Research article

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Engineering multimodal spectrum of Cayley tree fractal meta-resonator supercells for ultrabroadband terahertz light absorption

<https://doi.org/10.1515/nanoph-2019-0391>

Received September 26, 2019; revised December 9, 2019; accepted December 13, 2019

Abstract: Self-similar fractals provide a degree of freedom for varying the resonance frequency due to the multiscale geometric features involved and are an ideal candidate for ultrabroadband absorbing devices – especially in the terahertz (THz) band where there is a lack of natural absorbing materials. Metasurface-based THz absorbers often suffer from poor broadband performance, whereas strongly absorbing broadband devices are typically complex multilayer structures. Here, we numerically demonstrate an ultrabroadband, ultrathin, polarization-insensitive, wide-angle, single-layer planar metasurface THz absorber by integrating different Cayley tree fractal resonators into one supercell based on the frequency shifting and multi-resonance bands of different fractal orders. In terms of physics, we have exploited the self-similar nature of fractal geometry to engineer the multimodal spectrum of this system. With increasing fractal order N , an increasing number of modes can be excited with certain degeneracies where each mode corresponds to plasmon oscillations at different geometric scales inside fractal. As a result, broad, multi-peaked spectra with large degeneracy numbers can be achieved with larger N . Finally, by placing fractals of different order N into one supercell,

the coupling and superposition of the neighboring resonances exhibit the desired ultrabroadband response. The proposed absorber provides a wide incident wave angle with a full-width half-maximum absorption bandwidth of more than one octave, i.e. 3.88 THz. Greater than 80% absorption is achieved over a frequency range of 3 THz. Owing to its performance, this work is a step forward in realizing perfect blackbody absorbers that can be easily integrated with bolometric sensing technology to make high-efficient THz-sensing devices.

Keywords: metasurface; absorber; Cayley tree fractals; degeneracies.

1 Introduction

There is a lack of naturally occurring materials with valuable electromagnetic (EM) properties in the terahertz (THz) region of the EM spectrum lying between millimeter waves and the infrared band, known as “terahertz gap,” ranging from 100 GHz to 10 THz [1, 2]. Technological advancements in the THz domain have given rise to many potential applications such as medical and security imaging [3, 4], explosive detection [5], biological sensing [6], wireless communication [7], and spectroscopy [8]. Although a lot of research has been centered on THz materials and novel techniques for the construction of devices, the demand of broadband and highly efficient devices is still unsatisfied owing to the limitations of natural materials.

Metamaterials (MMs) came into limelight in the past decade and have ever since become a perfect candidate for THz high-performance devices owing to their exotic behavior in response to EM radiation that could not have been observed via conventional materials [9, 10]. These artificially engineered materials have overcome the problem of poor interaction of THz radiation with naturally occurring materials. Research conducted on two-dimensional (2D) planar MMs, called meta-surfaces, has illustrated many intriguing phenomena and useful applications of

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MMs, in nearly every spectral range from microwave to optical, such as perfect lensing [11], invisibility cloaking [12], negative refractive index [13], and many more [14–18]. Most of these applications have centered around the real part of effective medium parameters; however, there are many applications where material loss characterized by imaginary part of effective parameters is also important. One example of such application is resonant absorber. Landy et al. [19] proposed the first MM absorber in 2008 based on split ring resonators. Since then, various MM absorbers have been designed in every frequency regime from microwave to optical [19–21]. These MM absorbers are of significant importance at THz frequencies where it is hard to discover conventional materials with high absorption coefficients. Although single-band [19], dual-band [22–24], triple-band [25], and multiband [26–31] perfect absorbers with narrow operation bandwidth have been proposed, there is a dire need for structures with unity absorption over larger bandwidth for applications such as high-efficiency detectors [32], sensing [25, 33–36], THz imaging [37, 38] and stealth technology [39]. To overcome this problem, a sophisticated design approach has been adapted by stacking multiple layers of metal resonators and dielectrics [30, 40–42], but such designs are complex, costly, and difficult to fabricate. Instead of multiple layers, another way is to make a 2D planar arrangement by combining resonant structures of various sizes with different resonant frequencies into one unit cell [43–51]. Such an arrangement is much better because of their simplistic ultrathin design, but there are still imperfections in it due to the trade-off between maximum bandwidth and peak absorption. An alternative approach is to increase effective parameter of absorbing layer by introducing self-similar fractal resonant structures. The advantage of employing fractals is that it gives multiple resonances from the same element structure due to the multiscale nature of its geometry [48, 52–54]. The concept of fractals and fractional dimensions in EMs has been a focal point of a few contemporary studies. In the recent literature, fractal geometry-based dual-band [55–57], triple-band [58, 59], and wideband [60] metasurface absorbers, not limited to THz band, have been studied for their multiresonance and bandwidth enhancement advantage. In THz regime, recently a broadband absorption of greater than 80% from 2.82 THz to 5.15 THz is achieved by combining different fractals levels into one supercell, having relative absorption bandwidth of 65% [48]. The potential of various other fractal resonators in the design of single-layer MM absorber still needs to be investigated. In this article, we propose an ultrathin, single-layer, broadband, polarization-insensitive, wide-angle THz

absorber based on Cayley tree fractal resonators on top of the dielectric layer with metallic ground plate. We are able to achieve ultrabroadband absorption by combining unit cell of different fractal orders into supercell. The full-width half-maximum (FWHM) bandwidth of device is 3.88 THz (ranging from 2.24 to 6.13 THz), which is more than one octave along with 80% above absorption bandwidth of 3 THz (from 2.4 to 5.4 THz). The performance of our proposed design has been compared with the state of the art. Our proposed design methodology for ultrathin, ultrabroadband THz absorber can be used in important applications such as a metasurface-based microbolometer sensor for THz imaging [32, 37, 38], where broadband MM absorbers are needed to be coupled with microbolometer sensors to have a complete THz detector.

2 Design and simulation

Our proposed device is a metal–insulator–metal (MIM) absorber with a Cayley tree metal resonator as a top layer, a dielectric interface, and a bottom layer of metallic ground plane for perfect reflection as shown in Figure 1. The Cayley tree resonator [52] is a Y-shaped structure formed by a joint connection of three microstrip-line branches. The length and width of each branch are constant, i.e. $L_1 = 12\ \mu\text{m}$ and $w = 1\ \mu\text{m}$, respectively. Each branch is spaced apart with 120° rotation angle maintaining the threefold rotational symmetry ensuring polarization insensitivity. The resonator structure is symmetric along y - z plane. To introduce fractals into our design, with every terminal branch of previous fractal order two separate arms are included. The angle between each arm and the arm widths are the same as that of level 1. The arm lengths in subsequent levels are $L_2 = 6\ \mu\text{m}$, Figure 1B, and $L_3 = 5\ \mu\text{m}$, Figure 1C. However, in level 3, the outermost arm lengths were shortened to $L_{3/2}$ to prevent overlapping between neighboring structures. All these fractal order arm lengths are optimized in such a way that when combined they give a broadband response. The period of unit cell is $P_1 = 34\ \mu\text{m}$. The absorption is also dependent on thickness of the substrate “ h ,” which controls the narrowband and broadband absorption response. Polyimide is used as a dielectric layer with refractive index of $n = 1.68 + 0.06i$ [48]. The metal used for resonator and ground plane is gold (Au) with frequency-independent conductivity of $\sigma = 4.07 \times 10^7\ \text{S/m}$. The height of the top and bottom layers is $t_1 = t_2 = 100\ \text{nm}$, as shown in Figure 1E.

The numerical simulations are performed at normal incidence using the commercially available full-wave

Figure 1: Fractal broadband absorbers.

(A) Fractal level 1, (B) fractal level 2, (C) fractal level 3. (D) Supercell arrangement obtained by combining all three fractal orders. (E) Side view of supercell design. The arm length of level 1 is $L_1 = 12 \mu\text{m}$, level 2 is $L_2 = 6 \mu\text{m}$, and level 3 is $L_3 = 5 \mu\text{m}$, whereas the outer arm length of level 3 is $L_3/2$. The arm width $w = 1 \mu\text{m}$. The period of supercell is $P = 68 \mu\text{m}$.

solver CST Microwave Studio [61]. In simulations, unit-cell boundaries are applied in x and y directions, while open (add space) is set in z direction. The absorption $A(\omega)$ is calculated by $A(\omega) = 1 - R(\omega) - T(\omega)$, where $R(\omega) = |S_{11}|^2$ is reflectance and $T(\omega) = |S_{21}|^2$ is transmission. S_{11} and S_{21} are coefficients of reflection and transmission, respectively. As transmission is zero due to the presence of metallic round plane with greater thickness as compared to the skin depth of incident THz wave, the absorption can be obtained only by reflectance, $A(\omega) = 1 - R(\omega)$. To maximize the absorption, the impedance of MM device must match to that of free space [19], i.e. $Z = \sqrt{\mu/\epsilon} = 1$ where μ and ϵ represent complex permeability and permittivity, respectively.

3 Results and discussions

3.1 Narrowband absorption of different fractal levels

The optimized thickness of substrate “ h ” for the near-unity narrowband absorption is $6 \mu\text{m}$. Figure 2 shows the absorption spectra of the first, second, and third iteration of the proposed Cayley tree resonating structure. For each fractal level, a narrowband absorption can be observed,

with absorption exceeding above 95%. The absorption peak for first-order ($6\text{-}\mu\text{m}$ polyimide thickness) Cayley tree occurs at 3.83 THz with absorption efficiency of 99.7%, and this frequency has shifted to 3.32 THz along with another resonance frequency, which occurs at 6.6 THz with efficiencies of 99% and 98%, respectively, as we move to second order ($6\text{-}\mu\text{m}$ polyimide thickness). Significantly, this red shifting was achieved just by increasing the effective perimeter of the structure. For third order ($6\text{-}\mu\text{m}$ polyimide thickness), the resonance occurs at $2.7, 4.9$, and 7.6 THz having absorption efficiency of above 95% for each peak. So, it follows that, for N th-order fractal geometry, we can have N distinct resonance peaks or dipolar modes with the shift in resonance mode to a lower frequency than the previous level, which means we can have broadband response just by adding additional iterations to the fractal geometry.

3.2 Surface Currents and E-field explanation

To gain insight into the resonance phenomena, the E-field profiles at corresponding peaks for two orthogonal polarizations for each fractal order are shown in Figure 3. Figure 3 shows that the electric field is mostly concentrated at the terminal ends of the arms along the polarization axis in each level. The red areas correspond to the concentrations

Figure 2: Absorption spectrum.

(A) absorption of fractal order 1. Inset: first-order Cayley tree structure, (B) absorption of fractal order 2. Inset: second-order Cayley tree structure, (C) absorption of fractal order 3. Inset: third-order Cayley tree structure.

Figure 3: Electric field maps of first-, second-, and third-fractal-order Cayley tree structures at each resonance peak.

The structure is illuminated with linearly polarized plane waves propagating in the z direction with the electric field vector in the x - y plane.

(A1, B1) First-order Cayley tree for TE and TM polarization, respectively. (A2, A3) Second-order Cayley tree for TE polarization. (B2, B3)

Second-order Cayley tree for TM polarization. (C1–C3) Third-order Cayley tree at various extinction peaks for TE polarization.

of positive charges, while the areas with less magnitude (blue regions) indicate the negative charges. This accumulation of opposite charges at the edges excites a localized surface plasmon giving rise to the dipolar resonance response under the influence of incident electric field. Figure 3A1 and B1 show the E-fields of first-order Cayley

tree at 3.83 THz for transverse electric (TE) and transverse magnetic (TM) polarization, respectively. Figure 3A2 and B2 show the E-field of second-order Cayley tree at lower resonance mode (3.32 THz) for TE and TM polarization, respectively, whereas Figure 3A3 and B3 show the E-fields of second-order structure at higher-frequency resonance

mode (6.6 THz) for vertically and horizontally polarized excitations, respectively. For higher frequency, i.e. 6.6 THz, the field is mainly concentrated at the tips, whereas for lower frequency, i.e. 3.32 THz, the field spreads out from center to outermost tips. For third-order Cayley tree, only vertically polarized E-field maps are shown in Figure 3C1–C3. It follows the similar trend as that of level 2. At lower frequencies, the field is distributed from center to outermost tips. Meanwhile, going toward the higher-frequency resonance mode, the field is concentrated only at the tips and is diminished in the center.

We further investigate the surface current distribution in each fractal level at their lowest resonant frequencies only. As shown in Figure 4A1–C1 and Figure 4A2–C2, the surface currents on the top and bottom layers of unit cell of fractal Cayley tree structure are mostly antiparallel. These different antiparallel currents induce strong magnetic resonance response. It can be seen from Figures 3 and 4 that the electric resonance response is more dominant at these frequencies, as a result of which we are able to achieve perfect absorption of greater than 95% at each resonant frequency for first, second, and third iteration of Cayley tree.

3.3 Mechanism of multiple resonance peaks

From the qualitative analysis done by Gottheim et al. [52] for mode degeneracies and origin of distinct peaks

in Cayley tree fractal geometry, there are $3 \cdot 2^{N-1}$ tips in each N th-order Cayley tree fractal that corresponds to $N_{\text{tips}} - 1$ resonance modes. As seen in Figure 3, the charges induced upon plasmon excitation are largely concentrated at the tips of the structures, whereas the induced charge density in the center of each structure remains very small. In Figure 5, the distribution of the charges is shown for the first, second, and third fractal orders; these distributions show the types of mode degeneracy. In the first-order structure, two alternatives are possible as reported in Figure 5A and C. Mode 1 shows the interarm oscillation of charges. For the first-order structure, there are two resonance modes, but only a single mode can be observed as both modes are degenerate because of symmetry. Similarly, the symmetric second-order structure consists of 5 modes, where the higher-frequency mode (mode 2, Figure 5D) is triply degenerate, whereas the lower-frequency mode (mode 1, Figure 5B) is doubly degenerate. In mode 2, there is an interbranch oscillation of charges as shown in Figure 5D. For the symmetric $N=3$ case, there exist 11 modes, but because of symmetry, only three resonances show up, as shown in Figure 2C. The mode degeneracies are 2, 3, and 6 from low to high frequency. The first and second mode will be the same as appeared in fractal orders 1 and 2, whereas mode 3 will be more like a global plasmon, where the charges oscillate between the branches as shown in Figure 5E.

Figure 4: Surface current distribution.

(A1–C1) Front of first-, second-, and third-fractal-order Cayley tree, respectively. (A2–C2) Bottom layer of the unit-cell structure at first resonance frequency for each fractal order under TE polarized incident wave: (A1, A2) first-order Cayley tree, $f_1 = 3.83$ THz, (B1, B2) second-order Cayley tree $f_1 = 3.32$ THz, (C1, C2) third-order Cayley tree $f_1 = 2.7$ THz, respectively. The solid arrow indicates the direction of E-field.

Figure 5: Schematic depicting the configuration of charges for different resonance modes of a Cayley tree fractal for the first, second, and third order.

In the first-order structure, (A) and (C), two cases with the charges being localized to the corners exist. For the second order, modes 1 and 2 both exist. (B) Highest-frequency dipole mode with charges concentrated only at the outermost tips. (D) Lowest-frequency mode where the charges are no more localized at the tips and are instead spread out. (E) Third-order structure with mode 3 where the charges are distributed over the whole structure and act like a global plasmon. This pictorial explanation is based on [52].

3.4 Broadband absorption

The broadband absorption is achieved by combining these fractal orders into one supercell as shown in Figure 1D. The structures are arranged in fourfold symmetry offering polarization insensitivity. Figure 6A shows the simulated absorption spectrum for supercell with substrate thickness of 6 μm . It can be seen that although the absorption efficiency is low, we are able to achieve broadband spectra. The thickness of substrate has significant impact on the absorption magnitude; hence, we have simulated the supercell for multiple thicknesses, and the best

optimized thickness for near-unity absorption broadband spectrum comes out to be 11 μm .

Figure 6B shows the broadband absorption of supercell at 11 μm . Peak absorption magnitude of above 99% is observed at multiple frequencies, i.e. 3.24, 4.58, and 4.85 to 5.1 THz. The numerically evaluated absorption spectrum shows broadband operation in the frequency range of 2.4 to 5.4 THz with above 80% absorption. The calculated average absorption of the device is 91.04%, defined by the mean value of above 75% minima in this broadband range. The figure of merit of an absorber is characterized by the layer thickness expressed in terms

Figure 6: Absorption spectrum of broadband supercell design. (A) Polyimide thickness $h = 6 \mu\text{m}$, (B) polyimide thickness $h = 11 \mu\text{m}$.

of wavelength with respect to lowest or center working frequency and by the relative absorption bandwidth, also known as fractional bandwidth (FBW), which is defined by the ratio of FWHM bandwidth “ Δf ” to that of center frequency “ f_c .” For our absorber, the FWHM bandwidth of $\Delta f = 3.88$ THz (from 2.24 to 6.13 THz) with center frequency $f_c = 4.185$ THz is achieved, which gives FBW of 92.7%. We compared the performance of our proposed design of metasurface absorber with some recently reported planar, single-layer absorbers. The absorption efficiencies of some already existing single-layer metasurface absorber in the literature and our Cayley tree design are given in Table 1.

Our proposed design covers a larger relative bandwidth as compared to other planar, single-layer metasurface absorbers shown in Table 1. Moreover, the device FWHM bandwidth is more than one optical octave, i.e. 1.45, where number of octave is $\log_2(f_{\max}/f_{\min})$. According to the Rozanov ultimate thickness limit [62] for a broadband absorbers, the thickness “ h ” of a proposed design must be larger than the theoretical limit. Equation (1) defines the relation between the thickness and bandwidth of the absorber:

$$h \geq \frac{1}{2\pi^2} \left| \int_0^\infty \ln |R(\lambda)| d\lambda \right| \quad (1)$$

The calculated theoretical limit comes out to be 5.16 μm , and the thickness of our designed absorber is larger than this limit, i.e. 11 μm . The corresponding electrical thickness of our device with respect to center frequency of

broadband operation is $< \lambda/6$. Thus, our proposed absorber design is ultrathin and acquires larger broadband bandwidth relative to other reported planar and multilayer THz absorbers [43–51].

3.5 Impedance matching

In order to characterize our thin single-layer broadband absorber, we retrieved the effective impedance using the coefficients of reflection and transmission achieved by numerical simulations given by [63]:

$$z'(\omega) = \pm \sqrt{\frac{(1 + r'(\omega))^2 + t'(\omega)^2}{(1 - r'(\omega))^2 - t'(\omega)^2}} \quad (2)$$

where $z'(\omega)$ is the complex effective impedance, and $r'(\omega)$ and $t'(\omega)$ are complex reflection and complex transmission coefficient, respectively. The effective impedance normalized to free space impedance is shown in Figure 7. The solid green lines in Figure 7 indicate the frequencies with maximum absorption magnitude. To maximize the absorption, impedance must match that of free space; i.e. the real and imaginary parts of complex impedance should be equal to 1 and 0, respectively. It can be seen from Figure 7 that our proposed absorber design results in a nearly perfect impedance match to free space, and thus we obtained high absorption amplitude at frequencies 3.24, 4.58, and 4.85 to 5.1 THz as observed in Figure 6B.

Table 1: Absorption performances of recently reported THz broadband, multilayer and single-layer MM absorbers.

Reference	Resonating structure	Layers	Thickness (approx.) with respect to center wavelength	Relative absorption bandwidth >50% or FBW	Bandwidth absorption >80% THz
[40]	Multiple stacked bars resonators	Multilayer	$\lambda/11$	48%	0.8–1.38 (0.58 THz)
[41]	Nested circle rings	Multilayer	$\lambda/12$	61.2%	1.6–2.6 (1 THz)
[42]	Stacked cross resonators	Multilayer	$\lambda/4$	97.7%	7.1–8 (0.9 THz)
[43]	Multiple I-shaped resonators	Single layer	$\lambda/37$	19.1%	0.88–0.98 (0.1 THz)
[44]	Multiple square patches resonators	Single layer	$\lambda/34$	17.9%	6.23–7.08 (0.85 THz)
[45]	Multiple circular patches resonators	Single layer	$\lambda/22$	25.2%	5.3–6.4 (1.1 THz)
[46]	Sectional asymmetric structure	Single layer	$\lambda/36$	14%	(0.55 THz)
[47]	Circular dishes	Single layer	$\lambda/26$	11.3%	13.6–14.79 (1.19 THz)
[48]	Fractal crosses resonators	Single layer	$\lambda/6$	65.1%	3.01–4.84 (1.83 THz)
[49]	Hybrid structure (rhombus and circular patches)	Single layer	$\lambda/12$	56%	(0.13135 THz)
[50]	Multiple square rings resonators	Single layer	$\lambda/6$	77.6%	0.6–1.25 (0.65 THz)
Our design	Fractal Cayley tree resonators	Single layer	$\lambda/6$	92.7%	2.4–5.4 (3 THz)

Figure 7: Normalized effective impedance spectrum of the broadband supercell at $11\ \mu\text{m}$. The dashed gray lines mark the impedance with amplitude 1 and 0, and the solid green lines mark the resonance frequency with maximum absorption.

3.6 E-fields of broadband absorption

To better understand the broadband absorption of the supercell design at $11\ \mu\text{m}$, we further investigate the 2D electric field profiles at six prominent resonance peaks of absorption spectra as shown in Figure 8. To provide an immediate correlation between resonance strengths, all electric field profiles were normalized on the same scale. In Figure 8A–F, it can be seen that for peak P1 (2.54 THz)

and peak P6 (5.1 THz), the dipolar resonance occurs solely because of third-order Cayley tree and is near the first resonance frequency of level 3 unit cell. Similarly, for peak P2 (2.87 THz) (Figure 8B), second-order structure appears to resonate strongly as its E-field strength is stronger than the others. Figure 8C shows that first-order structure resonance appears to dominate at peak P3 (3.24 THz), whereas for peaks P4 and P5 as shown in Figure 8D and E, respectively, the electric response is complex because no single fractal level is dominant in these peaks. Therefore, they are assumed to be due to the coupling and superposition of the neighboring resonances. In conclusion, strong resonance frequency is mainly due to the fractal orders first, second, and third or because of the interaction between two levels.

3.7 Interference theory

So far, we have analyzed our MM device as a coupled system considering only the impedance matching and the local EM resonances between the resonators and the ground plane. However, we may also employ a quantitative analysis method known as interference theory to further analyze the physical mechanism for high-level absorption. The interference theory treats the MM absorber as a Fabry–Perot resonance cavity. We decoupled our system into two tuned interfaces, where the top resonators and ground plane are taken as nonthickness surfaces, and the coupling and near-field magnetic interactions between them are neglected. Top and bottom metal layers are only linked through multiple inner reflections, which then destructively interfere with the air–spacer reflection

Figure 8: Numerically simulated E-field profiles for broadband peaks. (A–F), Two-dimensional E-field profiles for six different peaks of a broadband supercell design at normal incident wave. (G) Simulated broadband absorption spectrum showing the six distinct peaks. Inset: supercell design.

resulting in minimum total reflection and maximum absorption [64].

As shown in Figure 9A, when light is incident at air–spacer interface, it is partially reflected by the top metal layer into air with a reflection coefficient $r'_{12} = r_{12}e^{i\varphi_{12}}$, and remaining is transmitted into the spacer with transmission coefficient $t'_{12} = t_{12}e^{i\varphi_{12}}$, which keeps on traveling until it meets the ground plane with complex propagation phase $\beta = -\sqrt{\varepsilon_r}k_0h$, where k_0 is the free space wavenumber. After its interaction with the ground plane, the wave completely reflects back into the spacer with reflection coefficient -1 and again undergoes reflection and transmission with coefficients $r'_{21} = r_{21}e^{i\varphi_{21}}$, and $t'_{21} = t_{21}e^{i\varphi_{21}}$, respectively. These multiple inner reflections constitute a phase shift that results into destructive interference with the reflection of incident wave from air–spacer interface and thus traps the waves inside our MIM structure giving us the high absorption amplitude. The total reflection is written as [64]:

$$r = r'_{12} - \frac{t'_{12}t'_{21}e^{i2\beta}}{1 + r'_{21}e^{i2\beta}}$$

- (3) Apart from matching impedance, another metric of a good absorber is its polarization and incidence angle

The absorption is then calculated through $A = 1 - |r|^2$. The magnitude and phase of reflection and transmission coefficients obtained by simulation are shown in Figure 9B. The corresponding calculated absorption spectrum is in good consensus with the simulated results as shown in Figure 9C. Thus, the interference theory model adequately explains the reason for high-level absorption of our MM absorber device. Our simulated design takes into account the physical limitations (feature size, thickness, and material selection) of a typical fabrication process and provides improved efficiency in comparison with state of the art. In light of the published literature discussing planar metasurface absorbers [43, 48, 50, and references therein], the full-wave simulations are expected to match experimental results with minimal tolerance once fabricated.

3.8 Polarization and incidence angle dependence

Figure 9: Multiple reflection and interference theory mechanism.

(A) Fabry–Perot cavity model of our proposed supercell design at $h = 11 \mu\text{m}$ based on interference theory. (B) Simulated magnitude of the complex coefficients of transmission and reflection at the interface. (C) Numerical results of absorption of broadband supercell design obtained from simulation and Fabry–Perot interference theory.

Figure 10: Numerically simulated angle-dependent absorption of broadband supercell design at thickness $11\ \mu\text{m}$. (A) For different polarization angle ($\varphi = 0^\circ$ – 90°). The polarization angle is rotating from TE polarization (0°) to TM polarization (90°). (B, C) For different incidence angle ($\theta = 0^\circ$ – 60°) for TE and TM modes, respectively.

insensitivity. Therefore, we also study the absorption characteristic of our device for different polarization and incidence angles. Figure 10A and B show the numerically simulated absorption spectrum of supercell over different polarization angles (φ). It can be seen that when φ changes from 0° to 90° the absorption remains unchanged for both TE and TM modes. This is due to the rotational fourfold symmetry of our device. It means that the absorber is insensitive to polarization angles. The absorptivity of supercell at oblique incidence angles for TE and TM polarizations is displayed in Figure 10C and D, respectively. The results show that our structure is highly optimized for wide angle of incidence for both TE and TM polarized waves. The broadband absorption spectrum remains unchanged for angle as high as 50° . Beyond that, the magnitude of the absorption starts to decrease gradually for TE polarization due to decrease in induced magnetic flux with increase in incidence angle. Thus, we can say that our proposed broadband THz absorber is polarization insensitive and has good absorption performance over wide incident angle.

4 Summary

In summary, we have proposed an ultrabroadband, thin metasurface absorber, based on fractal geometry resonators. Utilizing the intriguing property of fractals of frequency shifting and having N resonant peaks for the N th-order structure, we designed a broadband THz absorber by integrating different fractal orders into one supercell. All the levels of Cayley tree fractal geometry individually at polyimide thickness $5\ \mu\text{m}$ exhibit narrowband near-unity absorption response due to strong EM

resonances and when merged into supercell of thickness $11\ \mu\text{m}$ give broadband absorption with maximum absorption magnitude of above 99% at various frequencies. The simulated FWHM absorption bandwidth of our proposed device is more than one octave, i.e. 3.88 THz. Greater than 80% absorption is achieved over the bandwidth of 3 THz. The numerically calculated relative absorption bandwidth of the device is 92.7%. The broadband simulated spectra of a supercell with $11\text{-}\mu\text{m}$ polyimide thickness agree well with the theoretical results established on the basis of interference theory, which shows that the strong absorption at a thickness of $11\ \mu\text{m}$ is due to multiple inner reflection and destructive interference that traps the wave reflection into the air. Furthermore, supercell absorber design is polarization insensitive due to its fourfold rotational symmetry and provides wide incident wave angle of about 50° for both TE and TM waves. We have shown that fractal resonators give a considerable degree of freedom in frequency shifting and broadband response. It is a step forward to design perfect THz blackbody absorbers to meet different applications such as sensing (by integrating them with bolometric sensing technology). A preliminary version of this work was presented in [65].

References

- [1] Williams GP. Filling the THz gap – high power sources and applications. Rep Prog Phys 2006;69:301–26.
- [2] Tonouchi M. Cutting-edge terahertz technology. Nat Photonics 2007;1:97–105.
- [3] Woodward RM, Wallace VP, Pye RJ, et al. Terahertz pulse imaging of ex vivo basal cell carcinoma. J Invest Dermatol 2003;120:72–8.
- [4] Lee WM, Qin Q, Kumar S, Williams BS, Hu Q. Real-time terahertz imaging over a standoff distance (>25 meters). Appl Phys Lett 2006;89:141125.

- [5] Liu HB, Chen Y, Bastiaans GJ, Zhang XC. Detection and identification of explosive RDX by THz diffuse reflection spectroscopy. *Opt Express* 2006;14:415–23.
- [6] Yoshida H, Ogawa Y, Kawai Y. Terahertz sensing method for protein detection using a thin metallic mesh. *Appl Phys Lett* 2007;91:253901.
- [7] Federici J, Moeller L. Review of terahertz and sub terahertz wireless communications. *J Appl Phys* 2010;107:111101.
- [8] Beard MC, Turner GM, Schmuttenmaer CA. Terahertz spectroscopy. *J Phys Chem B* 2002;106:7146–59.
- [9] Zheludev NI. The road ahead for metamaterials. *Science* 2010;328:582–3.
- [10] Liu Y, Zhang X. Metamaterials: a new frontier of science and technology. *Chem Soc Rev* 2011;40:2494–507.
- [11] Pendry JB. Negative refraction makes a perfect lens. *Phys Rev Lett* 2000;85:3966–9.
- [12] Pendry JB, Schurig D, Smith DR. Controlling electromagnetic fields. *Science* 2006;312:1780–2.
- [13] Shelby RA, Smith DR, Schultz S. Experimental verification of a negative index of refraction. *Science* 2001;292:77–9.
- [14] Li Z, Kim I, Zhang L, et al. Dielectric metaholograms enabled with dual magnetic resonances in visible light. *ACS Nano* 2017;11:9382–9.
- [15] Mei S, Mehmood MQ, Hussain S, et al. Flat helical nanosieves. *Adv Funct Mater* 2016;26:5255–62.
- [16] Mehmood M, Mei S, Hussain S, et al. Visible-frequency metasurface for structuring and spatially multiplexing optical vortices. *Adv Mater* 2016;28:2533–9.
- [17] Li Z, Dai Q, Mehmood MQ, et al. Full-space cloud of random points with a scrambling metasurface. *Light Sci Appl* 2018;7:63.
- [18] Ansari MA, Kim I, Lee D, et al. Spin-encoded all-dielectric metahologram for visible light. *Laser Photonics Rev* 2019;13:1900065.
- [19] Landy NI, Sajuyigbe S, Mock JJ, Smith DR, Padilla WJ. Perfect metamaterial absorber. *Phys Rev Lett* 2008;100:207402.
- [20] Rana S, Mehmood MQ, Jeong H, Kim I, Rho J. Tungsten-based ultrathin absorber for visible regime. *Sci Rep* 2018;8:2443.
- [21] Kim I, So S, Rana AS, Mehmood MQ, Rho J. Thermally robust ring-shaped chromium perfect absorber of visible light. *Nanophotonics* 2018;7:1827–33.
- [22] Astorino MD, Frezza F, Tedeschi N. Ultra-thin narrow-band, complementary narrow-band, and dual-band metamaterial absorbers for applications in the THz regime. *J Appl Phys* 2017;121:063103.
- [23] Janneh M, De Marcellis A, Palange E, Tenggara AT, Byun D. Design of a metasurface-based dual-band Terahertz perfect absorber with very high Q-factors for sensing applications. *Opt Comm* 2018;416:152–9.
- [24] Wen QY, Zhang HW, Xie YS, Yang QH, Liu YL. Dual band terahertz metamaterial absorber: design, fabrication, and characterization. *Appl Phys Lett* 2009;95:241111.
- [25] Wang X, Wang GZ, Sang T. Simple design of novel triple-band terahertz metamaterial absorber for sensing application. *J Phys D Appl Phys* 2016;49:165307.
- [26] Meng HY, Wang LL, Zhai X, Liu GD, Xia SX. A simple design of a multi-band terahertz metamaterial absorber based on periodic square metallic layer with T-shaped gap. *Plasmonics* 2018;13:269–74.
- [27] Mao Z, Liu S, Bian B, et al. Multi-band polarization-insensitive metamaterial absorber based on Chinese ancient coin-shaped structures. *J Appl Phys* 2014;115:204505.
- [28] Wang X, Zhai X, Wang GZ, Huang WQ, Wang LL. Design of a four-band and polarization-insensitive terahertz metamaterial absorber. *IEEE Photonics J* 2015;7:1–8.
- [29] Cheng Y, Huang M, Chen H, Guo Z, Mao X, Gong R. Ultrathin six-band polarization-insensitive perfect metamaterial absorber based on a cross-cave patch resonator for terahertz waves. *Materials* 2017;10:591.
- [30] Liu S, Zhuge J, Ma S, et al. A bi-layered quad-band metamaterial absorber at terahertz frequencies. *J Appl Phys* 2015;118:245304.
- [31] Wang BX, Wang GZ, Sang T, Wang LL. Six-band terahertz metamaterial absorber based on the combination of multiple-order responses of metallic patches in a dual-layer stacked resonance structure. *Sci Rep* 2017;7:41373.
- [32] Grant J, Escorcia-Carranza I, Li C, McCrindle IJ, Gough J, Cumming DR. A monolithic resonant terahertz sensor element comprising a metamaterial absorber and micro-bolometer. *Laser Photonics Rev* 2013;7:1043–8.
- [33] Alves F, Grbovic D, Kearney B, Lavrik NV, Karunasiri G. Bi-material terahertz sensors using metamaterial structures. *Opt Express* 2013;21:13256–71.
- [34] Wen Y, Jia D, Ma W, et al. Photomechanical meta-molecule array for real-time terahertz imaging. *Microsyst Nanoeng* 2017;3:17071.
- [35] Hu X, Xu G, Wen L, et al. Metamaterial absorber integrated microfluidic terahertz sensors. *Laser Photonics Rev* 2016;10:962–9.
- [36] Tan S, Yan F, Wang W, Zhou H, Hou Y. Ultrasensitive sensing with three-dimensional terahertz metamaterial absorber. *J Opt* 2018;20:055101.
- [37] Escorcia Carranza I, Grant JP, Gough J, Cumming D. Terahertz metamaterial absorbers implemented in CMOS technology for imaging applications: scaling to large format focal plane arrays. *IEEE J Sel Top Quantum Electron* 2017;23:1–8.
- [38] Cumming RS, Grant JP, McCrindle IJH. Terahertz radiation detector, focal plane array incorporating terahertz detector, multispectral metamaterial absorber, and combined optical filter and terahertz absorber. U.S. Patent 9,513,171 (June 12, 2016).
- [39] Iwaszczuk K, Strikwerda AC, Fan K, Zhang X, Averitt RD, Jepsen PU. Flexible metamaterial absorbers for stealth applications at terahertz frequencies. *Opt Express* 2012;20:635–43.
- [40] Liu S, Chen H, Cui TJ. A broadband terahertz absorber using multi-layer stacked bars. *App Phys Lett* 2015;106:151601.
- [41] Pan W, Yu X, Zhang J, Zeng W. A novel design of broadband terahertz metamaterial absorber based on nested circle rings. *IEEE Photonic Tech L* 2016;28:2335–8.
- [42] Jia D, Xu J, Yu X. Ultra-broadband terahertz absorption using bi-metasurfaces based multiplexed resonances. *Opt Express* 2018;26:26227–34.
- [43] Huang L, Chowdhury DR, Ramani S, et al. Experimental demonstration of terahertz metamaterial absorbers with a broad and flat high absorption band. *Opt Lett* 2012;37:154–6.
- [44] Cheng Y, Nie Y, Gong R. A polarization-insensitive and omnidirectional broadband terahertz metamaterial absorber based on coplanar multi-squares films. *Opt Laser Technol* 2013;48:415–21.

- [45] Wang D, Liu MH, Hu XW, Kong LH, Cheng LL, Chen ZQ. Broadband and ultra-thin terahertz metamaterial absorber based on multi-circular patches. *Eur Phys J B* 2013;86:304.
- [46] Gong C, Zhan M, Yang J, et al. Broadband terahertz metamaterial absorber based on sectional asymmetric structures. *Sci Rep* 2016;6:32466.
- [47] Dang L, Nguyen VC, Le DH, et al. Broadband metamaterial perfect absorber obtained by coupling effect. *J Nonlinear Opt Phys Mater* 2017;26:1750036.
- [48] Kenney M, Grant J, Shah YD, Escorcia-Carranza I, Humphreys M, Cumming DR. Octave-spanning broadband absorption of terahertz light using metasurface fractal-cross absorbers. *ACS Photonics* 2017;4:2604–12.
- [49] Lu Y, Li J, Zhang S, Sun J, Yao JQ. Polarization-insensitive broadband terahertz metamaterial absorber based on hybrid structures. *Appl Opt* 2018;57:6269–75.
- [50] Shen G, Zhang M, Ji Y, Huang W, Yu H, Shi J. Broadband terahertz metamaterial absorber based on simple multi-ring structures. *AIP Advances* 2018;8:075206.
- [51] Cheng Y, Zou H, Yang J, Mao X, Gong R. Dual and broadband terahertz metamaterial absorber based on a compact resonator structure. *Opt Mater Express* 2018;8:3104–14.
- [52] Gottheim S, Zhang H, Govorov AO, Halas NJ. Fractal nanoparticle plasmonics: the Cayley tree. *ACS nano* 2015;9:3284–92.
- [53] Hegde RS, Khoo EH. Broadband optical response in ternary tree fractal plasmonic nanoantenna. *Plasmonics* 2016;11:465–73.
- [54] Krzysztofik WJ. Fractal geometry in electromagnetics applications-from antenna to metamaterials. *Microwave Rev* 2013;19:3–14.
- [55] Edries M, Mohamed HA, Hekal SS, El-Morsy MA, Mansour HA. Simulation design of dual band metamaterial absorber based on the fractal structure. In: *IEEE International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting*, Atlanta, GA, USA, IEEE, 2019:1335–6.
- [56] Yu D, Liu P, Dong Y. A novel miniaturized metamaterial absorber based on the modified Minkowski fractal structure. In: *2016 Asia-Pacific International Symposium on Electromagnetic Compatibility (APEC)*, Shenzhen, IEEE, Vol. 1. 2016:567–9.
- [57] Heydari S, Jahangiri P, Sharifi A, Zarrabi FB, Saeed Arezomand A. Fractal broken-cross with Jerusalem load absorber for multi-band application with polarization independence. *Microw Opt Techn Lett* 2017;59:1942–7.
- [58] Faruque MRI, Hasan MM, Islam MT. Tree-shaped fractal metasurface with left-handed characteristics for absorption application. *Appl Phys A* 2018;124:127.
- [59] Amiri M, Tofigh F, Shariati N, Lipman J, Abolhasan M. Miniature tri-wideband Sierpinski–Minkowski fractals metamaterial perfect absorber. *IET Microw Antenna P* 2019;13:991–6.
- [60] Wen Y, Liu S, Zhang H, Wang L. A design for band enhanced dielectric absorber based on fractal-like structure. *Plasmonics* 2018;13:1767–73.
- [61] CST, Microwave Studio. Available at: <https://www.cst.com>.
- [62] Rozanov KN. Ultimate thickness to bandwidth ratio of radar absorbers. *IEEE Trans. Antennas Propag* 2000; 48:1230.
- [63] Smith DR, Schultz S, Markoš P, Soukoulis CM. Determination of effective permittivity and permeability of metamaterials from reflection and transmission coefficients. *Phys Rev B* 2002;65:195104.
- [64] Chen HT. Interference theory of metamaterial perfect absorbers. *Opt Express* 2012;20:7165–72.
- [65] Zubair A, Mehmood MQ, Zubair M. “Design of a Fractal Metasurface Based Terahertz Broadband Absorber”, Paper presented at the 41st Progress in Electromagnetics Research Symposium (PIERS), Rome, Italy, 2019.