullin proposed a classification of the main types of dynamics inverse problems and developed methods for solving them in the class of ODEs. Dynamics inverse problems for Ito stochastic differential equations are studied in [8–14]. The present paper is organized as follows. In Section 2, we study the reconstruction problem in the class of second-order Ito stochastic differential equations by given properties of motion, when control is included in the drift coefficient. The separation method is used to determine the form of control parameters that provides sufficient conditions for the existence of a given integral manifold. Section 2.1 deals with the general nonlinear case. The general linear case is considered in Sections 2.2 and 2.3 deals with the scalar nonlinear case with drift control. Section 3 presents a solution of Meshchersky's stochastic problem, which is one of the dynamics inverse problems and, according to Galiullin [1], refers to the inverse reconstruction problem.

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2 Stochastic reconstruction problem

2.1 The nonlinear general case of the reconstruction problem

We consider the second-order Ito stochastic differential equation

$$\ddot{x} = f(x, \dot{x}, t) + D(x, \dot{x}, t)u + \sigma(x, \dot{x}, t)\dot{\xi}, \quad x \in R^n, \quad \xi \in R^k. \quad (1)$$

It is required to determine a vector-function $u = u(x, \dot{x}, t) \in R^r$ included in the drift coefficient, given the integral manifold

$$\Lambda(t) : \lambda(x, \dot{x}, t) = 0, \quad \lambda = \lambda(x, \dot{x}, t) \in C_{x\dot{x}t}^{121}, \quad \lambda \in R^m, \quad (2)$$

where $C_{x\dot{x}t}^{121}$ is the set of functions $\gamma(x, \dot{x}, t)$ that are continuously differentiable with respect to x and t and twice continuously differentiable with respect to $\dot{x}$.

In other words, given f, D, σ , and λ , we need to determine the control parameter u so that the set (2) is the integral manifold of equation (1).

Suppose that the n -dimensional vector-functions $f(x, \dot{x}, t)$, $(n \times r)$ -matrix $D(x, \dot{x}, t)$ and $(n \times k)$ -matrix $\sigma(x, \dot{x}, t)$ satisfy the following conditions:

- (i) $f(x, \dot{x}, t)$, $D(x, \dot{x}, t)$, $\sigma(x, \dot{x}, t)$ are continuous in t and Lipschitz continuous in x and $\dot{x}$ in the whole space $R^{2n} \ni z = (x^T, \dot{x}^T)^T$,
- (ii) $f(x, \dot{x}, t)$, $D(x, \dot{x}, t)$, and $\sigma(x, \dot{x}, t)$ satisfy the condition of linear growth $\|f(z, t)\|^2 + \|D(z, t)\|^2 + \|\sigma(z, t)\|^2 \leq L(1 + \|z\|^2)$, which ensures the existence and uniqueness up to stochastic equivalence of the solution $z(t)$ of equation (1) in R^{2n} with the initial condition $z(t_0) = z_0$ that is a strictly Markov process continuous with probability one [15, p. 39]. We will say that a function $p(x, \dot{x}, t)$ belongs to a class K , $p \in K$ if it satisfies conditions (i) and (ii).

Here $\{\xi_1(t, \omega), \dots, \xi_k(t, \omega)\}$ is a system of random processes with independent increments that can be represented as a sum of processes [16]: $\xi = \xi_0 + \int c(y)P^0(t, dy)$. ξ_0 is a Wiener process, P^0 is a Poisson process, $P^0(t, dy)$ is the number of jumps of the process P^0 jumps in the interval $[0, t]$ that fall on the set dy ; $c(y)$ is a vector function mapping the space R^{2n} to the space R^k of values of the process $\xi(t)$ for all t .

In this paper, we use the separation method [3, p. 12] to solve the stochastic reconstruction problem.

In order to solve the posed problem, we apply the Ito stochastic differentiation rule [16, p. 204] and construct the equation of perturbed motion

$$\dot{\lambda} = \frac{\partial \lambda}{\partial t} + \frac{\partial \lambda}{\partial x} \dot{x} + \frac{\partial \lambda}{\partial \dot{x}} f + \frac{\partial \lambda}{\partial \dot{x}} Du + \frac{\partial \lambda}{\partial \dot{x}} \sigma \dot{\xi}_0 + S_1 + S_2 + S_3, \quad (3)$$

where

$$S_1 = \frac{1}{2} \frac{\partial^2 \lambda}{\partial \dot{x}^2} : \sigma \sigma^T, \quad S_2 = \int \left[\lambda(x, \dot{x} + \sigma c(y), t) - \lambda(x, \dot{x}, t) - \frac{\partial \lambda}{\partial \dot{x}} \sigma \dot{x} c(y) \right] dy, \\ S_3 = \int [\lambda(x, \dot{x} + \sigma c(y), t) - \lambda(x, \dot{x}, t)] \dot{P}_0(t, dy),$$

and by $\frac{1}{2} \frac{\partial^2 \lambda}{\partial \dot{x}^2} : D$, following [16], we mean the vector $\left(\frac{\partial^2 \lambda}{\partial \dot{x}^2} : D \right)^T = \left(\text{tr} \left(\frac{\partial^2 \lambda_1}{\partial \dot{x}^2} D \right), \dots, \text{tr} \left(\frac{\partial^2 \lambda_m}{\partial \dot{x}^2} D \right) \right)^T$, $D = \sigma \sigma^T$.

Let us introduce arbitrary Yerugin functions [7]: an m -dimensional vector-function A and a $(m \times k)$ -matrix B such that $A(0, x, \dot{x}, t) \equiv 0$, $B(0, x, \dot{x}, t) \equiv 0$ and

$$\dot{\lambda} = A(\lambda, x, \dot{x}, t) + B(\lambda, x, \dot{x}, t)\dot{\xi}. \quad (4)$$

By comparing equations (3) and (4), we get the relations

$$\frac{\partial \lambda}{\partial \dot{x}} Du = A - \frac{\partial \lambda}{\partial t} - \frac{\partial \lambda}{\partial x} \dot{x} - \frac{\partial \lambda}{\partial \dot{x}} f - S_1 - S_2 - S_3, \quad (5)$$

$$\frac{\partial \lambda}{\partial \dot{x}} \sigma = B, \quad (6)$$

from which we need to determine the control u and the matrix σ .

Let $\tilde{A}$ and $\tilde{D}$ stand for the r -vector $\tilde{A} = A - \frac{\partial \lambda}{\partial t} - \frac{\partial \lambda}{\partial x} \dot{x} - \frac{\partial \lambda}{\partial \dot{x}} f - S_1 - S_2 - S_3$ and the $(m \times r)$ -matrix $\tilde{D} = \frac{\partial \lambda}{\partial \dot{x}} D$, respectively. To solve the problem, we use the separation method. Supposing that $m \leq r$, we represent the matrix $\tilde{D}$ in the form $\tilde{D} = (\tilde{D}_1, \tilde{D}_2)$, where the matrices $\tilde{D}_1$ and $\tilde{D}_2$ are of dimensions $(m \times m)$ and $(m \times (r - m))$, respectively. The control parameter is represented as $u = (\tilde{u}_1^T, \tilde{u}_2^T)^T$, where $\tilde{u}_1 \in R^m$, $\tilde{u}_2 \in R^{r-m}$. We then can rewrite (5) as $\tilde{D}u = \tilde{A}$, or

$$\tilde{D}_1 \tilde{u}_1 + \tilde{D}_2 \tilde{u}_2 = \tilde{A}. \quad (7)$$

Suppose that $\det \tilde{D}_1 \neq 0$. Then from (7) we have

$$\tilde{u}_1 = \tilde{D}_1^{-1} (\tilde{A} - \tilde{D}_2 \tilde{u}_2) \quad (8)$$

for an arbitrary $\tilde{u}_2$ that belongs to the class K .

Furthermore, let us consider the relation $N\sigma = B$, where $N = \frac{\partial \lambda}{\partial x}$ is an $(m \times r)$ -matrix. We represent the matrix N and the $(n \times k)$ -matrix σ as $N = (N_1, N_2)$ and $\sigma = \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}$, respectively. Here N_1 , N_2 , σ_1 , and σ_2 are the matrices of dimensions $(m \times m)$, $(m \times (n - m))$, $(m \times k)$, and $((n - m) \times k)$, respectively. We then rewrite (6) in the form

$$N_1 \sigma_1 + N_2 \sigma_2 = B. \quad (9)$$

Assuming $\det N_1 \neq 0$, from (9) we have

$$\sigma_1 = N_1^{-1} (B - N_2 \sigma_2) \quad (10)$$

for an arbitrary σ_2 from the class K .

Theorem 1. Let $m \leq r$ and let the matrices $\tilde{D} = (\tilde{D}_1, \tilde{D}_2)$ and $N = (N_1, N_2)$ be such that $\det \tilde{D}_1 \neq 0$ and $\det N_1 \neq 0$. Then, for the second-order differential equation of Ito type (1) to have a given integral manifold (2), it is sufficient that the m -dimensional part $\tilde{u}_1$ of the control parameter $u = (\tilde{u}_1^T, \tilde{u}_2^T)^T$ has the form (8) for arbitrary $\tilde{u}_2$ from the class K , and the $(m \times m)$ -submatrix σ_1 of the diffusion matrix $\sigma = \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}$ has the form (10) for arbitrary σ_2 from the class K .

Remark 1. The posed inverse reconstruction problem was previously solved by the quasi-inversion method [17] in the presence of random perturbations from the class of independent Wiener processes, which are known to be a particular case of random processes with independent increments.

2.2 The linear case of the stochastic problem with drift control

For the given second-order Ito stochastic differential equation, linear in drift,

$$\ddot{x} = E_1(t)x + E_2(t)\dot{x} + D(t)u + l_1(t) + T(x, \dot{x}, t)\dot{\xi}, \quad (11)$$

it is required to determine the vector control function by the given linear integral manifold

$$\Lambda(t) : \lambda \equiv Gx + F\dot{x} + l_2(t) = 0. \quad (12)$$

That is, given the $(m \times n)$ -matrices $G(t)$, $F(t)$ and m -dimensional function $l_2(t)$, the $(n \times n)$ -matrices $E_1(t)$, $E_2(t)$, $(n \times r)$ -matrix $D(t)$ and n -dimensional function $l_1(t)$, it is required to determine the vector

function $u = u(x, \dot{x}, t) \in R^r$ and the $(n \times k)$ -matrix $T(x, \dot{x}, t)$ so that for the constructed equation (11) the given properties (12) are an integral manifold.

In the problem under consideration, the equation of perturbed motion (3) is of the form

$$\dot{\lambda} = \dot{G}x + \dot{F}\dot{x} + \dot{l}_2(t) + \dot{G}\dot{x} + F(E_1(t)x + E_2(t)\dot{x} + D(t)u + l_1(t)) + FT\xi_0. \quad (13)$$

On the other hand, by Yerugin's method, for an arbitrary vector function $A = A_1(t)\lambda$ and a matrix function B_1 such that $B_1(0, x, \dot{x}, t) \equiv 0$ we have

$$\dot{\lambda} = A_1(t)\lambda + B_1(\lambda, x, \dot{x}, t)\xi. \quad (14)$$

Hence, relations (13) and (14) imply the equalities

$$\begin{cases} FDu = (A_1G - \dot{G} - FE_1)x + (A_1F - F - G - FE_2)\dot{x} + A_1l_2 - Fl_1 - \dot{l}_2, \\ FT = B_1. \end{cases} \quad (15)$$

To solve the linear problem by the separation method, let us first introduce the following notation: $\hat{A} = (A_1G - \dot{G} - FE_1)x + (A_1F - F - G - FE_2)\dot{x} + A_1l_2 - Fl_1 - \dot{l}_2$. Under the assumption $m \leq r$, we represent the matrix $\hat{D} = FD$ as $\hat{D} = (\hat{D}_1, \hat{D}_2)$, where the matrices $\hat{D}_1$ and $\hat{D}_2$ are of dimensions $(m \times m)$ and $(m \times (r - m))$, respectively. The control parameter u is represented in the form $u = (\hat{u}_1^T, \hat{u}_2^T)^T$, where $\hat{u}_1 \in R^m$, $\hat{u}_2 \in R^{r-m}$.

We further represent the $(n \times k)$ -matrix T as $T = \begin{pmatrix} \hat{T}_1 \\ \hat{T}_2 \end{pmatrix}$, where the submatrices $\hat{T}_1$ and $\hat{T}_2$ are of dimensions $(m \times k)$ and $((n - m) \times k)$, respectively. The $(m \times n)$ -matrix F is represented as $F = (F_1, F_2)$, with the submatrices F_1 and F_2 of dimensions $(m \times m)$ and $(m \times (n - m))$, respectively.

The equalities (15) then take the form $\hat{D}u = \hat{A}$, $FT = B_1$, or

$$\hat{D}_1\hat{u}_1 + \hat{D}_2\hat{u}_2 = \hat{A}, \quad (16)$$

$$F_1\hat{T}_1 + F_2\hat{T}_2 = B_1. \quad (17)$$

Hence, assuming $\det \hat{D}_1 \neq 0$ and $\det F_1 \neq 0$, from (16) and (17) we have the following relations:

$$\hat{u}_1 = \hat{D}_1^{-1}(\hat{A} - \hat{D}_2\hat{u}_2), \quad (18)$$

$$\hat{T}_1 = F_1^{-1}(B_1 - F_2\hat{T}_2). \quad (19)$$

Thus, the following statement holds.

Theorem 2. Let $m \leq r$ and let the matrices $\hat{D} = (\hat{D}_1, \hat{D}_2)$ and $F = (F_1, F_2)$ be such that $\det \hat{D}_1 \neq 0$ and $\det F_1 \neq 0$. Then, for the second-order equation linear in drift Ito stochastic differential (11) to have a given linear integral manifold (12), it is sufficient that the m -dimensional part $\hat{u}_1$ of the control parameter $u = (\hat{u}_1^T, \hat{u}_2^T)^T$ is of the form (18) for arbitrary $\hat{u}_2 \in K$, and the $(m \times m)$ -submatrix $\hat{T}_1$ of the diffusion matrix $T = \begin{pmatrix} \hat{T}_1 \\ \hat{T}_2 \end{pmatrix}$ is of the form (19) for arbitrary $\hat{T}_2 \in K$.

Remark 2. In the linear setting $S_1 \equiv S_2 \equiv S_3 \equiv 0$ takes place as opposed to the nonlinear case.

2.3 The scalar case of the recovery problem with drift controls

Given the second-order Ito stochastic differential equation

$$\ddot{x} = q(x, \dot{x}, t) + \gamma_1(x, \dot{x}, t)u_1 + \gamma(x, \dot{x}, t)\xi, \quad (20)$$

it is required to determine a scalar function by the given integral manifold

$$\lambda_2(x, \dot{x}, t) = 0, \quad \lambda_2 \in R^1. \quad (21)$$

In other words, by the given g, γ, γ_1 , and λ_2 , we determine the control parameter u_1 so that the set (21) is the integral manifold of equation (20).

By using the stochastic differentiation rule, we compose the equation of perturbed motion

$$\dot{\lambda}_2 = \frac{\partial \lambda_2}{\partial t} + \frac{\partial \lambda_2}{\partial x} \dot{x} + \frac{\partial \lambda_2}{\partial \dot{x}} q + \frac{\partial \lambda_2}{\partial \ddot{x}} \gamma_1 u_1 + \tilde{S}_1 + \tilde{S}_2 + \tilde{S}_3 + \frac{\partial \lambda_2}{\partial \ddot{x}} \gamma \dot{\xi}_0, \quad (22)$$

where

$$\begin{aligned} \tilde{S}_1 &= \frac{1}{2} \frac{\partial^2 \lambda_2}{\partial \ddot{x}^2} \gamma^2, \quad \tilde{S}_3 = \int [\lambda_2(x, \dot{x} + \gamma c(y), t) - \lambda_1(x, \dot{x}, t)] dP^0(t, dy), \\ \tilde{S}_2 &= \int \left[\lambda_2(x, \dot{x} + \gamma c(y), t) - \lambda_2(x, \dot{x}, t) - \frac{\partial \lambda_2}{\partial \ddot{x}} \gamma c(y) \right] dy. \end{aligned}$$

We introduce arbitrary scalar Yerugin's functions $a = a(\lambda_2, x, \dot{x}, t)$ and $b = b(\lambda_2, x, \dot{x}, t)$ such that $a(0, x, \dot{x}, t) \equiv b(0, x, \dot{x}, t) \equiv 0$ and

$$\dot{\lambda}_2 = a(\lambda_2, x, \dot{x}, t) + b(\lambda_2, x, \dot{x}, t) \dot{\xi}. \quad (23)$$

From (22) and (23), we obtain

$$\frac{\partial \lambda_2}{\partial x} \gamma_1 u_1 = a - \frac{\partial \lambda_2}{\partial t} - \frac{\partial \lambda_2}{\partial x} \dot{x} - \frac{\partial \lambda_2}{\partial \dot{x}} q - \tilde{S}_1 - \tilde{S}_2 - \tilde{S}_3, \quad (24)$$

$$\frac{\partial \lambda_2}{\partial x} \gamma = b. \quad (25)$$

Then, by (24) and (25), we determine the control parameter u_1 and the diffusion coefficient

$$u_1 = \left(\frac{\partial \lambda_2}{\partial \ddot{x}} \gamma_1 \right)^{-1} \left[a - \frac{\partial \lambda_2}{\partial t} - \frac{\partial \lambda_2}{\partial x} \dot{x} - \frac{\partial \lambda_2}{\partial \dot{x}} q - \frac{1}{2} \frac{\partial^2 \lambda_2}{\partial \ddot{x}^2} \gamma^2 - \tilde{S}_2 - \tilde{S}_3 \right], \quad (26)$$

$$\gamma = \left(\frac{\partial \lambda_2}{\partial \dot{x}} \right)^{-1} b. \quad (27)$$

We thus proved the following statement.

Theorem 3. For the scalar second-order Ito differential equation (20) to have a given scalar integral manifold (21), it is sufficient that the control parameter and the diffusion coefficient are of the form (26) and of (27), respectively.

Thus, we used the separation method to establish sufficient conditions for the solvability of the reconstruction problem with drift control in the presence of random perturbations from the class of processes with independent increments in the general nonlinear and linear cases, as well as in the scalar nonlinear case.

As an illustration of the stochastic reconstruction problem, let us consider the Meshchersky problem in the presence of random perturbations.

3 Meshchersky stochastic problem

The problem statement: Find the law of change in the mass of a particle that moves along a given trajectory under the action of given external forces [18, p. 19].

We consider the problem of realization of the motion of a heavy particle of variable mass $m(t)$ in a homogeneous gravity field, namely, a vertical ascent according to the laws of change in the range y and height z

$$\Lambda(t) : \begin{cases} \lambda_1(t) \equiv y - \varphi(t) = 0, \\ \lambda_2(t) \equiv z - \psi(t) = 0. \end{cases} \quad (28)$$

Taking into account the action of random perturbing forces, the equations of motion of a point have the following form [1, pp. 16–17]:

$$\begin{cases} m\ddot{y} = \dot{m}(\mu - 1)\dot{y} - mf(z, v)\frac{\dot{y}}{v} - \sigma_1(y, z, t)\dot{\xi}, \\ m\ddot{z} = \dot{m}(\eta - 1)\dot{z} - mf(z, v)\frac{\dot{z}}{v} - mg - \sigma_2(y, z, t)\dot{\xi}, \end{cases} \quad (29)$$

where $f(z, v)$ is the resistance of the medium per unit mass; $v = \sqrt{\dot{y}^2 + \dot{z}^2}$ is the point speed; $\mu = \mu(t)$, $\eta = \eta(t)$ are the ratios of the velocity projections of a changing mass and the mass of the point itself onto the coordinate y and z axes.

It is required to reconstruct the equations of motion (29) (i.e., to determine the laws of change of the quantities μ and ν) so that they admit the given particular motion (28).

The equations of perturbed motion have the form

$$\begin{cases} \ddot{\lambda}_1 \equiv \ddot{y} - \ddot{\varphi}(t) = \frac{\dot{m}}{m}(\mu - 1)\dot{y} - f(z, v)\frac{\dot{y}}{v} - \frac{\sigma_1}{m}\dot{\xi} - \ddot{\varphi}(t), \\ \ddot{\lambda}_2 \equiv \ddot{z} - \ddot{\psi}(t) = \frac{\dot{m}}{m}(\eta - 1)\dot{z} - f(z, v)\frac{\dot{z}}{v} - g - \frac{\sigma_2}{m}\dot{\xi} - \ddot{\psi}(t). \end{cases} \quad (30)$$

Following Yerugin's method [7], we introduce the functions $A_1 = A_1(\lambda_1, \dot{\lambda}_1, \lambda_2, \dot{\lambda}_2, y, z, t)$, $A_2 = A_2(\lambda_1, \dot{\lambda}_1, \lambda_2, \dot{\lambda}_2, y, z, t)$, $B_1 = B_1(\lambda_1, \dot{\lambda}_1, \lambda_2, \dot{\lambda}_2, y, z, t)$, and $B_2 = B_2(\lambda_1, \dot{\lambda}_1, \lambda_2, \dot{\lambda}_2, y, z, t)$ such that $A_1(0, 0, 0, 0, y, z, t) \equiv A_2(0, 0, 0, 0, y, z, t) \equiv B_1(0, 0, 0, 0, y, z, t) \equiv B_2(0, 0, 0, 0, y, z, t) \equiv 0$, and

$$\begin{cases} \ddot{\lambda}_1 = A_1 + B_1\dot{\xi}, \\ \ddot{\lambda}_2 = A_2 + B_2\dot{\xi}. \end{cases} \quad (31)$$

If we exclude strictly vertical and strictly horizontal motions (i.e., if $\dot{\varphi}$ and $\dot{\psi}$ are not identically zero), the comparison of systems (30) and (31) leads to the relations that solve the posed Meshchersky stochastic problem:

$$\begin{cases} \mu = 1 + \frac{m}{\dot{m}} \left[\frac{A_1}{\dot{y}} + \frac{f}{v} + \frac{\ddot{\varphi}}{\dot{y}} \right], \\ \eta = 1 + \frac{m}{\dot{m}} \left[\frac{A_2}{\dot{z}} + \frac{f}{v} + \frac{g}{\dot{z}} + \frac{\ddot{\psi}}{\dot{y}} \right], \\ \sigma_{1j} = mB_{1j}, \\ \sigma_{2j} = mB_{2j}. \end{cases} \quad (32)$$

In particular, in the case $\sigma_{ij} \equiv 0$ ($i, j = 1, 2$) and for $A_1 \equiv A_2 \equiv B_1 \equiv B_2 \equiv 0$, conditions (32) coincide with those in the class of the second-order ODEs [1, p. 17].

4 Conclusion

We solved the inverse problem of reconstruction second-order Ito stochastic differential equations by the given properties of motion when control is included in the drift coefficient. By using the separation method,

we determined the form of control parameters that provides sufficient conditions for the existence of a given integral manifold for the nonlinear, linear, and scalar cases.

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