Research article

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Landau quantisation of photonic spin Hall effect in monolayer black phosphorus

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Abstract: The photonic spin Hall effect (PSHE) is a promising candidate for controlling the spin states of photons and exploiting next-generation photonic devices based on spinoptics. Herein, the influences of a perpendicular magnetic field on the PSHE appearing on the surface of monolayer black phosphorus (BP) are investigated. Results reveal that both the in-plane and transverse spin-dependent shifts are quantised and show an oscillating pattern due to the splitting of Landau levels (LLs) induced by the external magnetic field B . And the oscillation period of spin Hall shifts gradually increases with strengthening B because of the increase of LL spacings. By contrast, for a fixed magnetic field, as the LL spacings become smaller and smaller with increasing the LL index, the oscillation period of spin Hall shifts gradually decreases as the photonic energy increases. Moreover, it is possibly due to the synergistic role of intrinsic anisotropy, high crystallinity, and quantisation-incurred localised decreases in beating-like complex conductivities of the BP film, giant spin Hall shifts, hundreds of times of the incident wavelength, are obtained in both transverse and in-plane directions. These unambiguously confirm the strong impact of the external magnetic field on the PSHE and shed important insights into understanding the rich magneto-optical transport properties in anisotropic two-dimensional atomic crystals.

1 Introduction

The photonic spin Hall effect (PSHE) is a photonic counterpart of the spin Hall effect in electronic systems, in which the electron spin and electric potential gradient are replaced by the optical helicity of incident photons and the refractive index gradient, respectively [1–4]. It is regarded as a consequence of the spin–orbit interaction of light on propagation. In 2004, Onoda et al. [1] theoretically described this effect via taking into account the geometric Berry phase and the angular momentum conservation. In 2008, Hosten and Kwiat [2] first employed a weak measurement technique to detect and verify this phenomenon for photons passing through an air–glass interface. Since then, the PSHE has been igniting tremendous enthusiasm for both scientific research interests and technological upgrading on an emerging research area–spinoptics (an optical version of spintronics in solids) [3–6]. In particular, integrating novel two-dimensional (2D) materials into photonic spin Hall devices could cause many exotic optical characteristics and applications. For instance, tailoring the spin–orbit coupling of light has been utilised to ameliorate the photoluminescence efficiency of 2D MoS_2 [7]. The PSHE modified by graphene has shown great potentials in refractive index sensors and precision metrology for nanostructures of 2D materials [8, 9].

In comparison with those isotropic 2D materials including MoS_2 and graphene, atomically thin-layered black phosphorus (BP), which was successfully exfoliated in laboratory in 2014, not only is privileged for its own exclusive hallmarks, but also has versatile performances and applications as a 2D material for manufacturing various devices [10–12]. For example, BP possesses a layer-dependent direct bandgap, tunable from ~ 0.3 eV in its bulk to approximately 1.5–2.0 eV in monolayer [11]. Black phosphorus is endowed with an intriguing in-plane anisotropy, yielding two highly asymmetric crystal directions within the BP lattice: armchair (AC) and zigzag (ZZ)

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[12]. Recently, Lin et al. [13] theoretically investigated the PSHE on a monolayer of BP in the terahertz region and found that the anisotropic nature results in a robust asymmetric spin splitting of photons in both transverse and in-plane directions. Zhang et al. [14] demonstrated that photonic spin Hall shifts on the surface of 2D BP are sensitive to the orientation of optical axis, doping concentration, and interband transitions. Nonetheless, it may be due to the relative delay of rediscovering BP as a new 2D material, investigations on the PSHE occurring in 2D BP atomic layers are still in the preliminary stage, and comprehensive researches are severely lacking. For instance, to our knowledge, none of previous studies on the PSHE in BP atomic crystal involve the external magnetic field [13–15]. In fact, the latest research on graphene shows that 2D electron gases being exposed to a magnetic field may result in an exceptionally high infrared magneto-optical transitions between the Landau levels (LLs) [16]. It is accompanied by a colossal amendment of magneto-optical conductivity which is a key parameter to tailor the PSHE occurring at a planar dielectric interface [16, 17]. Even though the effects of the magnetic field on the PSHE along with quantised spin Hall shifts have been recently reported in graphene [17–20], they are either confined to the terahertz region (even made excessive approximations in theoretical model) [17–19] or only considering the graphene-like planar microcavity [20]. Moreover, all these related researches on isotropic graphene are mainly limited to the analyses on the relationship between the magnetic field and PSHE [17–20]. The inmost mechanism of LL transitions underlying the magneto-optical response of 2D materials has not yet been connected to the PSHE. The in-depth magneto-optical transport properties in anisotropic 2D atomic crystals remain elusive.

In addition, the spin-dependent shifts arising from the PSHE could be accompanied by both spin-independent Goos–Hänchen (GH) and Imbert–Fedorov (IF) shifts [13]. In theory, great GH and IF shifts more than 1000 times of the incident wavelength λ_0 can be obtained in artificial microconfigurations [21–23]. Nowadays, how to efficiently amplify the PSHE has also come to the fore of researches [24, 25]. Spurred by advances in the PSHE and novel 2D materials, in this work, we theoretically investigate the PSHE in a monolayer of BP which is exposed to an external magnetic field and is illuminated by an excitation source in the direct bandgap energies of monolayer BP. Impacts of LL splitting induced via the magnetic field on both in-plane and transverse spin Hall shifts are studied in detail. Moreover, giant spin Hall shifts as well as the possible enhancement mechanism are also presented.

2 Model and theory

To describe the PSHE in a general model, let us consider the situation depicted in Figure 1A. Monolayer BP is deposited upon a nonmagnetic and homogeneous substrate. One Gaussian wave packet with monochromatic frequency ω impinges from air upon the surface of the BP film with an incident angle θ_i . The x and y axes of laboratory Cartesian coordinate (x, y, z) correspond to the AC and ZZ directions of the BP layer plane, respectively. And a uniform static magnetic field $\mathbf{B} = -B\hat{\mathbf{z}}$ is applied along the negative z axis. Additionally, the coordinate frames (x_i, y_i, z_i) and (x_r, y_r, z_r) are utilised to denote the central wave vectors of incident and reflected wave packets, respectively.

At present, there are two main models to be used to simulate the optical properties of 2D materials, i.e. the

Figure 1: The energy of incident photon is 1.8 eV.

(A) Schematic representation of the photonic spin splitting of one Gaussian beam reflected from a BP–substrate interface. An imposed static magnetic field B is applied along the negative z axis in a Cartesian coordinate system. (B) The Fresnel reflection coefficients with $B=0$ and 10 T.

surface current model (SCM) and the classical slab model (CSM) [26, 27]. In 2019, Song et al. [28] compared the SCM and the CSM in evaluating the complex optical conductivity of 2D material from the point of view of experiments. They demonstrated that the complex optical conductivity of monolayer 2D material obtained from the CSM coincides with that of the SCM. Here, the SCM is adopted, and monolayer BP is treated as a strictly 2D material without thickness. The magneto-optical response of BP is characterised by a conductivity tensor, which can be written in the usual manner as [26, 29, 30]

$$\sigma(\omega) = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{yy} \end{pmatrix} \quad (1)$$

with

$$\sigma_{jk} = i\sigma_0 \frac{\hbar^2}{l_B^2} \times \sum_{n', n, c, v} \frac{[f(E_{c,n'}) - f(E_{v,n})] \langle c, n' | v_j | v, n \rangle \langle v, n | v_k | c, n' \rangle}{(E_{c,n'} - E_{v,n})(E_{c,n'} - E_{v,n} + \hbar\omega + i\Gamma)} \quad (2)$$

where $j, k \in \{x, y\}$, n (or n') is the LL index of valence (or conduction) band; $\sigma_0 = e^2/\pi\hbar$, $l_B = (\hbar/eB)^{1/2}$, $f(E_\xi) = [\exp[(E_\xi - E_F)/k_B T] + 1]^{-1}$ is the Fermi–Dirac distribution function with Boltzmann constant k_B and temperature T ; $E_{v,n}$ and $E_{c,n'}$ are the LLs of valence and conduction bands, respectively; and $v_{j/k}$ are the components of group velocities. The sum runs over all states $\zeta = v, n$ and $\zeta' = c, n'$ with $\zeta \neq \zeta'$ [29, 30]. In this work, the level broadening factor Γ and temperature T are set to be 0.15 meV and 5 K, respectively. Besides, we take the Fermi energy $E_F = 0$ such that the contribution from intraband transitions can be ignored. Figure S1 in the Supplementary Material presents the LLs as a function of the magnetic field B for the first 40 LLs. More detailed descriptions on calculations of LLs and the transition matrix elements of $v_{j/k}$ can be found from Equations (S1) to (S8) in the Supplementary Material.

Universally, the incident plane of photons makes an angle φ with the AC axis of BP crystal. Therefore, the conductance matrix connecting the surface current and electromagnetic field is given by [14]

$$\sigma_M = \begin{pmatrix} \cos\varphi & \sin(-\varphi) \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} \cos\varphi & \sin\varphi \\ \sin(-\varphi) & \cos\varphi \end{pmatrix} = \begin{pmatrix} \sigma_{pp} & \sigma_{ps} \\ \sigma_{sp} & \sigma_{ss} \end{pmatrix} \quad (3)$$

where

$$\sigma_{pp} = \sigma_{xx} \cos^2\varphi + \sigma_{yy} \sin^2\varphi \quad (4a)$$

$$\sigma_{ss} = \sigma_{xx} \sin^2\varphi + \sigma_{yy} \cos^2\varphi \quad (4b)$$

$$\sigma_{ps} = (\sigma_{xx} - \sigma_{yy}) \sin\varphi \cos\varphi + \sigma_{xy} \quad (4c)$$

$$\sigma_{sp} = (\sigma_{xx} - \sigma_{yy}) \sin\varphi \cos\varphi - \sigma_{xy} \quad (4d)$$

It is clear that the cross conductivities σ_{ps} and σ_{sp} , which induce the couplings between p - and s -polarised waves, are no longer equal to each other due to the emergence of magnetic field-induced Hall conductivity σ_{xy} .

According to the electromagnetic boundary condition, the Fresnel reflection coefficients of BP–substrate are given as [31]

$$r_{pp} = (\beta_+ \alpha_- + \chi) / (\beta_+ \alpha_+ + \chi) \quad (5a)$$

$$r_{ss} = -(\beta_- \alpha_+ + \chi) / (\beta_+ \alpha_+ + \chi) \quad (5b)$$

$$r_{sp} = -\Lambda(\sigma_{\text{sym}} + \sigma_{xy}) / (\beta_+ \alpha_+ + \chi) \quad (5c)$$

$$r_{ps} = \Lambda(\sigma_{\text{sym}} - \sigma_{xy}) / (\beta_+ \alpha_+ + \chi) \quad (5d)$$

with $\alpha_\pm = (k_{z1}\varepsilon_2 \pm k_{z2}\varepsilon_1 + k_{z1}k_{z2}\sigma_{pp}/\omega)/\varepsilon_0$, $\beta_\pm = k_{z2} \pm k_{z1} + \omega\mu_0\sigma_{ss}$, $\chi = \mu_0 k_{z1}k_{z2}(\sigma_{xy}^2 - \sigma_{\text{sym}}^2)/\varepsilon_0$, $\Lambda = 2k_{z1}k_{z2}(\mu_0/\varepsilon_0)^{1/2}$, $\sigma_{\text{sym}} = (\sigma_{xx} - \sigma_{yy}) \sin\varphi \cos\varphi$, $k_{z1} = k_1 \cos\theta_i$, $k_{z2} = k_0(\varepsilon_2/\varepsilon_0 - \sin^2\theta_i)^{1/2}$. Here, ε_0 and μ_0 are permittivity and permeability in vacuum; and ε_1 and ε_2 are the permittivities of incident and refractive media. In the case where the incident medium is air, $\varepsilon_1 = \varepsilon_0$ and $k_1 = k_0 = \omega(\varepsilon_0\mu_0)^{1/2}$.

If the Gaussian beam is illuminated on the BP surface with horizontal $|H(k_i)\rangle$ and vertical $|V(k_i)\rangle$ polarisation states, the polarisation states of reflected beam can be written as [32]

$$\begin{pmatrix} |H(k_r)\rangle \\ |V(k_r)\rangle \end{pmatrix} = \begin{pmatrix} r_{pp} - \frac{k_{ry}(r_{ps} - r_{sp}) \cot\theta_i}{k_0} & r_{ps} + \frac{k_{ry}(r_{pp} + r_{ss}) \cot\theta_i}{k_0} \\ r_{sp} - \frac{k_{ry}(r_{pp} + r_{ss}) \cot\theta_i}{k_0} & r_{ss} - \frac{k_{ry}(r_{ps} - r_{sp}) \cot\theta_i}{k_0} \end{pmatrix} \begin{pmatrix} |H(k_i)\rangle \\ |V(k_i)\rangle \end{pmatrix} \quad (6)$$

In the above equation, the boundary conditions $k_{rx} = -k_{ix}$ and $k_{ry} = k_{iy}$ have been introduced, where k_{ix} and k_{iy} (k_{rx} and k_{ry}) represent the wave-vector components of incident (reflected) beam along x_i and y_i (x_r and y_r) axes, respectively. The PSHE manifests itself as spin-dependent

splitting, which appears in both transverse and in-plane directions. In the spin basis set, $|H\rangle$ and $|V\rangle$ can be decomposed into two orthogonal spin components

$$|H\rangle = (|+\rangle + |-\rangle) / \sqrt{2} \quad (7a)$$

$$|V\rangle = i(|-\rangle - |+\rangle) / \sqrt{2} \quad (7b)$$

where $|+\rangle$ and $|-\rangle$ stand for the left- and right-circular polarisation components, respectively. The angular spectrum of the incident beam in momentum space can be specified by

$$|\Phi\rangle = w_0 / \sqrt{2\pi} \exp[-w_0^2(k_x^2 + k_y^2) / 4] \quad (8)$$

where w_0 is the width of wave function. Then, the total wave function consists of the packet spatial extent and the polarisation state.

Based on Equations (6–8) and taking into account the paraxial approximation, the spin Hall shifts of the reflected wave packet can be mathematically calculated. For simplicity, only the horizontally polarised incident beam is considered below. The corresponding in-plane and transverse spin Hall shifts for two spin components ($|+\rangle$ and $|-\rangle$), as illustrated in Figure 1A, can be written by the following expressions [14]:

$$\langle \Delta x_{\pm} \rangle = \mp \frac{1}{k_0} \operatorname{Re} \left(\frac{r_{pp}}{r_{pp}^2 + r_{sp}^2} \frac{\partial r_{sp}}{\partial \theta_i} - \frac{r_{sp}}{r_{pp}^2 + r_{sp}^2} \frac{\partial r_{pp}}{\partial \theta_i} \right) \quad (9)$$

$$\langle \Delta y_{\pm} \rangle = \mp \frac{\cot \theta_i}{k_0} \operatorname{Re} \left(\frac{r_{pp} + r_{ss}}{r_{pp}^2 + r_{sp}^2} r_{pp} - \frac{r_{ps} - r_{sp}}{r_{pp}^2 + r_{sp}^2} r_{sp} \right) \quad (10)$$

It is seen from the above equations that the external magnetic field can be used to modulate the PSHE on the surface of 2D BP atomic film due to its decisive impact on the conductivity tensor. Hereafter, unless explicitly specified, the rotation angle φ is set to be zero. The beam waist of Gaussian wave function is $w_0 = 30 \mu\text{m}$. The permittivity ε_2 is set as $2.13\varepsilon_0$, matching with that value of amorphous SiO_2 substrate at visible and near-infrared wavelengths. All the numerical simulation is implemented in MATLAB R2019a.

3 Results and discussion

We first consider the condition that the energy of incident photon is $\hbar\omega = 1.8 \text{ eV}$. When both rotation angle φ and magnetic field B are equal to zero, the nondiagonal reflection coefficients $|r_{ps}|$ and $|r_{sp}|$ are zero. For $|r_{pp}|$, it gives the smallest value at $\theta_i = 56.6^\circ$, corresponding to Brewster's angle, as shown in Figure 1B. After a static magnetic field with $B = 10 \text{ T}$ is imposed, a nonzero $|r_{sp}|$ (and $|r_{ps}| = |r_{sp}|$) emerges, and Brewster's angle shifts to 55.6° . The emergence of $|r_{sp}|$ or $|r_{ps}|$ signifies that the reflected wave becomes elliptically polarised, and both orientation angle and ellipticity of reflected polarisation state change with the incident angle [13].

Figure 2A and B (a partially magnified version can be found in Figure S2 in the Supplementary Material) present variations of the Fresnel reflection coefficient $|r_{pp}|$ with respect to the photonic energy $\hbar\omega$ and the magnetic field B , respectively. By changing the value of $\hbar\omega$ or B , sheet-like reflection coefficients are clearly observed, indicating that the $|r_{pp}|$ and Brewster's angle are quantised. Because the reflection coefficient is closely related

Figure 2: The magnetic field B in (A) and the photonic energy in (B) are set as 10 T and 1.8 eV, respectively. Variations of the Fresnel reflection coefficient $|r_{pp}|$ with respect to the (A) photonic energy and (B) magnetic field B at different incident angles.

to the conductance matrix [see Equations (3–5)], this quantised behaviour can be explained by the Landau quantisation induced by the magneto-optical response of monolayer BP. First, charged particles in the BP film can only occupy orbits with discrete energy values, i.e. LLs, under an external magnetic field. These quantised energy levels are directly responsible for oscillations in complex conductivities of the BP film, as shown in Figures S3–S7 in the Supplementary Material. Second, by matching with LLs in Figure S1 in the Supplementary Material, we found that complex conductivities mainly result from optical transitions when the index differences between LLs are even numbers. This is in conformity with previous results on magneto-optical conductivity of a 20-layer BP film [29]. In order to more clearly view the beating-like conductivities in Figures S3 and S4 in the Supplementary Material, Figure 3A shows the absorption parts of partially magnified complex conductivities, namely the real or imaginary part of σ_{xx} (σ_{yy}) or σ_{xy} in the energy range from 1.78 to 1.85 eV. One can observe well-resolved five-peak structures (one peak is negative in $\text{Im}\sigma_{xy}$) in a period. From left to right, these five peaks correspond to interband transitions when the LL index changes $\Delta n = -4, -2, 0, 2, 4$. Note that transitions with higher Δn values corresponding to more peaks in conductivity spectra may occur if the photonic energy is large enough. Figure 3B gives the absorption parts of complex conductivities as a function of the magnetic field B when $\hbar\omega = 1.8$ eV, and a similar five-peak

pattern is also observed. Submitting these multi-peak conductivities into Equation (5), it could be reasonable to obtain discrete (quantised) Fresnel reflection coefficients by changing $\hbar\omega$ or B .

In virtue of the Landau quantisation of magneto-optical conductivities, we can further find from Figure 4 that photonic spin Hall shifts on the surface of monolayer BP also exhibit quantised characteristics. According to Figure 4, extreme values of spin Hall shifts appear at the photonic energies away from conductivity peaks. For example, the in-plane spin-dependent shift $\langle\Delta x_{\pm}\rangle$ gives local maximum and minimum values around 1.801 eV (see the inset in Figure 4A), while the conductivities ($\text{Re}\sigma_{xx}$, $\text{Re}\sigma_{yy}$ and $\text{Im}\sigma_{xy}$) approach zero at 1.801 eV (see Figure 3A). $\langle\Delta x_{\pm}\rangle$ tends to be zero, whereas the conductivities give peak values at about 1.821 eV. Moreover, the oscillation period of $\langle\Delta x_{\pm}\rangle$ gradually decreases as the photonic energy increases. As shown in the inset in Figure 4A, the energy separation between two neighbouring dips decreases from 8.02 to 7.91 meV as the photonic energy increases from 1.784 to 1.849 eV. The incident photon with a large energy can give rise to high-order quantum transitions. Thus, the decrease of oscillation period in spin Hall shifts is in accordance with the rule that LL spacings gradually decrease with the increasing of LL index, as depicted in Figure 3C. As for the transverse spin-dependent shift $\langle\Delta y_{\pm}\rangle$ in Figure 4B, even if its pattern is different from that of $\langle\Delta x_{\pm}\rangle$, its oscillation

Figure 3: Influence of magnetic field B on LL spacings $E_{v,n} - E_{v,n+1}$ (black solid and broken lines), for clarity, only conditions with LL index $n = 0, 10$, and 20 are shown.

Real part of the magneto-optical conductivity σ_{xx} (or σ_{yy}) and imaginary part of the Hall conductivity σ_{xy} as a function of the (A) photonic energy and (B) magnetic field B . The magnetic field B in (A) and the photonic energy in (B) are set as 10 T and 1.8 eV, respectively. (C) LL spacings of conduction or valence band (in unit of $E_{c,1} - E_{c,0}$ or $E_{v,0} - E_{v,1}$) as a function of LL index when $B = 10$ T (red symbols).

Figure 4: (A) In-plane and (B) transverse spin-dependent shifts on the surface of monolayer BP film as a function of the photonic energy when $B = 10$ T and $\theta_i = 55.6^\circ$. The insets show partially magnified photonic spin Hall shifts for clarity. The arrow pointing top and bottom indicates that both $\langle \Delta x_+ \rangle$ and $\langle \Delta y_+ \rangle$ give one of their extreme values at about 1.980 eV.

period is similar to that of $\langle \Delta x_+ \rangle$, i.e. $\langle \Delta x_+ \rangle$ and $\langle \Delta y_+ \rangle$ show peak values at nearly the same energy. For instance, both $\langle \Delta x_+ \rangle$ and $\langle \Delta y_+ \rangle$ give one of their extreme values at about 1.980 eV, as indicated by the arrow pointing top and bottom in Figure 4.

Figure 5A and B present the in-plane and transverse spin Hall shifts as a function of the photonic energy and the incident angle when $B = 10$ T. In general, the in-plane and transverse spin Hall shifts exhibit extreme values near Brewster's angles [13]. As a result, quantised $\langle \Delta x_+ \rangle$ and $\langle \Delta y_+ \rangle$ have a similar change tendency with the minimum values of $|r_{pp}|$ in Figure 2A. When both the factors of φ and B are zero, the in-plane spin Hall shift $\langle \Delta x_+ \rangle$ equals zero. This stems from the fact that conductivities of σ_{sym} and σ_{xy} become zero; accordingly, the reflection coefficient r_{sp} in Equation (9) becomes zero. In order to make a comparison with the PSHE when $B = 0$, Figure S10 in the Supplementary Material presents the values of $\langle \Delta x_+ \rangle$ and $\langle \Delta y_+ \rangle$ as a function of the photonic energy and the incident angle when $B = 0$ T and $\varphi = 30^\circ$. Meanwhile, the transverse spin Hall shift $\langle \Delta y_+ \rangle$ when $B = 0$ T and $\varphi = 0^\circ$ is also shown in Figure S9A

in the Supplementary Material. It is seen that both $\langle \Delta x_+ \rangle$ and $\langle \Delta y_+ \rangle$ are continuously changed and their values are at the wavelength levels, being consistent with previous results observed in the BP film when $B = 0$ T [14]. In marked contrast, after a strong magnetic field is implemented (Figure 5), spin Hall shifts become quantised, and their values are enhanced by two orders of magnitude (even three orders of magnitude in several special points for $\langle \Delta x_+ \rangle$).

Without the external magnetic field, the absorption parts of σ_{xx} and σ_{yy} ($\text{Re}\sigma_{xx} = 1.7\sigma_0$, $\text{Re}\sigma_{yy} = 3.7\sigma_0$, see Figure S8 in the Supplementary Material) are comparable with the peak values of $\text{Re}\sigma_{xx}$ in Figures 3A and S3A. However, as aforementioned, remarkably enhanced spin Hall shifts generally appear at the photonic energies away from conductivity peaks. Under the condition of $B = 0$ T, we arbitrarily divided the complex conductivities of σ_{xx} and σ_{yy} by 500. Then, enhanced spin Hall shifts of $\langle \Delta x_+ \rangle$ and $\langle \Delta y_+ \rangle$ by two orders of magnitude are also observed, as shown in Figures S9B and S11 in the Supplementary Material. Therefore, giant spin Hall shifts in Figure 5 could be partially attributed to quantisation-induced localised decreases in complex conductivities.

Figure 6 illustrates the spin Hall shifts of $\langle \Delta x_+ \rangle$ and $\langle \Delta y_+ \rangle$ as a function of the magnetic field B when $\hbar\omega = 1.8$ eV and $\theta_i = 55.6^\circ$. Similarly, the extreme values of spin Hall shifts appear at the magnetic field B away from conductivity peaks. For example, $\langle \Delta x_+ \rangle$ gives local maximum and minimum values ($\langle \Delta y_+ \rangle$ gives local maximum), whereas the conductivities ($\text{Re}\sigma_{xx}$, $\text{Re}\sigma_{yy}$, and $\text{Im}\sigma_{xy}$) tend to be zero around 8.464 T. Besides, $\langle \Delta x_+ \rangle$ and $\langle \Delta y_+ \rangle$ have a similar oscillation period, i.e., they give peak values at nearly the same magnetic field B . For example, both $\langle \Delta x_+ \rangle$ and $\langle \Delta y_+ \rangle$ give one of their extreme values at about 10.582 T, as indicated by the arrow pointing top and bottom in Figure 6. However, differing from that variation tendency with increasing the photonic energy, the oscillation period of $\langle \Delta x_+ \rangle$ or $\langle \Delta y_+ \rangle$ gradually increases as B increases. As shown in the inset in Figure 6A, the difference between two neighbouring peaks increases from 0.164 to 0.218 T as B increases from 7.340 to 8.700 T. The increase of oscillation period could be ascribed to the broadening of LL spacings induced by the magnetic field. This is based on the fact that the stronger the magnetic field is, the larger the LL spacings become, as shown in Figure 3C. As the distribution of LLs is nearly continuous at a very weak magnetic field [29, 30], the spin Hall shifts of $\langle \Delta x_+ \rangle$ and $\langle \Delta y_+ \rangle$ are continuously changed with the external magnetic field at $B < 3.5$ T. Thus, photonic spin Hall shifts at lower values of B are not shown in Figure 6.

Figure 5: Quantised and giant spin Hall shifts are observed in both $\langle \Delta x_{\pm} \rangle$ and $\langle \Delta y_{\pm} \rangle$. They have a similar change tendency with the minimum values of $|r_{pp}|$ in Figure 2A.

(A) In-plane and (B) transverse spin-dependent shifts on the surface of monolayer BP film as a function of the photonic energy and the incident angle when $B = 10$ T.

Figure 6: (A) In-plane and (B) transverse spin-dependent shifts on the surface of monolayer BP film as a function of the magnetic field B when $\hbar\omega = 1.8$ eV and $\theta_i = 55.6^\circ$.

The insets show partially magnified photonic spin Hall shifts for clarity. The arrow pointing top and bottom indicates that both $\langle \Delta x_{\pm} \rangle$ and $\langle \Delta y_{\pm} \rangle$ give one of their extreme values at about 10.582 T.

Figure 7 presents the spin Hall shifts of $\langle \Delta x_{\pm} \rangle$ and $\langle \Delta y_{\pm} \rangle$ as a function of the magnetic field B and the incident angle when $\hbar\omega = 1.8$ eV. Quantised and significantly enhanced spin Hall shifts are also observed. Similarly, both $\langle \Delta x_{\pm} \rangle$ and $\langle \Delta y_{\pm} \rangle$ have a similar change tendency with the minimum values of $|r_{pp}|$ in Figure 2B; it is demonstrated that the spin Hall shifts are sensitive to the variance of Brewster's angle. Besides, after multiplying the complex conductivities of σ_{xx} and σ_{yy} [calculated via Equation (2)] by 500, as shown in Figure S12 in the Supplementary Material, $\langle \Delta x_{\pm} \rangle$ tends to be zero ($\times 10^{-4} \lambda_0$), and $\langle \Delta y_{\pm} \rangle$ decreases to the wavelength scales. This further confirms that dramatically enhanced spin Hall shifts induced by an external magnetic field are closely associated with quantisation-incurred localised decreases in beating-like complex conductivities.

Even so, it has to be pointed out that the local minimisation of the conductivity may be only parts of the possible reasons as such giant shifts are usually unavailable on the interface without conductive sample. In our opinion, the intrinsic anisotropy and perfect crystallinity in the BP film are also necessary to obtain giant spin Hall shifts. The better the crystallinity is, the smaller the level broadening factor Γ is. Nonetheless, the appearance of disorder, defects, and impurities in the BP film can amplify the broadening factor Γ in Equation (2). Especially, a completely disordered BP layer may be similar to an isotropic 2D atomic film on the substrate. To exemplify this, we also calculate the spin Hall shifts of $\langle \Delta x_{\pm} \rangle$ and $\langle \Delta y_{\pm} \rangle$ with $\Gamma = 6$ meV, as shown in Figure S13 in the Supplementary Material. All

Figure 7: Quantised and giant spin Hall shifts are observed in both $\langle \Delta x_{\perp} \rangle$ and $\langle \Delta y_{\perp} \rangle$. They have a similar change tendency with the minimum values of $|r_{pp}|$ in Figure 2B.

(A) In-plane and (B) transverse spin-dependent shifts on the surface of monolayer BP film as a function of the magnetic field B and the incident angle when $\hbar\omega = 1.8$ eV.

the other parameters are identical to those of Figure 4. It is seen that the optical spectra are approximately continuous, and the quantisation is nearly vanishing for a large Γ value. Meanwhile, $\langle \Delta x_{\perp} \rangle$ tends to be zero, and $\langle \Delta y_{\perp} \rangle$ decreases to the wavelength scales.

4 Conclusions

To summarise, the Landau quantisation of PSHE appearing on the surface of monolayer BP has been studied in detail. Because of the energy-level splitting induced by an external magnetic field, the BP film presents beating-like complex conductivities. In consequence, the in-plane and transverse spin-dependent shifts are quantised and oscillated with changing the photonic energy or the magnetic field B . As the photonic energy increases, the oscillation period of spin Hall shifts gradually decreases because of the narrowing of LL spacings. Nevertheless, for a fixed photonic energy, the oscillation period of spin Hall shifts gradually increases with increasing the B as a stronger magnetic field will widen the LL spacings. Besides, spin Hall shifts are sensitive to the variance of Brewster's angle such that they have a similar change tendency with the minimum values of $|r_{pp}|$. Furthermore, it may be due to the synergistic role of intrinsic anisotropy, high crystallinity, and quantisation-incurred localised decreases in complex conductivities of monolayer BP, quantised spin Hall shifts are enhanced by two orders of magnitude (even

three orders of magnitude in several special $\hbar\omega$ or B values for $\langle \Delta x_{\perp} \rangle$). These findings provide insight into the fundamental properties of the spin-orbit interaction of light in 2D atomic crystals under an external magnetic field and may offer new opportunities for designing novel tunable quantum-photonics devices.

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