



## Research article

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# Topologically protected broadband rerouting of propagating waves around complex objects

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**Abstract:** Achieving robust propagation and guiding of electromagnetic waves through complex and disordered structures is a major goal of modern photonics research, for both classical and quantum applications. Although the realization of backscattering-free and disorder-immune guided waves has recently become possible through various photonic schemes inspired by topological insulators in condensed matter physics, the interaction between such topologically protected guided waves and free-space propagating waves remains mostly unexplored, especially in the context of scattering systems. Here, we theoretically demonstrate that free-space propagating plane waves can be efficiently coupled into topological one-way surface waves, which can seamlessly flow around sharp corners and electrically large barriers and release their energy back into free space in the form of leaky-wave radiation. We exploit this physical mechanism to realize topologically protected wave-rerouting around an electrically large impenetrable object of complex shape, with transmission efficiency exceeding 90%, over a relatively broad bandwidth. The proposed topological wave-rerouting scheme is based on a stratified structure composed of a topologically nontrivial magnetized plasmonic material coated by a suitable isotropic layer. Our results may open a new avenue in the field of topological photonics and electromagnetics, for applications that require engineered interactions between guided waves and free-space propagating waves, including for complex beam-routing systems and advanced stealth technology. More generally, our work may pave the way for robust defect/damage-immune scattering and radiating systems.

**Keywords:** photonic topological materials; nonreciprocity; magnetized plasmas; surface plasmon-polaritons; leaky waves.

## 1 Introduction

The application of relevant ideas of topological physics from solid-state electronic systems to the realm of photonics and electromagnetics is opening a new landscape of opportunities to realize optical structures that are robust under defects, discontinuities, and deformations [1–3]. A feature of particular relevance in these systems is the emergence of robust one-way surface states, which have been shown to exhibit unprecedented optical properties, such as disorder-immune propagation without back-reflection even in the presence of large scatterers and sharp bends [4–9]. However, with few exceptions [10, 11], thus far, topological surface waves have been typically studied in “closed” systems, in which the surface states lie within the bulk-mode bandgaps of the surrounding media and cannot couple to free-space radiation. In such closed systems, topological surface waves cannot be excited directly by incident propagating waves without additional coupling schemes; instead, they are typically accessed through near-field excitation mechanisms [5, 6, 12] or external waveguides [13, 14]. In many scenarios, however, it would be highly desirable to bridge the gap between free-space propagating waves (the continuous plane-wave spectrum supported by the surrounding environment) and topological surface states confined and guided on a given structure. This possibility may extend the realm of applications of topological wave physics to radiating and scattering devices and systems.

Indeed, the coupling between free-space radiation and guided waves propagating on the surface of an object is one of the most fundamental electromagnetic and photonic processes. Several open wave-guiding structures, such as leaky-wave antennas and optical couplers, have been designed in the past several decades to control and exploit this coupling in a variety of settings [15]. Interestingly, however, this process is quite fragile when it occurs

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on the surface of objects made of conventional materials since guided surface waves do not typically have a preferential direction in time and/or space, namely, wave propagation is time-reversal (TR) symmetric and spatially bidirectional (the modal dispersion relation is symmetrical,  $\omega(k)=\omega(-k)$ , where  $\omega$  is the angular frequency and  $k$  is the wavevector). As a direct consequence, incident electromagnetic waves coupled into a guided mode propagating along the surface of an object will typically scatter backward and reradiate if the object's surface has a complex nonsmooth shape with defects and discontinuities, hence wasting electromagnetic energy in unwanted directions and, perhaps, giving away relevant information to an unwanted receiver.

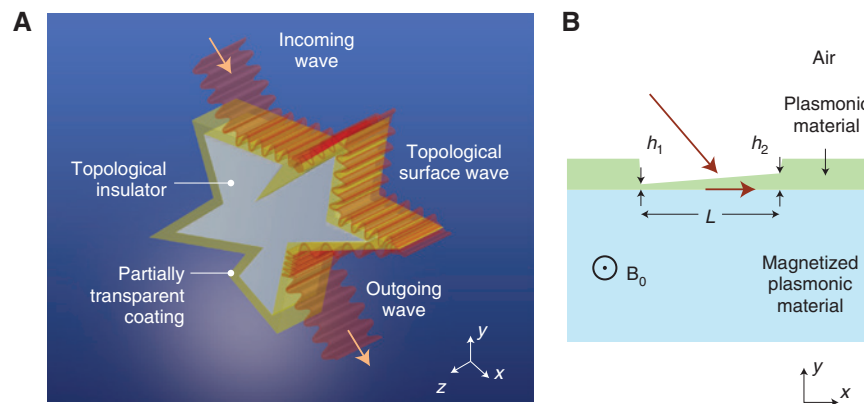
In this paper, we propose a general strategy to couple an incident propagating plane wave to a surface wave with topologically nontrivial properties and with controllable radiation loss. We show that such a surface wave can flow around sharp edges and large scatterers without inducing any back-scattering. Furthermore, we also show that the topological surface wave can be coupled back into free space, in the form of leaky-wave radiation, thus providing a platform for topologically protected wave-rerouting around an arbitrarily shaped impenetrable object, as sketched in Figure 1A.

We would like to stress that the proposed idea of wave-rerouting via topological surface waves is completely different compared to any scheme based on metamaterial invisibility devices, which are fundamentally limited to simple small objects and narrow bandwidths, as recently

demonstrated in different papers [16–18]. Our goal here is not to minimize the total scattering cross-section of a given object but rather to reroute electromagnetic energy around a complex shape without back-reflections even if the shape is modified or damaged. The proposed strategy is also drastically different from any schemes based on artificial impedance surfaces, holographic surfaces, and metasurfaces supporting surface waves, which have been used to guide waves around certain smooth objects (e.g. Ref. [19]) but are typically inefficient and inadequate in the presence of large surface defects or sharp corners and bends, yielding strong back-scattering due to the inherent modal bidirectionality.

## 2 Design and results

Our idea of topologically protected wave-rerouting and reradiation around a complex impenetrable object is illustrated in Figure 1. The design includes an impenetrable arbitrarily shaped object composed of a material acting as a photonic topological insulator (for example, a magnetically biased plasmonic material) separated by the surrounding free space by an isotropic partially transparent thin cover (composed of, for example, an isotropic nonmagnetized plasmonic material). It is assumed that the structure is translationally invariant along the  $z$ -axis, which is taken as the bias direction. As discussed in the



**Figure 1:** Topologically protected wave-rerouting around a large complex object.

(A) Concept figure illustrating the idea of topologically protected wave-rerouting and reradiation around a complex impenetrable object.

(B) Structural view, on the  $xy$ -plane, for a possible implementation of the idea in A, including details of the in-coupling/out-coupling region. A magnetically biased plasma, which behaves as a homogeneous topological photonic insulator, is coated with a topologically trivial isotropic plasmonic layer. Topologically protected unidirectional leaky surface waves exist on the surface of the magnetized plasma, and they can propagate around the object without any back-scattering independently of the shape of the surface in the  $xy$ -plane. To efficiently couple incident propagating waves into the leaky surface waves (and vice versa), we defined coupling regions with optimized coating thickness to minimize reflections and reradiation. To this aim, the thickness of the in-coupling and out-coupling regions is linearly increased and decreased, respectively, with  $h_1=0$  and  $h_2=0.46\lambda_p$ , over a coupling length equal to  $L=11.51\lambda_p$ , where  $\lambda_p$  is the free-space wavelength at  $\omega_p$ . The thickness of the plasmonic coating outside the coupling regions has been set to  $1.44\lambda_p$ .

following, the surface waves supported by this structure are unidirectional and topologically protected, and their coupling to radiation can be controlled by suitably engineering the isotropic cover.

The topological classification of nonreciprocal materials is based on a topological invariant number known as Chern number, corresponding to an integral of the Berry curvature of the photonic bands over momentum space, which makes this class of topological materials the photonic equivalent of quantum Hall insulators [1–3]. Although topological properties have been studied extensively for periodic structures with discrete translational symmetry [1–7, 11–14, 20], it has been recently shown that Chern numbers can also be defined for continuous media, such as a homogenous magnetized plasma [21], unveiling the topological nature of certain classes of unidirectional plasmonic surface states [22].

For magnetized plasmas, the permittivity tensor and its frequency-dispersive elements are given in Eqs. (1) and (2), respectively:

$$\varepsilon = \begin{pmatrix} \varepsilon_t & i\varepsilon_g & 0 \\ -i\varepsilon_g & \varepsilon_t & 0 \\ 0 & 0 & \varepsilon_a \end{pmatrix}, \quad (1)$$

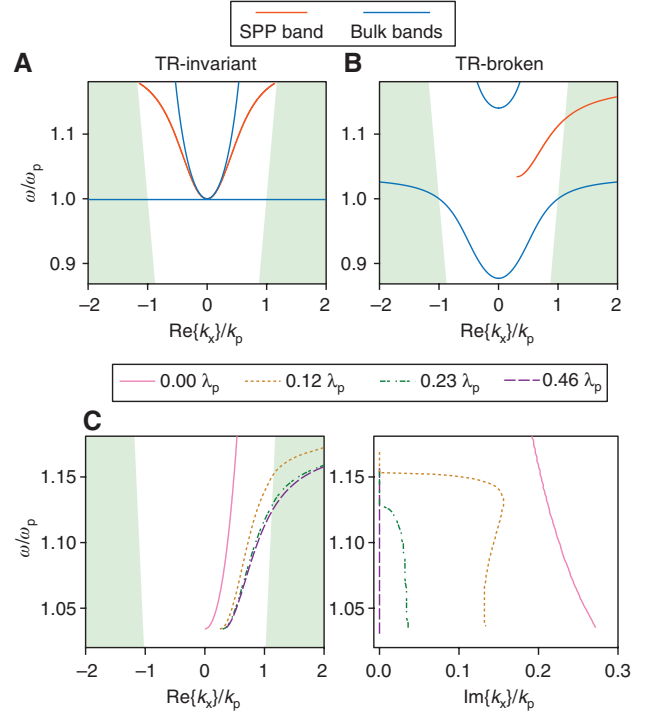
$$\varepsilon_t = 1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2}, \quad \varepsilon_g = \frac{\omega_p^2 \omega_c}{\omega(\omega_c^2 - \omega^2)}, \quad \varepsilon_a = 1 - \frac{\omega_p^2}{\omega^2}, \quad (2)$$

where  $\omega_p$  and  $\omega_c$  are the plasma and cyclotron frequencies, respectively.

The dispersion equation for the transverse-magnetic (TM) bulk modes in the plane orthogonal to the bias (magnetic field of the bulk mode parallel to the bias) can be written as

$$k_x^2 + k_y^2 = \frac{\varepsilon_t^2 - \varepsilon_g^2}{\varepsilon_t} k_0^2, \quad (3)$$

which gives the TM bulk-mode band structure plotted in Figure 2 (blue curves). By following the approach presented in Refs. [23, 24], the TM bulk modes can be shown to have nonzero Chern number, which originates from lifting the degeneracy at  $k=0$  between the bulk modes in the unbiased case (Figure 2A) by breaking TR symmetry. In particular, the bulk-mode gap Chern number for the magnetized plasma (Figure 2B) is found to be  $C_{\text{gap}} = -1$  (the sum of the Chern numbers of the bands below the gap). We restrict our attention to the TM modes since, for the transverse-electric (TE) mode, the electric field is along the bias; hence, the motion of the electrons is unaffected by the external magnetic field. In this case,



**Figure 2:** Dispersion diagrams of bulk and surface modes. (A and B) Analytically calculated band structures for the TM bulk modes of a biased/unbiased plasma with plasma frequency  $\omega_p$  (blue curves) and the TM surface modes (SPPs) on the interface between the same biased/unbiased plasma and an isotropic plasmonic cover with plasma frequency  $1.40 \omega_p$  and coating thickness of  $1.44 \lambda_p$  (solid red curves). (A) Unbiased isotropic plasma (TR-invariant, topologically trivial) and (B) magnetically biased gyrotropic plasma (TR-broken, topologically nontrivial) with cyclotron frequency  $\omega_c = 0.26 \omega_p$ . The dispersion diagram in B is calculated for the plane orthogonal to the bias. (C) Complex dispersion curves of the leaky surface modes for various plasmonic coating thicknesses used in the coupling regions (see Figure 1B). The green shaded areas denote the regions outside the light cone, which are not accessible with free-space propagating-wave excitation. The imaginary part of the wavenumber in B has been omitted as it was found to be negligibly small.

the mode becomes trivial as its properties are bias independent.

The different topological properties of the magnetized plasma and the isotropic cover produce one-way surface waves on their interface. As usually done for simple transversely invariant layered structures, the dispersion curves for the surface plasmon-polariton (SPP) modes (red curves in Figure 2) can be calculated analytically by considering the general form of the fields in each layer and imposing the relevant electromagnetic boundary conditions at the interfaces. The resulting homogenous system of equations yields the eigenmodes of the structure as well as their dispersion relation. Figure 2A shows that, when a semi-infinite nonmagnetized plasmonic material with plasma

frequency  $\omega_p$  is interfaced with an isotropic plasmonic cover (following a Drude-like permittivity with plasma frequency equal to  $1.40 \omega_p$ ), the resulting SPP dispersion curve is perfectly symmetrical,  $\omega(k) = \omega(-k)$ , as expected. Instead, if TR symmetry is broken through an out-of-plane static magnetic field applied to the semi-infinite plasma region, a unidirectional SPP mode emerges within the plasma band-gap (see Figure 2B). The surface mode dispersion is not just asymmetrical,  $\omega(k) \neq \omega(-k)$ ; in this case, the backward mode actually does not exist. As the number of one-way edge states is determined by the difference of the gap Chern numbers between interfaced materials (bulk-edge correspondence [3, 21]), such surface waves are also topologically protected, implying unidirectional propagation without any back-scattering even in the presence of large disorder.

Two important observations should be made at this point. (1) A one-way topological surface mode can exist, without any bulk-mode propagation, over a moderately large continuous bandwidth, corresponding to the width of the topological bandgap, which can be controlled, in our case, via the magnitude of the static bias field. As discussed in the following, this enables a broadband wave-rerouting effect (clearly, the bandwidth is also determined by the frequency-dispersive properties of the isotropic cover, which should be opaque, with negative permittivity, throughout the bulk-mode bandgap). (2) As seen in Figure 2B, the dispersion of the one-way surface mode lies within the light cone (white area) of the surrounding space (continuum of radiation modes, i.e. plane waves, in free space). Hence, if the isotropic cover is not too opaque or too thick, the topological surface mode will couple to radiation modes, becoming a “leaky” wave with complex modal wavenumber  $k$  (its imaginary part representing radiation loss) [15]. Although the topological properties have been defined for the closed system with no radiation, namely, with a sufficiently thick cover, if radiation loss is small and can be treated as a perturbation, we can still safely treat any weakly-leaky mode as a topologically protected state.

The thickness of the isotropic cover can be used to control the coupling of the topological surface mode to outgoing plane waves and vice versa. Indeed, as seen in Figure 2C, the cover thickness directly controls the imaginary part of the modal wavenumber; in addition, the real part of  $k$  is only marginally affected if the cover is sufficiently thick. This fact allows tailoring the radiation and propagation properties of the topological mode independently, as initially predicted in Ref. [10]. Here, we use this behavior to design a structure able to fully “capture” an incident propagating wave, converting it

into a topological surface wave, which then propagates seamlessly around an arbitrarily complex shape. The surface wave can then be made to release its energy, in the form of leaky-wave radiation, at the opposite side. Due to transverse-momentum conservation for the fields on a translationally invariant surface, however, if an impinging plane wave can couple to a leaky surface mode, the excited surface mode will also couple to radiation in the same spatial region, with the same efficiency, losing a significant portion of its energy right after the excitation [15]. The translational invariance of the structure should be broken to avoid this behavior. Instead, we note that breaking reciprocity does not break the equality between the in-coupling and out-coupling efficiencies for a guided leaky mode on a transversely invariant structure, which is just a consequence of transverse-momentum conservation. We therefore define a coupling region, as shown in Figure 1B, where the thickness  $h$  of the isotropic cover is suitably varied along the  $x$ -axis to maximally couple the impinging energy into a surface mode that does not radiate back to free space.

The complex dispersion properties of the surface mode for various thicknesses,  $k_x(\omega, h)$ , are reported in Figure 2C, confirming that an increase in the thickness of the opaque coating decreases the leakage rate of the surface wave monotonically. Considering the out-coupling region, at a given frequency, the amplitude of the wave emerging from the coating surface can be approximated as

$$I(x) \propto e^{-\int_0^x \text{Im}[k_x(h(x'))] dx'} e^{-\text{Im}[k_y(h(x))] h}, \quad (4)$$

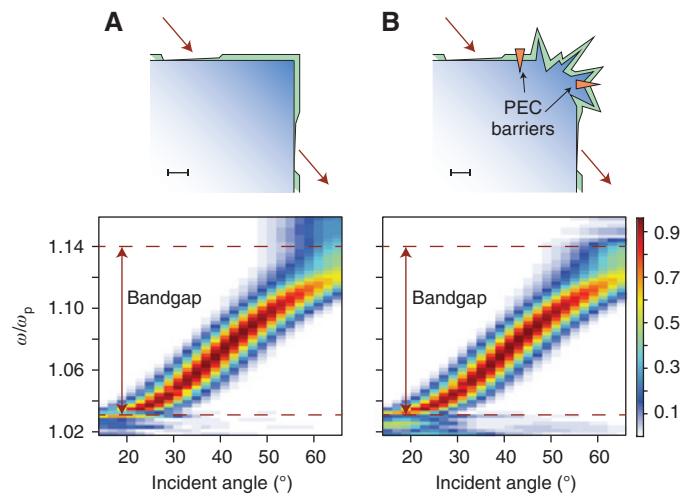
where  $h(x)$  is the coating thickness profile,  $k$  is the total wavenumber in the coating material, and  $k_y(h) = \sqrt{k^2 - k_x^2(h)}$ . Physically, the first exponential represents the decay, along the  $x$ -axis, of the surface wave due to radiation loss, whereas the second exponential represents the decay, along the  $y$ -axis, of evanescent waves within the opaque coating before they reach the surface. The first exponential function always decays as a function of  $x$ , whereas the second exponential decays or grows as a function of  $x$  depending on whether  $h(x)$  increases or decreases, respectively. Heuristically, therefore, if we consider a monotonically decreasing ramp for the out-coupling region, the radiated wave emerging from the structure has maximum intensity at the center of the ramp, tailing off at the sides as the leakage is very small at the near end (where the coating is thick) and the surface wave is almost completely radiation-damped at the far end. Thus, the leaky-wave radiation profile would resemble a Gaussian plane-wave beam. Furthermore, an incident beam of the same type is expected to efficiently



couple into the surface mode if the in-coupling region is designed in the same way (due to TR symmetry, with flipped bias). For simplicity, we have considered linearly increasing and decreasing ramps, as sketched in Figure 1B, for the in-coupling and out-coupling regions, respectively, and we have optimized the ramp properties (angle and length) for an incident and transmitted Gaussian plane-wave beam. The optimized values are indicated in the caption of Figure 1. Thanks to the unidirectional nature of the leaky surface mode in our structure, no back-scattering is expected along these inhomogeneous coupling regions. In addition, we observe in Figure 2C that, when the thickness of the isotropic plasmonic coating is larger than approximately  $0.5 \lambda_p$ , the imaginary part of the longitudinal wavenumber of the SPP mode becomes negligible. Hence, the SPP mode becomes fully confined to the interface, without any coupling to radiation modes.

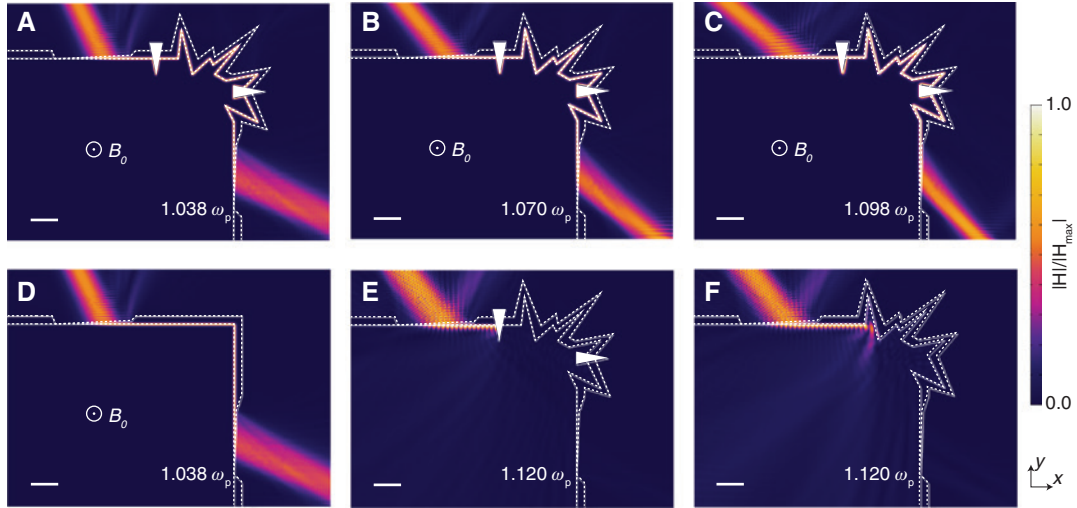
To verify our predictions, we have numerically modeled and simulated the proposed structure using a commercial software (COMSOL Multiphysics) [25] based on the finite-element method. As shown in Figures 3 and 4, we consider an incident wave impinging on the surface of an impenetrable magnetized-plasma corner at a certain angle and at a frequency within the plasma bulk-mode bandgap. The object is covered by an isotropic plasmonic layer with in-coupling and out-coupling regions as in Figure 1B. To demonstrate topologically-enabled wave-rerouting around this electrically-large object, we calculated the transmission of the propagating wave on the opposite side of the corner. To quantitatively assess

the impact of disorder, we carried out two comparative calculations: the transmission spectra, as a function of angle and frequency, for a surface having a simple right corner (Figure 3A) and for a highly disordered surface (Figure 3B) that includes sharp features and perfectly conducting (PEC) bullet-like barriers, which may model defects and damages to the structure. Rather remarkably, these results show that an incident wave can indeed be rerouted around the complex impenetrable object with efficiencies above 90%, within a relatively broad bandwidth, due to the possibility of efficiently exciting topological surface waves over the entire width of the bulk-mode bandgap. Even more interestingly, the comparison of the results in Figure 3A and B indicates that the transmission is only marginally affected by the large imperfections and deformations: the two transmission spectra are basically identical at all frequencies within the bandgap, confirming the topological robustness of the proposed wave-rerouting effect. As expected, for every operational frequency, maximal transmission is obtained for a different incident angle, determined by the momentum-matching condition with the surface mode. Indeed, the locus of the maximum transmission points in Figure 3 closely follows the topological surface mode dispersion in Figure 2B. Nevertheless, at each frequency, relatively high transmission is achieved within a range of about  $10^\circ$  around the maximum. Additional features in the transmission spectra at frequencies outside the bandgap are associated with the excitation of the plasma bulk modes.



**Figure 3:** Transmission efficiency of the topological wave-rerouting process.

Angle-resolved transmitted power spectra for (A) a structure as in Figures 1 and 2 with a simple right corner and (B) a similar structure but with sharp features, discontinuities, and PEC obstacles. The comparison of the results in A and B indicates that the transmission is almost completely unaffected by large imperfections and disorder at all frequencies within the bulk-mode bandgap of the topological material. The red dashed lines indicate the bandgap boundaries for the considered magnetized plasma. Scale bars in the top illustrations correspond to  $5 \lambda_p$ .



**Figure 4:** Steady-state amplitude distributions of the out-of-plane magnetic field for different cases of wave-rerouting around a complex electrically large object.

(A–D) The inner material is a topological photonic insulator (a magnetically biased plasma with the same properties as in Figure 2), which supports unidirectional topologically protected surface waves that can propagate around sharp corners and PEC obstacles (white areas). (E and F) The inner material is topologically trivial and reciprocal (the static bias has been switched off) to show the field response for the case of non-topological surface waves. The operational angular frequencies for each case are as follows: (A and D)  $1.038 \omega_p$ , (B)  $1.070 \omega_p$ , (C)  $1.098 \omega_p$ , and (E and F)  $1.120 \omega_p$ . The white dashed lines denote the boundaries of the different layers and the scale bars correspond to  $5 \lambda_p$ . Animated movie files for B and F are included as Supplementary Materials and provide further details on the field propagation characteristics.

To get detailed insight into the topologically protected propagation characteristics, we provide numerically calculated steady-state magnetic field distributions for various cases of interest in Figure 4. Figure 4A–D reports the field distributions for the topological (magnetically biased) structure, illuminated by a plane-wave Gaussian beam, for different frequencies and surface shapes. The broadband operation of the topological wave rerouting scheme can be appreciated in Figure 4A–C: high transmission is obtained at all frequencies within the bulk-mode bandgap for angles at which the dominant transverse wavenumber of the incident plane-wave beam matches the momentum of the topological surface mode. Furthermore, a comparison of Figure 4A and D clearly confirms that the presence of disorder, in the form of discontinuities, sharp features, and even PEC scatterers, does not induce any back-reflections or propagation losses. We also note that reflection, diffraction, and reradiation is minimized in the in-coupling region thanks to the suitable design of the isotropic cover thickness, as indicated in Figure 1A and as discussed above. Finally, to further confirm the crucial importance of topological robustness, Figure 4E and F shows the field distributions for a topologically trivial case, in which the static magnetic bias has been switched off, converting the magnetized plasma into a reciprocal isotropic plasmonic material. As seen in the dispersion diagram in Figure 2A,

the interface between the nonmagnetized plasma and the isotropic cover still supports surface waves at frequencies above the plasma frequency  $\omega_p$ , but such surface waves are bidirectional and topologically trivial. It is evident in Figure 4E and F that the non-topological surface waves are unable to overcome the complex-shaped surface and the PEC barriers. Instead, the surface waves are reflected and radiate back into free space (notice the standing wave formed along the direction of the incident field) or scatter into the bulk of the nonmagnetized plasmonic material. Negligible electromagnetic energy is transmitted to the other side of the corner.

### 3 Conclusion

Translating topological effects to the realm of photonics and electromagnetics has already led to radical innovation in the way we design structures and devices with increased robustness against imperfections and disorder. Within this context, in this manuscript, we have shown how topological properties can be exploited to reroute an incident propagating wave around an electrically large impenetrable object of complex shape with sharp corners and highly scattering features, with transmission efficiency exceeding 90%. Moreover, such

topological wave-rerouting effect has been shown to be relatively broadband, depending on the size of the bulk-mode bandgap, and almost completely independent of the shape of the object. We have discussed how this exciting wave-rerouting behavior is enabled by the ability to control the propagation and radiation properties of one-way, topologically protected, leaky surface waves, supported by the surface of a topological photonic insulator, realized here in the form of a magnetically biased plasma. Although natural magneto-optical effects are typically weak at optical frequencies, different material platforms may be employed for the experimental verification of our proposed designs in different regions of the electromagnetic spectrum, including magnetized  $n$ -doped semiconductors (such as  $n$ -type InSb) operating at terahertz frequencies [26, 27], magnetized ferrites at gigahertz frequencies [28], and also structures with suitable spatiotemporal modulations that mimic the effect of the magnetic bias [29, 30]. The isotropic plasmonic cover may be coated onto the gyrotropic plasmonic material, with controllable thickness, by employing standard micro-coating or nanocoating techniques, such as molecular beam epitaxy (e.g. Ref. [31]). We would also like to note that, as the SPP modes supported by a magnetized plasmonic material are not accessible through a TE-polarized incident wave, we have considered only the case of TM polarization. However, the proposed concept of topological wave-rerouting is general, and it may be extended to different topological platforms (e.g. Ref. [3]) that also support TE-polarized surface waves.

Our findings may have important practical implications, especially in areas such as stealth technology, robust signal rerouting, and full-duplex communications. The proposed topological platforms would be inherently robust to fabrication imperfections and to the unavoidable damages that they would suffer in realistic operating conditions.

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## References

[1] Haldane FDM, Raghu S. Possible realization of directional optical waveguides in photonic crystals with broken time-reversal symmetry. *Phys Rev Lett* 2008;100:013904.

- [2] Raghu S, Haldane FDM. Analogs of quantum-Hall-effect edge states in photonic crystals. *Phys Rev A At Mol Opt Phys* 2008;78:033834.
- [3] Lu L, Joannopoulos JD, Soljačić M. Topological photonics. *Nat Photonics* 2014;8:821.
- [4] Wang Z, Chong YD, Joannopoulos JD, Soljačić M. Reflection-free one-way edge modes in a gyromagnetic photonic crystal. *Phys Rev Lett* 2008;100:013905.
- [5] Wang Z, Chong Y, Joannopoulos JD, Soljačić M. Observation of unidirectional backscattering-immune topological electromagnetic states. *Nature* 2009;461:772.
- [6] Rechtsman MC, Zeuner JM, Plotnik Y, et al. Photonic Floquet topological insulators. *Nature* 2013;496:196.
- [7] Hafezi M, Demler EA, Lukin MD, Taylor JM. Robust optical delay lines with topological protection. *Nat Phys* 2011;7:907.
- [8] Gangaraj SAH, Monticone F. Topological waveguiding near an exceptional point: defect-immune, slow-light, and loss-immune propagation. *Phys Rev Lett* 2018;121:093901.
- [9] Gangaraj SAH, Monticone F. Coupled topological surface modes in gyrotropic structures: Green's function analysis. *IEEE Antennas Wireless Propag Lett* 2018;17:1993–7.
- [10] Gangaraj SAH, Monticone F. Topologically-protected one-way leaky waves in nonreciprocal plasmonic structures. *J Phys Condens Matter* 2018;30:104002.
- [11] Gorlach MA, Ni X, Smirnova DA, et al. Far-field probing of leaky topological states in all-dielectric metasurfaces. *Nat Commun* 2018;9:909.
- [12] Skirlo SA, Lu L, Igarashi Y, Yan Q, Joannopoulos J, Soljačić M. Experimental observation of large Chern numbers in photonic crystals. *Phys Rev Lett* 2015;115:253901.
- [13] Bahari B, Ndao A, Vallini F, El Amili A, Fainman Y, Kanté B. Non-reciprocal lasing in topological cavities of arbitrary geometries. *Science* 2017;358:636–40.
- [14] Harari G, Bandres MA, Lumer Y, et al. Topological insulator laser: theory. *Science* 2018;359:eaar4003.
- [15] Monticone F, Alù A. Leaky-wave theory, techniques, and applications: from microwaves to visible frequencies. *Proc IEEE* 2015;103:793–821.
- [16] Hashemi H, Qiu C-W, McCauley AP, Joannopoulos JD, Johnson SG. Diameter-bandwidth product limitation of isolated-object cloaking. *Phys Rev A* 2012;86:13804.
- [17] Monticone F, Alù A. Invisibility exposed: physical bounds on passive cloaking. *Optica* 2016;3:718–24.
- [18] Monticone F, Alù A. Do cloaked objects really scatter less? *Phys Rev X* 2014;3:041005.
- [19] Achouri K, Caloz C. Space-wave routing via surface waves using a metasurface system. *Sci Rep* 2018;8:7549.
- [20] Fukui T, Hatsugai Y, Suzuki H. Chern numbers in discretized Brillouin zone: efficient method of computing (spin) Hall conductances. *J Phys Soc Jpn* 2005;74:1674–7.
- [21] Silveirinha MG. Chern invariants for continuous media. *Phys Rev B Condens Matter Mater Phys* 2015;92:125153.
- [22] Seshadri SR. Excitation of surface waves on a perfectly conducting screen covered with anisotropic plasma. *IRE Trans Microw Theory Tech* 2016;10:573–8.
- [23] Gangaraj SAH, Nemilentsau A, Hanson GW. The effects of three-dimensional defects on one-way surface plasmon propagation for photonic topological insulators comprised of continuum media. *Sci Rep* 2016;6:30055.

- [24] Gangaraj SAH, Silveirinha MG, Hanson GW. Berry phase, Berry connection, and Chern number for a continuum bianisotropic material from a classical electromagnetics perspective. *IEEE J Multiscale Multiphys Comput Techn* 2017;2:3–17.
- [25] COMSOL Multiphysics version 5.4, 2018. COMSOL AB, Stockholm. Available at: <http://comsol.com>.
- [26] Palik ED, Kaplan R, Gammon RW, Kaplan H, Wallis RF, Quinn JJ. Coupled surface magnetoplasmon-optic-phonon polariton modes on InSb. *Phys Rev B* 1976;13:2497.
- [27] Moncada-Villa E, Fernández-Hurtado V, García-Vidal FJ, García-Martín A, Cuevas JC. Magnetic field control of near-field radiative heat transfer and the realization of highly tunable hyperbolic thermal emitters. *Phys Rev B Condens Matter Mater Phys* 2015;92:125418.
- [28] Pozar DM. *Microwave engineering*. 4th ed. Hoboken, New Jersey, John Wiley & Sons, 2012.
- [29] Fang K, Yu Z, Fan S. Realizing effective magnetic field for photons by controlling the phase of dynamic modulation. *Nat Photonics* 2012;6:782.
- [30] Khanikaev AB, Fleury R, Mousavi SH, Alù A. Topologically robust sound propagation in an angular-momentum-biased graphene-like resonator lattice. *Nat Commun* 2015;6:8260.
- [31] Pudell J, Maznev AA, Herzog M, et al. Layer specific observation of slow thermal equilibration in ultrathin metallic nanostructures by femtosecond X-ray diffraction. *Nat Commun* 2018;9:3335.

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**Supplementary Material:** The online version of this article offers supplementary material (<https://doi.org/10.1515/nanoph-2019-0075>).