Research article

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Diffraction-limited axial double foci and optical traps generated by optimization-free planar lens

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Abstract: Axial diffraction-limited multiple foci are a kind of investigated focal field for trapping multiple nano-particles. We first experimentally generated diffraction-limited axial double foci by optimization-free binary planar lens and theoretically demonstrated it, which can be applied in multi-particle trapping. The proposed binary planar lens was analytically designed. The BPL has a numerical aperture of 0.9 and a focal length of 150 μm . The focal field of the binary planar lens, which is composed of diffraction-limited axial double foci, was first experimentally validated. The measured maximum lateral full widths at half maximum of the two generated focal spots were diffraction-limited and consistent with the theoretical. The axial double foci formed two stable optical traps that can trap two Rayleigh dielectric particles simultaneously. The radial, azimuthal and axial optical

forces of the double optical traps are in good uniformity, which are 0.98, 0.99 and 0.96, respectively.

Keywords: subwavelength focusing; double foci; planar lens; optical trap; optical force.

1 Introduction

Optical tweezers can manipulate nano-particles without physical contact, providing a significant tool in many scientific fields, such as biology [1, 2], biophysics [3, 4], photonics [5, 6], chemistry [7–10] and measurement [11–13]. Multiple nano-particles trapping was more and more desired in the optical tweezer field [14–17]. Axial diffraction-limited multiple foci are a kind of investigated focal field for multiple nano-particles trapping [18–30]. To achieve three-dimensional diffraction-limited multiple foci, iterative phase-retrieval algorithms have been proposed to create the modulation phase patterns [22, 24]. However, the design processes of phase-retrieval algorithms require many iterations. The designed phase patterns are random. Meanwhile, amplitude or phase pupil filters were proposed to achieve axial diffraction-limited multiple foci [19, 21]. However, some design parameters of pupil filters need to be optimized by optimization algorithms. There was also non-iterative pupil filter to be proposed to achieve axial diffraction-limited multiple foci [25]. The mentioned optical systems are bulky, because of these optical systems composed of the objective and the additional modulation. Compared with the conventional refractive lenses, the planar lenses are thin, light, and compatible [31–34]. Recently, metasurfaces were designed to achieve multi-foci focusing or extended depth of focus. However, for optical metasurfaces, the size of the nano fin is as small as tens of nanometers [35, 36]. Therefore, for the visible light, design and fabrication of a metasurface with thousands of nano fin is always challenging. However, to our best knowledge, there is still no experimental research on achieving axial diffraction-limited multiple foci by a single optimization-free planar lens. In our work, each focus of the designed

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planar is diffraction limited, which has advantage in trapping precision. Some studies reported the transverse multiple foci generated by planar lens. These foci are not applicable for the situation of manipulating the nano wire or sorting particles along the optical axis.

Here, we first experimentally achieved the diffraction-limited axial double foci by an optimization-free planar lens and theoretically demonstrated its ability in multi-particle trapping. The designed planar lens has the binary amplitude transmittance. Its design doesn't involve iteration or optimization. Using the binary planar lens (BPL), the diffraction-limited axial double foci with an intensity uniformity of 0.9 were experimentally validated. Meanwhile, the concentric belts have prominent fabrication advantage. The optical force of the generated axial double foci was theoretically demonstrated. It is stable three-dimensional optical traps that can trap two Rayleigh dielectric particles simultaneously. Our work provides a scheme to achieve diffraction-limited axial double foci for nano-particles trapping.

2 Methods

2.1 Design

To generate the diffraction-limited axial double foci for nano-particles, an optimization-free BPL was designed as shown in Figure 1A. A cosine synthesized filter with the transmittance function of $\cos(2\pi \cos \theta) \cos[(r/R_{\max})^2]$ was applied to implement the amplitude modulation to achieve the axial double foci. Here, θ is the convergence angle, r is the radial coordinate in the plane of the BPL, and $R_{\max} = 309.7 \mu\text{m}$ is the maximum radius of the BPL. The principle of the cosine synthesized filter to achieve axial multi-focus was presented in our previous study [18]. The cosine synthesized filter hasn't the focusing function. To achieve the focusing function, a focusing phase function $\exp\{-2\pi i[(f^2 + r^2)^{1/2} - f]/\lambda\}$ was employed, where f is the focal length, and λ is the wavelength in the medium of the focal volume with a refractive index n_1 . In this article, $n_1 = 1$ (since the BPL is worked in the air), $f = 150 \mu\text{m}$, and $\lambda = 632.8 \text{ nm}$. For the BPL, the geometric relation $r = f \tan \theta$ is satisfied since the BPL is a planar element obeying the Helmholtz condition [37]. Therefore, a transmittance function that could achieve axial double foci was generated, which is expressed as:

$$T(\theta) = \cos(2\pi M \cos \theta) \cos\left[\left(\frac{\tan \theta}{\tan \alpha}\right)^2\right] \exp\left(-2\pi i f \frac{\sqrt{1 + \tan^2 \theta} - 1}{\lambda}\right) \quad (1)$$

where $\alpha = \arcsin(\text{NA}/n_1)$ is the maximum of θ . According to Euler transformation, one focal point is splitted into two points that are located at $z - \lambda M$ and $z + \lambda M$, respectively. Here, $M = 1$ represents the foci distance is designed to be 2λ , and $z = f = 150 \mu\text{m}$ represents the geometrical focal distance. According to the maximum radius and focal length of the BPL, the numerical aperture (NA) of the BPL is 0.9. Although the transmittance function $T(\theta)$ can achieve axial double foci, both its modulus $|T(\theta)|$ and argument $\angle T(\theta)$ are continuous. Obviously, $T(\theta)$ is difficult to be generated by one physical device. A computer-generated hologram can restructure a complex function with the continuous modulus and argument [38]. Therefore, we created a computer-generated hologram to handle $T(\theta)$ and generated a binary transmittance function $T_1(\theta)$ as following:

$$T_1(\theta) = \begin{cases} 1 & \text{for } \cos[\angle T(\theta)] - \cos q \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

where $q = \arcsin[|T(\theta)|]$. The binary transmittance function $T_1(\theta)$ can restructure the complex function $T(\theta)$ to achieve axial double foci. Based on calculated results, $T_1(\theta)$ has some concentric belts with widths less than 100 nm. To facilitate fabricating, the belts with widths less than 100 nm were removed from $T_1(\theta)$, and the generated transmittance function $T_B(\theta)$ is expressed as:

$$T_B(\theta) = \begin{cases} 1 & \text{for } r \geq 290 \mu\text{m} \text{ or } 105 \mu\text{m} \leq r \leq 173 \mu\text{m} \\ T_1(\theta) & \text{otherwise} \end{cases} \quad (3)$$

$T_B(\theta)$ is the final transmittance function of the BPL, which has about 417 belts. The narrowest belt width is 100 nm. The updated transmittance function of $T_B(\theta)$ is shown in Figure 1B.

By introducing (1), the double foci are analytically realized without optimization. Compared to the conventional Fresnel zone plates and the metasurface, the focusing and multi foci will be simultaneously achieved by the single planar lens, which is optimization-free. The finite element method (FEM) or finite differences in time domain (FDTD) method aren't employed. Therefore, the designing of the BPL is convenient.

2.2 Sample fabrication

The designed BPL was fabricated by electron beam lithography to experimentally achieve diffraction-limited axial double foci. First, a 10 nm Chromium conductive layer was deposited on the fused silica wafer substrate using FHR (GCEMarket MSI50x6-L). Then, the electron beam

Figure 1: BPL schematic.

(A) Design process and schematic of the BPL to achieve diffraction-limited axial double foci. (B) Transmittance of the BPL. Insets are the enlarged displays of the transmittance.

lithography (EBL) method (JEOL JBX5500ZA, 50kV) was applied to generate the annulus pattern on a 400 nm thickness e-beam resist (ALLRESIST AR-N 7520.18). Then, a metal layer with a thickness of 110 nm was deposited on the substrate using the electron beam evaporating (ULVAC Ei-5z). Finally, the sample was fabricated after removing the resist.

2.3 Experimental Setup

A linearly polarized He-Ne laser has a light beam with a wavelength of 632.8 nm. The neutral density filter (NDF)

and the linear polarized (LP) were applied to adjust the intensity and polarization of the laser beam. A quarter-wave plate (QWP) converted the linearly polarized beam into a circularly polarized beam. After passing through a spatial filter composed of two lenses (L_{1-2}) and a pinhole (PH), a filtered Gaussian incident beam was generated. Two Mirrors (M_{1-2}) reflected the Gaussian beam on the BPL, which was mounted on a piezoelectric transducer (PZT) stage (PI-735.1CD). A self-built microscope was employed to measure the focal electric energy density generated by the BPL, which consisted of a microscope objective (MO) with a NA of 0.95 (Olympus, MPLAPON 100X), an

achromatic lens (L_3) and a CCD camera. In this experiment, the PZT stage carried the BPL moving along the z -axis near the position $z=0$ in step of 50 nm. The measured light intensity distribution was calculated from 10 measurements averaged profile of the normalized focal electric field density.

3 Calculation

Here, the incident beam on the BPL is circularly polarized. According to the vector diffraction theory [39, 40], the focal electric field components and magnetic field components of the BPL can be calculated by:

$$\begin{cases} E_x(\rho, \phi, z) = I_0 - I_2 \exp(-i2\phi) \\ E_y(\rho, \phi, z) = -i[I_0 + I_2 \exp(-i2\phi)] \\ E_z(\rho, \phi, z) = -2iI_1 \exp(-i\phi) \end{cases}$$

and $\begin{cases} H_x(\rho, \phi, z) = i[I_0 - I_2 \exp(-i2\phi)]n_1\epsilon_0 c \\ H_y(\rho, \phi, z) = [I_0 + I_2 \exp(-i2\phi)]n_1\epsilon_0 c \\ H_z(\rho, \phi, z) = 2I_1 \exp(-i\phi)n_1\epsilon_0 c \end{cases}$ (4)

where (ρ, ϕ, z) are the cylindrical coordinates and the origin locates at the geometrical focus of the BPL, as shown in Figure 1A. ϵ_0 is the vacuum dielectric constant and c is the speed of light in a vacuum. The light intensity I_0 , I_1 and I_2 are expressed as:

$$\begin{cases} I_0 = -iA \int_0^\alpha P(\theta) \sin\theta (\cos\theta + 1) J_0(k\rho \sin\theta) \exp(ikz \cos\theta) d\theta \\ I_1 = -iA \int_0^\alpha P(\theta) \sin^2\theta J_1(k\rho \sin\theta) \exp(ikz \cos\theta) d\theta \\ I_2 = -iA \int_0^\alpha P(\theta) \sin\theta (\cos\theta - 1) J_2(k\rho \sin\theta) \exp(ikz \cos\theta) d\theta \end{cases}$$
 (5)

where $A = k f l_0 / (2\sqrt{2})$, $P(\theta) = \cos^{-3/2}(\theta) \cdot T_B(\theta) L(\theta) e^{i\Phi(\theta)}$ and $J_m(x)$ is the first kind of m -th order Bessel function. $k = 2\pi/\lambda$ is the wave number and l_0 is the amplitude factor. The circularly polarized incident beam is a Gaussian beam and its amplitude is $L(\theta) = \exp[-\beta^2(\tan\theta/\tan\alpha)^2]$, where β is the ratio between the pupil radius and the beam waist [41]. $\Phi(\theta) = k f (1 - 1/\cos\theta)$ is the phase difference between the converging wave front and the ideal spherical wave front.

The optical forces on a particle can be derived from the electromagnetic theory. The forces exerted on an object are entirely determined by the Maxwell's stress tensor $\mathbf{T}$. The Rayleigh scattering optical force is derived as the approximation of Maxwell's stress tensor. For a Rayleigh dielectric particle, whose radius a is much smaller than the wavelength λ (usually $a < \lambda/20$), the

optical forces exerted on the particle by the focal field of the BPL is [42–44]:

$$\vec{F} = \frac{1}{4} \text{Re}(\sigma) \nabla |\vec{E}|^2 + \frac{k_0}{\epsilon_0 c} \text{Im}(\sigma) \langle \vec{S} \rangle + \frac{k_0 c}{\epsilon_0} \text{Im}(\sigma) \nabla \times \langle \vec{L}_s \rangle \quad (6)$$

where $\sigma = \frac{\sigma_0}{1 - i\sigma_0 k_0^3 / (6\pi\epsilon_0)}$ and $\sigma_0 = 4\pi\epsilon_0 a^3 \frac{n_2^2/n_1^2 - 1}{n_2^2/n_1^2 + 2}$.

$k_0 = k/n_1$ is the wave number in a vacuum. n_2 is the refractive index of the Rayleigh dielectric particle. $\langle \vec{S} \rangle = \text{Re}(\vec{E} \times \vec{H}^*)/2$ is the time averaged Poynting vector and $\langle \vec{L}_s \rangle = \epsilon_0 (\vec{E} \times \vec{E}^*)/(4ik_0 c)$. The asterisk indicates the complex conjugate. The first term in (6) is the gradient force, which traps nano-particles [42, 45]. The second term in (6) is the radiation pressure pushing nano-particles away from the focus. The third term in (6) is the spin density force [43].

4 Results and discussion

The scanning electron microscope (SEM) micrographs of the fabricated BPL are shown in Figure 2. The widest two belts of the BPL expressed by $105 \mu\text{m} \leq r \leq 173 \mu\text{m}$ and $290 \mu\text{m} \leq r \leq R_{\text{max}}$ in (3) are distinct in Figure 2A. The narrowest belts and the central zone of the BPL are both precisely fabricated, as shown in Figure 2B, C. Compared with the fabrication of a metalens, the fabrication process of the designed BPL is more convenient [31, 46].

As shown in Figure 3A and B, an experimental setup was built up to characterize the focal performance of the BPL, as presented in Section 2.

Firstly, the generated Gaussian beam illuminating on the BPL was recorded by a CCD camera, as shown in Figure 4. The measured electric field density distribution in the x - y plane of the Gaussian beam is shown in Figure 4A. The average electric field density profiles of 10 experimental measurements for the generated Gaussian beam along the x - and y -axes are shown in Figure 4B. According to the measured data, the parameter β of the Gaussian beam was 0.25 derived by theoretical fitting. The fitting electric energy density profile is marked by the solid curve in Figure 4B. Obviously, the generated Gaussian beam is good beam quality. The shadow area in Figure 4B is the area of the Gaussian beam illuminating on the BPL. To demonstrate that the generated Gaussian beam is circularly polarized, the laser power of the Gaussian beam after passing through a polarization analyzer with different transmittance angles was recorded by an optical power meter (Thorlabs PM100D), as shown in Figure 4C. The maximum peak-valley value of the optical power curve shown in Figure 4C was $0.02 \mu\text{W}$,

Figure 2: SEM graph of the fabricated BPL with $NA=0.9$.

(A) Entire SEM micrograph of the fabricated BPL. Diameter of the fabricated BPL is $2 \times R_{\max} = 619.4 \mu\text{m}$. (B) Narrowest structure of the BPL. (C) Central zone of the BPL.

Figure 3: Experimental setup to characterize the focal performance of the BPL.

(A) Schematic of the experiment. (B) Physical experimental setup.

which was only 1.04% of the average optical power $1.93 \mu\text{W}$. The result demonstrated that the generated Gaussian beam is well circularly polarized.

4.1 Focal electric field

For the incident circularly polarized Gaussian beam with $\beta=0.25$, the theoretically calculated normalized focal electric energy density in the yoz plane of the BPL is shown in Figure 5A. Obviously, there are two focal spots along the z -axis. Figure 5B, C are the calculated lateral distributions of the focal electric energy densities for the front and back focal spots on the planes $z=1.24\lambda$ and $z=-1.24\lambda$ (for the sake of simplicity, represented as plane 1 and plane 2), respectively. The intensity distributions of the x , y and z polarization components are respectively same around the two foci (see Figures S1 and S2 in the Supplementary Material). These results demonstrate the feasibility of the designed BPL to achieve axial double foci. The experimental axial double foci are shown in Figure 5D–F. In the experiment, the PZT stage carried the BPL moving along the z -axis near the position $z=0$ in step of 50 nm . As shown in Figure 5D, the measured result presented axial double foci clearly. The measured lateral distributions of the focal electric field densities on plane 1 and plane 2 are shown in Figure 5E, F, respectively, which are consistent with the theoretical results. These measured results experimentally demonstrate the feasibility of the designed BPL to achieve axial double foci.

To further validate the experimental and theoretical results, the 10 measurements averaged profile of the normalized focal electric field density and the theoretically calculated result along z -axis are shown in Figure 5G. The

Figure 4: Experimentally generated circularly polarized Gaussian beam illuminating on the BPL.

(A) Normalized electric energy density distribution of the Gaussian beam in the x-y plane. Scale bar, 500 μm. (B) Average electric energy density profiles of 10 measurements of the Gaussian beam along the x- and y-axes, and the theoretical fitting profile. (C) Laser power of the Gaussian beam after passing through a polarization analyzer with different transmission angles.

side-lobes of the measured z-axis profile are higher than the calculated profile. However, the main-lobe profile of the experimentally generated double foci almost completely overlaps the theoretical profile. The maximal deviation between the experimental and theoretical z-axis main-lobe normalized electric field densities on plane 1 is as small as 0.068, as shown in Figure 5G. Meanwhile, the experimental distance between the double foci (DBF) is 2.53λ (1.60 μm), which only deviates 1.6% from the theoretical DBF of 2.49λ (1.58 μm). The difference of the normalized experimental electric field densities maximum of double foci is 0.098. Therefore, the axial intensities of the experimental double foci are uniform. To quantitatively describe the uniformity of the two focal spots, the uniformity $1 - |A_1 - A_2| / |A_1 + A_2|$ was employed. A_1 and A_2 are the electric field densities of the foci at $z = 1.24\lambda$ and $z = -1.24\lambda$, respectively. The uniformity of two foci is 0.95. As shown in Figure 5H, the measured full widths at half maximum (FWHMs) of the lateral normalized density profiles on plane 1 are 0.67λ (424 nm) and 0.68λ (430 nm) along the x- and y-axes, respectively. The difference between the two measured FWHMs is as small as 0.01λ (6 nm). The generated front focal spot is well circularly symmetric. Moreover, the maximal deviation

between the two measured FWHMs and the theoretical FWHM of 0.59λ (373 nm) on plane 1 is 0.09λ (57 nm). The deviation caused by the error in the experiment and the point spread function of the self-built microscope. The lateral normalized electric field density profiles of the experimentally generated back focal spot are shown in Figure 5I. The measured lateral FWHMs on plane 2 are 0.65λ (411 nm) and 0.67λ (424 nm) along the x- and y-axes, respectively. The generated back focal spot is a well circular symmetry. The differences of the measured front and back foci's FWHMs are 0.02λ (13 nm) along the x-axes and 0.01λ (6 nm) along the y-axes. These tiny differences demonstrate that the lateral profiles of the experimentally generated double foci are well consistent. Moreover, although the theoretical and experimental double foci are both larger than the diffraction limit with a lateral FWHM of $0.51\lambda/\text{NA}$ (0.57λ , 359 nm) [32], the maximal deviation between the measured FWHMs and the diffraction limit is about 0.11λ (71 nm), which is still minimal. Meanwhile, the calculated focusing efficiencies of BPL at the focal plane 1 and 2 are 72.5% and 71.4%, respectively. Obviously, the most transmitted light is focused at the desired focal spots. Therefore, the generated axial double foci are diffraction-limited. These measured results demonstrate

Figure 5: Normalized intensity distribution of the diffraction-limited axial double foci generated by the BPL. Scale bar, λ . (A) Calculated focal intensity distribution on the yoz plane. (B) and (C) Calculated intensity distributions on plane 1 ($z = 1.24\lambda$) and plane 2 ($z = -1.24\lambda$), respectively. (D) Measured intensity distributions on the yoz plane. (E) and (F) Measured intensity distributions of the front and back focal spots. (G) Average profile of 10 measurements of the focal intensity along the z -axis and the theoretically calculated intensity profile. (H) and (I) Average lateral intensity profiles of 10 measurements for the front and back focal spots along the x - and y -axes, and the theoretically calculated intensity profiles.

the feasibility of the designed BPL to achieve diffraction-limited axial double foci.

4.2 Optical traps

To validate the ability of trapping nano-particles of the axial double foci generated by the BPL, we calculated the optical forces exerted on a Rayleigh dielectric particle by the axial double foci. The Rayleigh dielectric particle is assumed $a = 30$ nm and $n_2 = 1.6$. The power of the laser illuminating on the BPL is 5 W.

The radial and azimuthal optical forces on the Rayleigh particle located in the focal planes are shown in Figure 6. In the focal planes, the circularly symmetric electric field density distribution causes the null azimuthal gradient force. The radial gradient force on plane 1 ranges from -0.44 pN to 0.01 pN, as shown in Figure 6A. The radial radiation pressure ranges from -0.04×10^{-3} pN to 0.13×10^{-3} pN, as shown in Figure 6B. The radial spin density force on plane 1 is null. The total radial optical

force is the sum of the radial gradient force and the radial radiation pressure. Figure 6E is the density profiles along x -axis of the total radial optical force, the radial gradient force and the radial radiation pressure. At $x = 0$ on plane 1, the total radial optical force is zero. In $|x| < 0.61$ μm , the total radial optical force is negative and up to -0.44 pN as shown in Figure 6E. Therefore, the total radial optical force on plane 1 points to the center. The nano particle can be trapped to the focal spot center. The azimuthal spin density force and the azimuthal radiation pressure ranges from -0.14×10^{-3} pN to 3.10×10^{-3} pN and from -4.00×10^{-3} pN to 0.11×10^{-3} pN, as shown in Figure 6C and D, respectively. Since the azimuthal gradient force is null, the total azimuthal optical force is the sum of the azimuthal spin density force and the azimuthal radiation pressure. Figure 6F is the density profiles along x -axis of the total azimuthal optical force, the azimuthal spin density force and the azimuthal radiation pressure on plane 1. At $x = 0$ on plane 1, the total azimuthal optical force is null. In $0 < |x| < 0.78$ μm , the total azimuthal optical force is negative and up to -1.00×10^{-3} pN as shown in Figure 6F, which

Figure 6: Lateral optical forces on plane 1.

(A) Radial gradient force. (B) Radial radiation press. (C) Azimuthal spin density force. (D) Azimuthal radiation press. (E) and (F) Total radial and azimuthal optical forces along x -axis on plane 1. Insets are the total radial and azimuthal forces on plane 2. F_{grad} represents the gradient force, F_{rp} represents the radiation pressure, and F_{sd} represents the spin density force.

means the total azimuthal optical force points along the clockwise direction around the center point of plane 1.

The axial optical forces on the Rayleigh particle on plane 2 are shown in Figure 7. As displayed in Figure 7A–C, the axial gradient force, radiation pressure and spin density force ranges from -7.90×10^{-3} pN to 1.30×10^{-3} pN, from -0.11×10^{-3} pN to 13.30×10^{-3} pN, and from -4.10×10^{-3} pN to 0.80×10^{-3} pN, respectively. Figure 7D shows the total axial force, the axial gradient force, the axial radiation pressure and the axial spin density force along x -axis on plane 1. In $|x| < 0.39$ μm , the total axial optical force is positive and up to 9.18×10^{-3} pN as shown in Figure 7D. As the maximal azimuthal and axial forces are $1/438.56$ and $1/47.76$ of the maximal radial force on the particle, the total force in the focal plane can trap the particle to the center. Therefore, a lateral optical trap is formed near the focal spot.

The electric field density distributions of the axial double foci are uniform. The optical forces in plane 2 are like that in plane 1. For the sake of simplicity, we presented the optical forces along x -axis at $z = -1.24\lambda$ as shown in the insets of Figures 6E and F and 7D. At $x = 0$ on plane 2, the total radial optical force is zero. In $|x| < 0.60$ μm , the total radial optical force is negative and up to -0.43 pN, which is almost contributed by the radial gradient force, as shown in the inset of Figure 6E. The maximal total

azimuthal force in plane 2 is -0.97×10^{-3} pN as is shown in the inset of Figure 6F. The total axial force ranges from -2.13×10^{-3} pN to 9.75×10^{-3} pN, as shown in the inset of Figure 7D. In Figure 7D, the distribution of the total axial forces is different. The reason is the opposite directions of the double foci's axial gradient force. As a particle approaching the focal center, the axial gradient force decays to zero, and the axial radiation press and the axial spin density force dominate the total axial force. On the contrary, as a particle moving away from the focal center, the axial gradient force increases rapidly, and the other two forces decay fast. Considering that the maximal total azimuthal and axial optical forces are about $1/437$ and $1/44$ of the maximal total radial force, the total force in plane 2 also traps the particle to the center. Another lateral optical trap is formed near the focal spot.

The optical forces distributions along z -axis are shown in Figure 8. Along z -axis, the radial and azimuthal optical forces are null. Therefore, the optical forces along z -axis only have the axial gradient force, the axial radiation press and the axial spin density force. Figure 8A showed the distributions of the three kinds of axial optical forces and the total force. At the focal centers, the axial gradient force is null. Therefore, the axial gradient forces at $z = 1.24\lambda$ and $z = -1.24\lambda$ are zero. In Figure 8A, the maximal axial radiation pressure and the axial spin density force

Figure 7: Axial optical forces on plane 1.

(A) Axial gradient force. (B) Axial radiation pressure. (C) Axial spin density force. (D) Total axial optical forces along x-axis on plane 1. Insets are the total axial forces on plane 2. F_{grad} represents the gradient force, F_{rp} represents the radiation pressure, and F_{sd} represents the spin density force.

are 13.34×10^{-3} pN and -4.15×10^{-3} pN, respectively. They can be neglected compared with the axial gradient force. More clearly, the axial radiation pressure and axial spin density force on z-axis are shown in Figure 8B. The total axial optical force on the z-axis ranges from -0.19 pN to 0.21 pN, which almost comes from the axial gradient force, as shown in Figure 8A. Moreover, near the axial positions of $z = 1.24\lambda$ and $z = -1.24\lambda$, the total axial optical force has two equilibrium points. The equilibrium points are both displaced less than 12 nm from the foci centers, which are all smaller than the particle's radius 15 nm. Therefore, the axial double foci generated by the BPL form double axial optical traps to manipulate two nano-particles simultaneously.

Meanwhile, the radial, azimuthal and axial optical forces on plane 1, plane 2 and yoz plane are shown in

Figure 9. The units of forces in Figure 9 are Pico newton. No azimuthal gradient force is in the focal field. The coordinate system diagram is shown as Figure 9B2. In Figure 9, the coordinate ranges are $x \in [-2\lambda, 2\lambda]$, $y \in [-2\lambda, 2\lambda]$, $z \in [-5\lambda, 5\lambda]$. The radial spin density force in the two focal planes is null, thus only the yoz distribution was shown in Figure 9A4. From Figure 9A1–A4 and Figure 9C1–C4, the total radial and axial optical forces are almost same as the radial and axial gradient forces. Obviously, the gradient force is dominating in the radial and axial optical forces around the equilibrium points. Without the gradient force, the total azimuthal force has the same order of magnitude of the azimuthal radiation pressure and the azimuthal spin density force, as shown in Figures 9B1, B3, and B4. Comparing the total radial, azimuthal and axial optical forces shown in Figures 9A1, B1, and C1,

Figure 8: Axial optical forces on the 30 nm radius dielectric particle with $n_2 = 1.6$ located along the optics axis.

(A) Total axial optical force. (B) Zoomed radiation pressure and spinning density force. F_{grad} represents the gradient force, F_{rp} represents the radiation pressure, and F_{sd} represents the spin density force.

the total azimuthal force is negligible compared with the other two forces. The reason is that the gradient force is dominating around the focal spots. Therefore, two three-dimension optical traps are formed near the axial double foci generated by the designed BPL. To evaluate the property of optical force, $1 - |A_1 - A_2| / |A_1 + A_2|$ was employed to describe the uniformity of the optical force. A_1 and A_2 are the radial, azimuthal, or axial total force of the foci at the desired transverse plane, respectively. The uniformity of two foci's radial, azimuthal and axial optical forces is 0.98, 0.99 and 0.96, respectively. The high uniformity of double foci's optical forces shows the potential application in the field of multi-particle operating and nanowires controlling.

For the stability of trapping a Rayleigh dielectric particle, the Brownian motion of the particle was investigated. The criterion of stability of a particle in the optical trap is $R_{\text{thermal}} = e^{-U_{\text{max}}/k_B T} \ll 1$, which guarantees the potential well of the gradient force much larger than the kinetic energy of the particle. Here, k_B is the Boltzmann constant, T is the Kelvin temperature, and the maximal depth of the potential

well of the gradient force is $U_{\text{max}} = \pi \epsilon_0 n_1^2 a^3 \left| \frac{n_2^2/n_1^2 - 1}{n_2^2/n_1^2 + 2} \right| |\vec{E}|_{\text{max}}^2$

[47]. Here, T is 300 K, a is 30 nm, n_1 is 1, n_2 is 1.6, and $|\vec{E}|_{\text{max}}$ is 2.08×10^7 V/m. R_{thermal} of the nanoparticle is calculated 2.05×10^{-12} , which satisfies $R_{\text{thermal}} \ll 1$. Therefore, the three-dimensional optical trap is stable enough for the particle's Brownian motion.

5 Conclusions

We designed and fabricated a BPL to achieve diffraction-limited axial double foci. The BPL has a NA of 0.9 and an

ultra-long focal length of 150 μm , which is approximately 237 times of the incident wavelength of 632.8 nm. The effect of the evanescent wave is completely ignored in the focal field generated by the BPL. Furthermore, the ultra-long focal length provides enough work distance between the BPL and the specimen, which is significant for practical applications [32, 48]. Compared with the existed metalenses and diffractive lenses needing computer optimization, the design BPL is convenient. The focusing field composed of diffraction-limited axial double foci with a DBF of 2.53λ (1.60 μm) here was first experimentally generated by an optimization-free planar lens. Using the BPL, the diffraction-limited axial double foci with an intensity uniformity of 0.95 were experimentally generated. The generated double foci are in good axial and lateral uniformity. The differences between the measured front and back foci's FWHMs are 0.02λ (13 nm) along x -axes and 0.01λ (6 nm) along y -axes. The experimental and theoretical foci's consistency demonstrated the feasibility of the design method of the BPL.

The feasibility of the generated axial double foci to trap multi-particle was theoretically investigated to emphasize its potential application. The total axial optical forces near the focal spot $z = 1.24\lambda$ ranges from -0.19 pN to 0.21 pN, and the axial gradient force ranges from -0.19 pN to 0.21 pN. The total radial optical forces near the focal spot achieves up to -0.44 pN, and the radial gradient force is up to -0.44 pN. Without the gradient force, the total azimuthal optical force near the focus is negligible compared with the former two forces. The criterion of stability of the particle in the optical trap is satisfied by $R_{\text{thermal}} = 2.05 \times 10^{-12}$ ($R_{\text{thermal}} \ll 1$). For another focus $z = -1.24\lambda$, the axial, radial, and azimuthal optical forces distributions are like those of the former focus, showing a good axial and lateral uniformity. Two three-dimensional

Figure 9: Radial, azimuthal and axial forces on plane 1, plane 2, and yoz plane.

The ranges of x - and y -coordinates of all the volumes are $[-2\lambda, 2\lambda]$. The ranges of z -axis coordinates of the volumes are $[-5\lambda, 5\lambda]$. All units of forces are pN. (A1) Total radial force. (A2) Radial gradient force. (A3) Radial radiation pressure. (A4) Radial spin density force. (B1) Total azimuthal force. (B2) Coordinate system diagram of the forces. (B3) Azimuthal radiation pressure. (B4) Azimuthal spin density force. (C1) Total axial force. (C2) Axial gradient force. (C3) Axial radiation pressure. (C4) Axial spin density force.

optical traps were generated with radial, azimuthal and axial optical forces in good uniformity, which are 0.98, 0.99, and 0.96, respectively.

The function of the fabricated BPL is divided into two parts. The first is the amplitude modulation based on the cosine synthesized filter [18], which is crucial to shape the

generated focal field for achieving axial double foci. The second is the focusing phase modulation like a general focusing lens. Adjusting the parameters of the amplitude modulation filter, a BPL can be designed to capture the end points of a nanowire. With greater trapping stiffness, a longer trapped rod has smaller angular fluctuations [49]. We also designed BPL with a DBF of 20 μm by employing $M=15.8$ in (1). The light intensity and optical forces uniformity are as excellent as that in this article. The generated diffraction-limited axial double foci can be applied in multi-particle trapping and manipulation, delivery, self-assembling and multi-layer laser direct writing [50].

6 Supplementary material

The supplementary material about the polarization distribution around the foci is available.

Real/imagination part and intensity of polarized components along x , y and z axis near these two foci.

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