Research article

Minkyung Kim and Junsuk Rho*

Topological edge and corner states in a two-dimensional photonic Su-Schrieffer-Heeger lattice

<https://doi.org/10.1515/nanoph-2019-0451>

Received October 31, 2019; revised December 9, 2019; accepted January 2, 2020

Abstract: Implementation of topology on photonics has opened new functionalities of photonic systems such as topologically protected boundary modes. We theoretically present polarization-dependent topological properties in a 2D Su-Schrieffer-Heeger lattice by using a metallic nanoparticle array and considering the polarization degree of freedom. We demonstrate that when eigenmodes are polarized parallel to the plane of the 2D lattice, it supports longitudinal edge modes that are isolated from the bulk states and transverse edge modes that are overlapped with the bulk states. Also, the in-plane polarized modes support a second-order topological phase under an open boundary condition by breaking the four-fold rotational symmetry. This work will offer polarization-based multifunctionality in compact photonic systems that have topological features.

Keywords: topological photonics; higher order topological phase; corner modes; plasmonics; nanoparticles.

1 Introduction

Introduction of the concept of topology to band theory has not only enriched our understanding of phases of matter but also spawned a new field called topological band theory which has explained many anomalous behaviors such as the quantum Hall effect [1] and the quantum spin Hall effect [2]. Such exotic phenomena have been

extended and reproduced in classical systems by means of photonic/phononic crystals [3–7], metamaterials [8–12], and circuitry [13, 14]. Among the many models supporting the topological phases, the Su-Schrieffer-Heeger (SSH) model, a dimerized chain, is known as a long-standing and the simplest model [15]. The 1D SSH model and its generalization to 2D have been extensively investigated in various areas including electronic systems, photonics, and acoustics for ample physics such as higher order topological phase [16–20] and fractional charge [21].

Here, we present the polarization-dependent topological properties of a 2D photonic SSH model consisting of metallic nanoparticles (NPs). In general, for 2D models, eigenmodes that are polarized perpendicular to the plane of the 2D lattice are considered. However, polarization in photonics can provide an additional degree of freedom to independently control topological features such as transport phenomena without requiring structural modification. We demonstrate that the 2D SSH model possesses a topological phase characterized by a 2D Zak phase also for polarization parallel to the plane of the lattice, thereby supporting topological edge modes. Furthermore, by exploiting the polarization dependence to break the C_4 symmetry of the 2D SSH model, a second-order topological phase, which is a 0D corner mode in a band gap, is observed under an open boundary condition, whereas previous demonstrations of second-order topological features appear in topological SSH lattices enclosed by trivial lattices [16–18]. This relaxed condition of the second-order topological phases and polarization-based control of topological boundary modes will be advantageous in realizing compact photonic platforms such as photonic circuitry, and nanocavity for robust manipulation of light.

2 Bulk states

Four NPs constitute a unit cell of a 2D photonic SSH model (Figure 1A). The intracellular and intercellular distance will be represented as R_1 and R_2 , respectively. To build an eigenvalue problem, we use a coupled dipole approximation method [22, 23], which has been used to simulate

*Corresponding author: Junsuk Rho, Department of Mechanical Engineering, Pohang University of Science and Technology (POSTECH), Pohang 37673, Republic of Korea; and Department of Chemical Engineering, Pohang University of Science and Technology (POSTECH), Pohang 37673, Republic of Korea, e-mail: jsrho@postech.ac.kr
<https://orcid.org/0000-0002-2179-2890>

Minkyung Kim: Department of Mechanical Engineering, Pohang University of Science and Technology (POSTECH), Pohang 37673, Republic of Korea

Figure 1: 2D photonic SSH model and its bulk dispersion.

(A) Schematic of a 2D photonic SSH model composed of NPs and the first Brillouin zone with high-symmetry lines. Four sites of a unit cell are labelled as A, B, C, and D. Blue and red lines indicate intracellular and intercellular coupling, respectively. R_1 and R_2 represent the intracellular and intercellular distance, respectively. (B–G) Bulk states along high symmetry lines when (B–D) $R_1 = R_2$ and (E–G) $R_1 \neq R_2$. Left, center, and right column correspond to x-, y-, and z-polarized modes, respectively.

the band structure and topological properties of a 1D [24–26] or 2D [27] array of nanoparticles. The NPs at $\mathbf{R}$ can be described by a point dipole with a dipole moment $\mathbf{p}(\mathbf{R})$ if the radius of the NPs is small in comparison to the center-to-center distance d ($r \leq d/3$) [28]. Under linear and nonmagnetic assumptions, the dipole moment $\mathbf{p}_n$ of a particle positioned at $\mathbf{R}_n$ can be calculated by multiplying the polarizability by the sum of the total electric fields induced by other dipoles. Then, the self-consistent dipole of an array of particles can be expressed as [29]

$$\mathbf{p}_n = \alpha_E \sum_{m \neq n} G^0(\mathbf{R}_n - \mathbf{R}_m) \mathbf{p}_m, \quad (1)$$

where α_E is an electric polarizability of the NPs given as $\alpha_E = 4\pi\epsilon_0 r^3(\epsilon - \epsilon_g)/(\epsilon + 2\epsilon_g)$. Here, ϵ_g is the background permittivity, and the permittivity ϵ of the NPs follows the Drude model $\epsilon(\omega) = \epsilon_\infty - \omega_p^2/(\omega^2 + i\gamma\omega)$ for plasma

frequency ω_p and damping frequency γ . ϵ_∞ is the permittivity when frequency goes to infinity. Under the quasi-static approximation, where the wavelength of interest is much larger than the lattice constant, radiation loss and retardation effect can be ignored, and thus the dipole-dipole interaction tensor is given as [29]

$$G^0(\mathbf{R}) = \frac{1}{R^3}(-I + 3\hat{\mathbf{R}}\hat{\mathbf{R}}^T), \quad (2)$$

where I is a unity matrix, R is a magnitude of $\mathbf{R}$ and $\hat{\mathbf{R}}$ is a unit row vector of $\mathbf{R}$. Calculation without the quasi-static approximation can be found in Supplementary Material. Since the strength of the dipole-dipole interaction attenuates rapidly as the separation R increases ($\propto 1/R^3$), we consider only nearest neighbor coupling. It is noteworthy that including the next nearest neighbor interaction slightly alters the dispersion, but the general features

such as existence of edge and corner states remain. Bulk, edge, and corner states' dispersion when the next nearest neighbor interaction is included can be found in Supplementary Material. Under nearest neighbor coupling assumption, the difference $\mathbf{R}_n - \mathbf{R}_m$ in Eq. (1) has only an x -component or a y -component for any n and m . Therefore, we can further simplify Eq. (2) to $G^0(\mathbf{R}) = 2/R^3$ if $\mathbf{R}$ is parallel and $G^0(\mathbf{R}) = -1/R^3$ if $\mathbf{R}$ is perpendicular to the dipole moment direction. It means that the 3×3 matrix $G^0(\mathbf{R})$ has only diagonal elements. Therefore, the electric field induced by a dipole has a polarization parallel to the dipole moment. Because x -, y -, and z -polarized modes are fully independent, the eigenvalue problem can be decoupled into three equations of 4×4 matrices instead of one equation of a 12×12 matrix. This is a direct analogy to the tight-binding model, for which the bulk Hamiltonian can be written as

$$\begin{aligned} H_0 = & t_x (B_{m,n}^+ A_{m,n} + D_{m,n}^+ C_{m,n}) \\ & + t_y (A_{m,n}^+ C_{m,n} + B_{m,n}^+ D_{m,n}) \\ & + \bar{t}_x (A_{m+1,n}^+ B_{m,n} + C_{m+1,n}^+ D_{m,n}) \\ & + \bar{t}_y (C_{m,n+1}^+ A_{m,n} + D_{m,n+1}^+ B_{m,n}) + \text{h.c.}, \end{aligned} \quad (3)$$

where h.c. denotes the Hermitian conjugate. $A_{m,n}^+$ ($A_{m,n}$) is a creation (annihilation) operator on a site A at the (m, n) -th unit cell, and t_i ($\bar{t}_i$) is an intracellular (intercellular) coupling strength in the $i = x, y$ direction. For the x -polarized mode, the coupling strengths are $t_x = 2/R_1^3$, $t_y = -1/R_1^3$, $\bar{t}_x = 2/R_2^3$, and $\bar{t}_y = -1/R_2^3$. Coupling strengths for y -polarized mode can be obtained straightforwardly in a similar way. For the z -polarized mode, $t_x = t_y = -1/R_1^3$ and $\bar{t}_x = \bar{t}_y = -1/R_2^3$. Hereafter, we refer to x and y polarization (z polarization) as in-plane (out-of-plane) polarization. By applying the Bloch theorem, we can construct a 4×4 matrix of a bulk Hamiltonian for each polarization as

$$\begin{aligned} H = & \begin{pmatrix} 0 & H_{12} & H_{13} & 0 \\ H_{12}^* & 0 & 0 & H_{24} \\ H_{13}^* & 0 & 0 & H_{34} \\ 0 & H_{24}^* & H_{34}^* & 0 \end{pmatrix}, \\ H_{12} = H_{34} = & \bar{t}_x e^{-ik_x R_2} + t_x e^{ik_x R_1}, \\ H_{13} = H_{24} = & t_y e^{-ik_y R_1} + \bar{t}_y e^{ik_y R_2}. \end{aligned} \quad (4)$$

Bulk dispersion can be calculated by solving the eigenvalue problem $4\pi\epsilon_0\alpha_E^{-1}\mathbf{p}_i = H\mathbf{p}_i$, where $\mathbf{p}_i$ is a 4×1 vector consisting of the i -th component of the dipole moments of four sites in a unit cell for $i = x, y, z$ multiplied by position-dependent phase term

$$\mathbf{p}_i = \begin{pmatrix} \mathbf{p}_{A,i} \exp(ik \cdot \mathbf{R}_A) \\ \mathbf{p}_{B,i} \exp(ik \cdot \mathbf{R}_B) \\ \mathbf{p}_{C,i} \exp(ik \cdot \mathbf{R}_C) \\ \mathbf{p}_{D,i} \exp(ik \cdot \mathbf{R}_D) \end{pmatrix}.$$

Bulk dispersions of x , y , and z polarization along the high-symmetry line are calculated by solving the eigenvalue problem (Figure 1B–G). We examine the dispersion along Γ - X - M - Γ only, but dispersion along Γ - Y - M - Γ can be also inferred by considering the symmetry relation, as x and y polarizations are mirror images of each other under $x=y$ operation. The geometrical parameters are set as follows: lattice constant $a = 100$ nm, radius of NP $r = a/10$, $R_1 = 0.6a$ for $R_1 > R_2$ and $R_1 = 0.4a$ for $R_1 < R_2$, and $R_2 = a - R_1$. The eigenfrequency is normalized by the Dirac frequency of the NP, $\omega_D = \omega_p / \sqrt{2\epsilon_g + \epsilon_\infty}$. We use $\epsilon_g = 1$ for the background medium, and the Drude parameter of gold is taken from [30] for NPs: $\epsilon_\infty = 1$, $\omega_p = 2 \times \pi \times 2.07 \times 10^{15}$ rad/s, and $\gamma = 2 \times \pi \times 4.45 \times 10^{12}$ rad/s. Our system is non-Hermitian in a strict sense that it includes loss. However, because γ is more than two orders of magnitude smaller than ω_p ($\gamma \ll \omega_p$), all simulated results showed indistinguishable change under a lossless assumption, that is, $\gamma = 0$.

First we focus on cases where NPs are equally spaced ($R_1 = R_2$, Figure 1B–D). Geometrically, the 2D SSH model possesses C_4 symmetry in the xy plane. If we disregard symmetry-breaking due to the in-plane polarization, bulk states also have C_4 symmetry, and in such cases, spatial field distributions of bulk states at Γ are associated with s , p_x , p_y , and d_{xy} bands from the lower to the upper band in order. However, for x - and y -polarized modes, in-plane polarization breaks the symmetry, and this breaking leads to an inversion of such states. For x polarization, the lowest and the highest bands have p_x and p_y states, respectively, whereas the other bands have s and d_{xy} at Γ (Figure 1B). Such states become hybridized as the wave vector moves from Γ to X and thereby form two doubly degenerate states along X - M . The degeneracies along X - M when $R_1 = R_2$ are a consequence of the Brillouin zone (BZ) folding that can occur because the unit cell was set to be reducible. However, along M - Γ , the degeneracies are lifted as a result of the broken C_4 symmetry. Bulk states of the y polarization can be analyzed similarly with p_x and p_y switched (Figure 1C). In contrast, bulk states for z polarization show the same features as the conventional 2D SSH model, such as s , p_x , p_y , and d_{xy} from the lowest to the highest bands at Γ and degeneracy of p_x and p_y bands along the Γ - M line (Figure 1D). When $R_1 \neq R_2$, all bands are gapped except the z -polarized p_x and p_y bands, in which degeneracy is protected by the

C_4 symmetry (Figure 1E–G). We do not specify whether R_1 or R_2 is the largest because the sublattice symmetry guarantees the same bulk states when R_1 and R_2 are interchanged. However, parities of the spatial distribution of dipole moments change when R_1 and R_2 are switched, indicating a topological phase transition. The dipole moments at Γ when $R_1 > R_2$ are shown in Supplementary Material.

3 Edge and corner states

To simulate the edge states, we solve the eigenvalue problem of an array of 20 unit cells aligned along the x -axis, with periodic boundary assumption along the y -axis. In this case, the boundary is an interface between the NP arrays and air, i.e. parallel to the y -axis. When $R_1 < R_2$, the topological phase is trivial, and no edge states are found

(Figure 2A–C). In contrast, when $R_1 > R_2$, edge states appear in all three polarization (Figure 2D–F). Inversion symmetry makes the edge states doubly degenerate, with phases symmetric and antisymmetric along the aligned direction. Normalized electric field amplitudes of edge modes at $k_y = 0.5 \pi/a$ are localized at boundaries (Figure 2G–I).

x -Polarized edge states, whose polarization is transverse to the propagation direction, are overlapped with the projected bulk states (Figure 2D). On the other hand, y -polarized edge states, whose polarization is longitudinal to the propagation direction, are isolated from the bulk states' dispersion (Figure 2E). In short, the in-plane polarized 2D SSH model supports isolated longitudinal edge modes and hidden transverse edge modes. The conventional 2D SSH model, in which out-of-plane polarization is assumed, can also support edge modes in a band gap along either x - or y -axis by implementing anisotropy [17]. However, by considering the in-plane polarization, edge modes with specified propagating direction can be

Figure 2: Edge states dispersion and field profile.

(A–F) Projected bulk states and edge states of 20 unit cells aligned along x -axis. Left, center, and right column correspond to x -, y -, and z -polarized modes, respectively. (A–C) $R_1 < R_2$ and (D–F) $R_1 > R_2$. (G–I) Normalized electric field distributions of x -, y -, and z -polarized edge modes respectively.

selectively excited by using a specifically designed source depending on the polarization without needing to change the geometrical structures.

Interestingly, in-plane polarization in the 2D SSH model also supports a higher order topological phase. Conventional bulk-boundary correspondence states that an nD topological system holds $(n - 1)D$ boundary modes in a band gap. However, topological systems that do not obey the traditional bulk-boundary correspondence have boundary modes with lower dimensions such as $(n - 2)D$ in a band gap [31, 32]. This second-order topological phase has also been reported and experimentally verified in photonic crystals with a 2D SSH lattice [16–18]. In these systems, the existence of the corner modes requires an interface between topologically distinct crystals. However, the corner modes exist in the in-plane polarized 2D SSH model surrounded by vacuum: in other words, under an open boundary condition.

To further investigate the corner mode, we consider in-plane polarized eigenmodes of an array of 20×20 unit cells (Figure 3A). It has four projected bulk states and three lower dimensional states. Magnified views of three boundary states show doubly degenerate edge states (red and green boxes) and quadruply degenerate corner states (blue box). The normalized dipole moment distributions of edge and corner states are shown in Figure 3B and C. However, for out-of-plane polarization, the same array has edge states at the band gap, which have normalized dipole momentum localized in y -parallel (Figure 3E) and x -parallel (Figure 3F)

directions, but supports no corner mode that is isolated from the edge states.

For better understanding, spectra of the bulk, edge, and corner states of x and z polarization are plotted in Figure 4. The spectra of in-plane polarization mode clearly shows that the corner modes exist at the second band gap (Figure 4A). The edge modes are found at the other gaps. On the contrary, for out-of-plane polarization, although the corner modes exist, they are overlapped with the bulk or edge modes and are not observed in a band gap (Figure 4B). It implies that 2D SSH model under open boundaries supports second-order topological phase only for in-plane polarization (Figure 4C).

Two-dimensional photonic crystals with nonzero bulk quadrupole moment can possess topological corner modes under the open boundaries [33–36]. In contrast, the 2D SSH model has zero bulk quadrupole moment. Therefore, realization of the topological corner modes in the 2D SSH model with out-of-plane polarization necessarily requires boundaries between photonic crystals with distinct topological phase (Figure 4D). The absence of isolated corner mode of out-of-plane polarization originates from the C_4 symmetry which induces a degeneracy at Γ and M . The degeneracy between the second and third bands gives rise to an overlap of projected bulk states and prevents the corner mode from being isolated from the bulk states. Breaking of C_4 symmetry by in-plane polarization breaks the degeneracy and leads to gapped bulk and edge states and topological corner modes inside the band gap.

Figure 3: Eigenmodes of an array of 20×20 unit cells.

(A) Eigenfrequency of x polarization and magnified images of edge (red and green boxes) and corner (blue box) states. Normalized dipole distribution of (B) an edge mode and (C) a corner mode. (D) Eigenfrequency of z polarization and magnified views of edge states. (E) and (F) Normalized dipole distribution of two distinct edge modes.

Figure 4: Spectra of the bulk, edge and corner modes and conceptual illustration for second-order topological phase in 2D SSH model. (A, B) Spectra of the bulk, edge, and corner modes of (A) in-plane and (B) out-of-plane polarization. Gray: bulk, Orange: edge, Blue: corner. x-Axis is set as an arbitrary unit and normalized to 0.5. (C–D) Illustrations of 2D SSH model supporting second-order topological phase for (C) in-plane and (D) out-of-plane polarization. Yellow: 2D SSH model with $R_1 > R_2$ (topologically nontrivial), blue: 2D SSH model with $R_1 < R_2$ (topologically trivial).

4 Two-dimensional Zak phase

Because the 2D SSH model preserves both time-reversal and inversion symmetries, Berry curvature of the 2D SSH model is zero in the whole BZ. Instead, the topological phase of the 2D SSH model is characterized by the 2D Zak phase [37]. The Zak phase [38], associated with the shift of Wannier band, or bulk polarization, is quantized to 0 for the trivial case and to π for the nontrivial case. The extended 2D Zak phase θ can be calculated as

$$\theta_j = -\frac{1}{2\pi} \int_{BZ} d^2k \text{Tr}[A_j(\mathbf{k})], \quad j = x, y. \quad (5)$$

Here, the (m, n) th component of A_j is given as $(A_j)_{mn}(\mathbf{k}) = i \langle \mathbf{u}_{mk} | \partial k_j | \mathbf{u}_{nk} \rangle$, where $|\mathbf{u}_{nk}\rangle$ is the periodic part of the Bloch function of n th band. The 2D Zak phase is associated with the bulk polarization P in terms of $\theta = 2\pi P$. To numerically obtain the 2D Zak phase, we use a Wilson loop [17, 31, 39]

$$\begin{aligned} \theta_x &= \frac{a}{2\pi} \int dk_y v_x^n(k_y), \\ \theta_y &= \frac{a}{2\pi} \int dk_x v_y^n(k_x), \end{aligned} \quad (6)$$

where v_j^n is the n -th eigenvalue of the Wannier Hamiltonian

$$H_{W,j} = -i \log \Pi_{m=0}^M \left[F_{j,\mathbf{k}+m\Delta\mathbf{k}_j} \right] \quad (7)$$

for M satisfying $(M+1)\Delta k_j = 2\pi/a$ and $j = x, y$; the (m, n) -th component of $F_{j,\mathbf{k}}$ is $(F_{j,\mathbf{k}})_{mn} = \int \mathbf{u}_{mk}^*(\mathbf{r}) \cdot \varepsilon(\mathbf{r}) \mathbf{u}_{nk}(\mathbf{r}) d\mathbf{r}$ for m, n

$\in 1, 2, \dots, N$, where N is the number of bands below the band gap. The eigenvalues of the Wannier Hamiltonian of in-plane polarization are (π, π) for the first and the third band gap and $(0, 0)$ for the second and the fourth band gap (details can be found in Supplementary Material). The (π, π) change of the bulk polarization in each step confirms that the four photonic bands all have nontrivial Zak phase. It reflects the existence of both x - and y -polarized edge modes (Figure 2D–F). Similar calculation for out-of-plane polarization gives the same results, (π, π) jump in each step, which agrees well with previous publications [17, 37]. As if the bulk-edge correspondence associates the existence of edge states and the 2D Zak phase, or the bulk polarization, existence of the corner states is related to nonzero edge polarization [17, 39]. The edge polarization can be calculated by the nested Wilson loop [17, 31, 39]. Here, because the in-plane polarized modes are all separated by the band gap, the nested Wilson loop is reduced to the Wilson loop [17] and edge polarization becomes identical to the bulk polarization. The nonzero edge polarization is consistent with the existence of the corner modes.

In comparison to the quantum Hall or quantum spin Hall phase, the 2D Zak phase provides weaker robustness. The edge states have both positive and negative sign of group velocity (Figure 2), implying that their propagation is not robust against perturbations and also not unidirectional. Nevertheless, the nontrivial 2D Zak phase ensures that the existence of the edge states is robust against perturbations smaller than the difference between intercellular and intracellular coupling strength [37].

5 Discussion and conclusions

The topological phase and the existence of edge and corner modes can be represented schematically (Figure 5). Boxes correspond to arrays consisting of $N \times N$ unit cells under open boundary condition. Solid black lines indicate the absence of edge states. Solid blue lines denote the edge states in a band gap, while dashed blue lines represent edge states overlapped with the projected bulk states. Existence of the corner modes in a band gap are marked by blue dots at the corner. When $R_1 > R_2$, the topological phase is nontrivial ($\theta = (\pi, \pi)$), whereas it is trivial ($\theta = (0, 0)$) otherwise. The polarization direction does not affect the 2D Zak phase. However, the polarization determines the existence of isolated edge and corner states. When the polarization is perpendicular to the 2D crystals, boundaries along both the x - and y -axis support topological edge states that are isolated from the bulk states. For in-plane polarization, the same 2D SSH model supports edge states in all boundaries, but the transverse edge states are hidden by the projected bulk states. Such edge states can be excited by applying a source designed to selectively excite the edge states while leaving out the bulk states. These edge states are distinct from the topological edge states in a general sense in that they share the same frequency with the bulk states. On the other hand, the longitudinal edge states exist inside a bulk band gap.

It can be understood as a locking between polarization and propagation direction of isolated edge modes.

Figure 5: Diagram of topological phases and edge/corner modes. x -Axis represents the ratio between intercellular and intracellular distance (R_2/R_1). y -Axis distinguishes polarization of eigenmodes. Boxes and their boundaries correspond to arrays consisting of $N \times N$ unit cells and their edge modes under open boundary condition. Solid black line: no edge state; dashed blue line: edge state overlapped with bulk states; solid blue line: edge state in a band gap. Blue dots at the corner represent the existence of corner modes in a band gap.

This polarization-dependent topological transport phenomena provide a way to realize controllable photonic devices for information processing and optical communication. More specifically, the propagation direction of edge states can be controlled by the polarization of a source. For example, the edge mode propagates along either x - or y -axis or along both axes by setting the source at a corner as x -, y -, or z -polarized, respectively. Another possible application is a switchable nanocavity in which the existence of the cavity modes is determined by the polarization. As illustrated in Figure 5, the corner modes exist in a band gap only for in-plane polarization. The corner modes are tightly confined at the corner of the array within a volume smaller than $10^{-5}\lambda^3$ (see Supplementary Material for detail). The extreme confinement of the corner modes can be used to realize a nanocavity with high quality factor [19, 20] and to enhance light-matter interaction such as nonlinear responses. The localized cavity modes with polarization-dependent switchability can be also integrated with other photonic components such as a laser and a sensor.

In conclusion, we implemented a coupled dipole approximation to study the polarization-dependent topological phase of an array of metallic nanoparticles in a 2D SSH lattice. By considering in-plane polarization, the existence of edge and corner modes was investigated. An in-plane polarized 2D SSH model possesses isolated longitudinal edge modes propagating parallel to the polarization direction. We also demonstrated that the nontrivial 2D SSH model under an open boundary condition supports a second-order topological phase by showing in-plane polarized corner modes, whereas previously reported corner modes in 2D SSH model have been formed between topologically distinct lattices. Using polarization as a new degree of freedom to control topological features, multifunctional photonic devices with robust wave control will become possible.

Acknowledgments: This work was financially supported by the National Research Foundation (NRF) grants (NRF-2019R1A2C3003129, CAMM-2019M3A6B3030637, NRF-2019R1A5A8080290, NRF-2018M3D1A1058998) funded by the Ministry of Science and ICT (MSIT), Republic of Korea. M.K. acknowledges the Global Ph.D. fellowship (NRF-2017H1A2A1043204) from NRF-MSIT, Republic of Korea.

References

- [1] Klitzing KV, Dorda G, Pepper M. New method for high-accuracy determination of the fine-structure constant based on quantized Hall resistance. *Phys Rev Lett* 1980;45:494.

- [2] Kane CL, Mele EJ. Quantum spin Hall effect in graphene. *Phys Rev Lett* 2005;95:226801.
- [3] Wu LH, Hu X. Scheme for achieving a topological photonic crystal by using dielectric material. *Phys Rev Lett* 2015;114:223901.
- [4] He C, Ni X, Ge H, et al. Acoustic topological insulator and robust one-way sound transport. *Nat Phys* 2016;12:1124.
- [5] Yang Z, Gao F, Shi X, et al. Topological acoustics. *Phys Rev Lett* 2015;114:114301.
- [6] Wang Z, Chong Y, Ioannopoulos JD, Soljačić M. Reflection-free one-way edge modes in a gyromagnetic photonic crystal. *Phys Rev Lett* 2008;100:013905.
- [7] Yang Y, Gao Z, Xue H, et al. Realization of a three-dimensional photonic topological insulator. *Nature* 2019;565:622.
- [8] Khanikaev AB, Mousavi SH, Tse WK, Kargarian M, MacDonald AH, Shvets G. Photonic topological insulators. *Nat Mater* 2013;12:233.
- [9] Gao W, Lawrence M, Yang B, et al. Topological photonic phase in chiral hyperbolic metamaterials. *Phys Rev Lett* 2015;114:037402.
- [10] Yang B, Guo Q, Tremain B, et al. Direct observation of topological surface-state arcs in photonic metamaterials. *Nat Commun* 2017;8:97.
- [11] Kim M, Gao W, Lee D, et al. Extremely broadband topological surface states in a photonic topological metamaterial. *Adv Opt Mater* 2019;7:1900900.
- [12] Kim M, Lee D, Lee D, Rho J. Topologically nontrivial photonic nodal surface in a photonic metamaterial. *Phys Rev B* 2019;99:235423.
- [13] Imhof S, Berger C, Bayer F, et al. Topoelectrical-circuit realization of topological corner modes. *Nat Phys* 2018;14:925.
- [14] Ningyuan J, Owens C, Sommer A, Schuster D, Simon J. Time- and site-resolved dynamics in a topological circuit. *Phys Rev X* 2015;5:021031.
- [15] Su WP, Schrieffer JR, Heeger AJ. Solitons in polyacetylene. *Phys Rev Lett* 1979;42:1698–701.
- [16] Chen XD, Deng WM, Shi FL, Zhao FL, Chen M, Dong JW. Direct observation of corner states in second-order topological photonic crystal slabs. *Phys Rev Lett* 2019;122:233902.
- [17] Xie BY, Wang HF, Wang HX, et al. Second-order photonic topological insulator with corner states. *Phys Rev B* 2018;98:205147.
- [18] Xie BY, Su GX, Wang HF, et al. Visualization of higher-order topological insulating phases in two-dimensional dielectric photonic crystals. *Phys Rev Lett* 2019;122:233903.
- [19] Ota Y, Katsumi R, Watanabe K, Iwamoto S, Arakawa Y. Topological photonic crystal nanocavity laser. *Commun Phys* 2018;1:86.
- [20] Ota Y, Liu F, Katsumi R, et al. Photonic crystal nanocavity based on a topological corner state. *Optica* 2019;6:786–9.
- [21] Heeger AJ, Kivelson S, Schrieffer JR, Su WP. Solitons in conducting polymers. *Rev Mod Phys* 1988;60:781–850.
- [22] García de AFJ. Interaction of radiation and fast electrons with clusters of dielectrics: a multiple scattering approach. *Phys Rev Lett* 1999;82:2776–9.
- [23] Yurkin MA, Hoekstra AG. The discrete dipole approximation: an overview and recent developments. *J Quant Spectrosc Radiat Transf* 2007;106:558–89.
- [24] Downing CA, Weick G. Topological collective plasmons in bipartite chains of metallic nanoparticles. *Phys Rev B* 2017;95:125426.
- [25] Downing CA, Weick G. Topological plasmons in dimerized chains of nanoparticles: robustness against long-range quasistatic interactions and retardation effects. *Eur Phys J B* 2018;91:253.
- [26] Ling CW, Xiao M, Chan CT, Yu SF, Fung KH. Topological edge plasmon modes between diatomic chains of plasmonic nanoparticles. *Opt Express* 2015;23:2021–31.
- [27] Honari-Latifpour M, Yousefi L. Topological plasmonic edge states in a planar array of metallic nanoparticles. *Nanophotonics* 2019;8:799–806.
- [28] Park SY, Stroud D. Surface-plasmon dispersion relations in chains of metallic nanoparticles: an exact quasistatic calculation. *Phys Rev B* 2004;69:125418.
- [29] García de AFJ. Colloquium: light scattering by particle and hole arrays. *Rev Mod Phys* 2007;79:1267–90.
- [30] Blaber MG, Arnold MD, Ford MJ. Search for the ideal plasmonic nanoshell: the effects of surface scattering and alternatives to gold and silver. *J Phys Chem C* 2009;113:3041–5.
- [31] Benalcazar WA, Bernevig BA, Hughes TL. Quantized electric multipole insulators. *Science* 2017;357:61–6.
- [32] Serra-Garcia M, Peri V, Süssstrunk R, et al. Observation of a phononic quadrupole topological insulator. *Nature* 2018;555:342.
- [33] Noh J, Benalcazar WA, Huang S, et al. Topological protection of photonic mid-gap defect modes. *Nat Photon* 2018;12:408.
- [34] Mittal S, Orre VV, Zhu G, Gorlach MA, Poddubny A, Hafezi M. Photonic quadrupole topological phases. *Nat Photon* 2019;13:692–6.
- [35] El Hassan A, Kunst FK, Moritz A, Andler G, Bergholtz EJ, Bourennane M. Corner states of light in photonic waveguides. *Nat Photon* 2019;13:697–700.
- [36] Chen Y, Lin ZK, Chen H, Jiang JH. Plasmon-polaritonic quadrupole topological insulators. *Phys Rev B* 2020;101:041109(R).
- [37] Liu F, Wakabayashi K. Novel topological phase with a zero Berry curvature. *Phys Rev Lett* 2017;118:076803.
- [38] Zak J. Berry's phase for energy bands in solids. *Phys Rev Lett* 1989;62:2747–50.
- [39] Benalcazar WA, Bernevig BA, Hughes TL. Electric multipole moments, topological multipole moment pumping, and chiral hinge states in crystalline insulators. *Phys Rev B* 2017;96:245115.

Supplementary Material: The online version of this article offers supplementary material (<https://doi.org/10.1515/nanoph-2019-0451>).